Fluctuations of conserved charges in the nonlocal chiral quark model at finite temperatures

Andrey Radzhabov

Institute for System Dynamics and Control Theory
Irkutsk, Russia

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In collaboration with
D. Blaschke, M. Buballa, A.E. Dorokhov, M.K. Volkov, J. Wambach, A.S. Zhevlakov

Plan

1. Motivation
2. Nonlocal $SU(2)$ chiral quark model at mean field
3. Beyond mean field ($1/N_c$ corrections)
4. Finite temperature extension
5. NJL model $+$ Polyakov loop potential $=$ PNJL model
6. Nonlocal $SU(3)$ at finite T
7. Comparison with LQCD and HRG
8. Conclusion

Motivation. I

1. The well-developed methods of perturbative QCD are applicable for description of the hard processes, with small distances between quarks and gluons.
2. Most of the data deals with processes where the momentum transfer between the quarks and gluons is of the order 1 GeV or less $\rightarrow$ should be the subject of nonperturbative study.
3. The same situation takes place for the QCD phase diagram where the most interesting region of the small and moderate temperatures T (and chemical potentials μ) is also in nonperturbative regime.
4. The only nonperturbative *ab initio* calculations are performed in lattice QCD and (besides of some limitations) the theoretical interpretation, or interpolation to inaccessible region, of numerical simulations is need.
5. Effective theories based on symmetries in nonperturbative region are legitimate tools.

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Motivation. II

1. One of the successful effective model is the local Nambu–Jona-Lasinio model.
   NJL is one of the simplest models for description of "nonperturbative" dynamics. It provides for spontaneous chiral symmetry breaking and the formation of a quark condensate. The spectrum, low-energy dynamics, the main strong and electromagnetic decays of mesons have a reasonable explanation within this model.

Problems

- ultraviolet divergences: regularization scheme dependence, cut-off parameter(s)
- no quark confinement: quark-antiquark thresholds in quark loops, "free" quarks in hadronic phase

2. *Nonlocality*
   Eliminate ultraviolet divergences $\rightarrow$ unique extension beyond mean field
3. *Nonlocality + effective potential of Polyakov loops*
   Description of hadron phase, where the fluctuations plays major role

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Nonlocal $SU(2)$ chiral quark model at mean field. Lagrangian.

The quark part of Lagrangian of the nonlocal model of the NJL type has the form

$$\mathcal{L}_q = \bar{q}(x)(i\hat{\partial} - m_c)q(x) + \frac{G}{2}[J_\sigma(x)J_\sigma(x) + J_\pi^a(x)J_\pi^a(x)]$$

where m_c is the current quark mass and nonlocal quark currents are

$$J_I(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x - x_1)\,\Gamma_I\, q(x + x_2), \quad \Gamma_\sigma = \mathbf{1},\, \Gamma_\pi^a = i\gamma^5\tau^a$$

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$$Z(\bar\eta,\eta) = \frac{1}{N}\int \mathcal{D}\bar{q}\mathcal{D}q \exp\left(i\mathcal{S}(\bar{q},q) + i\int d^4x[\eta\bar{q} + \bar\eta q]\right)$$

$$\int \mathcal{D}\Phi \exp\left(i\int d^4x(\pm J\Phi - B\Phi^2)\right) = \frac{1}{N'}\exp\left(i\int d^4x \frac{J^2}{4B}\right)$$

$$Z(\bar\eta,\eta) = \frac{1}{N''}\int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}\tilde\sigma\mathcal{D}\pi \exp\left(i\mathcal{S}(q,\bar{q},\tilde\sigma,\pi) + i\int d^4x[\eta\bar{q} + \bar\eta q]\right)$$

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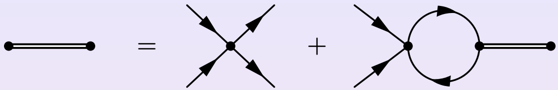
After linearization of the four-fermion vertices by introducing auxiliary scalar ($\tilde{\sigma}$) and pseudoscalar (π^a) meson fields the quark sector is described by the Lagrangian

$$\mathcal{L}_{q\pi\sigma} = \bar{q}(x)(i\hat{\partial} - m_c)q(x) - \frac{\pi_a^2 + \tilde{\sigma}^2}{2G} + J_\sigma(x)\tilde{\sigma}(x) + \pi^a(x)J_\pi^a(x).$$

The scalar field $\tilde{\sigma}$ has a nonzero vacuum expectation value: $\langle 0|\tilde{\sigma}|0\rangle = \sigma_0 \neq 0$. To arrive at the scalar field with zero expectation value, it is necessary to redefine the field: $\tilde{\sigma} = \sigma + \sigma_0$. As a result the chiral symmetry is spontaneously broken and quark acquires a dynamical mass $M(p^2) = m_c + m_d f^2(p^2)$, where $m_d = -\sigma_0$.

Mesons can be described via a Bethe-Salpeter equation.



The meson propagators are given by

$$\mathrm{D}_p^{\mathrm{M}} = \frac{1}{-G^{-1} + \Pi_p^{\mathrm{M}}},$$

where M is π or σ. $\Pi_{\mathrm{M}}(p^2)$ are the polarization operators defined by

$$\Pi_p^{\mathrm{M}} = -i \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[S_{k_-} \Gamma^{\mathrm{M}}_{k_-,k_+} S_{k_+} \Gamma^{\mathrm{M}}_{k_+,k_-}\right],$$

where $k_\pm = k \pm p/2$. In Euclidean space polarization loops takes the form

$$\Pi_p^{\mathrm{M}} = 4N_c N_f \int \frac{d_E^4 k}{(2\pi)^4} \frac{f_{k_+}^2 f_{k_-}^2}{D_{k_+} D_{k_-}} \left[(k_+ \cdot k_-) \pm m_{k_+} m_{k_-}\right],$$

here plus corresponds to π and minus to σ.

In order to have correspondence with QCD the quark mass should scale as N_c^0 for large number of colors

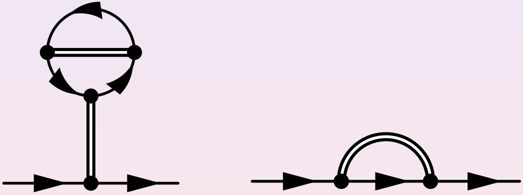
$$m_d = G N_c \cdot 8 \int \frac{d_E^4 k}{(2\pi)^4} f^2(k) \frac{m(k)}{k^2 + m^2(k)}$$

This means that four-quark coupling constant should scales as $G \sim 1/N_c$. As a result meson propagator leads to $1/N_c$ suppression of diagrams

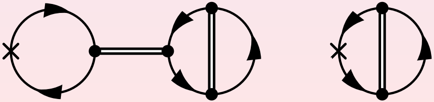
$$\mathrm{D}_p^{\mathrm{M}} = \frac{1}{-G^{-1} + \Pi_p^{\mathrm{M}}} \to \frac{1}{N_c},$$

Beyond the mean-field there are corrections to dynamical quark mass and renormalization functions, $\Sigma_p^{Nc} = \not{p} A_p + B_p$.

$$
\begin{aligned}
\left(S_p^{\mathrm{mf+Nc}}\right)^{-1} &= S_p^{-1} + \Sigma_p^{\mathrm{Nc}} \\
S_p^{\mathrm{mf+Nc}} &\approx S_p - S_p \Sigma_p^{\mathrm{Nc}} S_p + \ldots
\end{aligned}
$$

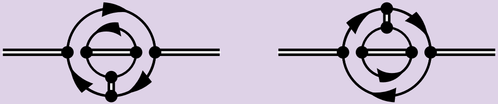


Corresponding corrections to the quark condensate are

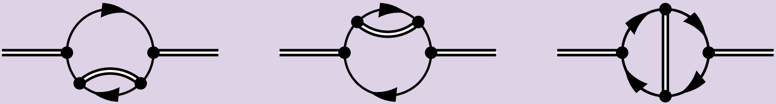


Beyond mean field. Meson propagator.

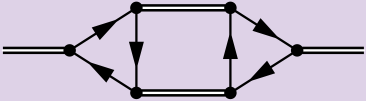
Type 1 $1/N_c$ corrections



Type 2 $1/N_c$ corrections

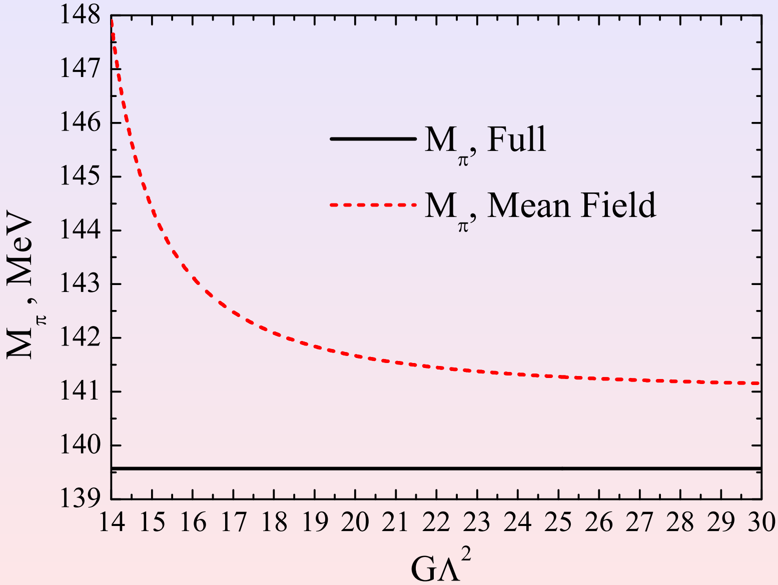


Type 3 $1/N_c$ corrections

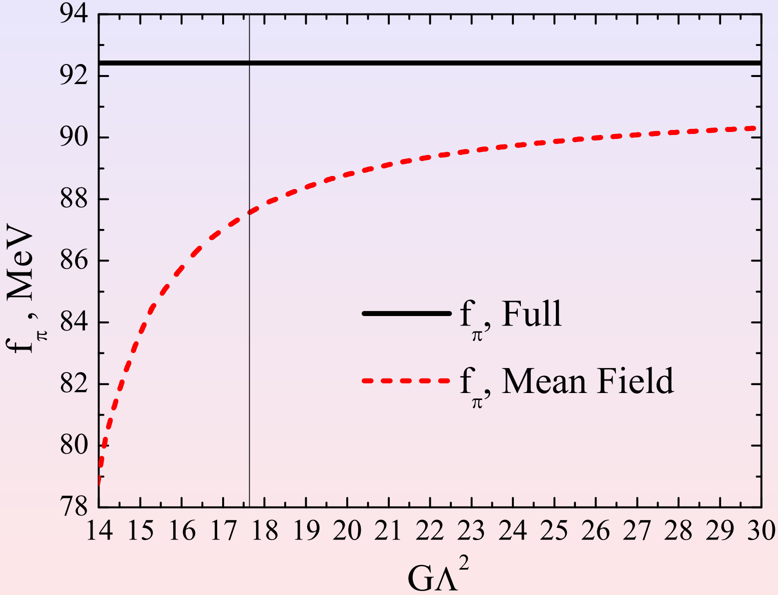


Fit model parameters using physical values for pion mass $M_{\pi^\pm} = 139.57$ MeV and weak pion decay constant $f_\pi = 92.42$ MeV.

N	Λ, MeV	m_c, MeV	m_c^{MF}, MeV	m_d, MeV	m_d^{MF}, MeV	$G\Lambda^2$	$G^{\mathrm{MF}}\Lambda^2$
1	1479.2	2.82	2.63	139.2	200	13.35	14.02
2	934.8	5.58	5.51	211.2	250	14.89	15.61
3	705.9	8.64	8.59	269.1	300	17.06	17.85
4	**670.3**	**9.38**	**9.31**	**281.9**	**311.7**	**17.64**	**18.45**
5	580.5	11.78	11.72	322.5	350	19.72	20.61
6	500.8	14.95	14.88	373.8	400	22.83	23.84
7	445.3	18.15	18.07	424.0	450	26.33	27.49
8	404.4	21.37	21.26	473.4	500	30.20	31.55


148
147
146
145
144
143
142
141
140
139
14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
M_π, MeV
$G\Lambda^2$
M_π, Full
M_π, Mean Field

Beyond mean field. Weak pion decay constant.



Finite T. Local NJL I

$$\mathcal{L}(x) \;=\; \bar{q}(x)(i\hat{\partial}_x - m_c)q(x) + \frac{G_1}{2}\left((\bar{q}(x)q(x))^2 + (\bar{q}(x)i\gamma^5\tau^a q(x))^2\right)$$

where $\bar{q}(x) = \{u(x), d(x)\}$ are the fields of u, d antiquarks, m_c is the current quark mass, G_1 is the four-quark coupling constant, τ^a are the Pauli matrices.
After bosonization the Lagrangian takes the form

$$\mathcal{L}(\bar{q}, q, \sigma, \pi) = \bar{q}(x)(i\hat{\partial}_x - m_c + \sigma(x) + i\gamma_5\tau_a\pi_a(x))q(x) - \frac{\sigma^2(x) + \pi_a^2(x)}{2G_1},$$

where $\sigma(x)$, $\pi_a(x)$ are the scalar and pseudoscalar meson fields.
Light current quarks transform to massive constituent quarks as a result of spontaneous chiral symmetry breaking($\langle\sigma\rangle_0 \neq 0$).

Gap equation

$$m = m_c - 8mGI_1(m)$$

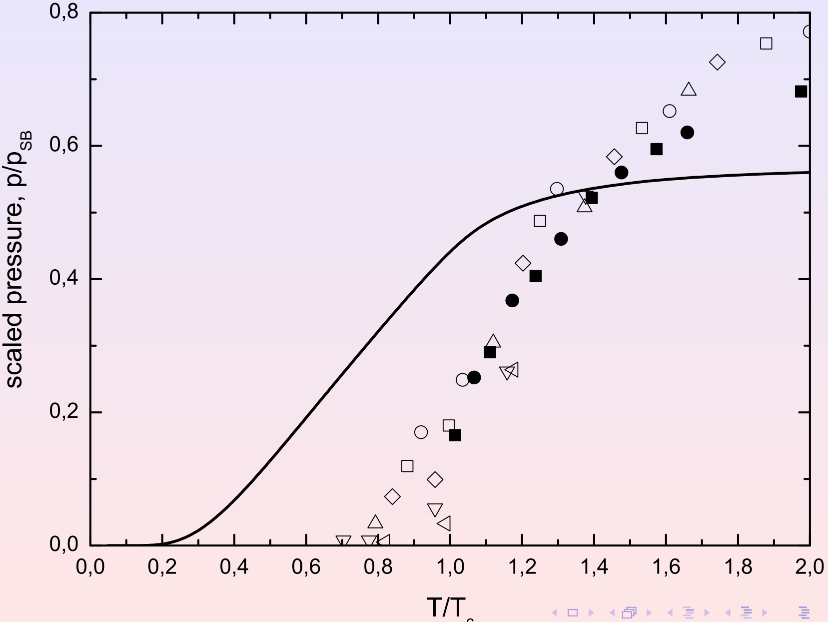
$I_1(m,T,\mu)$ is quadratically divergent integral. With help of cut-off over 3 momentum it takes form

$$I_1(m,T,\mu) = \frac{N_c}{(2\pi)^2}\int_0^\Lambda dp\frac{p^2}{E}\left(1-\eta(\vec{p},\mu)-\tilde{\eta}(\vec{p},\mu)\right),$$

where

$$\eta(\vec{p},\mu) = \left(1+\exp\frac{E-\mu}{T}\right)^{-1}, \quad \tilde{\eta}(\vec{p},\mu) = \left(1+\exp\frac{E+\mu}{T}\right)^{-1}$$

Finite T. Scaled pressure in local NJL model as a function of temperature.



Untraced Polyakov loop(Polyakov line)

$$L(\vec{x}) = \mathcal{P}\mathrm{exp}\left\{ i \int_0^\beta d\tau A_4(\tau, \vec{x}) \right\}$$

transforms under SU(3) gauge transformations as : $L \to U L U^\dagger$
Gauge invariant object

$$\Phi = \frac{1}{3}\mathrm{Tr}_c L \qquad \bar{\Phi} \;=\; \frac{1}{3}\mathrm{Tr}_c L^+$$

transform under global Z(3) transformations :

$$\Phi \to e^{i\frac{2\pi n}{3}}\Phi \qquad \bar{\Phi} \to e^{-i\frac{2\pi n}{3}}\bar{\Phi}$$

$$\langle \Phi \rangle = e^{-F_q/T}$$

confinement $\langle \Phi \rangle = 0$
deconfinement $\langle \Phi \rangle \neq 0$

Polyakov loop potential $\mathcal{U}$

U potential I

$$\frac{\mathcal{U}(\Phi,T)}{T^4}(\Phi,\bar\Phi) = -\frac{b_2(T)}{2}\bar\Phi\Phi - \frac{b_3}{6}(\Phi^3+\bar\Phi^3) + \frac{b_4}{4}(\Phi\bar\Phi)^2,$$

$$b_2(T) = a_0 + a_1\left[\frac{T_0}{T}\right] + a_2\left[\frac{T_0}{T}\right]^2 + a_3\left[\frac{T_0}{T}\right]^3$$

C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D **73** (2006) 014019.

U potential II

$$\frac{U(\Phi,\bar\Phi)}{T^4} = -\frac{1}{2}a(T)\,\bar\Phi\Phi + b(T)\,\ln\left[1-6\,\bar\Phi\Phi+4\left(\bar\Phi^3+\Phi^3\right)-3\left(\bar\Phi\Phi\right)^2\right]$$

with

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2, \qquad b(T) = b_3\left(\frac{T_0}{T}\right)^3.$$

S. Roessner, C. Ratti and W. Weise, Phys. Rev. D **75** (2007) 034007.

NJL model + Polyakov loop potential = PNJL model

$$\mathcal{L}(x) \quad = \quad \bar{q}(x)(i\hat{D}_x - m_c)q(x) + \frac{G_1}{2}\left((\bar{q}(x)q(x))^2 + (\bar{q}(x)i\gamma^5\tau^a q(x))^2\right) - U(\Phi, T)$$

A constant temporal background gauge field $\phi \equiv \langle A_4 \rangle = \langle iA_0 \rangle \phi_3\lambda_3 + \phi_8\lambda_8$ is minimally coupled to the quarks. Here $\Phi = \frac{1}{N_c}\,\mathrm{Tr}_c\, e^{i\phi/T}$ denotes the Polyakov loop expectation value and $\bar{\Phi}$ its conjugate. Equations of motion of mean fields are

$$\frac{\partial\Omega}{\partial m} = 0, \quad \frac{\partial\Omega}{\partial\Phi} = 0, \quad \frac{\partial\Omega}{\partial\bar{\Phi}} = 0$$

NJL model + Polyakov loop potential = PNJL model

$$I_1(m,T,\mu) = \frac{N_c}{(2\pi)^2}\int\limits_0^\Lambda dp\frac{p^2}{E}\left(1-\eta^\Phi(\vec{p},\mu)-\tilde{\eta}^\Phi(\vec{p},\mu)\right),$$

$$\eta^\Phi(\vec{p},\mu) = \frac{(\Phi+2\bar{\Phi}e^{-\frac{E-\mu}{T}})e^{-\frac{E-\mu}{T}}+e^{-3\frac{E-\mu}{T}}}{1+3(\Phi+\bar{\Phi}e^{-\frac{E-\mu}{T}})e^{-\frac{E-\mu}{T}}+e^{-3\frac{E-\mu}{T}}},$$

$$\tilde{\eta}^\Phi(\vec{p},\mu) = \frac{(\bar{\Phi}+2\Phi e^{-\frac{E+\mu}{T}})e^{-\frac{E+\mu}{T}}+e^{-3\frac{E+\mu}{T}}}{1+3(\bar{\Phi}+\Phi e^{-\frac{E+\mu}{T}})e^{-\frac{E+\mu}{T}}+e^{-3\frac{E+\mu}{T}}}$$

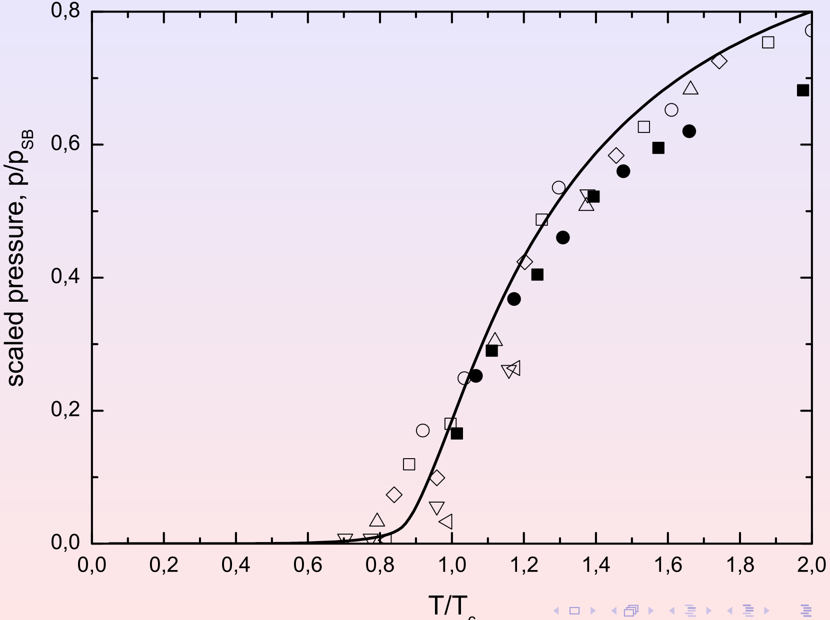
$\Phi=\bar{\Phi}=1$

$$\eta^\Phi(\vec{p},\mu)=\eta(\vec{p},\mu),\quad \tilde{\eta}^\Phi(\vec{p},\mu)=\tilde{\eta}(\vec{p},\mu)$$

$\Phi=\bar{\Phi}=0$

$$\eta^\Phi(\vec{p},\mu)=\left(1+e^{3\frac{E-\mu}{T}}\right)^{-1},\quad \tilde{\eta}^\Phi(\vec{p},\mu)=\left(1+e^{3\frac{E+\mu}{T}}\right)^{-1}$$

Scaled pressure in PNJL model as a function of temperature.

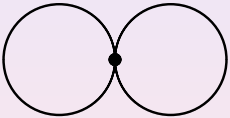


Finite T. Thermodynamic potential

The model can be easily extend to the finite temperature case using Φ-derivable model together with $1/N_c$ expansion. Thermodynamic potential per volume is

$$\Omega = i\mathrm{Tr}\ln(\mathbf{S}^{-1}) - i\mathbf{Tr}(\Sigma\mathbf{S}) + \Psi(\mathbf{S}) + U(\Phi,\bar{\Phi}) - \Omega_0$$

where $\mathbf{S}^{-1} = (S^c)^{-1} + \Sigma$ represents the full quark propagator, $\Sigma = \delta\Psi/\delta(i\mathbf{S})$, and $U(\Phi,\bar{\Phi})$ is the Polyakov loop potential.
The mean-field results can be obtained using “glasses” form of Ψ potential



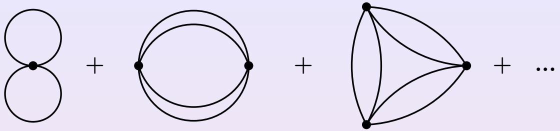
In the nonlocal model “glasses” potential takes the form

$$\Psi_{\text{glasses}} = -\sum_{\mathrm{M}=\pi,\sigma} \frac{G}{2}\left[-\mathbf{Tr}(\Gamma^{\mathrm{M}} i\mathbf{S})\right]^2,$$

Equations of motion can be obtain by minimization of the thermodynamic potential

$$\frac{\partial\Omega}{\partial\mathbf{S}} = 0 \Rightarrow \frac{\partial\Omega}{\partial m_d} = 0, \qquad \frac{\partial\Omega}{\partial\phi_3} = 0$$

The next step is to take into account also the “ring sum”



$$\Psi_{\text{ring}} = -\sum_{\text{M}=\pi,\sigma} \frac{d_{\text{M}}}{2} i\mathbf{Tr}\ln\left[1 - G\mathbf{\Pi}^{\text{M}}\right],$$

where $\mathbf{\Pi}^{\text{M}}$ is polarization loop consist of **full** quark propagators.
In strict $1/N_c$ expansion scheme we have to use mean-field equation for dynamical quark mass and modified equation for ϕ_3 and then calculate $1/N_c$ corrections to quark propagator.

$$\begin{aligned} \Omega &= \Omega_{\text{mf}} + \Omega_{\text{Nc}} \\ \Omega_{\text{mf}} &= i\text{Tr}\ln(S^{-1}) - i\text{Tr}(\Sigma S) - \frac{G}{2}\left[-\text{Tr}(\Gamma^{\sigma} iS)\right]^2 + U(\Phi, \bar{\Phi}) \\ \Omega_{\text{Nc}} &= -\sum_{M=\pi,\sigma} \frac{d_M}{2} i\text{Tr}\ln\left[1 - G\Pi^{\text{M}}\right] \end{aligned}$$

Nonlocal $SU(3)$ chiral quark model.

The Lagrangian of the $SU(3) \times SU(3)$ chiral quark model contains chirally symmetric four-quark interaction and six-quark interaction which brakes chiral symmetry

$$\mathcal{L} = \bar{q}(x)(i\hat{\partial} - m_c)q(x) + \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)]$$
$$- \frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_S^a(x)J_P^b(x)J_P^c(x)],$$

where $q\,(x)$ are the quark fields, m_c is the diagonal matrix of the quark current masses $m_{c,i}$ $(i = u, d, s)$, G and H are the four- and six-quark coupling constants. Second line in the Lagrangian represents the Kobayashi–Maskawa–t'Hooft determinant vertex with the structural constant

$$T_{abc} = \frac{1}{6}\epsilon_{ijk}\epsilon_{mnl}(\lambda_a)_{im}(\lambda_b)_{jn}(\lambda_c)_{kl},$$

where λ_a are the Gell-Mann matrices for $a = 1, .., 8$ and $\lambda_0 = \sqrt{2/3}I$. The nonlocal structure of the model is introduced via the nonlocal quark currents

$$J_M^a(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{q}(x - x_1)\,\Gamma_M^a q(x + x_2),$$

where $M = S, P$ and $\Gamma_S^a = \lambda^a$, $\Gamma_P = i\gamma^5\lambda^a$, and $f(x)$ is a form factor reflecting the nonlocal properties of the QCD vacuum.

Nonlocal $SU(3)$ chiral quark model.

The model can be bosonized using the stationary phase approximation which leads to the system of gap equations for the dynamical quark masses $m_{d,i} \quad (i = u, d, s)$

$$m_{d,i} + GS_i + \frac{H}{2} S_j S_k = 0, \quad S_i = -8N_c \int \frac{d_E^4 k}{(2\pi)^4} \frac{f^2(k^2) m_i(k^2)}{D_i(k^2)},$$

where $i \neq j \neq k$, $m_i(k^2) = m_{c,i} + m_{d,i} f^2(k^2)$, $D_i(k^2) = k^2 + m_i^2(k^2)$ is the dynamical quark propagator obtained by solving the Schwinger-Dyson equation, $f(k^2)$ is the nonlocal form factor in the momentum representation. We take Gaussian form factor, $f^2(k^2) = \exp(-k^2/\Lambda^2)$, in Euclidean space where Λ is the parameter of nonlocality.

The mean-field thermodynamic potential reads

$$\Omega_{\mathrm{mf}} = \mathcal{U}(\Phi, \bar{\Phi}) - 2T \sum_{f,i,n} \int \frac{d^3k}{(2\pi)^3} \ln\left[D_f((k^i_{n,f})^2)\right] - \\ - \frac{1}{2}\left(\frac{H}{2} S_u S_d S_s + \sum_f \left(m_{d,f} + \frac{G}{2} S_f\right) S_f\right),$$

where summations are of flavors ($f = u, d, s$), color ($i = 0, +, -$) and fermionic Matsubara frequencies.

$$N_c \int \frac{d^4_E k}{(2\pi)^4} I_f(k^2) \quad \rightarrow \quad \sum_i T \sum_n \int \frac{d^3k}{(2\pi)^3} I_f\left((k^i_{n,f})^2\right),$$

where $(k^i_{n,f})^2 = (\omega^i_n - i\mu_f)^2 + \vec{k}^2$. Due to the coupling to the Polyakov loop the fermionic Matsubara frequencies $\omega_n = (2n+1)\pi T$ are shifted

$$\omega^{\pm}_n = \omega_n \pm \phi_3, \quad \omega^0_n = \omega_n.$$

The order parameters (mean field values for m_d and ϕ_3) are obtained by minimization of the mean-field part of the thermodynamic potential

$$\frac{\partial \Omega_{\rm mf}}{\partial m_{d,u}} = 0, \quad \frac{\partial \Omega_{\rm mf}}{\partial m_{d,d}} = 0, \quad \frac{\partial \Omega_{\rm mf}}{\partial m_{d,s}} = 0, \quad \frac{\partial \Omega_{\rm mf}}{\partial \phi_3} = 0$$

After calculation of mean field values one can calculate $1/N_c$ correction to thermodynamic potential

$$\Omega_{\rm corr} \quad = \quad \frac{1}{2} \sum_m \int \frac{d^3 p}{(2\pi)^3} \ln\left[1 - \mathbf{G_{ch}} \mathbf{\Pi_{ch}}(\vec{p}, \nu_m)\right]$$

where the sum is over bosonic Matsubara frequencies ν_m and includes full set of mesons in the corresponding channel. Namely, $(\pi^{\pm}, K^{\pm}, K^0, \bar{K}^0, \pi^0, \eta, \eta')$ in the pseudoscalar channel and $(a_0^{\pm}(980), \kappa^{\pm}, \kappa^0, \bar{\kappa}^0, \pi^0, a_0^0(980), \sigma, f_0(980))$ in the scalar one.

1. Combination of nonstrange(l) and strange(s) quark condensates – subtracted quark condensate,

$$\boldsymbol{\Delta}_{l,s} = \frac{\langle \bar{q}q \rangle_T^l - \frac{m_c^l}{m_c^s} \langle \bar{q}q \rangle_T^s}{\langle \bar{q}q \rangle^l - \frac{m_c^l}{m_c^s} \langle \bar{q}q \rangle^s}.$$

2. pressure, $p = -\Omega$
3. interaction measure, $I = \epsilon - 3p = T\frac{\partial p}{\partial T} - 4p$
4. diagonal and nondiagonal moments of conserved charge fluctuations: baryon number μ_B, strangeness μ_S and electric charge μ_Q,

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}$$

Quark chemical potentials can be expressed in terms of chemical potentials corresponding to conserved charges:

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q,\ \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q,\ \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_s.$$

Fluctuations of conserved charges can be related with quark susceptibilities

$$\begin{aligned}
\chi_2^B &= \frac{1}{9}\left(\chi_2^s + 2\chi_2^u + 2\chi_{11}^{ud} + 4\chi_{11}^{us}\right)\\
\chi_2^Q &= \frac{1}{9}\left(\chi_2^s + 5\chi_2^u - 4\chi_{11}^{ud} - 2\chi_{11}^{us}\right)\\
\chi_2^S &= \chi_2^s\\
\chi_{11}^{SQ} &= \frac{1}{3}\left(\chi_2^s - \chi_{11}^{us}\right)\\
\chi_{11}^{SB} &= \frac{1}{3}\left(-\chi_2^s - 2\chi_{11}^{us}\right)\\
\chi_{11}^{QB} &= \frac{1}{9}\left(-\chi_2^s + \chi_2^u + \chi_{11}^{ud} - \chi_{11}^{us}\right)
\end{aligned}$$

We will perform comparison with lattice QCD simulations

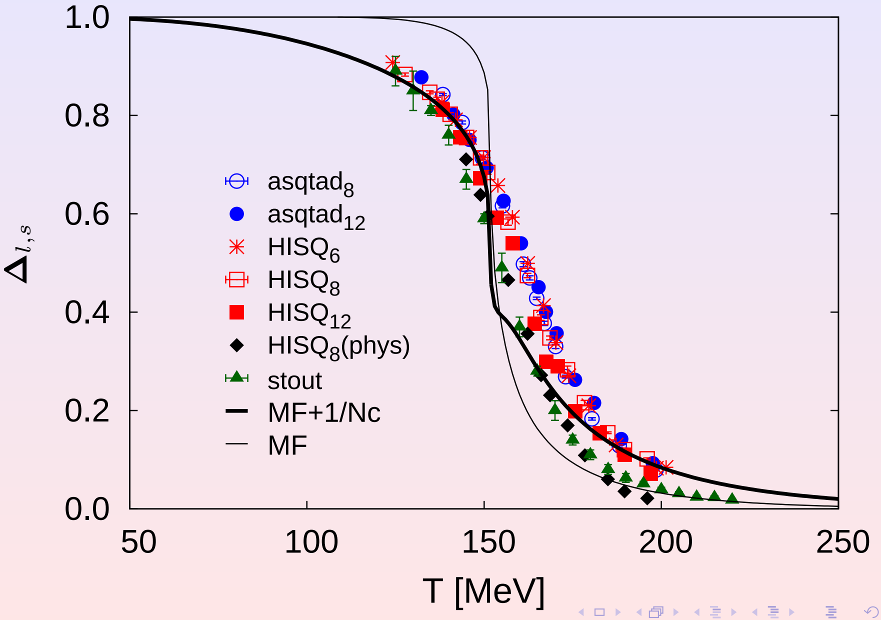
1. HISQ and asqtad actions, HotQCD Collaboration,
   A. Bazavov *et al.* Phys. Rev. D **86** (2012) 034509.
   A. Bazavov *et al.*, Phys. Rev. D **85** (2012) 054503.
2. Stout action, [Wuppertal-Budapest Collaboration],
   S. Borsanyi *et al.* JHEP **1009** (2010) 073.

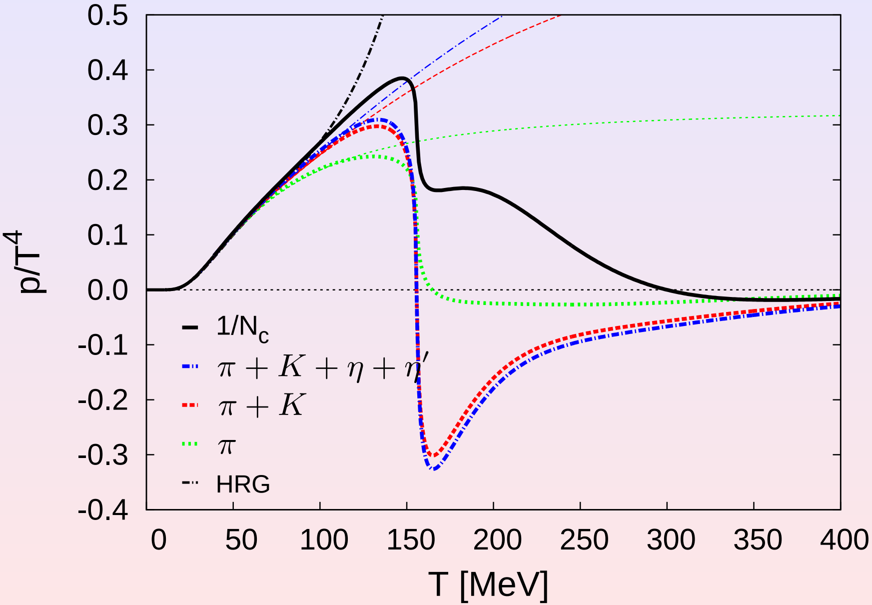
and Hadron Resonance Gas model. Thermodynamic potential for mesons and baryons in HRG model is

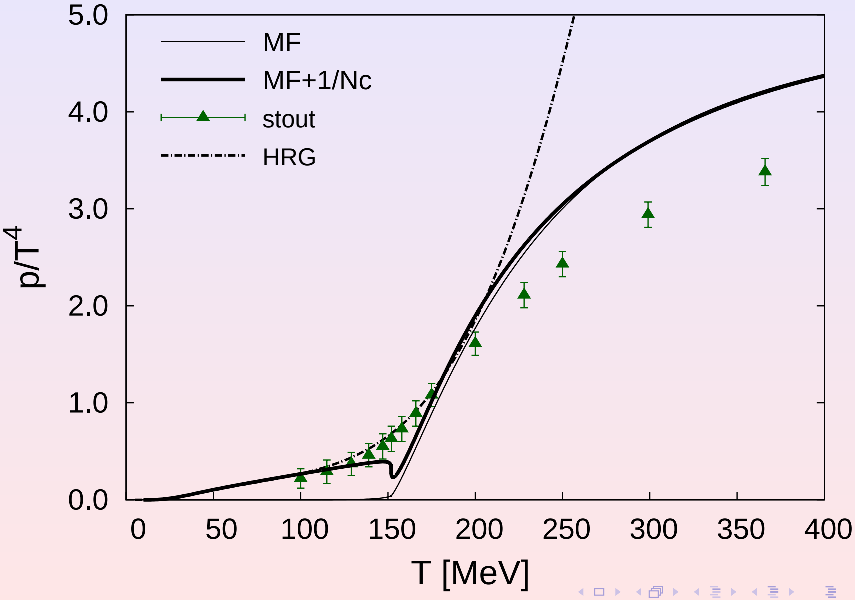
$$\Omega_i^M = +\frac{T d_i}{2\pi^2} \int dk\, k^2 \log\left[1 - e^{(B_i\mu_B + Q_i\mu_Q + S_i\mu_S)/T} e^{-E_i/T)}\right]$$
$$\Omega_i^B = -\frac{T d_i}{2\pi^2} \int dk\, k^2 \log\left[1 + e^{(B_i\mu_B + Q_i\mu_Q + S_i\mu_S)/T} e^{-E_i/T)}\right]$$

where $E_i = \sqrt{k^2 + M_i^2}$ and B_i, Q_i ,S_i, d_i are baryon number, charge, strangeness and degeneracy factor of hadron. We sum all hadrons from PDG with masses up to 2 GeV.

Subtracted quark condensate.




0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
p/T^4
0
50
100
150
200
250
300
350
400
T [MeV]
1/N_c
$\pi + K + \eta + \eta'$
$\pi + K$
π
HRG


MF
MF+1/Nc
stout
HRG
5.0
4.0
3.0
2.0
1.0
0.0
p/T^4
0
50
100
150
200
250
300
350
400
T [MeV]

Polyakov loop potential $\mathcal{U}$ modded.

U potential II

$$\frac{U(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\,\bar{\Phi}\Phi + b(T)\,\ln\left[1 - 6\,\bar{\Phi}\Phi + 4\left(\bar{\Phi}^3 + \Phi^3\right) - 3\left(\bar{\Phi}\Phi\right)^2\right]$$

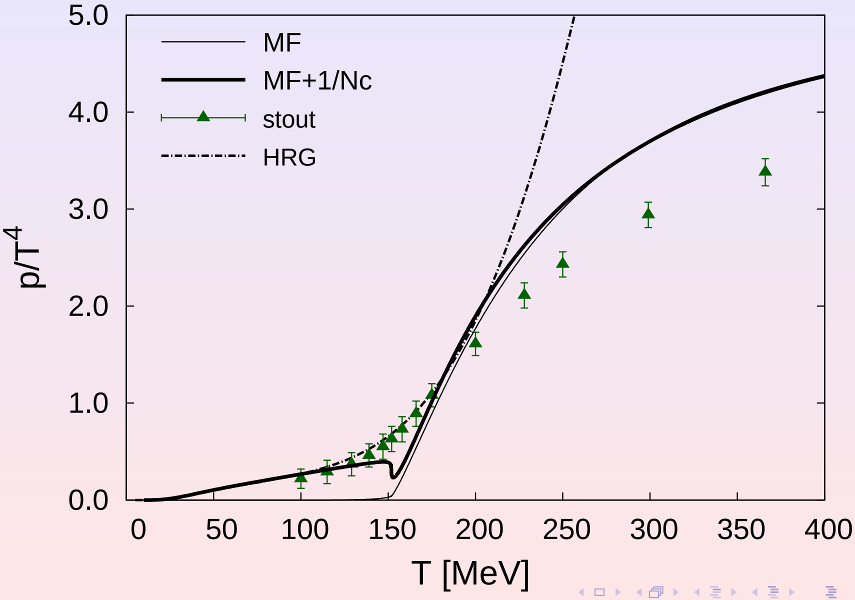
with

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2, \qquad b(T) = b_3\left(\frac{T_0}{T}\right)^3.$$

Naive modification

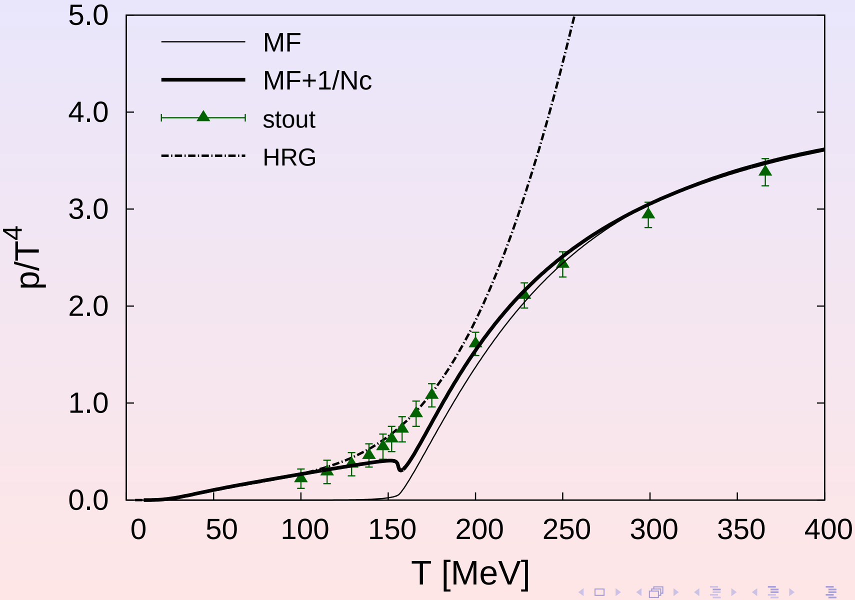
$$a_0' \to a_0/2$$

Pressure.

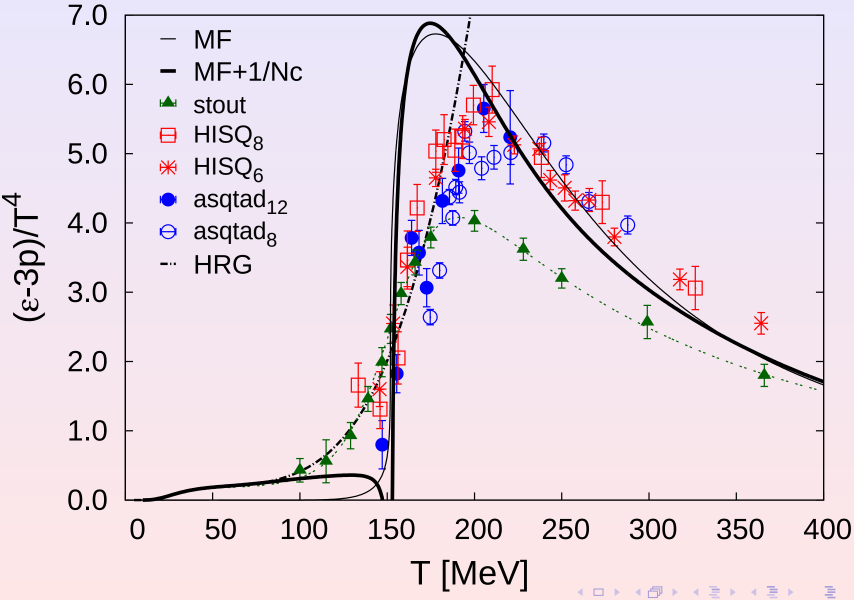


MF
MF+1/Nc
stout
HRG
p/T^4
5.0
4.0
3.0
2.0
1.0
0.0
0
50
100
150
200
250
300
350
400
T [MeV]

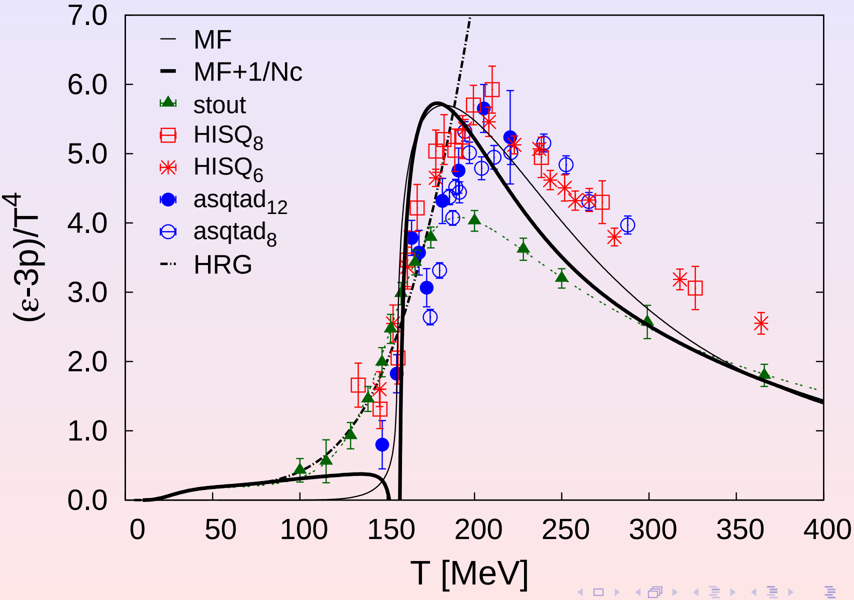
Pressure with modified PL potential.



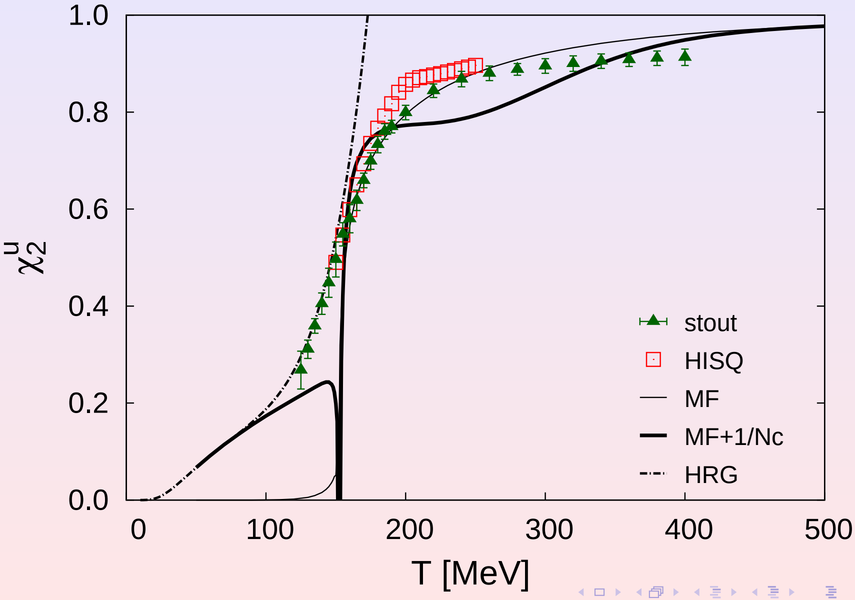
Interaction measure.



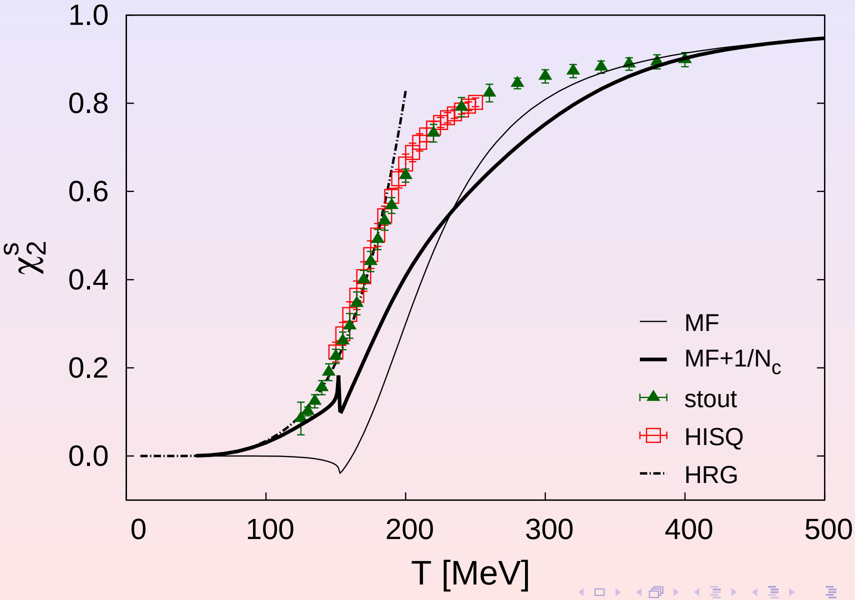
Interaction measure with modified PL potential.



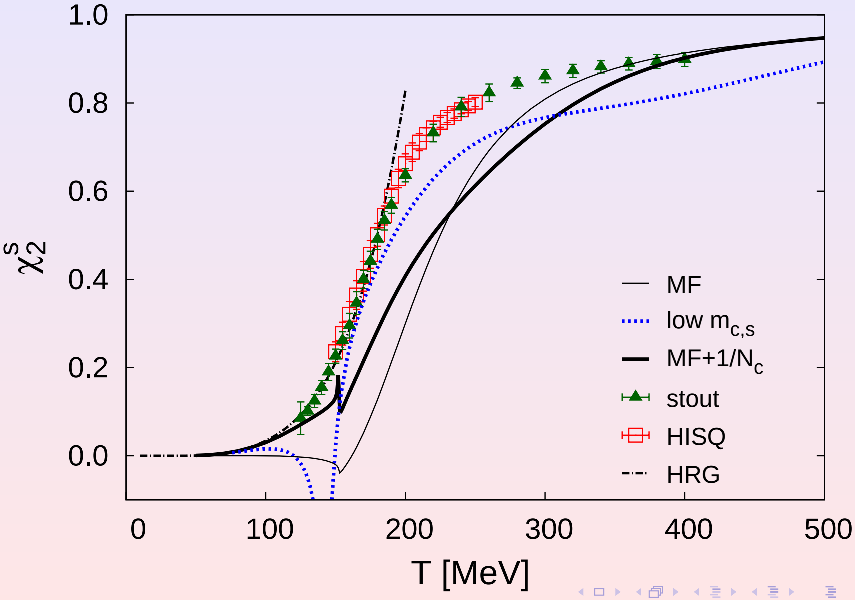
Nonstrange susceptibility.



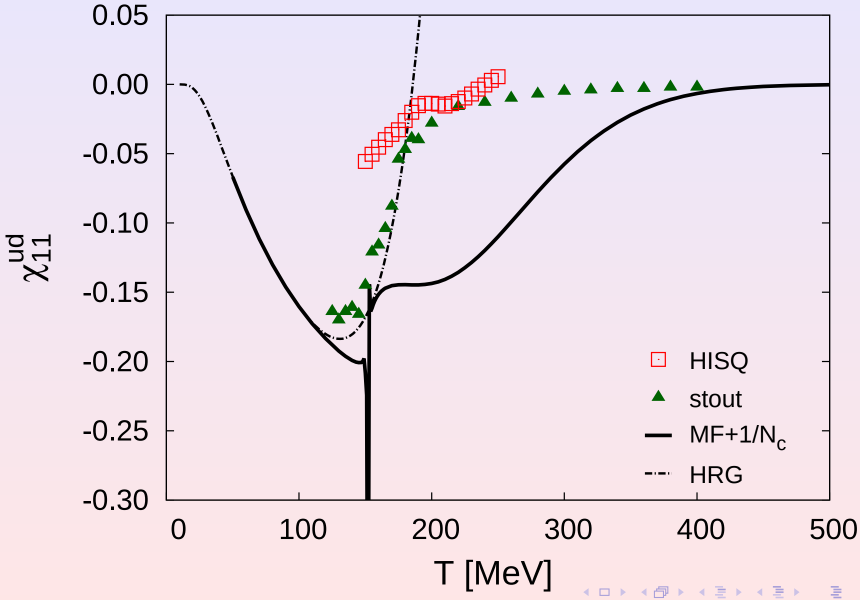
Strange susceptibility.

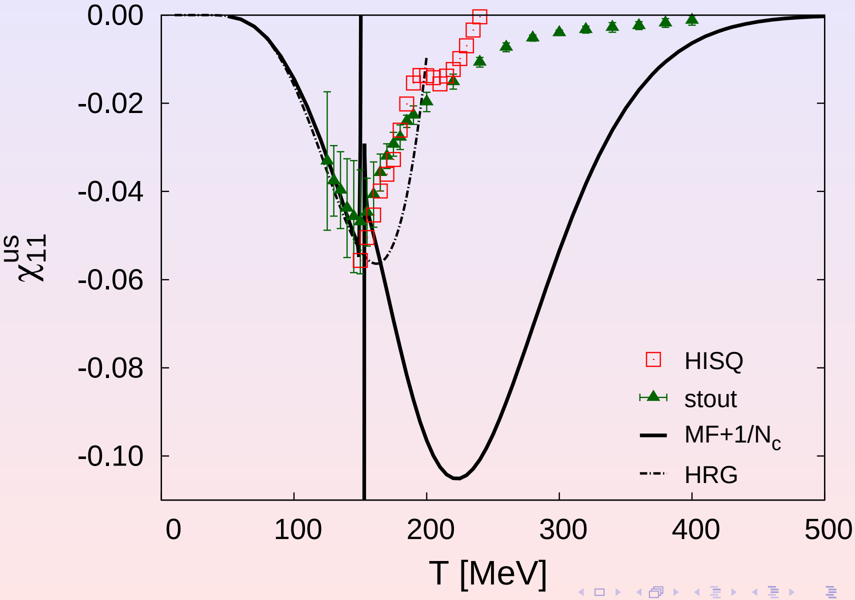


Strange susceptibility for low strange mass.

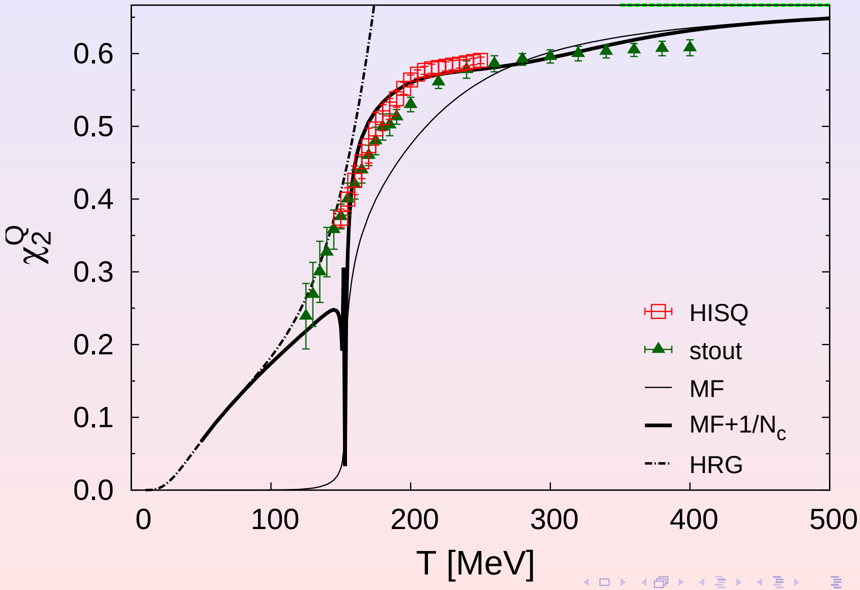


ud correlation.

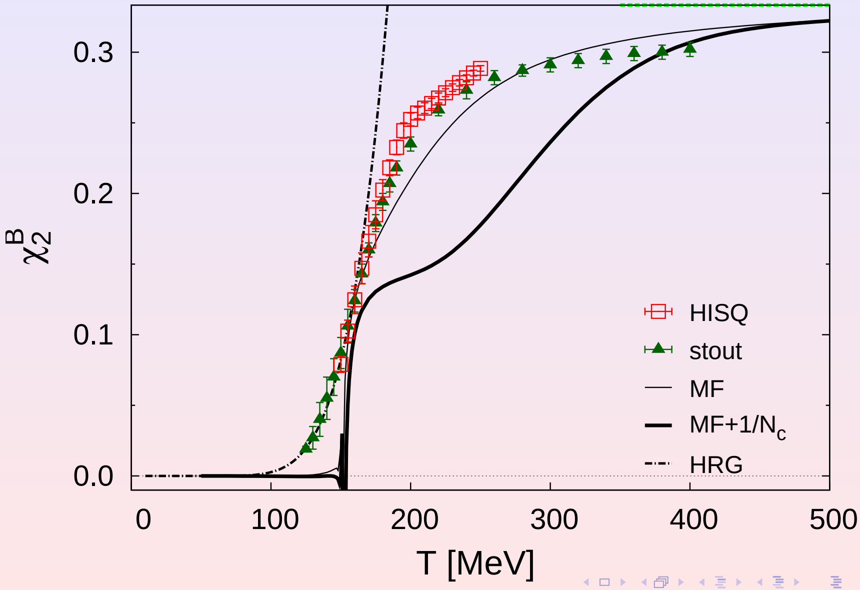


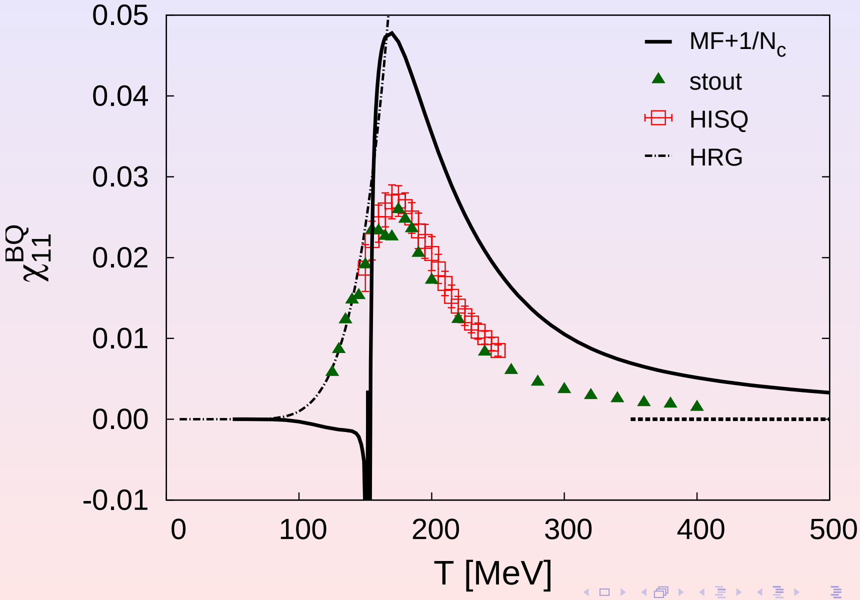

0.00
-0.02
-0.04
-0.06
-0.08
-0.10
χ_{11}^{us}
0
100
200
300
400
500
T [MeV]
HISQ
stout
MF+1/N$_c$
HRG

Charge fluctuation.

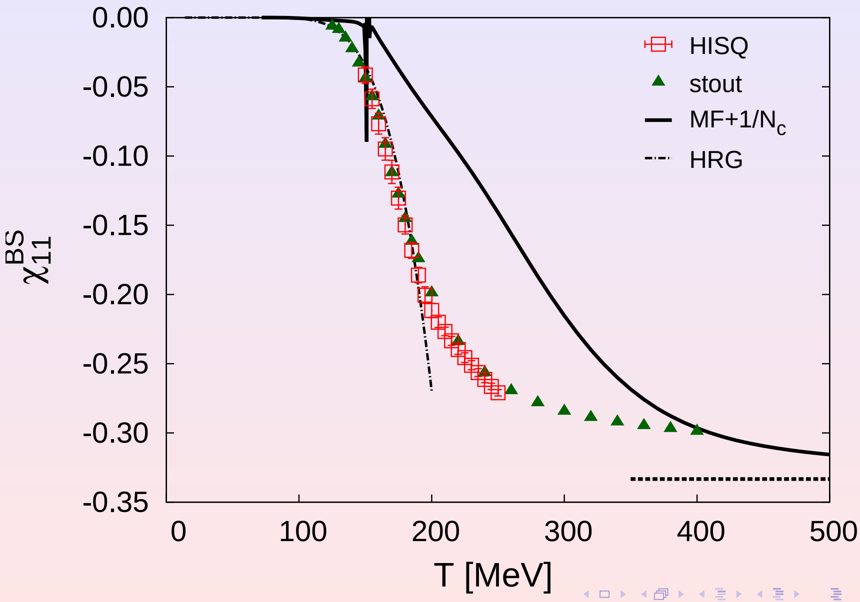


Baryon number fluctuation.

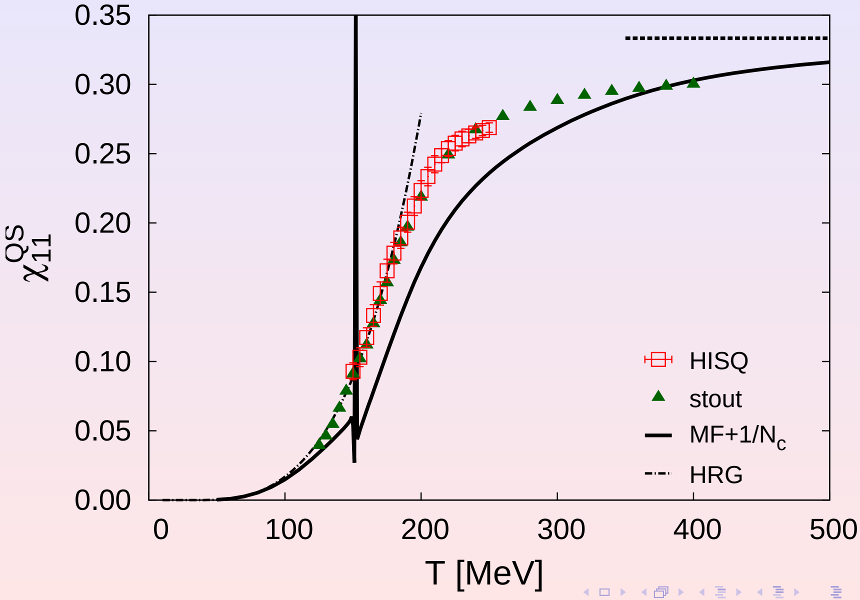


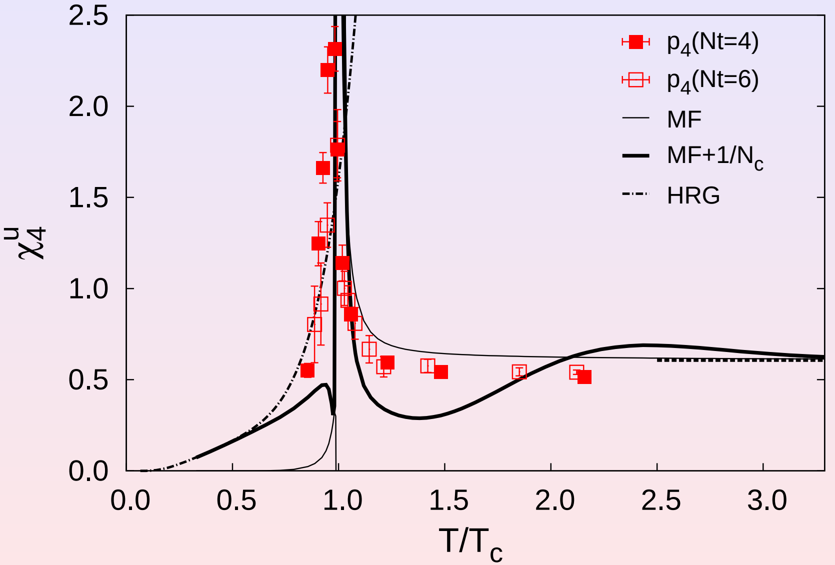

0.05
0.04
0.03
0.02
0.01
0.00
-0.01
χ_{11}^{BQ}
0
100
200
300
400
500
T [MeV]
MF+1/N$_c$
stout
HISQ
HRG

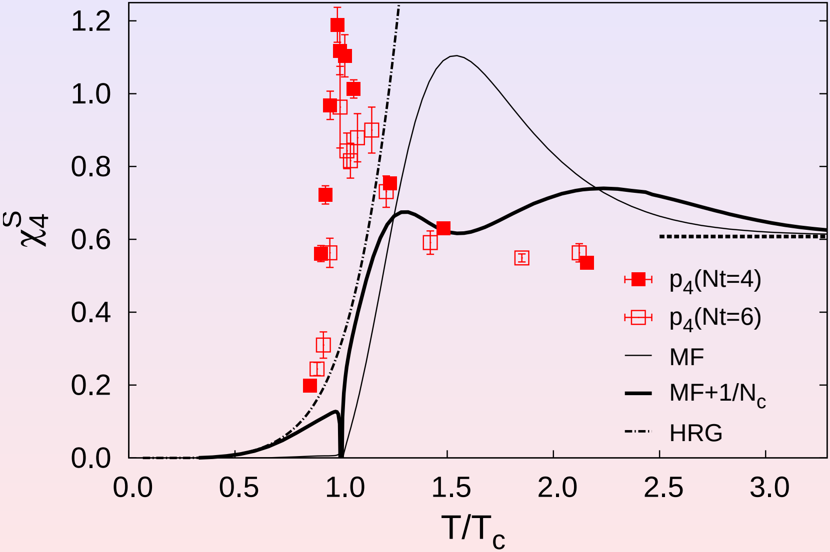
Baryon number-strangeness fluctuation.

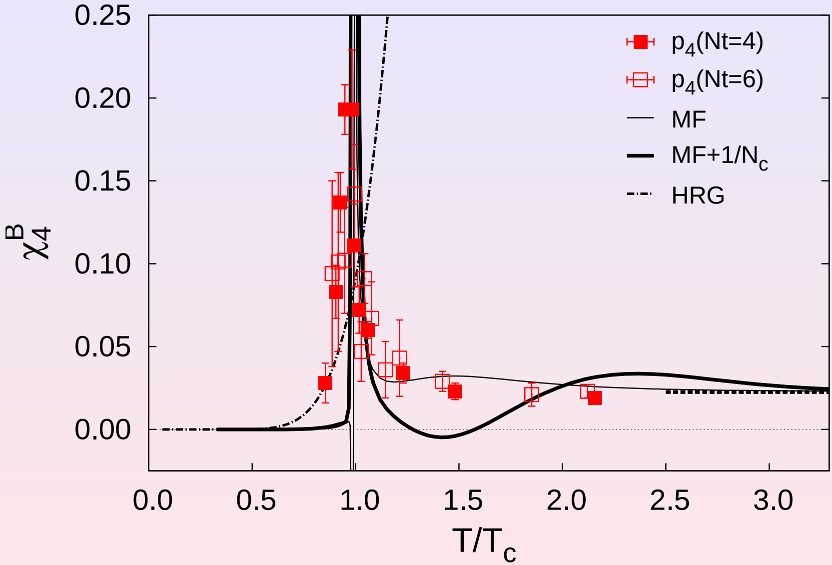


Charge-strangeness fluctuation.

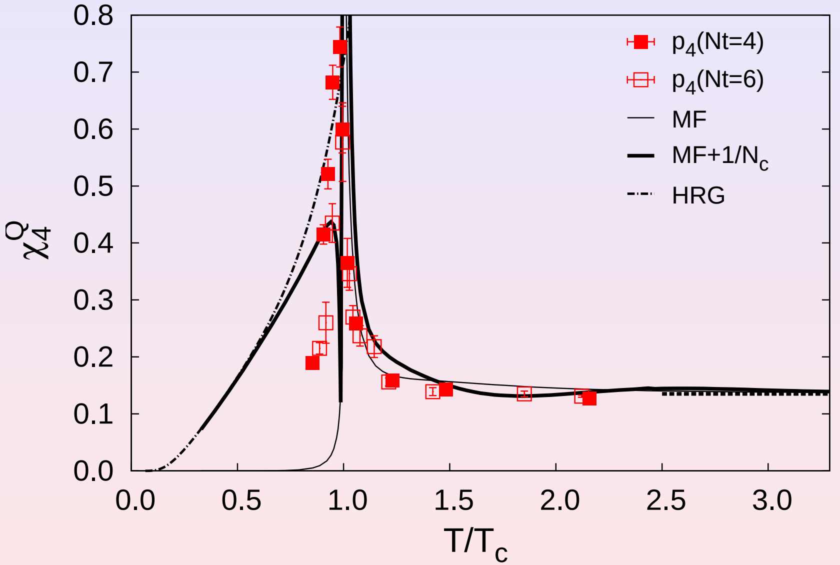



2.5
2.0
1.5
1.0
0.5
0.0
χ_4^u
0.0
0.5
1.0
1.5
2.0
2.5
3.0
T/T$_c$
p$_4$(Nt=4)
p$_4$(Nt=6)
MF
MF+1/N$_c$
HRG


1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
χ_4^S
T/T$_c$
p$_4$(Nt=4)
p$_4$(Nt=6)
MF
MF+1/N$_c$
HRG



0.25
0.20
0.15
0.10
0.05
0.00
χ_4^B
0.0
0.5
1.0
1.5
2.0
2.5
3.0
T/T$_c$
p$_4$(Nt=4)
p$_4$(Nt=6)
MF
MF+1/N$_c$
HRG


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
χ_4^Q
0.0
0.5
1.0
1.5
2.0
2.5
3.0
T/T$_c$
p$_4$(Nt=4)
p$_4$(Nt=6)
MF
MF+1/N$_c$
HRG

1. The nonlocal $SU(3)$ quark model is extended beyond mean field using a strict $1/N_c$ expansion scheme
2. Nonlocality provides an unambiguous way to consider a role of $1/N_c$ (mesonic) corrections.
3. Nonlocality+effective PL potential gives possibility to consider 'hadronic' phase with correct degrees of freedom.
4. At large temperatures $1/N_c$ corrections decreased (mesons 'melts').
5. No baryons $\rightarrow$ poor description of baryonic susceptibilities and mixed susceptibilities of different flavor.

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Thanks for your attention !

Nonlocal chiral quark model at mean field. External fields: Weak pion decay.

In order to obtain gauge-invariant interactions with axial external fields we introduce Schwinger phase factor

$$q(y) \to Q(x,y) = \mathcal{P}\exp\left\{ i\int\limits_x^y dz^\mu [V_\mu^a(z) + A_\mu^a(z)\gamma_5]T^a \right\} q(y),$$

$$\mathcal{L}_{q\pi\sigma VA} = \bar{q}(i\hat{\partial} - m_c - \hat{V} - \hat{A}\gamma_5)q - \frac{\pi_a^2 + \tilde{\sigma}^2}{2G} + J_\sigma\tilde{\sigma} + \pi^a J_\pi^a,$$

$$J_I(x) = \int d^4x_1 d^4x_2\, f(x_1) f(x_2)\, \bar{Q}(x - x_1, x)\, \Gamma_I\, Q(x, x + x_2)$$

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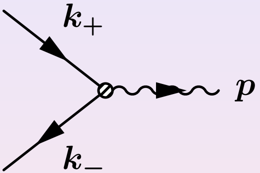
$$J_I(x) = \int d^4x_1 d^4x_2\, f(x_1)f(x_2)\,\bar{Q}(x-x_1,x)\,\Gamma_I\,Q(x,x+x_2)$$

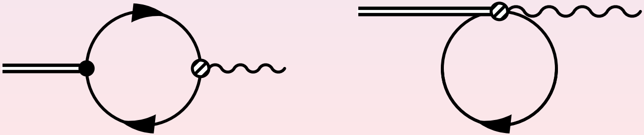
$$\frac{\partial}{\partial y^\mu}\int_x^y dz^\nu\ F_\nu(z) = F_\mu(y), \qquad \delta^{(4)}(x-y)\int_x^y dz^\nu\ F_\nu(z) = 0.$$

Nonlocal chiral quark model at mean field. External fields: Weak pion decay.

For weak pion decay we need vertexes with longitudinal projection of axial field with quark–anti-quark and quark–anti-quark–meson



As a result, the weak pion decay constant at mean field level contain two pieces



Sign of $1/N_c$ correction to quark condensate.

In nonlocal model $1/N_c$ correction to quark condensate is positive for all sets of model parameters. In local NJL model it was found that this correction is negative. In order to study transition from nonlocal to local models let us construct nonlocal model which contains three parameters, namely

1. parameter of nonlocality Λ (i.e. $f^2(p) = \exp(-p^2/\Lambda^2)$)
2. parameter of quark loop regularization Λ_q (Pauli-Villars regularization)
3. parameter of meson loop regularization Λ_M (3D regularization)

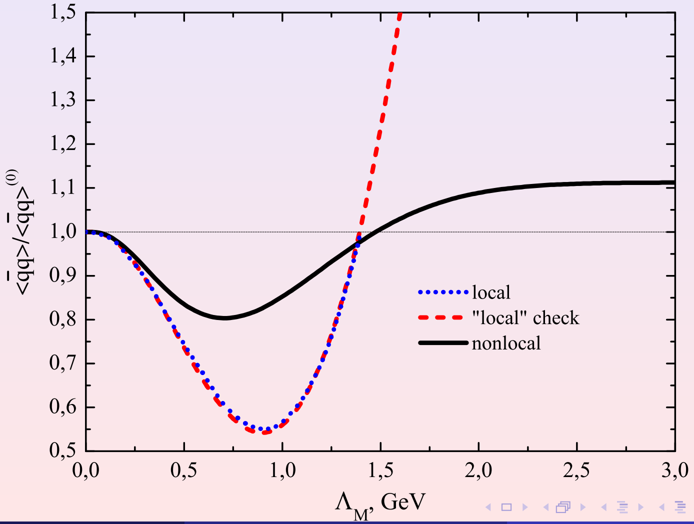
Local model can be obtained in the limit

$$\Lambda \to \infty \ , \quad \Lambda_q = \Lambda_q^{phys} \ , \quad \Lambda_M = \Lambda_M^{phys}$$

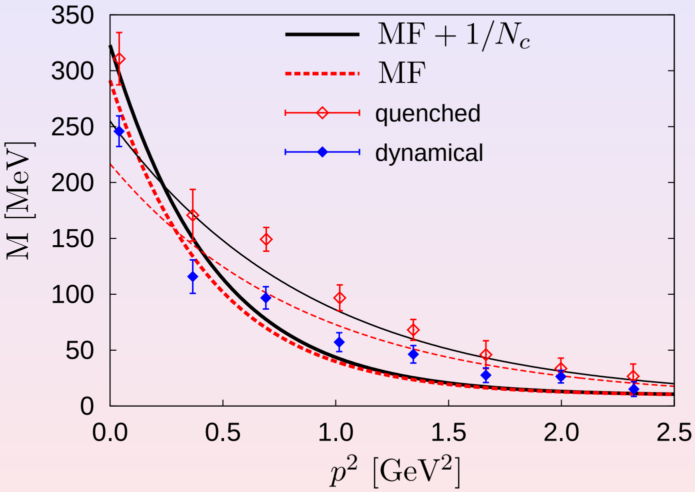
Usual nonlocal model without regularization can be obtained in the limit

$$\Lambda = \Lambda^{phys} \ , \quad \Lambda_q \to \infty \ , \quad \Lambda_M \to \infty$$

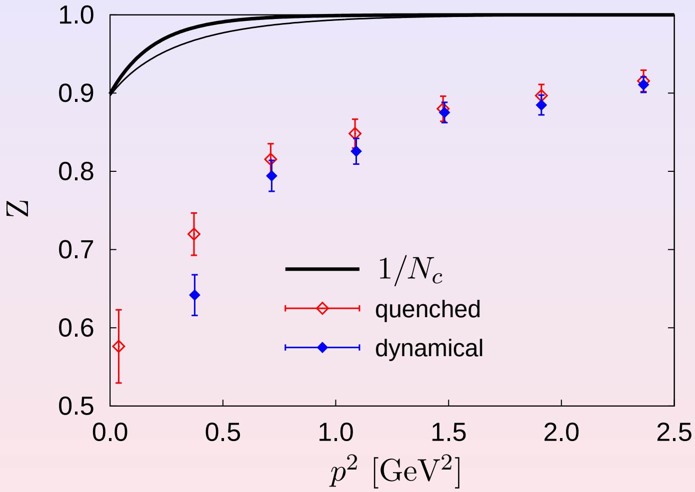
At mean field we have smooth transition from local model to nonlocal one. The next step is to consider $1/N_c$ corrections and investigate the role of mesonic 3D cut-off Λ_M.



Quark mass function.

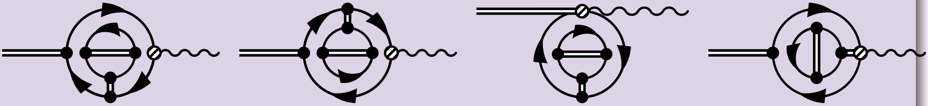


Quark renormalization function.

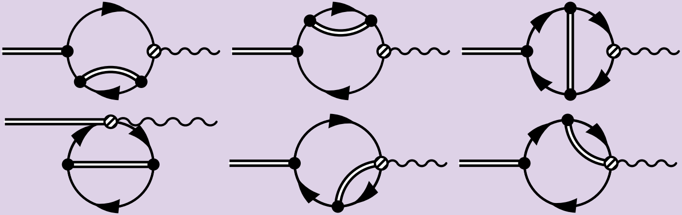


Beyond mean field. Weak pion decay.

Type 1 $1/N_c$ corrections for weak pion decay

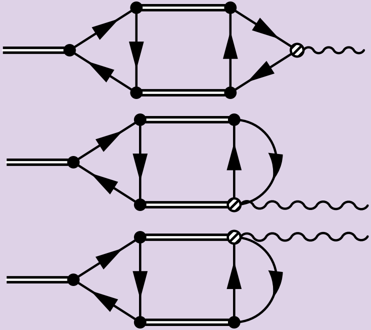


Type 2 $1/N_c$ corrections for weak pion decay



Beyond mean field. Weak pion decay.

Type 3 $1/N_c$ corrections for weak pion decay

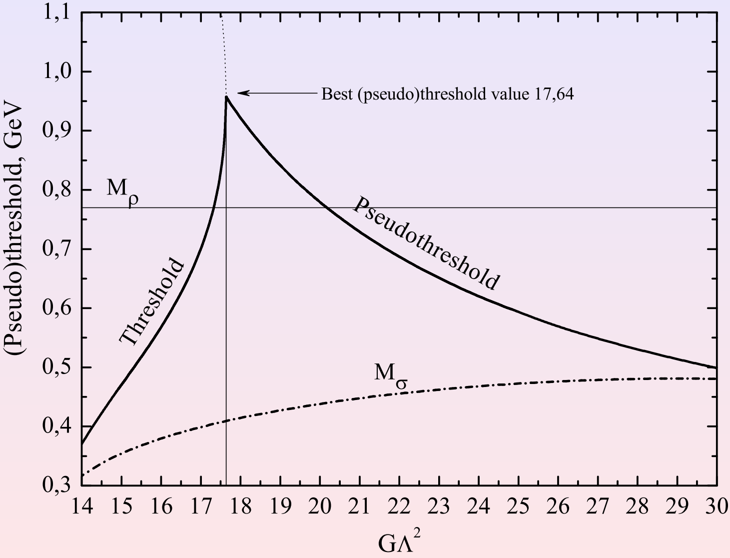


Beyond mean field. Properties of parameterizations: first pole(s) position and corresponding (pseudo)treshold.

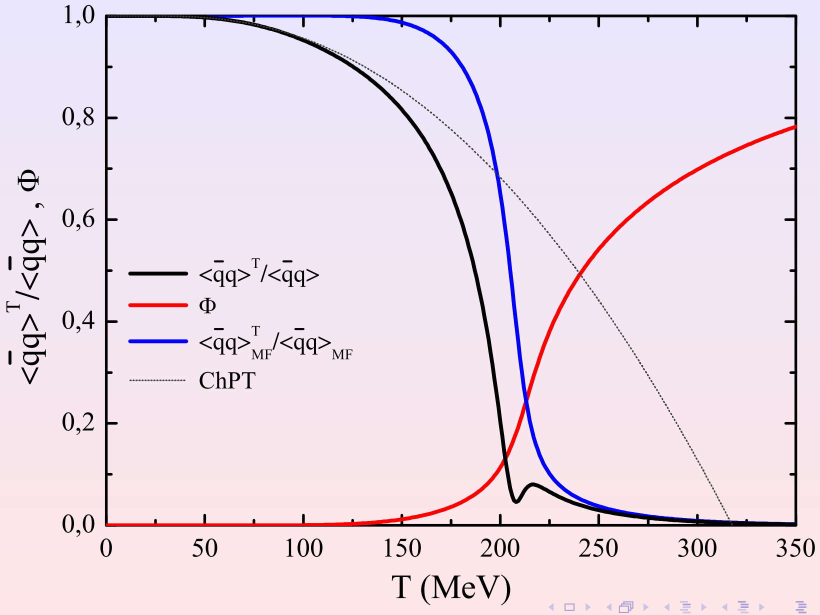
The important point is the region of applicability of the model. Practically, in the momentum region it is connected with the lowest singularities of quark propagator. For different parametrization quark can have first pole on real axis or pair of complex conjugated poles. First case correspond to the real threshold of quark diagrams (after $p > p_{\mathrm{Threshold}}$ they have imaginary part) whereas second one to pseudo-threshold (they pure real and have a cusp at $p_{\mathrm{Threshold}}$).

N	First pole(s) position, MeV	(pseudo)treshold, MeV
1	-0.0205502	286.7
2	-0.0528514	459.8
3	-0.1262370	710.6
4	$\mathbf{-0.108160 \pm i0}$	**957.3**
5	$-0.108160 \pm i0.176299$	793.7
6	$-0.028857 \pm i0.183204$	654.7
7	$0.011703 \pm i0.169223$	562.0
8	$0.034359 \pm i0.153559$	496.0

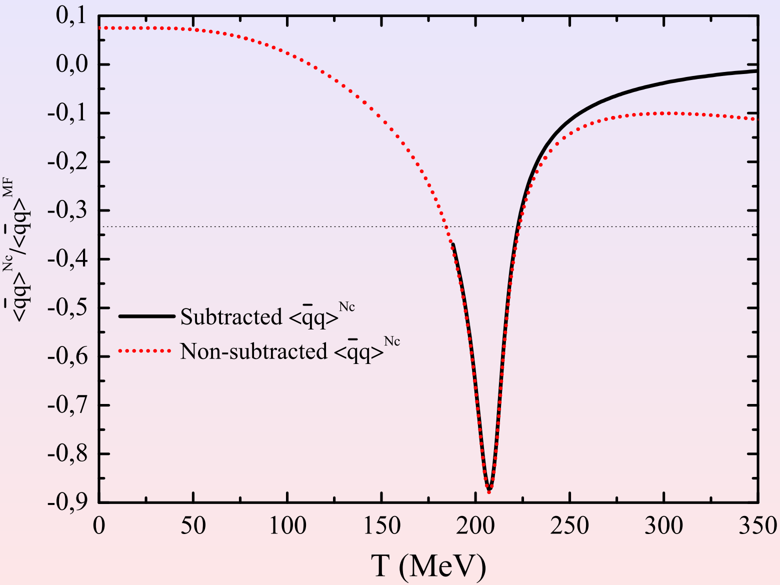
Beyond mean field. Fixing model parameters. Threshold.



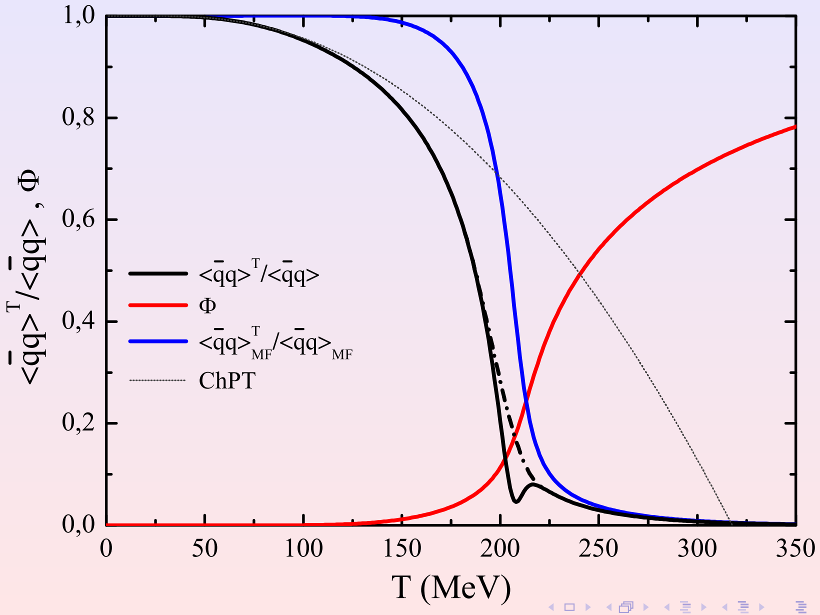
Finite T. Quark condensate.

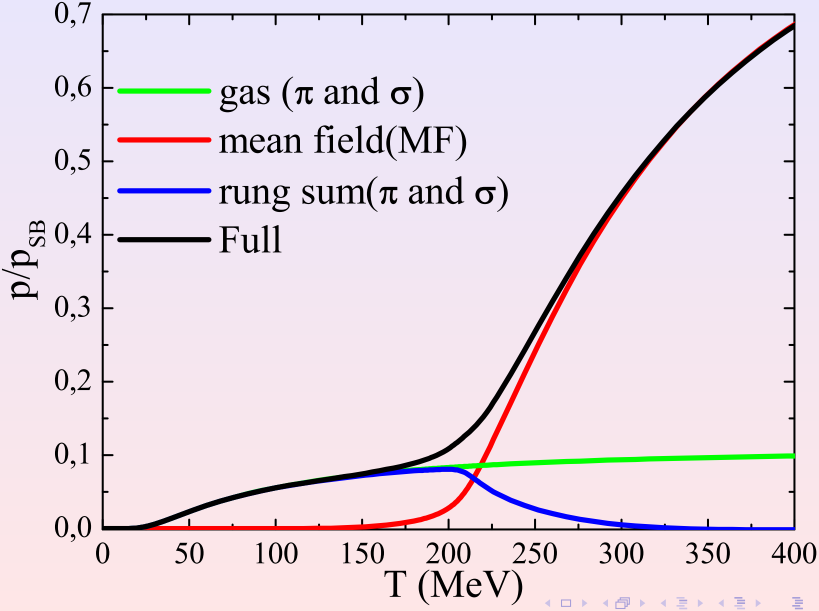


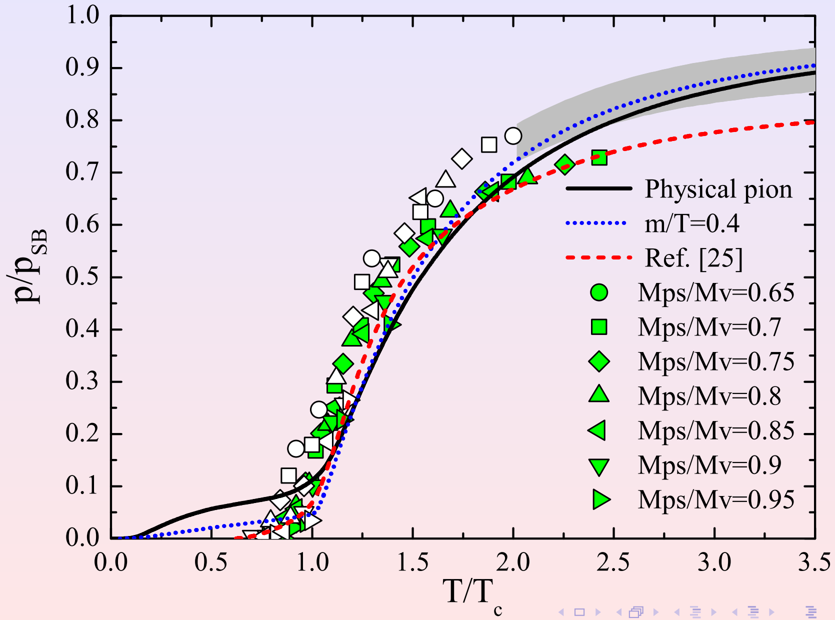
Finite T. Relative correction to the quark condensate.



Finite T. Quark condensate.




0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
p/p_{SB}
gas (π and σ)
mean field(MF)
rung sum(π and σ)
Full
0
50
100
150
200
250
300
350
400
T (MeV)


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
p/p_SB
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
T/T_c
Physical pion
m/T=0.4
Ref. [25]
Mps/Mv=0.65
Mps/Mv=0.7
Mps/Mv=0.75
Mps/Mv=0.8
Mps/Mv=0.85
Mps/Mv=0.9
Mps/Mv=0.95