Renormalization procedure for fermions with CP invariant interaction

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The 7th BLTP JINR-APCTP Joint Workshop «Modern problems in nuclear and elementary particle physics»
July 14-19, 2013

t-quark is a short-living particle because of the open channels with W on mass shell, so it may be considered as a resonance state. Similar to ordinary hadron resonances, the dressed t-quark propagator can be obtained as a result of Dyson summation of self-energy insertions or, equivalently, by solving the Dyson–Schwinger equation.

But for t-quark the vertex violates parity, so γ^5 takes part in this process and it generates nonstandard form of resonance — it differs from usual Breit-Wigner term.

This question was discussed earlier, see e.g. B.A. Kniehl and A. Sirlin. Phys.Rev. D**77** (2008) 116012, where the dressed fermion propagator was obtained in analogy with boson one, i.e. without separation positive and negative energy poles.

Here we make the next step: we build the projectors onto the positive (negative) energy poles and study the corresponding resonance factors. It is possibly to do for arbitrary form of interaction, the particular case is V-A vertex of Standard Model.

Standard Breit–Wigner formula in QFT

To obtain Breit–Wigner-like formula in QFT we need to solve the Dyson–Schwinger equation for the dressed propagator,

$$G = G_0 + G_0 \Sigma G, \quad \text{or} \quad G^{-1} = G_0^{-1} - \Sigma, \tag{1}$$

where G_0 and G are bare and dressed propagators and Σ is the self-energy. For bosons it gives

$$G_0 = \frac{1}{m_0^2 - s - \mathrm{i}\,\varepsilon} \quad \text{and} \quad G = \frac{1}{m_0^2 - s - \Sigma(s)} \sim \frac{1}{m^2 - s - \mathrm{i}\,\Gamma m},$$

and if Σ has imaginary part, the dressed propagator G should be compared with relativistic Breit–Wigner formula for renormalization.
For fermions all is similar:

$$G_0 = \frac{1}{\hat{p} - m_0} \quad \text{and} \quad G = \frac{1}{\hat{p} - m_0 - \Sigma(p)},$$

but to make this procedure more transparent, it is convenient to pass to off-shell projection operators.

Standard Breit–Wigner formula in QFT

Off-shell projection operators looks like:

$$\Lambda^{\pm} = \frac{1}{2}\Big(1 \pm \frac{\hat{p}}{W}\Big), \tag{2}$$

where $W = \sqrt{p^2}$ is invariant mass or rest-frame energy.
In this basis dressing looks like

$$\begin{gathered} G_0 = \frac{1}{\hat{p} - m_0} = \Lambda^+ \frac{1}{W - m_0} + \Lambda^- \frac{1}{-W - m_0} \Rightarrow \\ \Rightarrow G = \Lambda^+ \frac{1}{W - m_0 - \Sigma_1(W)} + \Lambda^- \frac{1}{-W - m_0 - \Sigma_2(W)}, \end{gathered} \tag{3}$$

where the self-energy is also decomposed in this basis

$$\Sigma(p) = A(p^2) + \hat{p}B(p^2) \equiv \Lambda^+\Sigma_1(W) + \Lambda^-\Sigma_2(W).$$

After it the positive energy pole should be compared with Breit–Wigner formula

$$\frac{1}{W - m_0 - \Sigma_1(W)} \sim \frac{1}{W - m + \mathrm{i}\,\Gamma/2}. \tag{4}$$

The above formulas correspond to parity conservation.

In case of parity violation the projection basis (2) must be supplemented by elements with γ^5, it is handy to choose the basis as

$$\mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^+\gamma^5. \tag{5}$$

Now the decomposition of a self-energy or a propagator has four terms

$$S = \sum_{M=1}^{4} S_M \mathcal{P}_M, \tag{6}$$

where coefficients S_M are followed by obvious symmetry properties

$$S_2(W) = S_1(-W), \quad S_4(W) = S_3(-W).$$

Let's denote by $S(p)$ and $S_0(p)$ the dressed and bare inverse propagators. With the use of decomposition (6), the Dyson–Schwinger equation (1) is reduced to the set of equations for scalar coefficients

$$S_M = (S_0)_M - \Sigma_M, \quad M = 1, \ldots, 4. \tag{7}$$

Considering the self-energy Σ as a known value, we obtain the dressed propagator

$$G = \sum_{M=1}^{4} G_M \mathcal{P}_M,$$

where the coefficients G_M are

$$G_1 = \frac{S_2}{\Delta}, \quad G_2 = \frac{S_1}{\Delta}, \quad G_3 = -\frac{S_3}{\Delta}, \quad G_4 = -\frac{S_4}{\Delta}, \tag{8}$$

and $\Delta = S_1 S_2 - S_3 S_4$.

In spite of simple answer (6), it is inconvenient because the positive and negative energy poles are not separated, compare with formula

$$G = \Lambda^{+} \frac{1}{W - m_0 - \Sigma_1(W)} + \Lambda^{-} \frac{1}{-W - m_0 - \Sigma_2(W)}.$$

We want to obtain the analog of formula

$$G = \Lambda^{+} \frac{1}{W - m_0 - \Sigma_1(W)} + \Lambda^{-} \frac{1}{-W - m_0 - \Sigma_2(W)},$$

for the parity non-conservation case.

In order to obtain the analog of above mentioned formula in case of parity violation, we use the spectral representation of inverse propagator

$$S = \lambda_1 \Pi_1 + \lambda_2 \Pi_2, \tag{9}$$

where Π_k are projectors, satisfying the eigenstate problem

$$S\Pi_k = \lambda_k \Pi_k. \tag{10}$$

Let's write the dressed propagator $S(p)$ as

$$S = \sum_{M=1}^{4} S_M \mathcal{P}_M,$$

with arbitrary coefficients and will look for the matrix Π in the same form with coefficients a_M.
It is easy to find that eigenvalues λ_i are roots of the equation

$$\lambda^2 - \lambda(S_1 + S_2) + (S_1 S_2 - S_3 S_4) = 0. \tag{11}$$

After some algebra one can find the projectors

$$\begin{aligned}\Pi_1 &= \frac{1}{\lambda_2-\lambda_1}\Big((S_2-\lambda_1)\mathcal{P}_1+(S_1-\lambda_1)\mathcal{P}_2-S_3\mathcal{P}_3-S_4\mathcal{P}_4\Big),\\ \Pi_2 &= \frac{1}{\lambda_1-\lambda_2}\Big((S_2-\lambda_2)\mathcal{P}_1+(S_1-\lambda_2)\mathcal{P}_2-S_3\mathcal{P}_3-S_4\mathcal{P}_4\Big),\end{aligned} \tag{12}$$

with desired properties:

- $S\Pi_k = \lambda_k \Pi_k,$
- $\Pi_k^2 = \Pi_k,$
- $\Pi_1\Pi_2 = \Pi_2\Pi_1 = 0,$
- $\Pi_1 + \Pi_2 = 1.$

The dressed propagator $G(p)$ is obtained by reversing of equation (9)

$$G = \frac{1}{\lambda_1}\Pi_1 + \frac{1}{\lambda_2}\Pi_2. \tag{13}$$

The determinant $\Delta(W)$ of S is

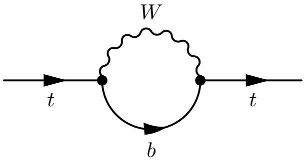
$$\Delta(W) = S_1 S_2 - S_3 S_4 = (W - m_0 - \Sigma_1)(-W - m_0 - \Sigma_2) - \Sigma_3\Sigma_4,$$

where $\Sigma_i(W)$ are self-energy components in the basis. Free propagator has poles at points $W = m_0$ and $W = -m_0$, the dressed one has them at $W = m$ and $W = -m$. On the other hand, $\Delta(W)$ is equal to product of eigenvalues

$$\Delta(W) = \lambda_1(W)\lambda_2(W). \tag{14}$$

Top Quark as Resonance

Consider the particular case of the above formulas, when vertex has $V - A$ structure. The main 1-loop contribution to self-energy in SM is



$$\Sigma(p) = -\,\mathrm{i}\,g^2|V_{\mathrm{tb}}|^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}\gamma^\mu(1-\gamma^5)\frac{\hat{p}-\hat{k}+m_{\mathrm{b}}}{(p-k)^2-m_{\mathrm{b}}^2}\times$$
$$\times\,\gamma^\nu(1-\gamma^5)\frac{g_{\mu\nu}-k_\mu k_\nu/m_{\mathrm{W}}^2}{k^2-m_{\mathrm{W}}^2}, \quad (15)$$

and generates only kinetic term

$$\Sigma(p) = \hat{p}(1-\gamma^5)\Sigma_0(W^2). \quad (16)$$

Its decomposition in the basis (5) has the following coefficients:

$$\Sigma_1 = W\Sigma_0(W^2),\ \Sigma_2 = -W\Sigma_0,\ \Sigma_3 = -W\Sigma_0,\ \Sigma_4 = W\Sigma_0.$$

Top Quark as Resonance

As a preliminary, let us forget about renormalization of self-energy and calculate the eigenvalues

$$\lambda_{1,2} = -m \pm W\sqrt{1 - 2\Sigma_0(W^2)}.$$

In analogy with OMS scheme let's subtract the real part of self-energy at resonance point

$$\lambda_{1,2} = -m \pm W\sqrt{1 - 2\big(\Sigma_0(W^2) - \mathrm{Re}\,\Sigma_0(m^2) - (\mathrm{Re}\,\Sigma_0(m^2))'(W^2 - m^2)\big)}.$$

As a result we have rather unusual resonance factor

$$\frac{1}{\lambda_1(W)} = \frac{1}{W\sqrt{1 + \mathrm{i}\,\dfrac{\Gamma}{m}} - m}, \tag{17}$$

which only at $\Gamma/m \ll 1$ returns to standard Breit–Wigner form,

$$\frac{1}{\lambda_1(W)} \simeq \frac{1}{W - m + \mathrm{i}\,W\dfrac{\Gamma}{2m}} \quad \text{at } \Gamma/m \ll 1.$$

OMS renormalization scheme

To analyse the obtained dressed propagator in more details, we need to renormalize it. We will use the OMS scheme of renormalization in order to compare with Breit–Wigner formula.
At first, let's suppose that self-energy does not have imaginary part. We put:

- Σ_1 has zero of second order at $W = m$
- Σ_3 has zeroes at $W = m$ and $W = -m$.

The Σ_2 and Σ_4 are defined by substitution $W \to -W$, so the OMS renormalization in this case is

$$\begin{aligned}
\Sigma_1^{\mathsf{r}}(W) &= \Sigma_1(W) - \Sigma_1(m) - \Sigma_1'(m)(W - m), \\
\Sigma_2^{\mathsf{r}}(W) &= \Sigma_1^{\mathsf{r}}(-W), \\
\Sigma_3^{\mathsf{r}}(W) &= -W\left(\Sigma_0(W^2) - \Sigma_0(m^2)\right), \\
\Sigma_4^{\mathsf{r}}(W) &= \Sigma_3^{\mathsf{r}}(-W).
\end{aligned}$$

OMS renormalization scheme

Eigenvalues in OMS scheme are

$$\lambda_{1,2}(W) = -mK \pm WK\sqrt{d}, \text{ where } d = 1 - 2\widetilde{\Sigma}/K \tag{18}$$

and $K = 1 + 2m^2\Sigma_0'(m^2)$, $\widetilde{\Sigma} = \Sigma_0(W^2) - \Sigma_0(m^2)$.
Let us write down the eigenvalues in vicinity of $W = m$

$$\lambda_1(W) = W - m + o(W - m),$$
$$\lambda_2(W) = -2mK - (W - m) + o(W - m),$$

and in vicinity of $W = -m$

$$\lambda_1(W) = -2mK - (-W - m) + o(-W - m),$$
$$\lambda_2(W) = -W - m + o(-W - m).$$

OMS renormalization scheme

Projectors on eigenstates have the form

$$\Pi_1 = \mathcal{P}_1 \frac{\sqrt{d} + (1 - \widetilde{\Sigma}/K)}{2\sqrt{d}} + \mathcal{P}_2 \frac{\sqrt{d} - (1 - \widetilde{\Sigma}/K)}{2\sqrt{d}} - \mathcal{P}_3 \frac{\widetilde{\Sigma}/K}{2\sqrt{d}} + \mathcal{P}_4 \frac{\widetilde{\Sigma}/K}{2\sqrt{d}},$$
$$\Pi_2 = \mathcal{P}_1 \frac{\sqrt{d} - (1 - \widetilde{\Sigma}/K)}{2\sqrt{d}} + \mathcal{P}_2 \frac{\sqrt{d} + (1 - \widetilde{\Sigma}/K)}{2\sqrt{d}} + \mathcal{P}_3 \frac{\widetilde{\Sigma}/K}{2\sqrt{d}} - \mathcal{P}_4 \frac{\widetilde{\Sigma}/K}{2\sqrt{d}}, \tag{19}$$

and dressed propagator is

$$G(p) = \frac{m_0 + \hat{p} - \hat{p}(1 + \gamma^5)\widetilde{\Sigma}/K}{K(W^2 d - m_0^2)}.$$

OMS renormalization scheme

The expressions for eigenvalues and projectors may be simplified in vicinity of $W^2 = m^2$, where $\widetilde{\Sigma}(W) \ll 1$ and we take into account only linear in $\widetilde{\Sigma}$ terms

$$\lambda_{1,2}(W) = K(-m \pm W) \mp W\widetilde{\Sigma}(W^2),$$

$$\Pi_1 = \mathcal{P}_1 - \mathcal{P}_3 \frac{\widetilde{\Sigma}}{2K} + \mathcal{P}_4 \frac{\widetilde{\Sigma}}{2K} = \Lambda^+ - \frac{\widetilde{\Sigma}(W^2)}{2K} \frac{\hat{p}\gamma^5}{W},$$

$$\Pi_2 = \mathcal{P}_2 + \mathcal{P}_3 \frac{\widetilde{\Sigma}}{2K} - \mathcal{P}_4 \frac{\widetilde{\Sigma}}{2K} = \Lambda^- + \frac{\widetilde{\Sigma}(W^2)}{2K} \frac{\hat{p}\gamma^5}{W}.$$

Let's consider the case when the self-energy $\Sigma(W)$ acquire the imaginary part. The formulas for eigenvalues and projectors, (18) and (19), remain the same, but in this case

$$\widetilde{\Sigma}(W^2) = \Sigma_0(W^2) - \mathrm{Re}\,\Sigma_0(m^2), \quad \text{and} \quad K = 1 + 2m^2(\mathrm{Re}\,\Sigma_0)'(m^2).$$

Resonance factor $1/\lambda_1$ in vicinity of $W = m$ practically coincides with naive expression (17)

$$\frac{1}{\lambda_1(W)} = \frac{1}{K\left(W\sqrt{1-2\widetilde{\Sigma}/K} - m\right)} \approx \frac{1}{K\left(W\sqrt{1+\mathrm{i}\,\frac{\Gamma(W)}{KW}} - m\right)}, \qquad (20)$$

if to introduce the energy-dependent width $\Gamma(W) = -2W\,\mathrm{Im}\,\Sigma_0(W^2)$.

OMS renormalization scheme

At small Γ resonance factor returns to standard form

$$\frac{1}{\lambda_1(W)} \simeq \frac{1}{W - m + \mathrm{i}\,\Gamma(W)/2} \quad \text{at } W \simeq m,\ \Gamma/m \ll 1.$$

Using the same approximations in projectors, we can write down a parametrization of dressed propagator in vicinity of $W = m$:

$$G = \frac{1}{W - m + \mathrm{i}\,\Gamma(W)/2}\left(\mathcal{P}_1 + \mathrm{i}\,\frac{\Gamma(W)}{4KW^2}\hat{p}\gamma^5\right) + \\ + \frac{1}{-2mK - (W - m) - \mathrm{i}\,\Gamma(W)/2}\left(\mathcal{P}_2 - \mathrm{i}\,\frac{\Gamma(W)}{4KW^2}\hat{p}\gamma^5\right). \quad (21)$$

Pole renormalization scheme

The pole renormalization scheme for fermion with parity non-conservation have been considered in detail in work B.A. Kniehl, A. Sirlin, Phys.Rev. **D77**, 116012 (2008). We will consider the pole scheme on the base of spectral representation. In this case it is sufficient to renormalize the single pole contribution $1/\lambda_1(W)$. It simplifies essentially the algebraic procedure and clarifies some aspects.

The inverse propagator has the form

$$S(p) = \hat{p} - m_0 - \Sigma(p) = \\ = \hat{p} - m_0 - \big(A(p^2) + \hat{p}B(p^2) + C(p^2)\gamma^5 + \hat{p}\gamma^5 D(p^2)\big). \quad (22)$$

In CP-symmetric theory $C(p^2) = 0$.

In terms of scalar functions the eigenvalues and corresponding projectors (12) have the form

$$\lambda_1(W) = -m_0 - A(W^2) + WR(W^2),$$
$$\lambda_2(W) = \lambda_1(-W),$$
$$\Pi_1(W) = \frac{1}{2}\Big[1 - \gamma^5 \frac{C(W^2)}{WR(W^2)} + \frac{\hat{p}}{W}\Big(\frac{1 - B(W^2)}{R(W^2)} - \gamma^5 \frac{D(W^2)}{R(W^2)}\Big)\Big],$$
$$\Pi_2 = \Pi_1(-W),$$

where we have introduced the notation

$$R(W^2) = \sqrt{\big(1 - B(W^2)\big)^2 - D^2(W^2) + C^2(W^2)/W^2}.$$

Let's $\lambda_1(W_1) = 0$, where $W_1 = M_p - \mathrm{i}\,\Gamma_p/2$:

$$-m_0 - A(W_1^2) + W_1 R(W_1^2) = 0.$$

Real part of this equality allows to get rid of m_0 in dressed propagator

$$S(p) = \hat{p} - \Big(\tilde{A}(p^2) + \hat{p}B(p^2) + \gamma^5 C(p^2) + \hat{p}\gamma^5 D(p^2)\Big),$$
$$\tilde{A}(p^2) = A(p^2) - A(W_1^2) + \big(W_1 R(W_1^2)\big).$$

The imaginary part of (21),

$$\mathrm{Im}\,\Big(-A(W_1^2) + W_1 R(W_1^2)\Big) = 0$$

gives relation between Γ_p and self-energy at pole point. In particular, in case of parity conservation it reduces to the obvious relation

$$\mathrm{Im}\,\Big(W_1 - \big(A(W_1^2) + W_1 B(W_1^2)\big)\Big) = 0, \quad \text{or} \quad \frac{\Gamma_p}{2} = -\,\mathrm{Im}\,\Sigma_1(W_1^2).$$

Pole renormalization scheme

Let's introduce wave function renormalization constants connecting bare and renormalized fields

$$\Psi = Z^{1/2}\Psi^{\mathrm{r}}, \quad \bar{\Psi} = \bar{\Psi}^{\mathrm{r}}\bar{Z}^{1/2}.$$

In case of parity violation $Z^{1/2}$, $\bar{Z}^{1/2}$ are matrices

$$Z^{1/2} = \alpha + \beta\gamma^5, \quad \bar{Z}^{1/2} = \bar{\alpha} + \bar{\beta}\gamma^5.$$

Renormalized inverse propagator

$$\begin{aligned} S^{\mathrm{r}}(p) = (\bar{\alpha} + \bar{\beta}\gamma^5)\Big[\hat{p} - \big(\tilde{A} + \hat{p}B + \gamma^5 C + \hat{p}\gamma^5 D\big)\Big](\alpha + \beta\gamma^5) = \\ = I\big[-\tilde{A}(\alpha\bar{\alpha} + \bar{\beta}\beta) - C(\bar{\alpha}\beta + \bar{\beta}\alpha)\big] + \\ + \hat{p}\big[(1 - B)(\alpha\bar{\alpha} - \beta\bar{\beta}) - D(\bar{\alpha}\beta - \bar{\beta}\alpha)\big] + \\ + \gamma^5\big[-C(\bar{\alpha}\alpha + \bar{\beta}\beta) - \tilde{A}(\bar{\alpha}\beta + \bar{\beta}\alpha)\big] + \\ + \hat{p}\gamma^5\big[-D(\bar{\alpha}\alpha - \bar{\beta}\beta) + (1 - B)(\bar{\alpha}\beta - \bar{\beta}\alpha)\big] \end{aligned} \tag{23}$$

allows to obtain the renormalized components of self-energy.

Looking at first term in spectral representation, we see that renormalization is divided into two parts: renormalization of eigenvalue and projector.
For stable fermion there is a physical requirement for projector. The projector at point $W = m$ has form

$$\Pi_1^{\mathsf{r}}(m) = \frac{1}{2}\Big[1 - \gamma^5 c + \frac{\hat{p}}{m}(b - \gamma^5 d)\Big],$$

where parameters b, d and c are related by $b^2 - d^2 + c^2 = 1$. However, if $c \neq 0$, $d \neq 0$ then $\Pi_1^{\mathsf{r}}(m)$ do not commutate with spin projector, what leads to spin flip for fermion on mass shell. Therefore there are requirements for renormalization of a stable fermion:

$$C^{\mathsf{r}}(m^2) = 0, \quad D^{\mathsf{r}}(m^2) = 0. \tag{24}$$

For unstable fermion, when pole is at point $W_1 = M_p - \mathrm{i}\,\Gamma_p/2$, there is some arbitrariness. The simplest generalization of (24) consists in:

$$C^{\mathsf{r}}(W_1^2) = 0, \quad D^{\mathsf{r}}(W_1^2) = 0. \tag{25}$$

The same relations arise from a principle, suggested in B.A. Kniehl, A. Sirlin, Phys.Rev. **D77**, 116012 (2008): the chiral components should have poles with unit absolute value of residue.

A few words about the relation between renormalization constants $Z^{1/2}$, $\bar{Z}^{1/2}$. The pseudo-hermiticity condition

$$\bar{Z}^{1/2} = \gamma^0 \big(Z^{1/2}\big)^\dagger \gamma^0, \tag{26}$$

is traditionally used in literature, which is reduced to $\bar{\alpha} = \alpha^*$, $\bar{\beta} = -\beta^*$. However, as it was noted in D. Espriu, J. Manzano, P. Talavera, Phys. Rev. **D66**, 076002 (2002), one should refused from this condition, if self-energy has absorptive parts. The same is seen from our renormalized propagator (23).

Assuming pseudo-hermiticity we calculate $D^{\mathrm{r}}(W^2)$ thus:

$$D^{\mathrm{r}}(W^2) = |\alpha|^2 \left\{ D(W^2)\left(1 + \frac{|\beta|^2}{|\alpha|^2}\right) - (1 - B(W^2))\left(\frac{\beta}{\alpha} + \frac{\beta^*}{\alpha^*}\right)\right\}. \qquad (27)$$

Because $D(W^2)$ and $B(W^2)$ contain physically different contributions we cannot provide the condition $D^{\mathrm{r}}(W_1^2) = 0$ for complex self-energy. So, the pseudo-hermiticity condition seems to be too restrictive for parity violating theory.

Let's consider below the case of CP conservative theory when component $C(p^2) = 0$. In order to avoid CP violation under renormalization it is necessary to require (see (23))

$$\bar{\alpha}\beta + \bar{\beta}\alpha = 0. \tag{28}$$

The pseudo-hermiticity condition (26) leads to (28) in case of real α, β (stable fermion). However, for resonance one have to refuse from pseudo-hermiticity, (26).

Pole renormalization scheme

Putting into account the condition (28) the renormalized inverse propagator becomes

$$\begin{aligned} S^{\mathsf{r}} = \alpha\bar{\alpha}\Big\{ &- \tilde{A}(W^2)(1-x^2) + \\ &+ \hat{p}\Big[(1-B(W^2))(1+x^2) - D(W^2)2x\Big] + \\ &+ \hat{p}\gamma^5\Big[- D(W^2)(1+x^2) + (1-B(W^2))2x\Big]\Big\}, \end{aligned} \tag{29}$$

where α, $\bar{\alpha}$ and $x = \beta/\alpha$ are complex numbers.
The condition at pole $D^{\mathsf{r}}(W_1^2) = 0$ defines

$$x \equiv \frac{\beta}{\alpha} = \frac{1 - B_1 - R_1}{D_1},$$

where $B_1 = B(W_1^2)$, $D_1 = D(W_1^2)$, $R_1 = R(W_1^2)$.

Substituting that into S^{r}, taking out common factor and denoting it by Z we get

$$\begin{aligned} S^{\mathsf{r}} = Z\Big\{ &-\tilde{A}(W^2)+ \\ &+\hat{p}\Big[(1-B(W^2))\frac{1-B_1}{R_1} - D(W^2)\frac{D_1}{R_1}\Big]+ \\ &+\hat{p}\gamma^5\Big[-D(W^2)\frac{1-B_1}{R_1} + (1-B(W^2))\frac{D_1}{R_1}\Big]\Big\} = \\ &= \hat{p} - \Sigma^{\mathsf{r}}, \end{aligned} \tag{30}$$

where renormalized components are given by

$$\begin{aligned} \tilde{A}^{\mathsf{r}}(W^2) &= Z\tilde{A}(W^2), \\ B^{\mathsf{r}}(W^2) &= 1 - Z\Big[(1-B(W^2))\frac{1-B_1}{R_1} - D(W^2)\frac{D_1}{R_1}\Big], \\ D^{\mathsf{r}}(W^2) &= Z\Big[D(W^2)\frac{1-B_1}{R_1} - (1-B(W^2))\frac{D_1}{R_1}\Big]. \end{aligned}$$

To determine Z factor we consider renormalized eigenvalue $\lambda_1^{\mathsf{r}}(W)$, its derivative at $W = W_1$ has to equal 1. It is easy to check that

$$R^{\mathsf{r}}(W^2) = \sqrt{(1 - B^{\mathsf{r}}(W^2))^2 - (D^{\mathsf{r}}(W^2))^2} = ZR(W),$$

and

$$\lambda_1^{\mathsf{r}}(W) = Z\lambda_1(W).$$

If to require $(\lambda_1^{\mathsf{r}})'(W_1) = 1$ it gives

$$Z = \frac{1}{R(W_1^2) + 2W_1^2 R'(W_1^2) - 2W_1 A'(W_1^2)}. \tag{31}$$

In case of unstable fermions, the right hand side of (31) is, generally speaking, complex. If we define

$$\lambda^{\mathsf{r}}_{1,2}(W) = |Z|\lambda_{1,2}(W), \tag{32}$$

we have the renormalized propagator with $\lambda_i(W)$ satisfying the Schwartz principle,

$$\lambda^{\mathsf{r}}_i(W^*) = \left(\lambda^{\mathsf{r}}_i(W)\right)^*. \tag{33}$$

So, λ^{r}_i has zeroes at complex conjugate points W_1, W_1^* with unit absolute value of residues.

Now we want to extend the described scheme on case of system of interacting fermions when CP is conserved. In this case it is convenient to use matrix notations,

$$G_0(p) = \big(G_{0ij}(p)\big),$$

is matrix of bare propagators, $G_{0ii}(p)$ is bare propagator of i fermion,

$$G(p) = \big(G_{ij}(p)\big),$$

is matrix of dressed propagators. Dyson–Schwinger equation for the system is similar to one for one fermion

$$G = G_0 + G_0 \Sigma G, \quad \text{or} \quad G^{-1} = G_0^{-1} - \Sigma,$$

where $\Sigma(p)$ is matrix self-energy and mixing terms:

Factorizing to positive and negative energy parts

To separate positive and negative energy contributions in matrix propagator we use some extension of spectral representation. Let's denote by $S(p)$ and $S_0(p)$ dressed and bare matrix propagators, we look for projectors Π_K such that

$$S\Pi_K = \lambda_K \Pi_K\,,$$

where λ_K are corresponding eigenvalues. Though the problem is very similar but as all objects are matrices there is significant difference. First, one may search for projector in form

$$\Pi_K = \sum_{M=1}^{4} A_M^K \mathcal{P}_M\,,$$

where matrices A_M^K are solutions of matrix equations

$$Q_1^K(W)A_1^K(W) = 0, \quad A_1^K(W)Q_2^K(W) = 0,$$

the rest matrices $(M = 2, 3, 4)$ are expressed in terms of A_1^K.

Eigenvalues and eigenvectors

The requirement

$$\det Q_1(W) = 0, \quad Q_1(W) = \big(S_2(W) - \lambda\big) S_3^{-1}(W) \big(S_1(W) - \lambda\big) - S_4(W),$$

is equation on eigenvalues. The vectors ψ_K defined as solutions of equation

$$Q_1^K(W)\psi_K(W) = 0,$$

gives answer for matrix A_1^K

$$A_1^K(W) = \psi_K(W)\psi_K^{\mathrm{t}}(W),$$

and with second vector φ_K

$$\varphi_K(W) = S_3^{-1}\big(S_1 - \lambda\big)\psi_K(W),$$

we know the rest matrices

$$A_2^K(W) = -\varphi_K(W)\varphi_K^{\mathrm{t}}(W), \; A_3^K(W) = \psi_K(W)\varphi_K^{\mathrm{t}}(W),$$
$$A_4^K(W) = -\varphi_K(W)\psi_K^{\mathrm{t}}(W).$$

Renormalization in system of fermions

In order to renormalize matrix of inverse propagators let's introduce renormalization "constants"

$$Z = \alpha + \beta\gamma^5, \quad \bar{Z} = \bar{\alpha} + \bar{\beta}\gamma^5,$$

where α, β, $\bar{\alpha}$ and $\bar{\beta}$ are some $n \times n$ matrices. The renormalized inverse propagator S^{r} is defined as

$$S^{\mathrm{r}} = \bar{Z} S Z = \sum_{M=1}^{4} \bar{Z} S_M Z \mathcal{P}_M .$$

Requirment that renormalized propagator have to be CP invariant leads to relations $\bar{\alpha} = \alpha^{\,\mathrm{t}}$ and $\bar{\beta} = -\beta^{\,\mathrm{t}}$.
The renormalization leads to redefinition of matrices A_M^K but choosing matrices α and β in form

$$\alpha = \Big(\psi_1(m_1), \psi_2(m_2), \ldots, \psi_n(m_n)\Big), \; \beta = -\Big(\varphi_1(m_1), \varphi_2(m_2), \ldots, \varphi_n(m_n)\Big),$$

where m_k is a pole for k particle.

That choice of matrices α and β gives desire property for matrix of dressed propagators: in vicinity of $W = m_k$ it has the only pole at (k, k) with unit residue

$$\left(G^{\mathsf{r}}_{ij}\right)\Big|_{W^2 \to m_k^2} = \begin{pmatrix} O(1) & \dots & \vdots & \dots \\ \dots & \dots & \dots & \dots \\ O(1) & \dots & \dfrac{1}{m_k - \hat{p}} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}.$$

Conclusions

- We studied in detail the dressing of fermion propagator in the case of the parity non-conservation. In contrast to previous works, we found the representation of propagator, where the positive and negative energy poles are separated from each other.

$$G(p) = \frac{1}{\lambda_1}\Pi_1 + \frac{1}{\lambda_2}\Pi_2.$$

- We found that in case of parity violation the resonance factor $1/\lambda_1(W)$ differs from Breit–Wigner-like formula. But in case of SM vertex the obtained resonance factor returns to the standard form for small width $\Gamma/m \ll 1$. Resonance factor $1/\lambda_1$ in vicinity of $W = m$:

$$\frac{1}{\lambda_1(W)} \approx \frac{1}{K\left(W\sqrt{1 + \mathrm{i}\,\frac{\Gamma(W)}{KW}} - m\right)} \approx \frac{1}{W - m + \mathrm{i}\,\Gamma(W)/2}$$

Conclusions

- For top quark $\Gamma/m \sim 10^{-2}$ is really a small parameter, so for SM its resonance factor will practically coincide with standard one. But looking at corresponding projectors, one can see that Π_k do not commutate with spin projectors $(1+\gamma^5\hat{s})/2$ and it can lead to non-trivial spin properties at the level of Γ/m.

$$\Pi_1(W) \approx \mathcal{P}_1 + \mathrm{i}\,\frac{\Gamma(W)}{4KW^2}\hat{p}\gamma^5$$

- We extended spectral representation on case of system of fermions and reviewed in new form renormalization for that system with interaction preserving CP invariance. In spectral representation approach the renormalization procedure has very clean and simple form.

Thank you for your attention!