Loop mixing of the opposite parity fermion fields and its manifestation in πN scattering

A.Kaloshin

(in collaboration with E.Kobeleva and V.Lomov)

Irkutsk State University

17 июля 2013 г.

Outlook

- Introduction
- Projection basis and matrix propagator
- Opposite parity fermion (OPF) fields mixing and K-matrix
- Descripting of PWA results for S_{11} and P_{11} waves
- Conclusions

Introduction

Mixing of states (fields) is a well-known phenomenon existing in the systems of neutrinos, quarks and hadrons. As for theoretical description of mixing phenomena, a general tendency with time and development of experiment consists in transition from a simplified quantum-mechanical description to the quantum field theory methods.

Mixing of fermion fields has some specifics as compared with boson case. Firstly, there exists γ-matrix structure in a propagator. Secondly, fermion and antifermion have the opposite P-parity, so fermion propagator contains contributions of different parities. As a result, besides a standard mixing of fields with the same quantum numbers, for fermions there exists a mixing of fields with opposite parities (OPF-mixing) at loop level, even if the parity is conserved in Lagrangian.

Below we say about non-standard effect of OPF-mixing and its manifestation in systems of baryon resonances.

Projection basis

We will use the **off-shell** projection operators $\Lambda^{\pm}$:

$$\Lambda^{\pm} = \frac{1}{2}\Big(1 \pm \frac{\hat{p}}{W}\Big), \qquad W = \sqrt{p^2},$$

where W is the rest-frame energy.
Main properties of projection operators are:

$$\Lambda^{\pm}\Lambda^{\pm} = \Lambda^{\pm}, \quad \Lambda^{\pm}\Lambda^{\mp} = 0, \quad \Lambda^{\pm}\gamma^5 = \gamma^5\Lambda^{\mp},$$

We can decompose propagator (self-energy) in this basis:

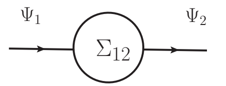
$$G = \sum_{M=1}^{2} \mathcal{P}_M G^M, \qquad \mathcal{P}_1 \equiv \Lambda^+, \quad \mathcal{P}_2 \equiv \Lambda^-. \tag{1}$$

If γ^5 takes part in game, it's convenient to do as:

$$G = \sum_{M=1}^{4} \mathcal{P}_M G^M, \qquad \mathcal{P}_3 \equiv \Lambda^+\gamma^5, \quad \mathcal{P}_4 \equiv \Lambda^-\gamma^5. \tag{2}$$

Non-diagonal loop

First of all, look at the non-diagonal self-energy:



Let parity is conserved in Lagrangian.
Mixing of fields with the same quantum numbers:

$$\begin{aligned}\Sigma_{12} &= A(p^2) + \hat{p}B(p^2) = \\ &= \Lambda^+ \ \left[A(W^2) + WB(W^2)\right] + \Lambda^- \ \left[A(W^2) - WB(W^2)\right]\end{aligned}$$

Mixing of fields with opposite parities:

$$\begin{aligned}\Sigma_{12} &= \gamma^5 C(p^2) + \hat{p}\gamma^5 D(p^2) = \\ &= \Lambda^+\gamma^5 \ \left[C(W^2) + WD(W^2)\right] + \Lambda^-\gamma^5 \ \left[C(W^2) - WD(W^2)\right]\end{aligned}$$

Main statement: $\Sigma_{12} \neq 0$ for mixing of opposite parities fields.
Fermion specifics !

Inverse propagator in this basis looks as:

$$S(p) = \mathcal{P}_1 \begin{pmatrix} W - m_1 - \Sigma^1_{11} & 0 \\ 0 & W - m_2 - \Sigma^1_{22} \end{pmatrix} + \\ + \mathcal{P}_2 \begin{pmatrix} -W - m_1 - \Sigma^2_{11} & 0 \\ 0 & -W - m_2 - \Sigma^2_{22} \end{pmatrix} + \\ + \mathcal{P}_3 \begin{pmatrix} 0 & -\Sigma^3_{12} \\ -\Sigma^3_{21} & 0 \end{pmatrix} + \mathcal{P}_4 \begin{pmatrix} 0 & -\Sigma^4_{12} \\ -\Sigma^4_{21} & 0 \end{pmatrix}, \quad (3)$$

where the indexes $i, j = 1, 2$ in the self-energy Σ^M_{ij} numerate dressing fermion fields and the indexes $M = 1, \ldots 4$ are refered to the γ-matrix decomposition (39).

Appendix: *Off-shell projection operators and fermion dressing*

Reversing of (40) gives the matrix dressed propagator:

$$
\begin{aligned}
G =& \mathcal{P}_1 \begin{pmatrix} \dfrac{-W - m_2 - \Sigma^2_{22}}{\Delta_1} & 0 \\ 0 & \dfrac{-W - m_1 - \Sigma^2_{11}}{\Delta_2} \end{pmatrix} + \\
&+ \mathcal{P}_2 \begin{pmatrix} \dfrac{W - m_2 - \Sigma^1_{22}}{\Delta_2} & 0 \\ 0 & \dfrac{W - m_1 - \Sigma^1_{11}}{\Delta_1} \end{pmatrix} + \\
&+ \mathcal{P}_3 \begin{pmatrix} 0 & \dfrac{\Sigma^3_{12}}{\Delta_1} \\ \dfrac{\Sigma^3_{21}}{\Delta_2} & 0 \end{pmatrix} + \mathcal{P}_4 \begin{pmatrix} 0 & \dfrac{\Sigma^4_{12}}{\Delta_2} \\ \dfrac{\Sigma^4_{21}}{\Delta_1} & 0 \end{pmatrix}.
\end{aligned} \tag{4}
$$

$$
\Delta_1 = \big(W - m_1 - \Sigma^1_{11}\big)\big(-W - m_2 - \Sigma^2_{22}\big) - \Sigma^3_{12}\Sigma^4_{21},
$$

$$
\Delta_2 = \big(-W - m_1 - \Sigma^2_{11}\big)\big(W - m_2 - \Sigma^1_{22}\big) - \Sigma^4_{12}\Sigma^3_{21} = \Delta_1\big(W \to -W\big).
$$

Where OPF-mixing can be seen?

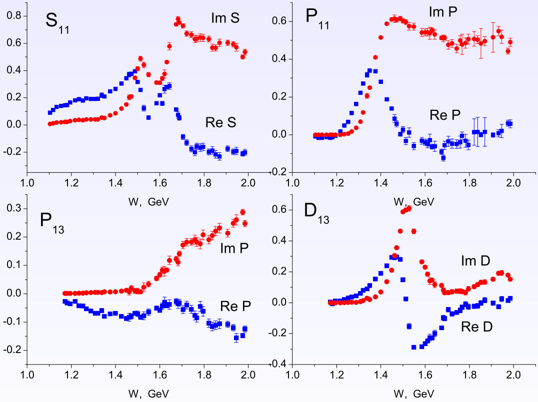
Below we will discuss manifestation of OPF-mixing in πN scattering. There are two places, where we can identify this effect:

1. Simplest one is the pair of partial waves P_{13}, D_{13}, where baryons $3/2^{\pm}$ are produced. It was discussed in: **A.Kaloshin, E.Kobeleva, V.Lomov, Int. J.of Mod.Phys. A26 (2011) 2307** on the base of the matrix propagator.
2. OPF-mixing in another pair: S_{11}, P_{11} ($J^P = 1/2^{\pm}$) is subject of paper:**A.Kaloshin, E.Kobeleva, V.Lomov, arXiv:1306.6171.** This required to develop a variant of K-matrix, which includes this effect.

I will say mainly about last item: OPF-mixing in partial waves S_{11}, P_{11}.

Partial wave analysis (PWA) of $\pi N \to \pi N$ with $I = 1/2$

R.A.Arndt et al. PR C74 (2006) 045205; (gwdac.phys.gwu.edu)



The pair of partial waves P_{13}, D_{13} looks as simplest case for identification of the discussed OPF-mixing effect.

OPF-mixing and K-matrix

We need to discuss the effect of OPF-mixing in amplitudes of πN scattering and its implementation in framework of K-matrix description. For a first step one may restrict oneself by a simplified case: two resonance states and two channels.

Effective Lagrangians $\pi N N'$ without derivatives and conserving the parity:

$$L_{\text{int}} = g_1 \bar{N}_1(x) N(x) \phi(x) + \text{h.c.}, \qquad \text{for } J^P(N_1) = 1/2^-, \qquad (5)$$

$$L_{\text{int}} = \imath g_2 \bar{N}_2(x) \gamma^5 N(x) \phi(x) + \text{h.c.}, \quad \text{for } J^P(N_2) = 1/2^+. \qquad (6)$$

Let us consider two baryon states of opposite parities with masses m_1 ($J^P = 1/2^-$), m_2 ($J^P = 1/2^+$) and two intermediate states πN, ηN. Using the effective Lagrangians we can calculate contributions of states N_1, N_2 to partial waves at tree level:

OPF-mixing and K-matrix

s-wave amplitudes:

$$f_{s,+}^{\text{tree}}(\pi N \to \pi N) = -\frac{(E_N^{(\pi)} + m_N)}{8\pi W}\left(\frac{g_{1,\pi}^2}{W - m_1} + \frac{g_{2,\pi}^2}{W + m_2}\right),$$

$$f_{s,+}^{\text{tree}}(\pi N \to \eta N) = -\frac{\sqrt{(E_N^{(\pi)} + m_N)(E_N^{(\eta)} + m_N)}}{8\pi W}\left(\frac{g_{1,\pi}g_{1,\eta}}{W - m_1} + \frac{g_{2,\pi}g_{2,\eta}}{W + m_2}\right),$$

$$f_{s,+}^{\text{tree}}(\eta N \to \eta N) = -\frac{(E_N^{(\eta)} + m_N)}{8\pi W}\left(\frac{g_{1,\eta}^2}{W - m_1} + \frac{g_{2,\eta}^2}{W + m_2}\right)$$

and p-wave amplitudes:

$$f_{p,-}^{\text{tree}}(\pi N \to \pi N) = \frac{(E_N^{(\pi)} - m_N)}{8\pi W}\left(\frac{g_{1,\pi}^2}{-W - m_1} + \frac{g_{2,\pi}^2}{-W + m_2}\right),$$

$$f_{p,-}^{\text{tree}}(\pi N \to \eta N) = \frac{\sqrt{(E_N^{(\pi)} - m_N)(E_N^{(\eta)} - m_N)}}{8\pi W}\left(\frac{g_{1,\pi}g_{1,\eta}}{-W - m_1} + \frac{g_{2,\pi}g_{2,\eta}}{-W + m_2}\right),$$

$$f_{p,-}^{\text{tree}}(\eta N \to \eta N) = \frac{(E_N^{(\eta)} - m_N)}{8\pi W}\left(\frac{g_{1,\eta}^2}{-W - m_1} + \frac{g_{2,\eta}^2}{-W + m_2}\right).$$

OPF-mixing and K-matrix

Here $W = \sqrt{s}$ is the total CMS energy and $E_N^{(\pi)}$ $(E_N^{(\eta)})$ is nucleon CMS energy of system πN (ηN)

$$E_N^{(\pi)} = \frac{W^2 + m_N^2 - m_\pi^2}{2W}. \tag{9}$$

Short notations for coupling constants, e.g. $g_{1,\pi} = g_{N_1 N \pi}$.
The tree amplitudes (5)–(6) contain poles with both positive and negative energy, originated from propagators of N_1 and N_2 fields of opposite parities. Accounting the loop transitions results in dressing of states and also in mixing of these two fields.
Note that $W \to -W$ replacement gives

$$E_N^{(\pi)} + m_N \to -\left(E_N^{(\pi)} - m_N\right), \tag{10}$$

so tree amplitudes (5)–(6) exhibit the MacDowell symmetry property **(S. W. MacDowell, Phys. Rev. 116 (1959) 774)**.

$$f_{p,-}(W) = -f_{s,+}(-W). \tag{11}$$

OPF-mixing and K-matrix

In K-matrix representation for partial amplitudes

$$f = K\big(1 - \imath PK\big)^{-1}, \tag{12}$$

diagonal matrix $\imath P$, constructed from CMS momenta, originates from imaginary part of a loop. Therefore, K-matrix here is simply a matrix of tree amplitudes that should be identified with amplitudes (5),(6). As the result we come to representation of partial amplitudes for s- and p-waves

$$f_s(W) = K_s(W)\big(1-\imath PK_s(W)\big)^{-1}, \quad f_p(W) = K_p(W)\big(1-\imath PK_p(W)\big)^{-1}, \tag{13}$$

where the matrices K_s, K_p (i.e. tree amplitudes (5),(6)), may be written in factorized form

$$K_s = -\frac{1}{8\pi}\rho_s\hat{K}_s\rho_s, \quad K_p = \frac{1}{8\pi}\rho_p\hat{K}_p\rho_p. \tag{14}$$

OPF-mixing and K-matrix

Here ρ_s, ρ_p are

$$\rho_s(W) = \begin{pmatrix} \sqrt{\dfrac{E_N^{(\pi)} + m_N}{W}}, & 0 \\ 0, & \sqrt{\dfrac{E_N^{(\eta)} + m_N}{W}} \end{pmatrix}, \qquad (15)$$

$$\rho_p(W) = \begin{pmatrix} \sqrt{\dfrac{E_N^{(\pi)} - m_N}{W}}, & 0 \\ 0, & \sqrt{\dfrac{E_N^{(\eta)} - m_N}{W}} \end{pmatrix}, \qquad (16)$$

and matrix P consists of CMS momenta as analytic functions of W. In this case "primitive"K-matrices contain poles with both positive and negative energy

OPF-mixing and K-matrix

$$\hat{K}_s(W) = \begin{pmatrix} \frac{g_{1,\pi}^2}{W - m_1} + \frac{g_{2,\pi}^2}{W + m_2}, & \frac{g_{1,\pi} g_{2,\eta}}{W - m_1} + \frac{g_{2,\pi} g_{2,\eta}}{W + m_2} \\ \frac{g_{1,\pi} g_{2,\eta}}{W - m_1} + \frac{g_{2,\pi} g_{2,\eta}}{W + m_2}, & \frac{g_{1,\eta}^2}{W - m_1} + \frac{g_{2,\eta}^2}{W + m_2} \end{pmatrix}, \tag{17}$$

$$\hat{K}_p(W) = \hat{K}_s(-W) = \begin{pmatrix} \frac{g_{1,\pi}^2}{-W - m_1} + \frac{g_{2,\pi}^2}{-W + m_2}, & \frac{g_{1,\pi} g_{2,\eta}}{-W - m_1} + \frac{g_{2,\pi} g_{2,\eta}}{-W + m_2} \\ \frac{g_{1,\pi} g_{2,\eta}}{-W - m_1} + \frac{g_{2,\pi} g_{2,\eta}}{-W + m_2}, & \frac{g_{1,\eta}^2}{-W - m_1} + \frac{g_{2,\eta}^2}{-W + m_2} \end{pmatrix} \tag{18}$$

Recall that m_1 is mass of $J^P = 1/2^-$ state and m_2 is mass of $J^P = 1/2^+$ one. Generalization of this construction for the case of more channels and states is obvious.
Since CMS momenta have the property $P(-W) = -P(W)$, the MacDowell symmetry property (9) is extended from tree amplitudes to unitarized K-matrix ones (11).

Naive expectations

Look again at tree partial amplitudes:

$$f_{s,+}^{\text{tree}}(\pi N \to \pi N) = -\frac{(E_N^{(\pi)} + m_N)}{8\pi W}\left(\frac{g_{1,\pi}^2}{W - m_1} + \frac{g_{2,\pi}^2}{W + m_2}\right)$$

$$f_{p,-}^{\text{tree}}(\pi N \to \pi N) = \frac{(E_N^{(\pi)} - m_N)}{8\pi W}\left(\frac{g_{2,\pi}^2}{-W + m_2} + \frac{g_{1,\pi}^2}{-W - m_1}\right)$$

From a common sense one can expect that negative energy pole should give a negligible effect in physical energy region. However, this is not the case if corresponding coupling constant is large $\mid g_{2,\pi} \mid \gg \mid g_{1,\pi} \mid$. One can compare decay widths of s- and p-states

$$\Gamma(N_1 \to \pi N) = g_{N_1 \pi N}^2 \Phi_s, \quad \Gamma(N_2 \to \pi N) = g_{N_2 \pi N}^2 \Phi_p, \qquad (19)$$

where Φ_s, Φ_p are corresponding phase volumes. For resonance states not far from threshold, with masses, e.g. 1.5–1.7 GeV, phase volumes differ greatly, $\Phi_s \gg \Phi_p$. If both resonances have typical hadronic width $\Gamma \sim 100$ MeV, then coupling constants differ dramatically too, $\mid g_{N_2 \pi N} \mid \gg \mid g_{N_1 \pi N} \mid$.

Inclusion of derivatives

Above we use the simplest effective Lagrangians (3)–(4) to derive tree amplitudes. However, it is well-known, that spontaneous breaking of chiral symmetry requires pion field to appear in Lagrangian only through derivative

$$\mathscr{L}_{\rm int} = f_2 \bar{N}_2(x)\gamma^5\gamma^\mu N(x)\partial_\mu\phi(x) + \text{h.c.}, \quad J^P = 1/2^+, \quad f_2 = \frac{g_2}{m_2+m_N}. \tag{20}$$

It is not difficult to understand how inclusion of derivative changes tree amplitudes and, hence K-matrix. Pole contribution $\pi(k_1)N(p_1) \to N_2(p) \to \pi(k_2)N(p_2)$ in that case takes the form:

$$T = f_2^2 \bar{u}(p_2)\gamma^5\hat{k}_2\frac{1}{\hat{p}-M}\gamma^5\hat{k}_1 u(p_1). \tag{21}$$

With use of equations of motion, we see that inclusion of derivative at vertex leads to the following modification of resonance contribution

$$g_2^2\frac{1}{\hat{p}-M} \to f_2^2(\hat{p}+m_N)\frac{1}{\hat{p}-M}(\hat{p}+m_N). \tag{22}$$

Inclusion of derivatives

Separation of the positive and negative energy poles is performed with the off-shell projector operators $\Lambda^{\pm} = 1/2\big(1 \pm \hat{p}/W\big)$

$$f_2^2(\hat{p}+m_N)\frac{1}{\hat{p}-m_N}(\hat{p}+m_N) = \Lambda^+\frac{f_2^2(W+m_N)^2}{W-M} + \Lambda^-\frac{f_2^2(W-m_N)^2}{-W-M}, \tag{23}$$

where the first term gives contribution to p-wave and second one to s-wave. Modification of the pole contributions in "primitive"K-matrices (15)–(16) is evident

$$g_2^2 \to f_2^2(W-m_N)^2, \quad \text{for } s\text{-wave}, \tag{24}$$
$$g_2^2 \to f_2^2(W+m_N)^2, \quad \text{for } p\text{-wave}. \tag{25}$$

One can expect that the inclusion of derivatives most strongly affects on threshold properties of s-wave due to dumping factor $(W-m_N)^2$.

Partial amplitudes of πN scattering

Note that our K-matrix differs from one used by other authors (**e.g. R.A.Arndt et al. PR C74 (2006) 045205**) by:

- Another form of phase-space factors (QFT calculations)
- Presence of the negative energy poles in $\hat{K}$

These two points together lead to MacDowell symmetry.

We will use our K-matrix for description of partial waves S_{11} and P_{11} of πN scattering in the energy region $W < 2$ GeV. Following to idea of **M. Batinic et al, PR C51(1995) 2310**, we will use three channels of reaction: πN, ηN and σN, where the last is "effective"channel, imitating different $\pi\pi N$ states.

"Primitive"$\hat{K}$-matrices have a form (15)–(16) but can contain several $J^P = 1/2^+$ and $J^P = 1/2^-$ states.

Fit of P_{11}

First of all, let us try to describe S_{11} and P_{11} waves separately. p-wave is described rather well by our formulas with derivative in vertex (22)–(23), see Fig. 1. In this case the s-wave states are missing in amplitudes, the p-wave K-matrix has two positive energy poles.

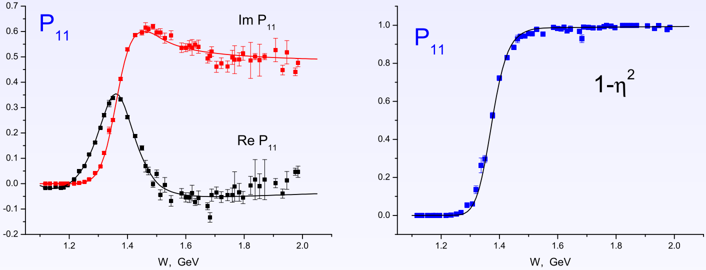


Рис. : The results of fitting of P_{11}-wave. Dots – results of PWA (**R.A.Arndt et al. PR C74 (2006) 045205**) solid lines represent our amplitudes (11)–(16) in the presence of derivative in vertex (22)–(23). K-matrix has only p-wave states. Partial wave normalization corresponds to **R.A.Arndt et al.**: $\mathrm{Im}\, f = |f|^2 + (1 - \eta^2)/4$.

Fit of P_{11}

Quality of description is defined by:

$$\chi^2/\mathrm{DOF} = 273/95. \tag{26}$$

The use of vertices without derivative leads to impairment of quality of description: $\chi^2 > 350$, again we need two poles with close masses.

Both variants give a negative background contribution to S_{11} wave, comparable in magnitude with other contributions, as it seen on Fig. 2. Variant without derivative in vertex gives a larger background contribution, rapidly changing near thresholds. It seems that description of P_{11} partial wave without derivative in vertices contradicts to data on S_{11}. On Fig. 2 there are shown some typical curves, there exist different variants with sharp behavior near thresholds. The presence of derivative in a vertex suppresses the threshold region in background contribution due to factor $(W - m_N)^2$, but in resonance region this is rather large contribution, see Fig. 2.

Background in s-wave

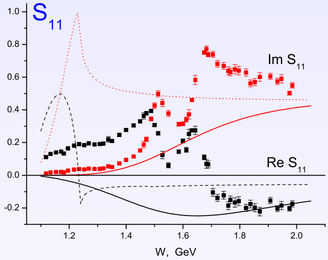


Рис. : Background contribution to s-wave, generated by p-wave states, i.e. in this case K-matrix for s-wave (15) has only negative energy poles. Solid lines represent variant with derivative in vertex (corresponding to curves on Fig. 1), dashed lines – variant without derivative in vertex.

Fit of S_{11}

Attempt to describe S_{11} without background has no success: it doesn't allow to reach even qualitative agreement with PWA.

As a next step, let us add the background contribution, arising from p-wave states (solid lines on Fig. 1) with fixed parameters of p-wave.

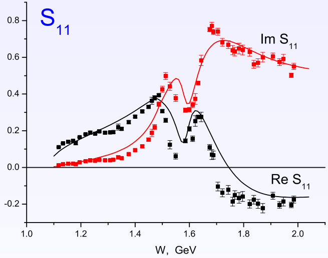


Рис. : Results of s-wave fitting with fixed parameters for p-wave states. Parameters of p-wave correspond to curves on Fig. 1, s-wave contains two states with K-matrix masses 1.55 and 1.75 GeV.

Fit of S_{11}

One can see from Fig. 3 that quality of description is unsatisfactory in this case but double-peak behavior is arisen in partial wave for the first time. It means that to describe S_{11} wave a background contribution is necessary and its value is close to solid line curves at Fig. 1

Joint fit of S_{11} and P_{11}

Let's perform the joint analysis of S_{11} and P_{11}, when resonance states in one wave generate background in other and vice versa. In this case $\hat{K}$-matrices have poles with both positive and negative energy: we use two s-wave and two p-wave poles. This leads to noticeable improvement of description, as it seen from Fig. 4:
$\chi^2/\mathrm{DOF} = 850/190$.

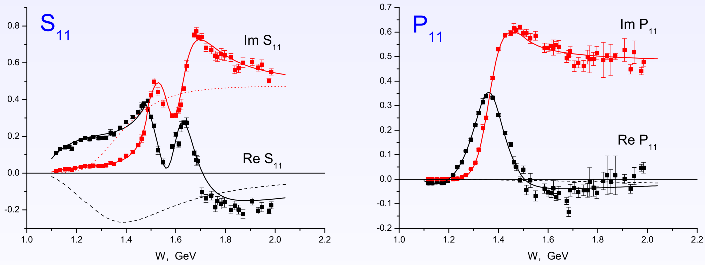


Рис. : Result of joint fitting of S_{11} and P_{11}-waves of πN scattering. Dashed lines show real and imaginary parts of (unitarized) background contribution.

Joint fit of S_{11} and P_{11}

At last, background can be generated not only by negative energy poles but by other terms. We accounted it by adding to elastic amplitudes $\pi N \to \pi N$ a smooth contributions of the form:

$$\hat{K}_s^B = A + B(W - m_N)^2, \quad \hat{K}_p^B = A + B(W + m_N)^2, \qquad (27)$$

which do not violate the MacDowell symmetry property. Note that we have quite good description $\chi^2/\mathrm{DOF} = 584/187$ and background contribution in S_{11} is close to simplest variant of Fig. 2.

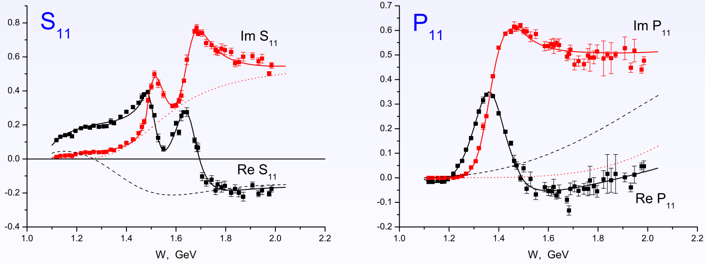


Рис. : Result of joint fitting of S_{11} and P_{11} waves of πN scattering.

Few remark after fit

- We used simplified description of πN partial waves (σN is some "quasi-channel") to recognize the effect of OPF-mixing in system of baryons $1/2^{\pm}$. Rather unexpectedly we obtained a good quality of description $\chi^2/\mathrm{DOF} = 584/187$, which is compartible with much more comprehensive analyses up to 6 channels.
- It seems that OPF-mixing may be introduced into dynamical models used for baryon physics, e.g. **H. Kamano, S. Nakamura, T.-S. Lee, and T. Sato, Phys.Rev. C81, 065207 (2010)**. Besides theoretical constrains it can have also some practical meaning.

Poles in complex plane

In Table 1 we present the pole masses and widths obtained by continuation of our amplitudes to complex W plane. As a whole, we see that our values for m_p, Γ_p are rather close to previously obtained. The only hint for disagreement is appearance at some sheets of a stable pole $1/2^+$ with $m_p \approx 1500$ MeV instead of generally accepted mass $m_p \approx 1365$ MeV.

Partial wave, PDG values	This work	Some other works
S_{11}, $1/2^-$ N(1535) (1510, 70) N(1650) (1655, 165)	(1507, 87) (1659, 149)	(1502, 95), (1648, 80) [?] (1519, 129), (1669, 136) [?]
P_{11}, $1/2^+$ N(1440) (1365, 190)	(1365, 194) (1500, 160)	(1359, 162) [?] (1385, 164) [?] (1387, 147) [?]

Таблица : Pole masses and widths (M_R, Γ_R) extracted from poles position in the complex plane W: $W_0 = M_R - \imath\Gamma_R/2$.

Conclusions

- Effect of mixing of fermion fields with opposite parity can be readily realized in the framework of K-matrix approach. It leads to well-known MacDowell symmetry

$$f_{l,+}(W) = -f_{l+1,-}(-W),$$

  connecting two partial waves.
  BUT: Taking OPF-mixing into account, MacDowell symmetry leads to practical consequences: resonance in one partial wave gives rise to background contribution in another and vice versa.
- This connection, as in case of $3/2^{\pm}$ resonances, works mainly in one direction: it generates large negative background in a wave with lower orbital momentum.
- As for practical use: we suppose that this connection may be of interest as a source of additional information about wave with higher orbital momentum (in our case about P_{11} and baryons $1/2^{+}$).

Appendix: *Off-shell projection operators and fermion dressing*

We will use the **off-shell** projection operators $\Lambda^{\pm}$:

$$\Lambda^{\pm} = \frac{1}{2}\Big(1 \pm \frac{\hat{p}}{W}\Big), \qquad W = \sqrt{p^2},$$

where W is the rest-frame energy.
Main properties of projection operators are:

$$\Lambda^{\pm}\Lambda^{\pm} = \Lambda^{\pm}, \quad \Lambda^{\pm}\Lambda^{\mp} = 0, \quad \Lambda^{\pm}\gamma^5 = \gamma^5\Lambda^{\mp},$$

$$\Lambda^{+} + \Lambda^{-} = 1, \quad \Lambda^{+} - \Lambda^{-} = \frac{\hat{p}}{W}.$$

Appendix: *Off-shell projection operators and fermion dressing*

Dyson–Schwinger equation for dressed propagator $G(p)$:

$$G(p) = G_0 + G\Sigma G_0, \tag{28}$$

where G_0 is a bare propagator and Σ is a self-energy.
We can expand all elements in eq. (26) in the basis of projection operators:

$$G = \sum_{M=1}^{2} \mathcal{P}_M G^M, \qquad \mathcal{P}_1 \equiv \Lambda^+, \quad \mathcal{P}_2 \equiv \Lambda^-. \tag{29}$$

After it Dyson–Schwinger equation is reduced to equations on scalar functions:

$$G^M = G_0^M + G^M \Sigma^M G_0^M, \qquad M = 1, 2, \tag{30}$$

or

$$\left(G^{-1}\right)^M = \left(G_0^{-1}\right)^M - \Sigma^M. \tag{31}$$

Appendix: *Off-shell projection operators and fermion dressing*

Decomposition of inverse dressed propagator:

$$G^{-1} = \mathcal{P}_1\ (W - m - \Sigma^1) + \mathcal{P}_2\ (-W - m - \Sigma^2). \tag{32}$$

Usual form of the self-energy is

$$\Sigma(p) = A(p^2) + \widehat{p}B(p^2), \tag{33}$$

and its decomposition in projection basis:

$$\Sigma^1 = A(W^2) + WB(W^2), \qquad \Sigma^2 = A(W^2) - WB(W^2). \tag{34}$$

Note the property of coefficients in the projection basis:

$$\Sigma^2(W) = \Sigma^1(-W).$$

Dressed propagator has a form:

$$G = \mathcal{P}_1\ \frac{1}{(W - m - \Sigma^1)} + \mathcal{P}_2\ \frac{1}{(-W - m - \Sigma^2)}. \tag{35}$$

Appendix: *Off-shell projection operators and fermion dressing*

When we have two fermion fields Ψ_i, the inclusion of interaction leads also to mixing of these fields. In this case the Dyson–Schwinger equation (26) acquire matrix indices:

$$G_{ij} = (G_0)_{ij} + G_{ik}\Sigma_{kl}(G_0)_{lj}, \quad i,j,k,l = 1,2. \tag{36}$$

Therefore we have the same equation, but all factor are matrices 2×2.

$$G(p) = G_0 + G\Sigma G_0, \tag{37}$$

The simplest variant is when the fermion fields Ψ_i have the same quantum numbers and the parity is conserved in the Lagrangian. Inverse propagator in this case:

$$\begin{aligned} G^{-1} &= \mathcal{P}_1 S^1(W) + \mathcal{P}_2 S^2(W) = \\ &= \mathcal{P}_1 \begin{pmatrix} W - m_1 - \Sigma^1_{11} & -\Sigma^1_{12} \\ -\Sigma^1_{21} & W - m_2 - \Sigma^1_{22} \end{pmatrix} + \mathcal{P}_2 S^1(-W). \end{aligned} \tag{38}$$

Appendix: *Off-shell projection operators and fermion dressing*

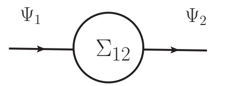
The matrix coefficients as before have the symmetry property $S^2(W) = S^1(-W)$. To obtain the matrix dressed propagator $G(p)$ one should reverse the matrix coefficients:

$$G(p) = \mathcal{P}_1(S^1(W))^{-1} + \mathcal{P}_2(S^2(W))^{-1} \tag{39}$$

We see that with use of projection basis the problem of fermion mixing is reduced to studying of the same mixing matrix as for bosons besides the obvious replacement $s - m^2 \to W - m$.

Appendix: *Off-shell projection operators and fermion dressing*

First of all, look at the non-diagonal self-energy:



Let parity is conserved in Lagrangian.
Mixing of fields with the same quantum numbers:

$$\begin{aligned} \Sigma_{12} &= A(p^2) + \hat{p}B(p^2) = \\ &= \Lambda^{+} \ \left[A(W^2) + WB(W^2)\right] + \Lambda^{-} \ \left[A(W^2) - WB(W^2)\right] \end{aligned}$$

Mixing of fields with opposite parities:

$$\begin{aligned} \Sigma_{12} &= \gamma^5 C(p^2) + \hat{p}\gamma^5 D(p^2) = \\ &= \Lambda^{+}\gamma^5 \ \left[C(W^2) + WD(W^2)\right] + \Lambda^{-}\gamma^5 \ \left[C(W^2) - WD(W^2)\right] \end{aligned}$$

Main statement: $\Sigma_{12} \neq 0$ for mixing of opposite parities fields.
Fermion specifics !

Appendix: *Off-shell projection operators and fermion dressing*

Let us consider the joint dressing of two fermion fields of opposite parities provided that the parity is conserved in a vertex. In this case the diagonal transition loops Σ_{ii} contain only I and $\hat{p}$ matrices, while the off-diagonal ones Σ_{12}, Σ_{21} must contain γ^5.
Projection basis should be supplemented by elements containing γ^5, it is convenient to choose the γ-matrix basis as:

$$\mathcal{P}_1 = \Lambda^+, \quad \mathcal{P}_2 = \Lambda^-, \quad \mathcal{P}_3 = \Lambda^+\gamma^5, \quad \mathcal{P}_4 = \Lambda^-\gamma^5. \tag{40}$$

In this case the γ-matrix decomposition has four terms:

$$S = \sum_{M=1}^{4} \mathcal{P}_M S^M, \tag{41}$$

where the coefficients S^M are matrices and have the obvious symmetry properties $S^2(W) = S^1(-W)$, $S^4(W) = S^3(-W)$.

Appendix: *Off-shell projection operators and fermion dressing*

Inverse propagator in this basis looks as:

$$S(p) = \mathcal{P}_1 \begin{pmatrix} W - m_1 - \Sigma^1_{11} & 0 \\ 0 & W - m_2 - \Sigma^1_{22} \end{pmatrix} + \\ + \mathcal{P}_2 \begin{pmatrix} -W - m_1 - \Sigma^2_{11} & 0 \\ 0 & -W - m_2 - \Sigma^2_{22} \end{pmatrix} + \\ + \mathcal{P}_3 \begin{pmatrix} 0 & -\Sigma^3_{12} \\ -\Sigma^3_{21} & 0 \end{pmatrix} + \mathcal{P}_4 \begin{pmatrix} 0 & -\Sigma^4_{12} \\ -\Sigma^4_{21} & 0 \end{pmatrix}, \tag{42}$$

where the indexes $i, j = 1, 2$ in the self-energy Σ^M_{ij} numerate dressing fermion fields and the indexes $M = 1, \ldots 4$ are refered to the γ-matrix decomposition (39).

Appendix: *Off-shell projection operators and fermion dressing*

Reversing of (40) gives the matrix dressed propagator:

$$
\begin{aligned}
G =&\mathcal{P}_1 \begin{pmatrix} \dfrac{-W - m_2 - \Sigma^2_{22}}{\Delta_1} & 0 \\ 0 & \dfrac{-W - m_1 - \Sigma^2_{11}}{\Delta_2} \end{pmatrix} + \\
&+\mathcal{P}_2 \begin{pmatrix} \dfrac{W - m_2 - \Sigma^1_{22}}{\Delta_2} & 0 \\ 0 & \dfrac{W - m_1 - \Sigma^1_{11}}{\Delta_1} \end{pmatrix} + \\
&+\mathcal{P}_3 \begin{pmatrix} 0 & \dfrac{\Sigma^3_{12}}{\Delta_1} \\ \dfrac{\Sigma^3_{21}}{\Delta_2} & 0 \end{pmatrix} + \mathcal{P}_4 \begin{pmatrix} 0 & \dfrac{\Sigma^4_{12}}{\Delta_2} \\ \dfrac{\Sigma^4_{21}}{\Delta_1} & 0 \end{pmatrix}.
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
\Delta_1 &= \big(W - m_1 - \Sigma^1_{11}\big)\big(-W - m_2 - \Sigma^2_{22}\big) - \Sigma^3_{12}\Sigma^4_{21}, \\
\Delta_2 &= \big(-W - m_1 - \Sigma^2_{11}\big)\big(W - m_2 - \Sigma^1_{22}\big) - \Sigma^4_{12}\Sigma^3_{21} = \Delta_1\big(W \to -W\big).
\end{aligned}
$$

Appendix: *Comparison with D.Arndt et al.*

Note that our K-matrix amplitudes (11) may be rewritten in other form, close to the one used in: **R. A. Arndt, J. M. Ford, and L. Roper, Phys.Rev. D32, 1085 (1985)**.

$$\begin{aligned} f_s(W) &= -\frac{1}{8\pi}\rho_s\hat{K}_s\left[1 + \imath\rho_s P\rho_s\hat{K}_s(W)/(8\pi)\right]^{-1}\rho_s, \\ f_p(W) &= \quad \frac{1}{8\pi}\rho_p\hat{K}_p\left[1 - \imath\rho_p P\rho_p\hat{K}_p(W)/(8\pi)\right]^{-1}\rho_p. \end{aligned} \tag{44}$$