7th BLTP JINR-APCTP Joint Workshop "Modern problems in nuclear and elementary particle physics",
July 14-19, 2013 , Russia, Irkutsk Region, Bolshiye Koty

Symmetry energy in the era of advanced gravitational wave detectors

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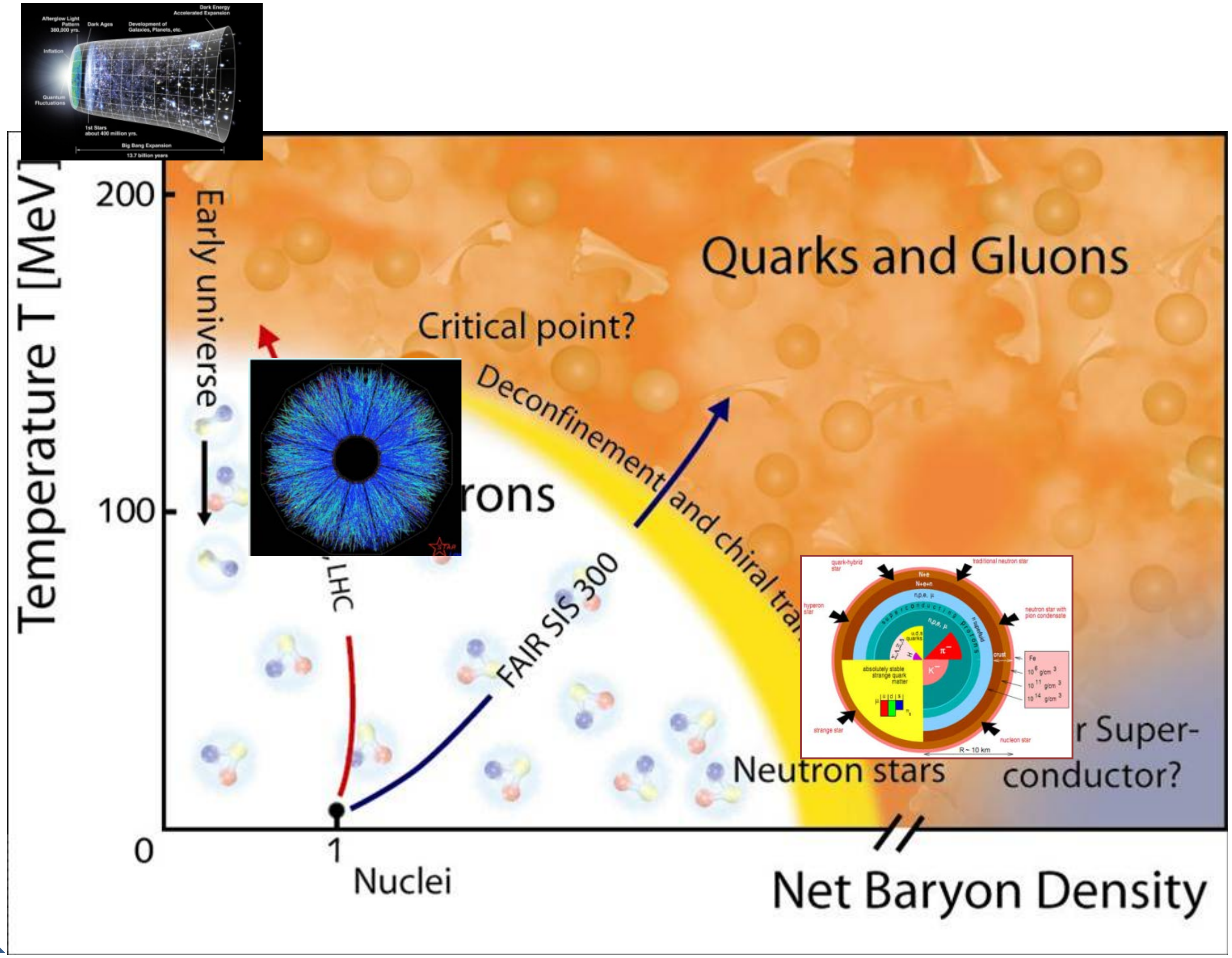
Hanyang University

I. Introduction
II. Symmetry energy
III. Dilaton in dense hadronic matter
IV. Half skyrmion and new scaling(BLPR)
V. Summary

I. Introduction

Temperature frontier

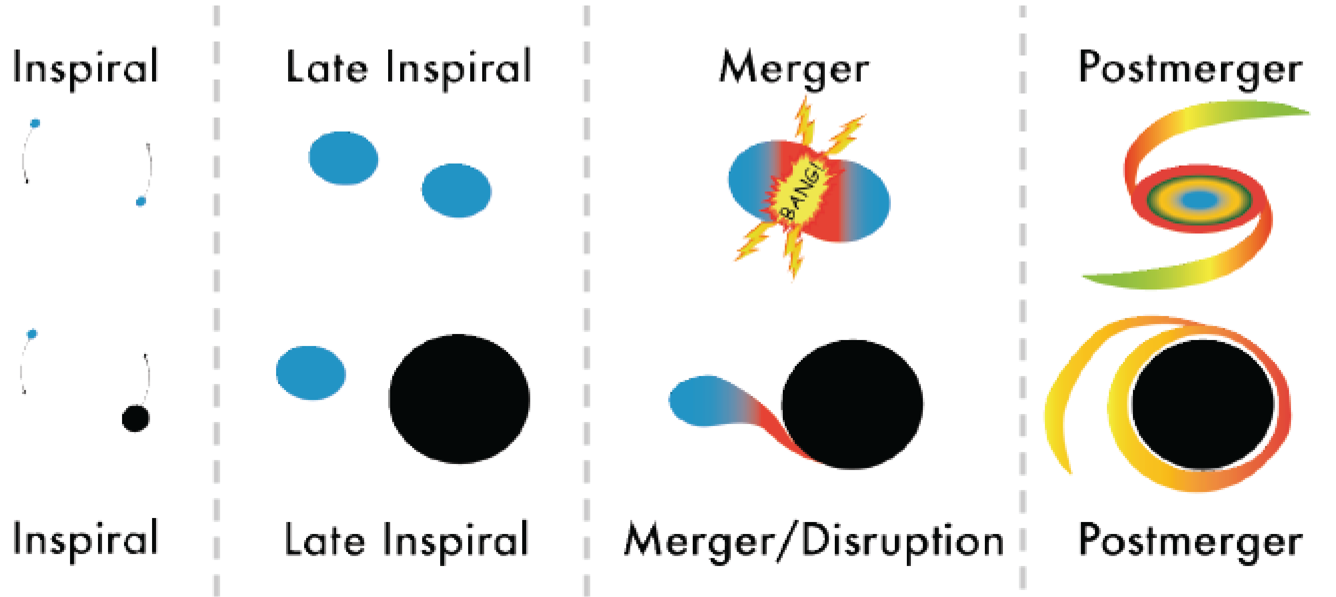


Density frontier

High baryon number density

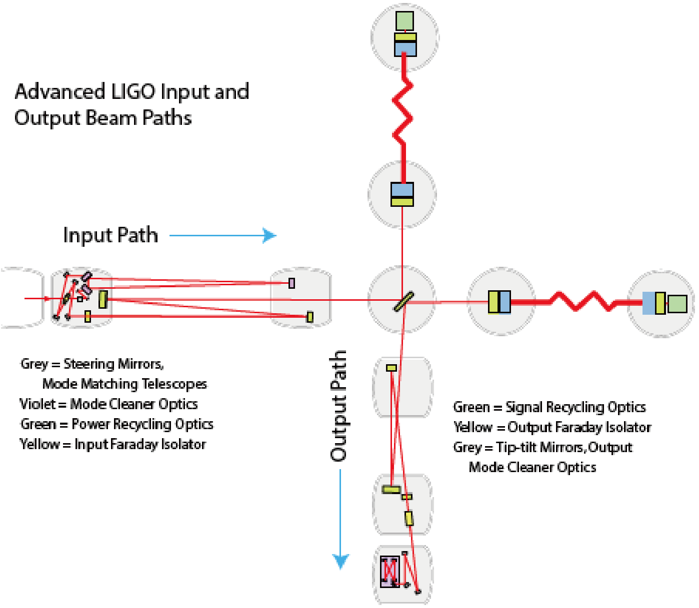
- RIB Machines(FAIR, FRIB, NICA, RAON, ..):
  $n > n_0$
- Neutron star
  1 solar mass inside 10km :
  $n \sim 10^{15} g/cm^3 \gg n_0$

Gravitational wave from cosmic collider
: binary(neutron star-neutron star/
neutron star-black hole) coalescence


Inspiral
Late Inspiral
Merger
Postmerger
BANG!
Inspiral
Late Inspiral
Merger/Disruption
Postmerger

C. Ott(2013)

I. Advanced LIGO/Virgo: 2015 →



The Advanced LIGO beam layout. Note the folded input and output paths.

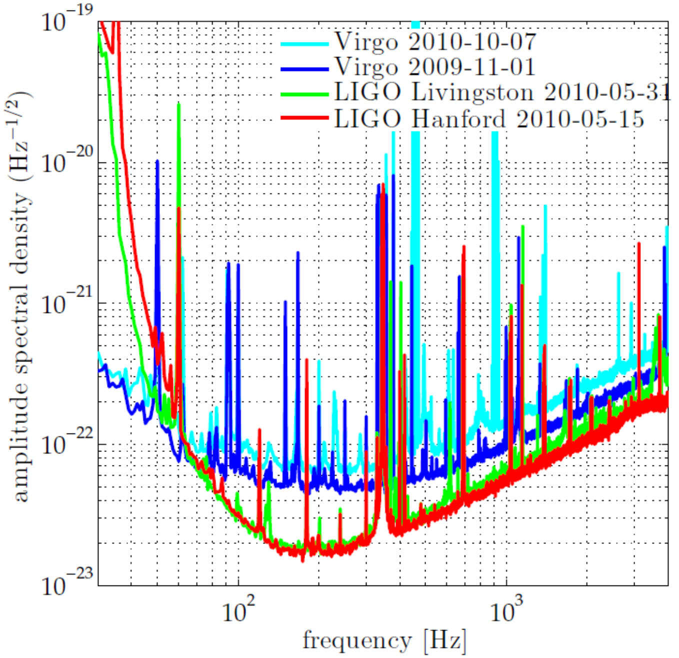
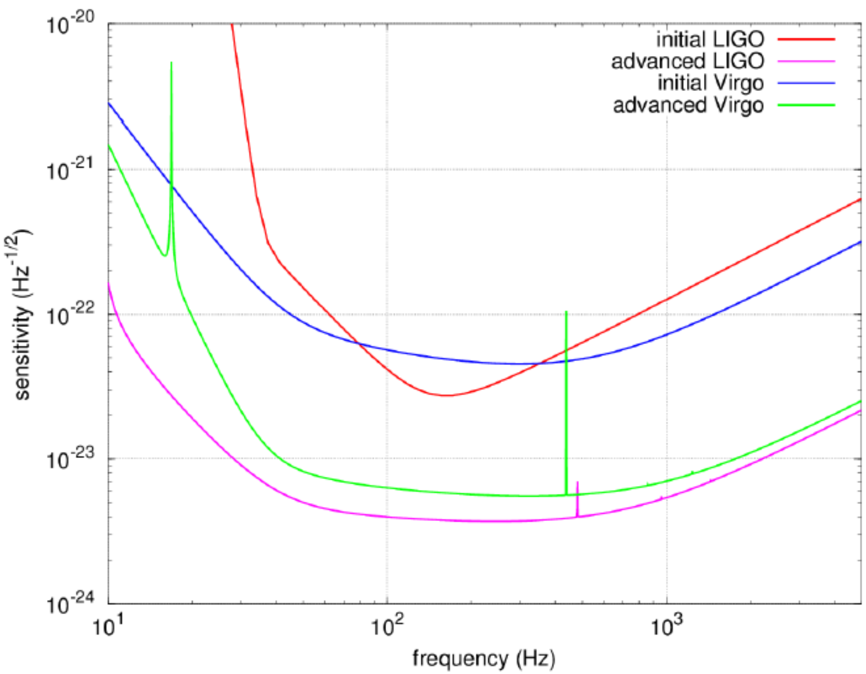


FIG. 1.— Best strain noise spectra from the LIGO and Virgo detectors during the 2009-2010 science runs.



LIGO, 1205.2216

Eric Chassande-Mottin, 1210.7173

Improvement of Sensitivity : 10-times

- Higher rate of GW detection

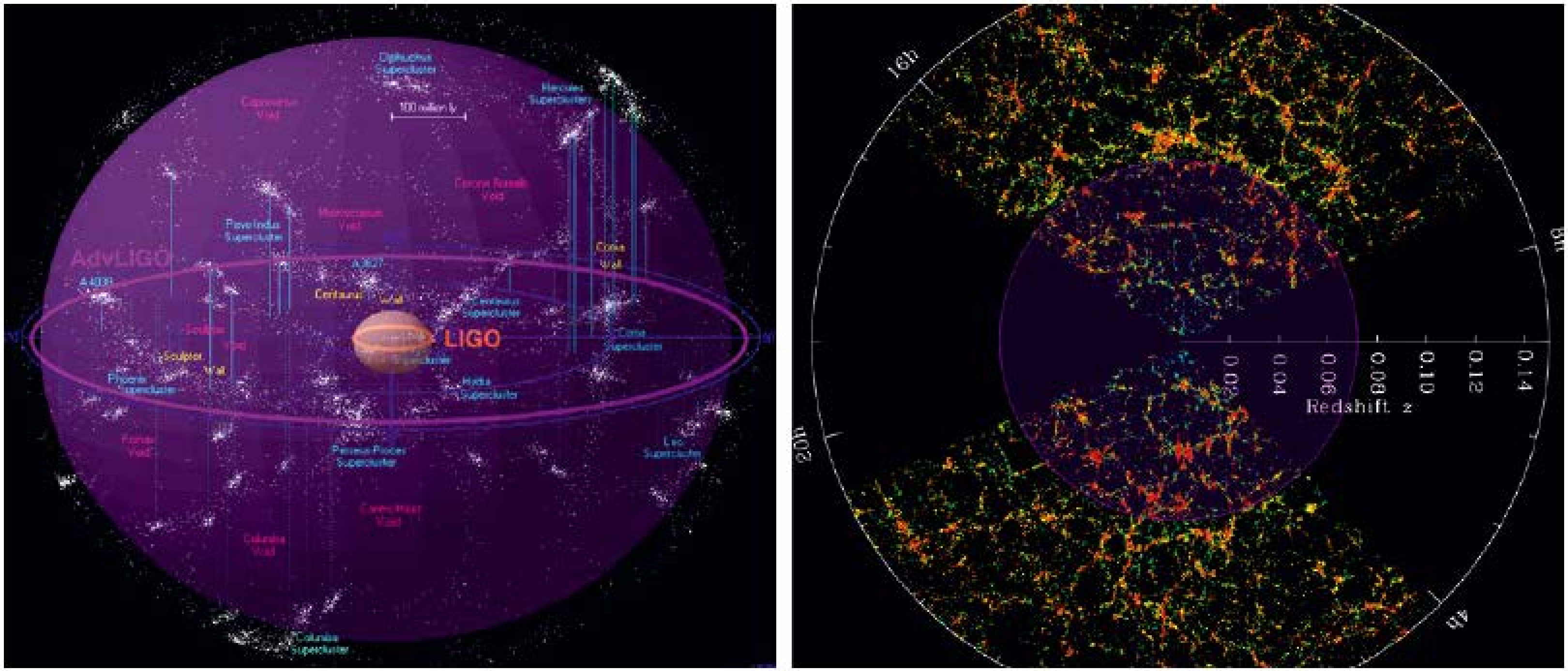
  NS-NS coalescence : 0.02/yr → 40/yr

Table 5. Detection rates for compact binary coalescence sources.

IFO	Source[a]	$\dot{N}_{low}$ yr^{-1}	$\dot{N}_{re}$ yr^{-1}	$\dot{N}_{high}$ yr^{-1}	$\dot{N}_{max}$ yr^{-1}
Initial	NS–NS	2×10^{-4}	0.02	0.2	0.6
	NS–BH	7×10^{-5}	0.004	0.1	
	BH–BH	2×10^{-4}	0.007	0.5	
	IMRI into IMBH			<0.001[b]	0.01[c]
	IMBH-IMBH			10^{-4}[d]	10^{-3}[e]
Advanced	NS–NS	0.4	40	400	1000
	NS–BH	0.2	10	300	
	BH–BH	0.4	20	1000	
	IMRI into IMBH			10[b]	300[c]
	IMBH-IMBH			0.1[d]	1[e]

J.Abadie, et al. Class. Quantum Grav. 27,173001(2010)

GW amplitude detection: increase of 10 times sensitivity
→ increase of 10^3 times volume for observation.

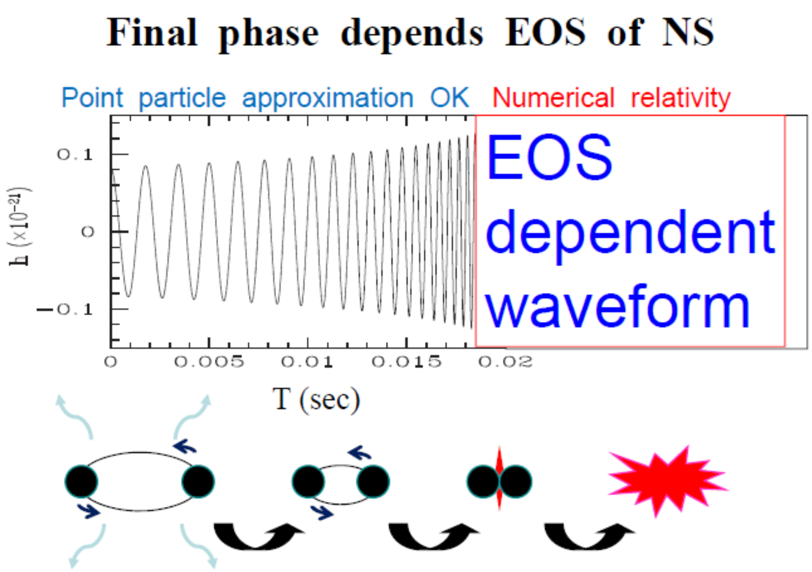


The superclusters within 300 Mpc (left) and the Sloan Digital Sky Survey (right)

- Current and future GW interferometric detectors

Eric Chassande-Mottin, 1210.7173

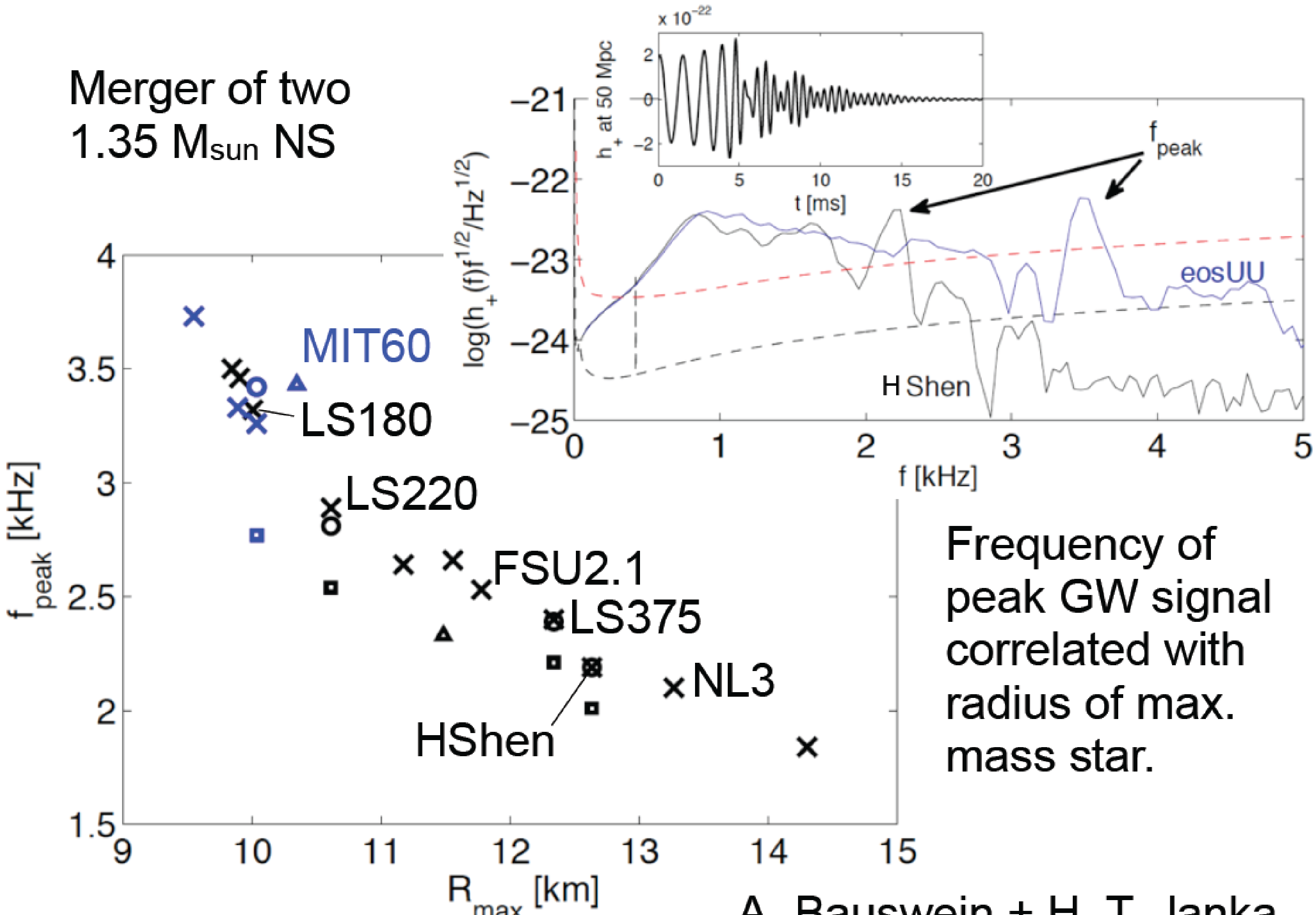
- EoS in Binary merger
  a. inspiral period: mass
  b. coalescing period : EoS (soft or stiff)



Shibata, 2011

Gravitational Waves from NS mergers and EOS

Merger of two 1.35 M_{sun} NS



Frequency of peak GW signal correlated with radius of max. mass star.

A. Bauswein + H. T. Janka

Effect of hyperons

Sekiguchi, Kiuchi, Kyutoku, Schibata   PRL 107, 211101(2011)

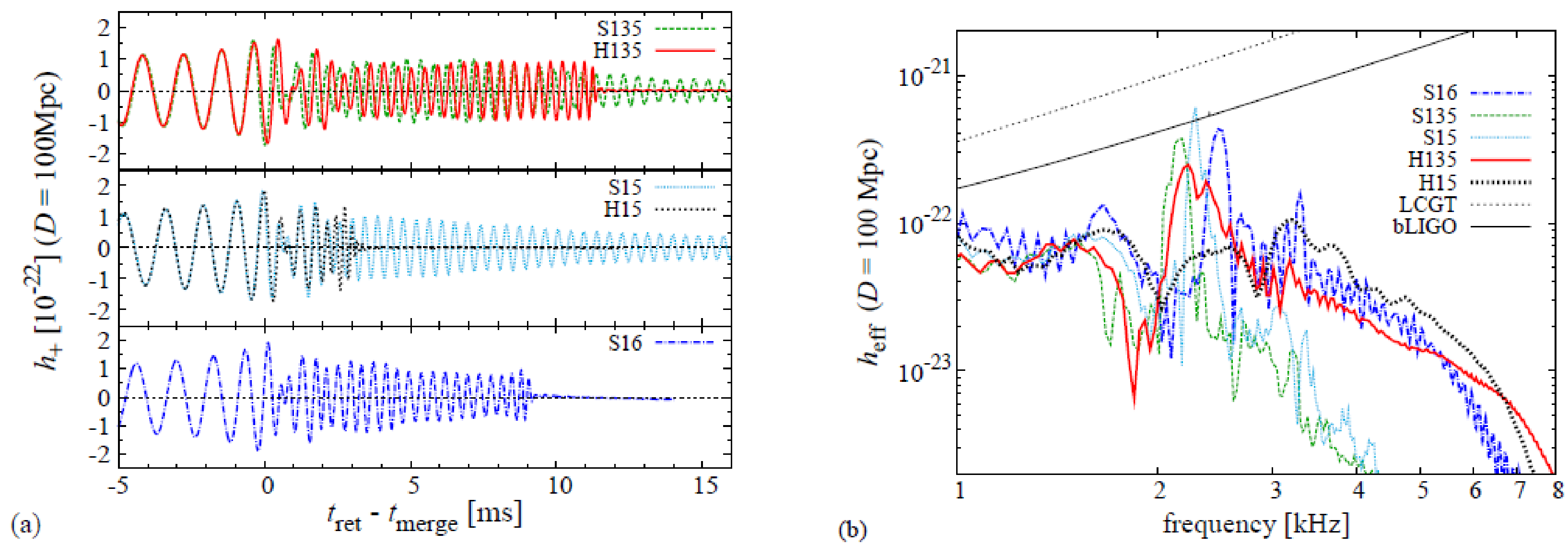
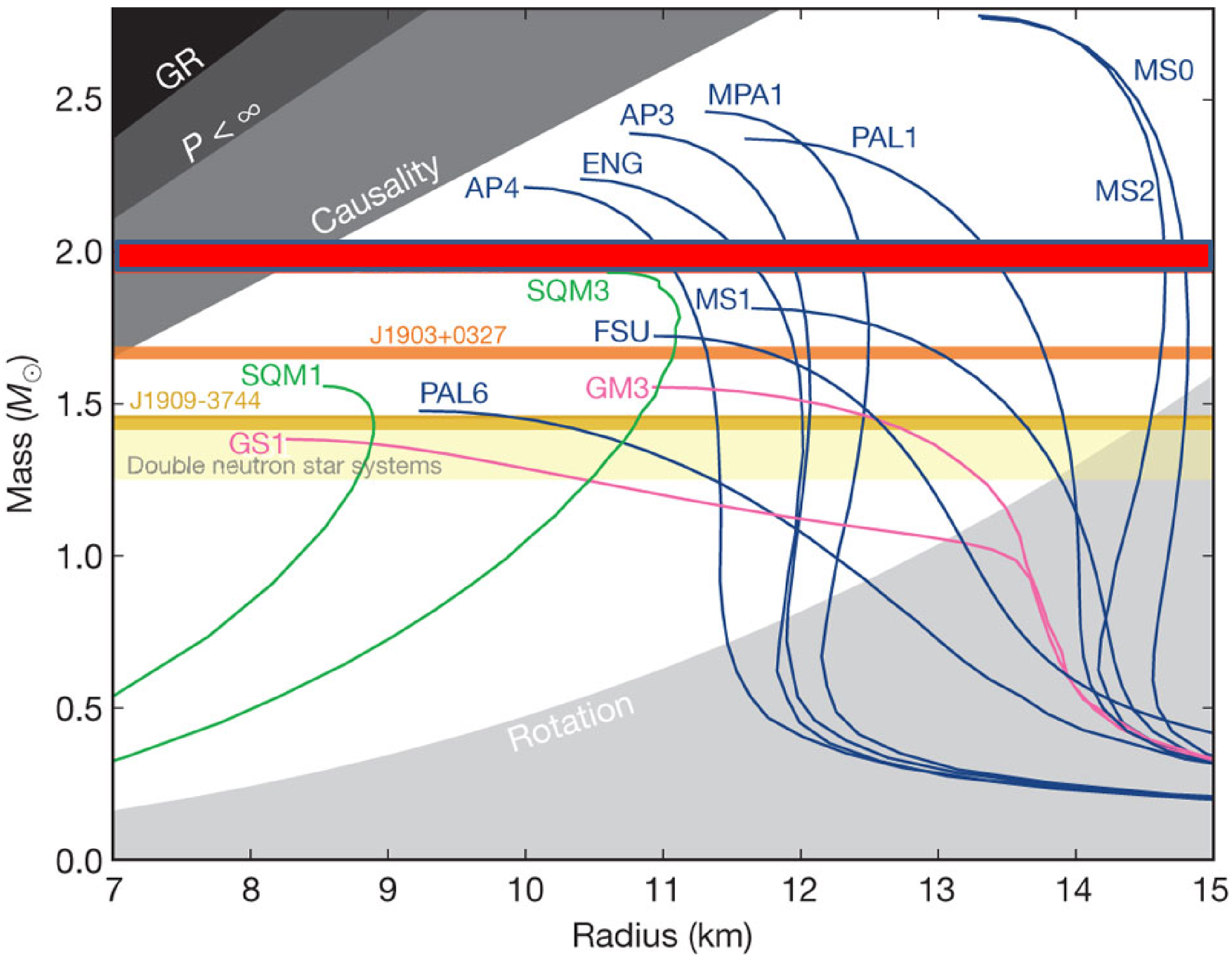


FIG. 4: (a) GWs observed along the axis perpendicular to the orbital plane for the hypothetical distance to the source $D = 100$ Mpc. (b) The effective amplitude of GWs defined by $0.4f|h(f)|$ as a function of frequency for $D = 100$ Mpc. The noise amplitudes of a broadband configuration of Advanced Laser Interferometer Gravitational wave Observatories (bLIGO), and Large-scale Cryogenic Gravitational wave Telescope (LCGT) are shown together.

constraint on EoS from NS observations



J0348+0432

J1614-2230

Equation of state, $E(N_n, N_p, V)$

- energy of the system for a given number of nucleon number, N, in a finite volume.
- comopsition of nuclear matter

  $n, \quad x = n_p/n$

  leptons: electrons, muons

  strange hadrons: kaon, hyperons

  dilatons, ...

  quarks,
- chemical potential of neutron and proton

→ Symmetry enrgy

$$\mu_n - \mu_p = 4(1-2x)S(n) \quad \Leftrightarrow \quad m_i(m_i^*)$$

II. Symmetry energy

Nuclei

Mass formula (Weizsäcker, 1935; Bethe and Bacher, 1936)
Binding energy of a nucleus ($A = Z + N$)

$$\mathcal{B}(N,Z) = b_{\rm vol}A - b_{\rm surf}A^{2/3} - \frac{1}{2}b_{\rm sym}\frac{(N-Z)^2}{A} - \frac{3}{5}\frac{Z^2e^2}{R_c}$$

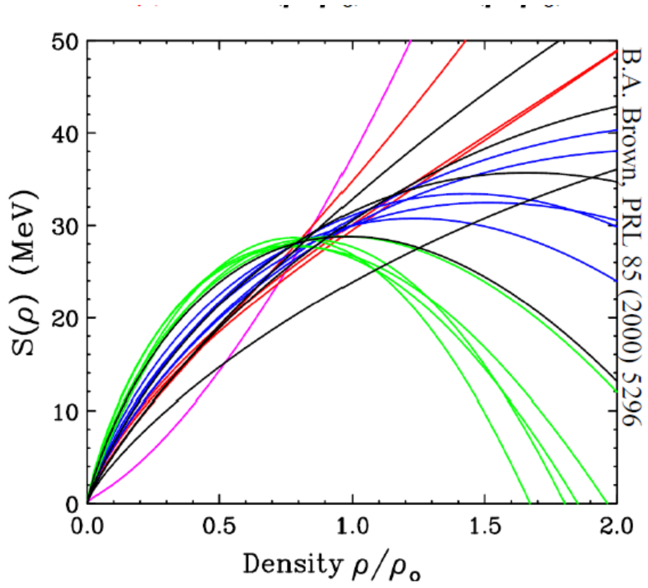
Nuclear matter

$$x = \frac{n_p}{n}$$

$$E(n,x) = m_N + \frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3} + S(n)(1-2x)^2 + V(n)$$

Symmetry energy:
measure of n-p asymmetry in nuclear matter

1. S(n) beyond n_0: ?



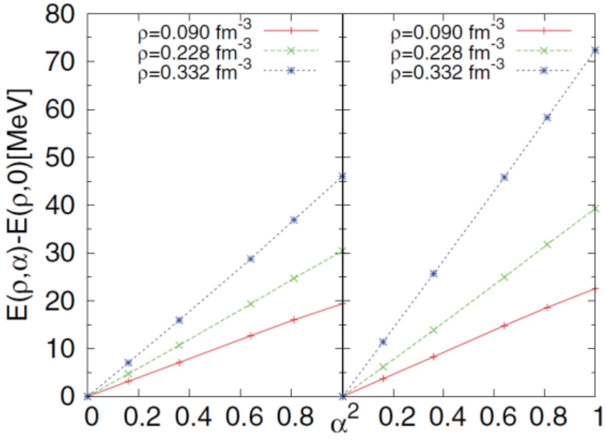
2. New physics encoded in S(n)

- Driving force toward symmetric matter

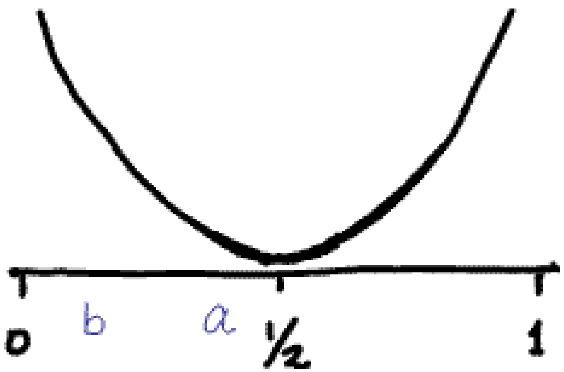
$$E(n,x) \cong E(n,x=1/2) + (1-2x)^2 S_{1/2}(n)$$

$$S_{1/2} = \frac{1}{8}\frac{\partial^2 E(n,x)}{\partial x^2}\Big|_{x=1/2}$$

$$E_{symm}(n,x) = (1-2x)^2 S(n)$$



J.M. Lattimer and M. Prakash, Phys. Rep. (2000),
H.Dong, T.T.S. Kuo, R. Machleitd, PRC 83(2011)



Iso-spin symmetry and symmetry energy

Consider a system of nucleon with N_p protons and N_n neutrons

$$|N_p, N_n >= \Sigma_I C_I |I : N_p, N_n >$$

where $|\vec{I}|^2 = I(I+1)$.
Energy of the state

$$\begin{aligned} E(N,x) =< N,x|H|N,x > &= \Sigma_I |C_I(n,x)|^2 E_I(n) \\ &= (1-2x)^2 S(n) + \cdots \end{aligned}$$

E_I is an reduced matrix element of the Hamiltonian, $H = H_0 + H_{int}$,

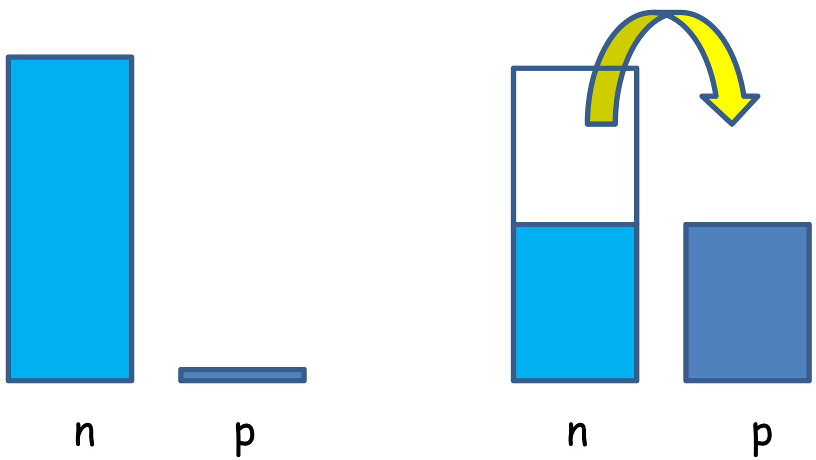
$$E_I =< I||H||I >$$

which is independent of I_3, and depends on the details of the strong interactions for each I-channel.

Examples: symmetric nuclear matter (I=0, …)
pure neutron matter(I ≠ 0, ….)

- simple example: free nucleon gas
  E_I(interaction) → 0, however

Pauli exclusion principle for nucleons

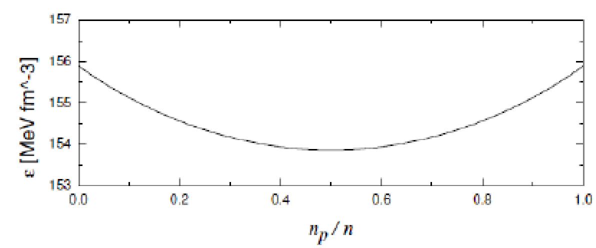


E(x~0) > E(x=1/2)

- Non-interacting n-p system

$$\epsilon_n = \frac{8\pi}{(2\pi)^3}\int_0^{p_F}\left(p^2+m_n^2\right)^{1/2}p^2dp$$

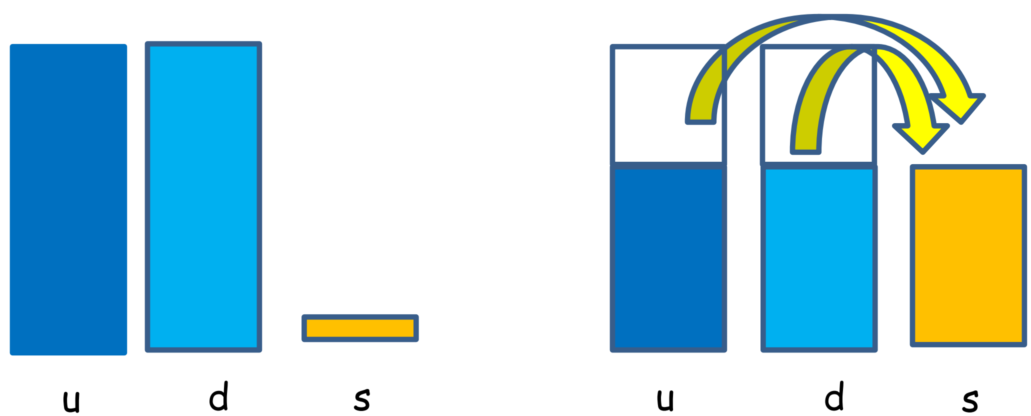
$$n_n = \frac{8\pi}{(2\pi)^3}\int_0^{p_F}p^2dp$$



$$S_{free}(n) = \left(2^{2/3}-1\right)\frac{3}{5}E_F^0\left(\frac{n}{n_0}\right)^{2/3}$$

→ additional degrees of freedom reduce energy of the baryonic matter

- strange quark matter (SQM)
  - more stable at higher density

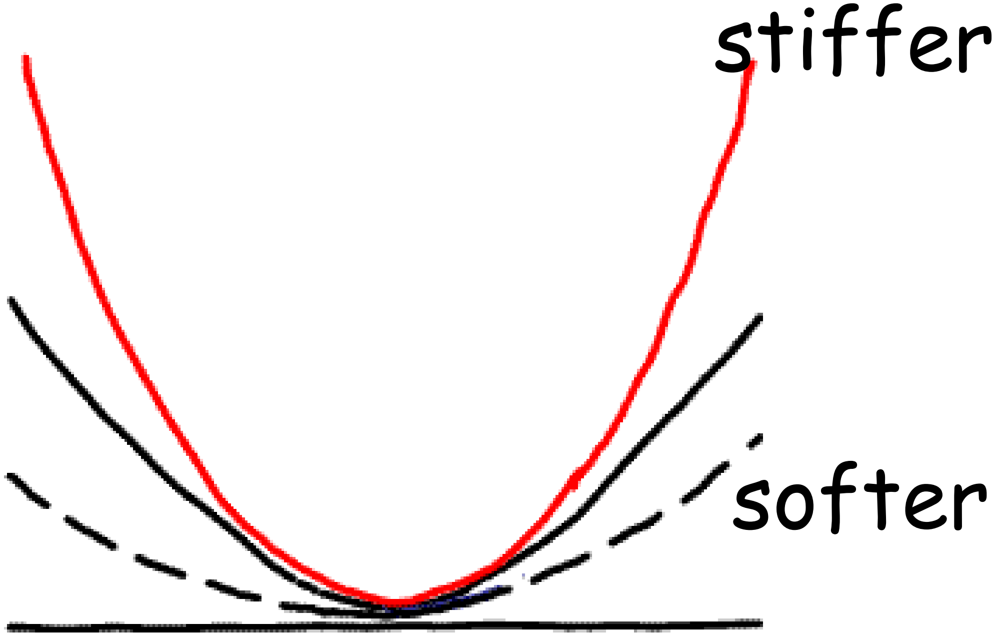


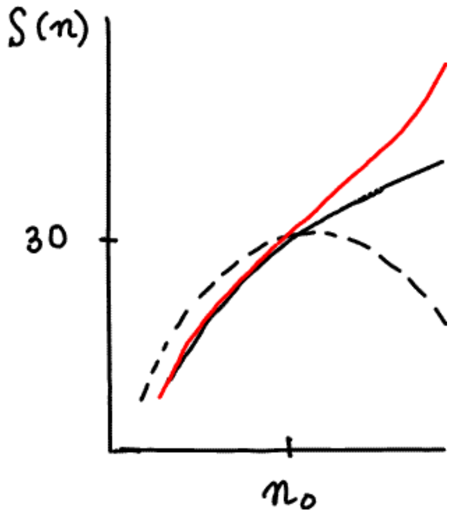
E(s=0) > E(s=1/3)

- Strong interactions: tensor force, …

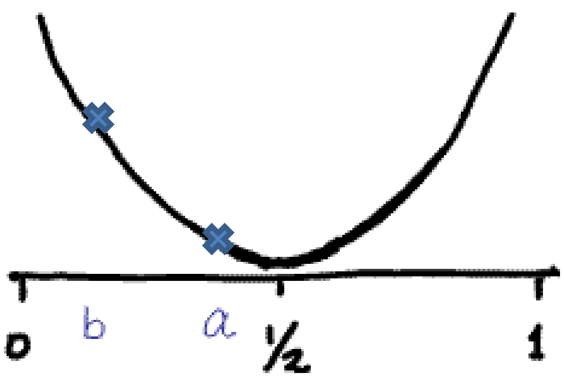
$$
\begin{aligned}
V_M^T(r) \quad &= S_M \frac{f_{NM}^2}{4\pi} m_M \tau_1 \cdot \tau_2 S_{12} \\
&\left(\left[\frac{1}{(m_M r)^3} + \frac{1}{(m_M r)^2} + \frac{1}{3 m_M r} \right] e^{-m_M r} \right)
\end{aligned}
$$

$n > n_0$



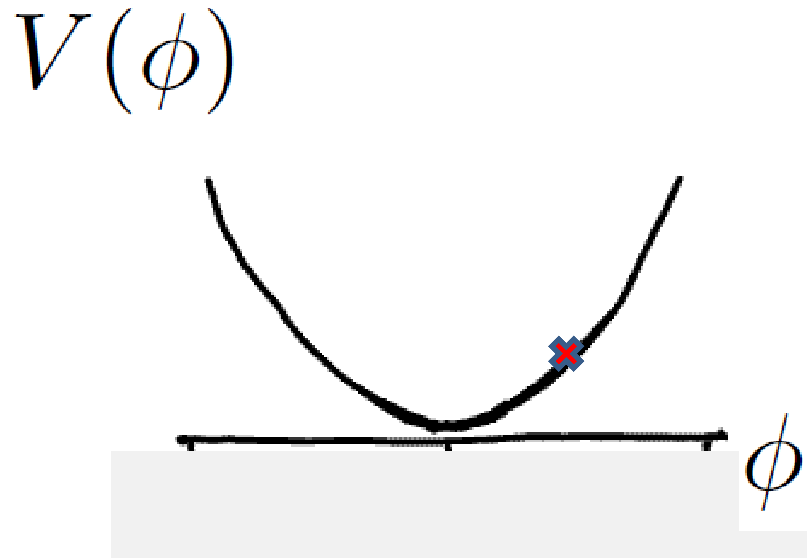


nuclei,
neutron star matter



$$x \neq 1/2$$

inflation
by scalar field



$$\phi \neq \phi_0$$

- Examples of asymmetric configuration

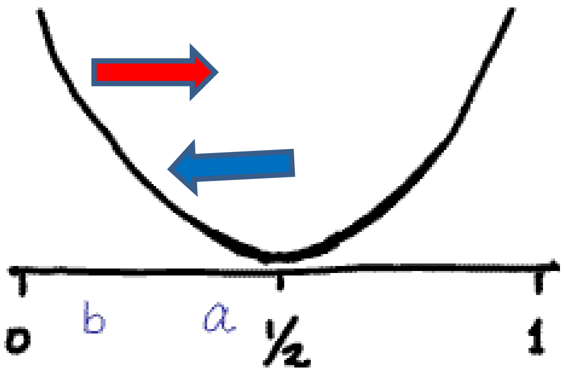
a. Heavier nuclei with N> Z

Coulomb interaction due to proton

→ less proton number preferred (asymmetric config.)

Strong interaction

symmetry energy → symmetric config.



x ~ 1/2

b. Neutron star matter

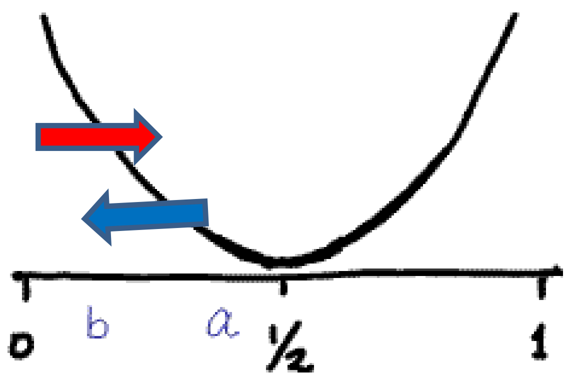
N >> Z → N ~ Z

Strong interaction

Symmetry energy → more protons

Charge neutrality and Weak interaction :

→ Weak equilibrium

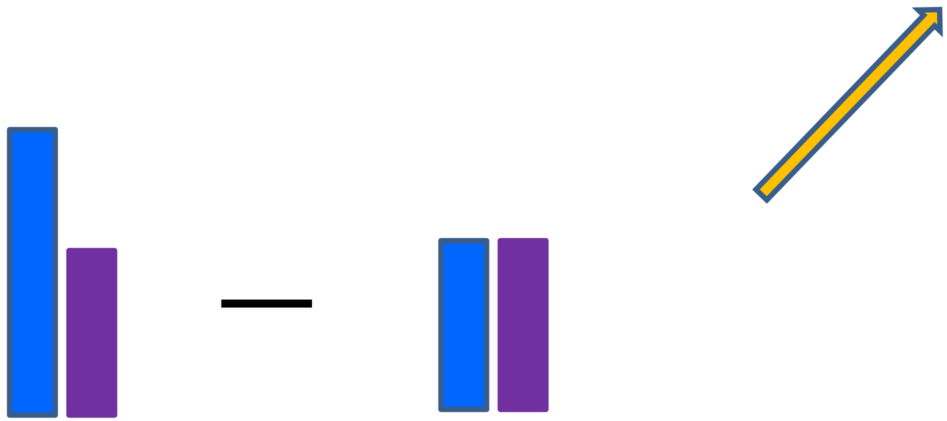


x ~ 0 - 1/2

- # Anatomy of symmetry energy

HKL and M. Rho, Int. J. Mod. Phys. E **22**, 1330005 (2013)

$$\epsilon(n,x) = \epsilon(n, x=1/2) + \epsilon_{sym}(n,x)$$



$$\epsilon(n,x) = \epsilon(n, x=1/2) + nS(n)(1-2x)^2$$

$$\Delta_{nn}(n) = \epsilon(n,0) - 2\epsilon(n/2, x=0)$$

$$\Delta_{np}(n) = \epsilon(n, x=1/2) - 2\epsilon(n/2, x=0)$$

$$\epsilon_{sym}(n,0) = \Delta_{nn}(n) - \Delta_{np}(n)$$

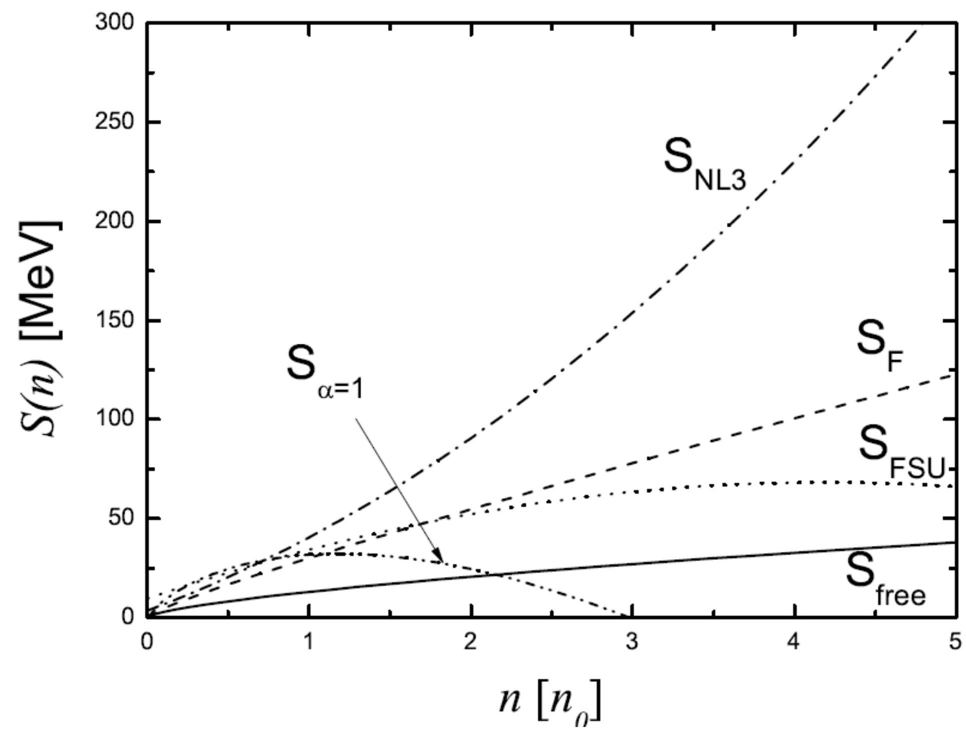
$$\boxed{S(n) = [\Delta_{nn}(n) - \Delta_{np}(n)]/n}$$

Phenomenological forms of symmetry energy

$$S_F(n) = (2^{2/3} - 1)\frac{3}{5}E_F^0 \left[\left(\frac{n}{n_0}\right)^{2/3} - F(n)\right] + S_0 F(n)$$

$$S_\alpha = (2^{2/3} - 1)\frac{3}{5}E_F^0 \left(\frac{n}{n_0}\right)^{2/3} + A(\alpha)\frac{n}{n_0} + [18.6 - A(\alpha)]\left(\frac{n}{n_0}\right)^{B(\alpha)}$$

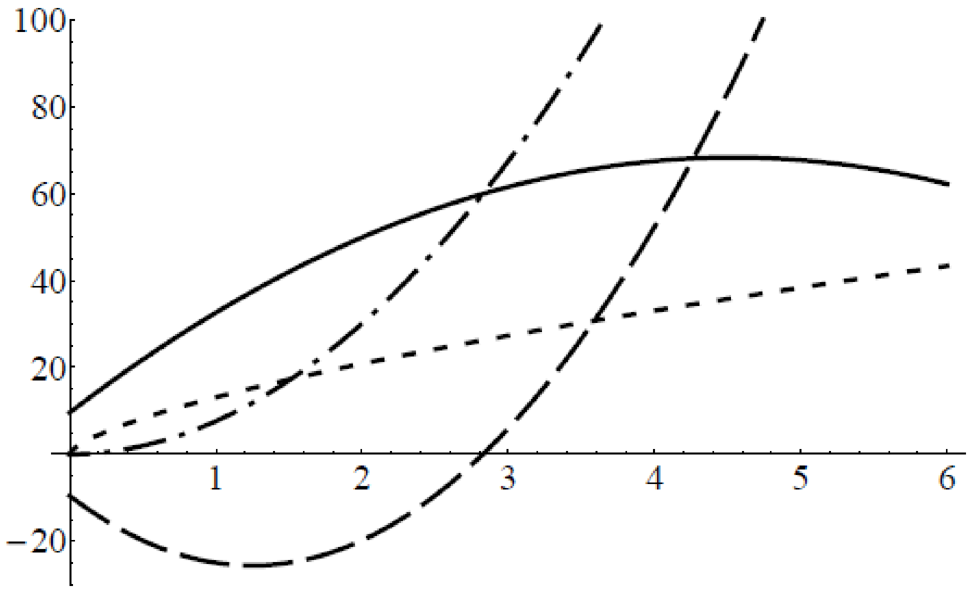
$$S_3(n) \simeq S_0^* + L\rho + \frac{1}{2}K\rho^2$$



Model	J	L	$K_{\rm sym}$	K_0	ϵ_0
FSU	32.59	60.5	−51.3	230	−16.30
NL3	37.29	118.2	100.9	271.5	−16.24

$$\Delta_{nn} = n\left[\frac{1}{2}K_0(\tilde{u}^2-\bar{u}^2)+L(\tilde{u}-\bar{u})+\frac{1}{2}K_{\rm sym}(\tilde{u}^2-\bar{u}^2)\right]$$

$$\Delta_{np} = n\left[\frac{1}{2}K_0(\tilde{u}^2-\bar{u}^2)-\left(J+L\bar{u}+\frac{1}{2}K_{\rm sym}\bar{u}^2\right)\right],$$



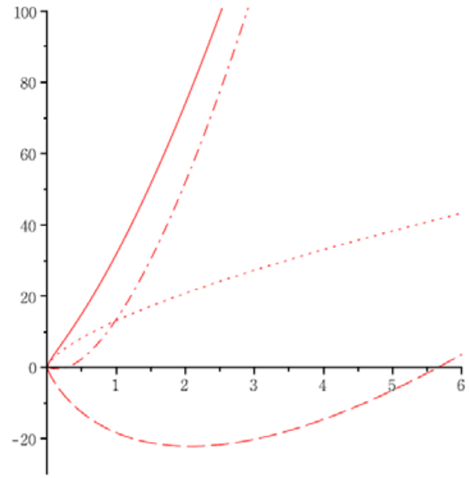


Figure 1: The symmetry energy factor $S(n)$ (thick line), $\Delta np/n$ (dashed), Δ_{nn}/n(dash-dotted) and the symmetry energy for non-interacting nucleons (dotted) for the LCK model with $\alpha = -1$.

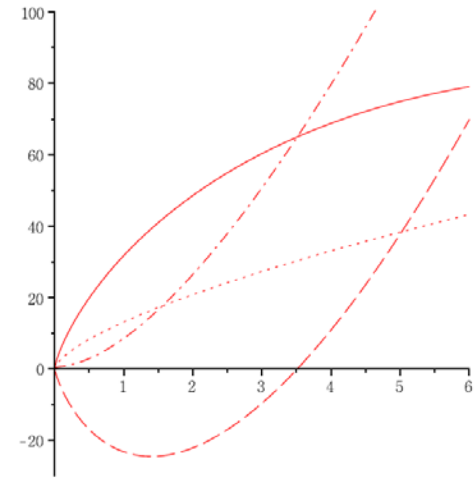


Figure 2: Same as Fig. 1 for the LCK model with $\alpha = 0$.

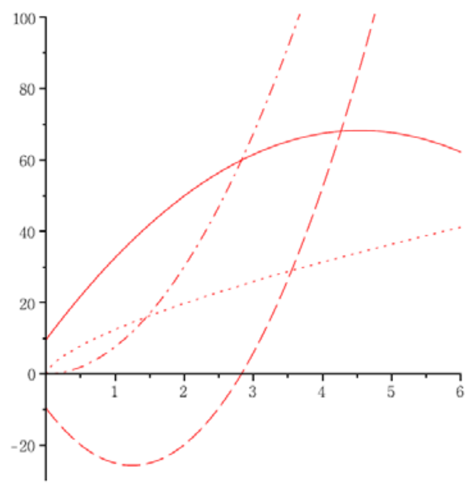


Figure 4: Same as Fig. 1 for the FSU model.

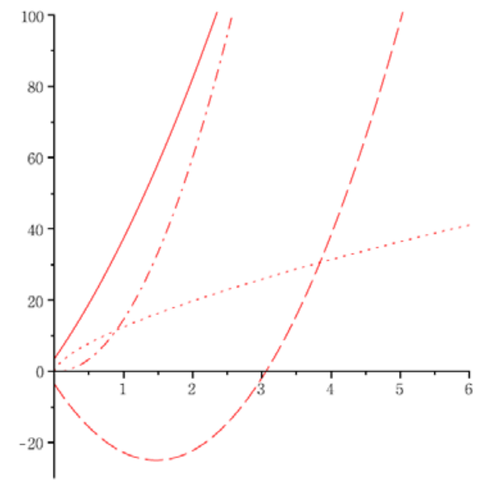


Figure 5: Same as Fig. 1 for the NL3 model.

- How can we explore dense hadronic matter other than doing simple extrapolations from low density models?
- Lattice QCD ?
- Effective theory at finite density?
- ……

skyrmions on the lattice
→ symmetry energy (density dependent)

III. Dilaton in dense hadronic matter

HKL, SCGT12(2012)

Walking Technicolor at LHC?

SSB of chiral symmetry → dilaton as GB

Explicit(spontaneous) breaking of scale symmetry
→ SSB of chiral symmetry

Chiral symmetry: nonlinear realization

$\pi \quad \chi$

Change of $< \chi >$ with baryon number density

SSB of scale symmetry: Freund –Nambu model

$$V(\psi,\phi) = V_a + V_b$$

$$\begin{aligned} V_a &= \frac{1}{2} f^2 \psi^2 \phi^2 \\ V_b &= \frac{\tau}{4}\left[\frac{\phi^2}{g^2} - \frac{1}{2}\phi^4 - \frac{1}{2g^4}\right] = \frac{\tau}{8g^4}(g^2\phi^2 - 1)^2 \end{aligned}$$

$$\chi = (g^2\phi^2 - 1)/(2g) \qquad \delta\chi = \epsilon(x\cdot\partial + 2)\chi + \frac{\epsilon}{g}$$

$$m_\psi^2 = f^2\phi_0^2, \qquad m_\chi^2 = \tau\phi_0^2$$

matter field　　　　dilaton

(approximate) SSB of scale symmetry:

only the mass of Goldstone boson(dilaton) must be small while all other fields can be arbitrary.

Freund & Nambu, 1968; B. Zumino, 1970

Explicit(spontaneous) breaking of scale symmetry → SSB of chiral symmetry

Let us consider the lagrangian of linear sigma model with $\Phi = \sigma + i\tau \cdot \vec{\phi}$ with potential of the following form

$$V(\phi) = \lambda(\frac{1}{2}Tr\Phi\Phi^\dagger - v^2)^2 = \lambda(\sigma^2 + \Sigma_i \phi_i^2 - v^2)^2. \quad (6)$$

The scale invariance is broken due to nonvanishing v^2. But it is the term which breaks chiral symmetry spontaneously. When we assign the vacuum expectation only to σ, $\phi_i, i = 1,2,3$ becomes Glodstone bosons related to the spontaneous chiral symmetry breaking and we call it pions, π. For $\lambda = 0$, there is no explicit breaking of scale symmetry regardless of the value of v^2. We introduce a scalar field, χ, and reformulate the above potential in the following form,

$$V(\phi, \chi) = \lambda(\frac{1}{2}Tr\Phi\Phi^\dagger - \chi^2)^2 + V(\chi) \quad (7)$$

The first term is analogous to V_a in eq.(2), which is scale invariant. The second term breaks scale symmetry. In general, V_χ is giving the nonvavinshing vacuum expectation value for χ, which can be of the following forms

$$V_\chi = \frac{\tau}{8}(\chi^2 - v^2)^2 \quad \text{or} \quad V_\chi = \tau\chi^4 \ln\frac{\chi}{e^{1/4}v} \quad (8)$$

Dilaton for scale symmetry

1. chiral symmetry breaking and partially for
2. QCD trace anomaly

→ Effective theory(talks by B.Y. Park and by Y. Oh)

1. pions,
2. dilaton

$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{tr}\left[\partial_\mu U \partial^\mu U^\dagger\right]$$

⇓

$$\mathcal{L}(U,\chi) = \frac{\chi^2}{4}\mathrm{tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] + V(\chi) + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi$$

3. vector mesons (SU(2) x U(1))
   (talks by B.Y. Park and by Y. Oh)

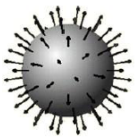
IV. Half skyrmion and new scaling(BLPR)

single hedgehog skyrmion

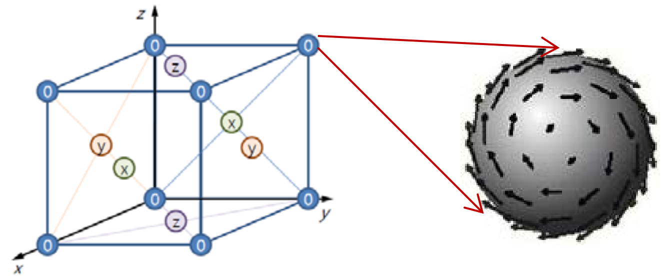
1960, T. H. R. Skyrme

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2$$

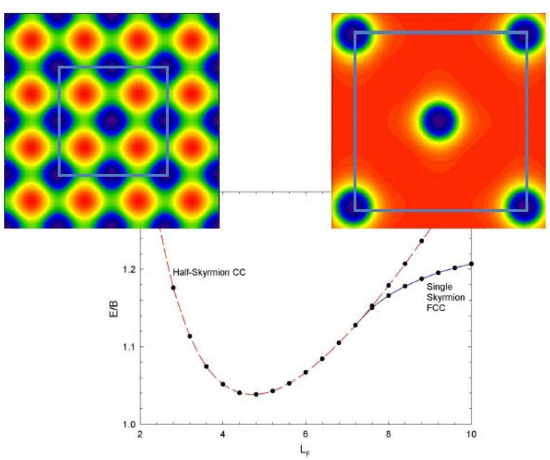
$U(\vec{x})$: mapping from R^3-{∞}$=S^3$ to $SU(2)=S^3$
→ topological soliton



R ~ 1 f m
M ~ 1.5 GeV → baryon ?

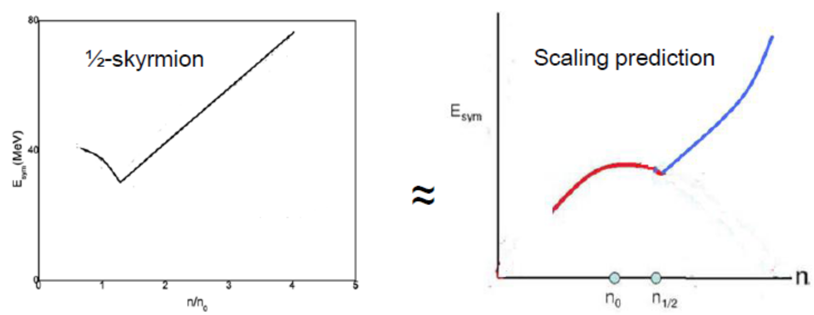


Skyrmion Crystal



Goldhaber and Manton(1987)
Byung-yoon Park et al. (2003), …

Effect on symmetry energy

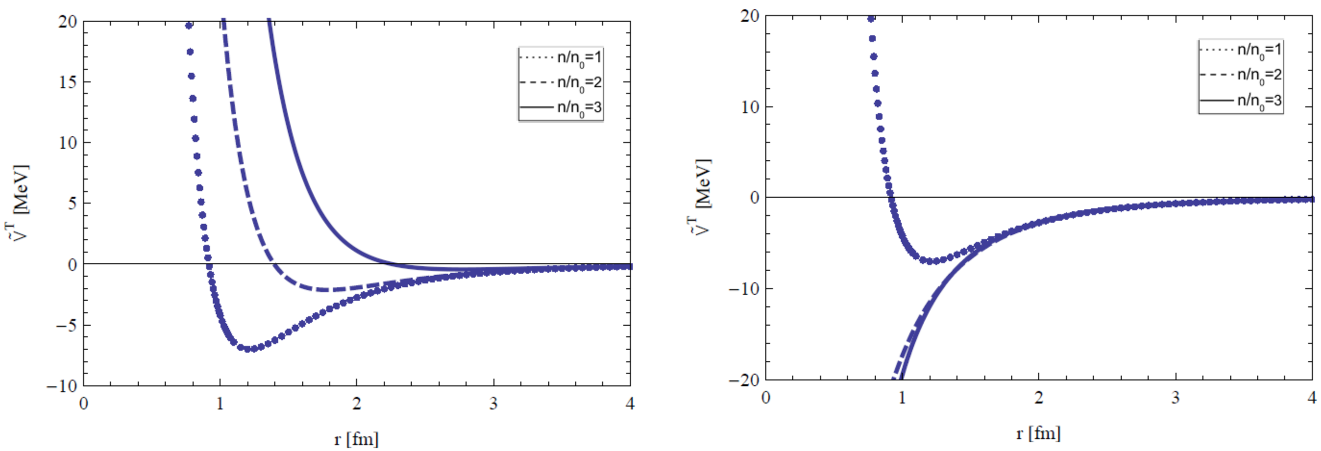


HKL, B.Y. Park and M. Rho, PRC83, 025206(2011)

(a) Suppression of rho-mediated tensor force

$$V_M^T(r) = S_M \frac{f_{NM}^2}{4\pi} m_M \tau_1 \cdot \tau_2 S_{12} \left(\left[\frac{1}{(m_M r)^3} + \frac{1}{(m_M r)^2} + \frac{1}{3 m_M r} \right] e^{-m_M r} \right)$$

$$M = \pi, \rho, \; S_{\rho(\pi)} = +1(-1)$$



Sum of π and ρ tensor forces in units of MeV for densities $n/n_0 = 1$ (dotted), 2 (dashed) and 3 (solid) with the "old scaling" $\Phi \approx 1 - 0.15 n/n_0$ and $R \approx 1$ for all n (upper panel) and with the "new scaling," $\Phi_I \approx 1 - 0.15 n/n_0$ with $R \approx 1$ for $n < n_{1/2}$ and $\Phi_{II} \approx \Phi_I$ and $R \approx \Phi_{II}^2$ for $n > n_{1/2}$, assuming $n_0 < n_{1/2} < 2n_0$ (right panel). For simplicity we set $F_\rho^*/F_\rho = 1$.

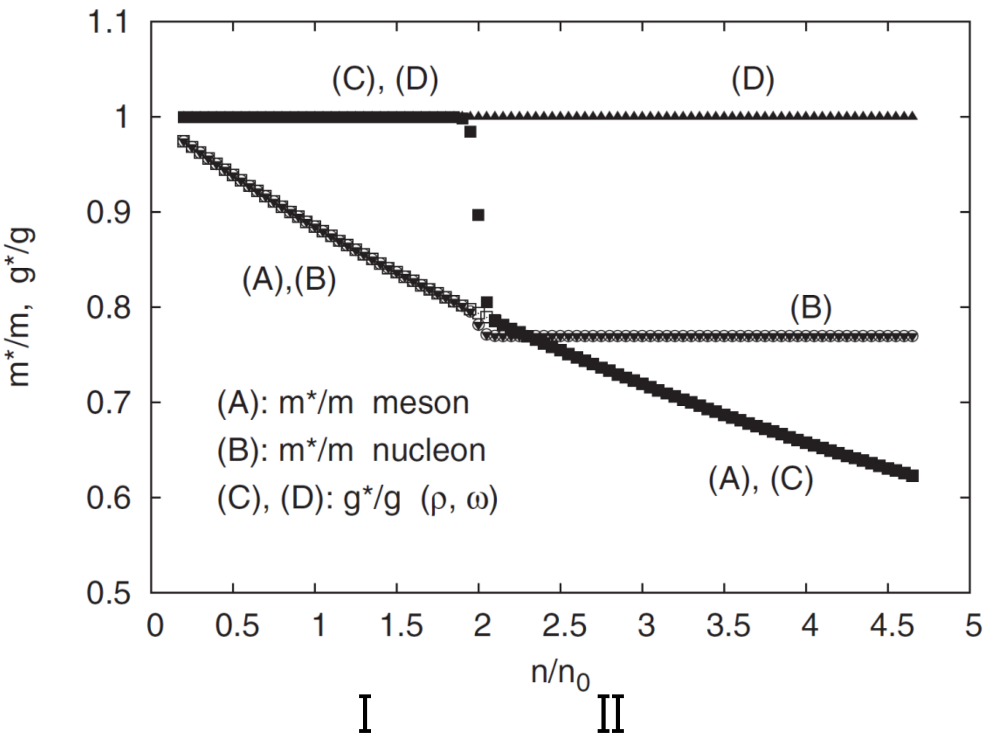
HKL & M.Rho, arXiv:1306.4584, **EPJA to be published**

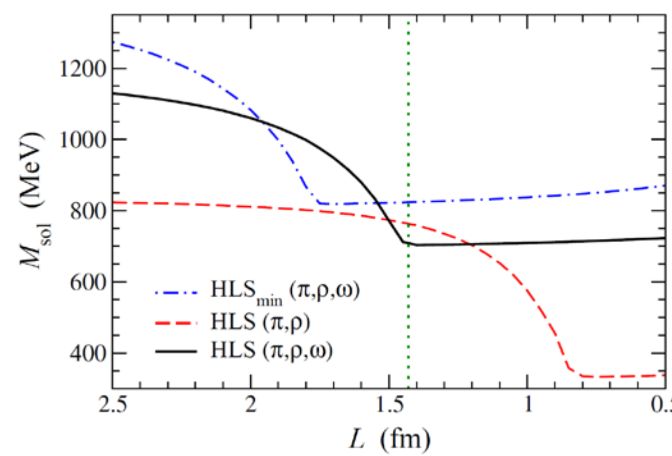
- New scaling (BLPR) for dense matter

HKL, B.Y. Park and M. Rho, PRC83, 025206(2011)

- old scaling (BR) for region I
- RG implemented EFT(Stony Brook)

H.Dong, T.T.Kuo, HKL, R.Machleidt, M.Rho, PRC(2013)



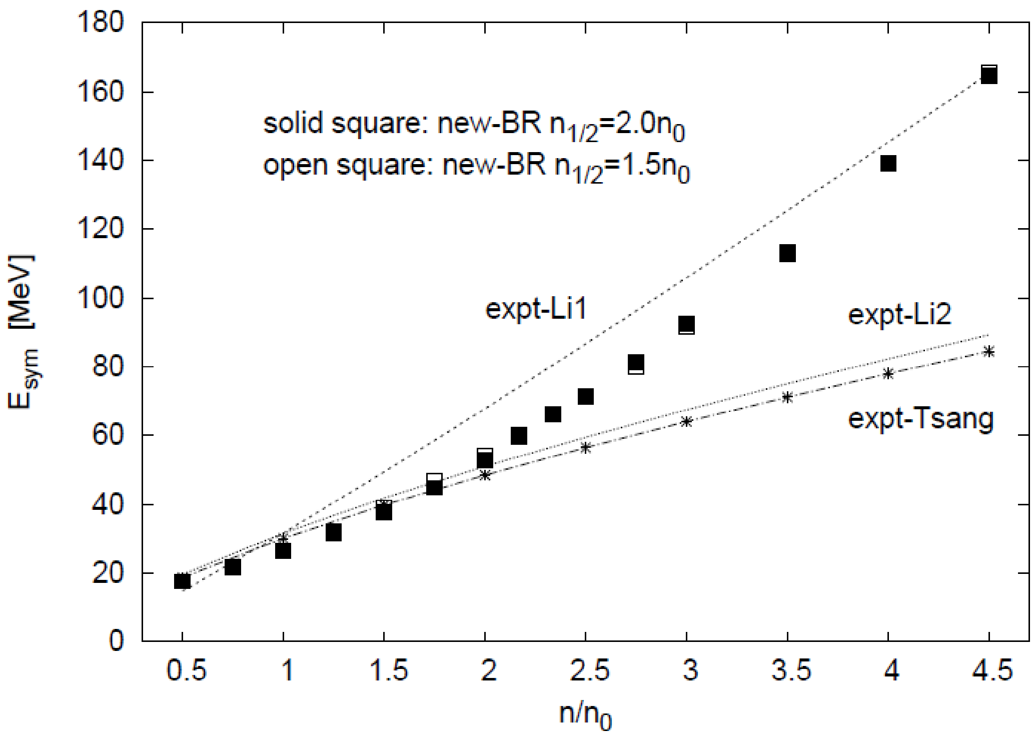


Y.L. Ma et al., PRD(2013)

(b) stronger attraction in n-p channel
→ stiffer symmetry energy   S(n)

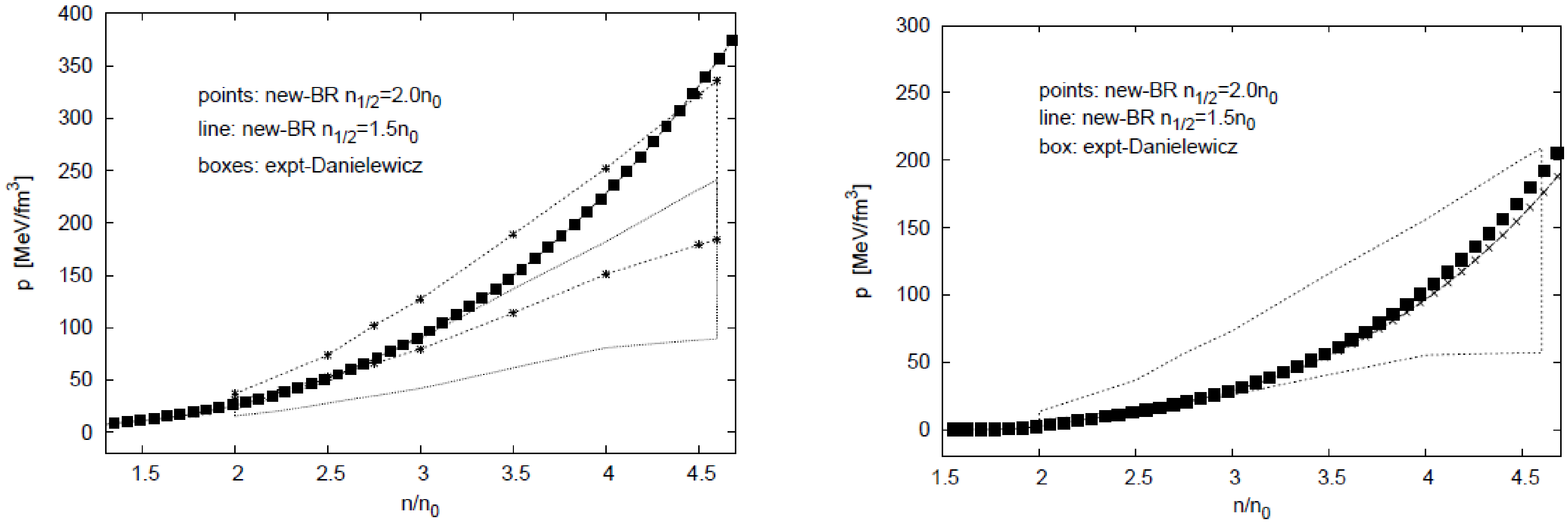
$$S(n) = E(n, n_p = 0) - E(n, n_n = n_p)$$

H.Dong, T.T.Kuo, HKL, R.Machleidt, M.Rho, PRC(2013)



(c) EoS with new scaling, BLPR

Dong, Kuo, HKL, Machleidt, Rho, 1207.0429

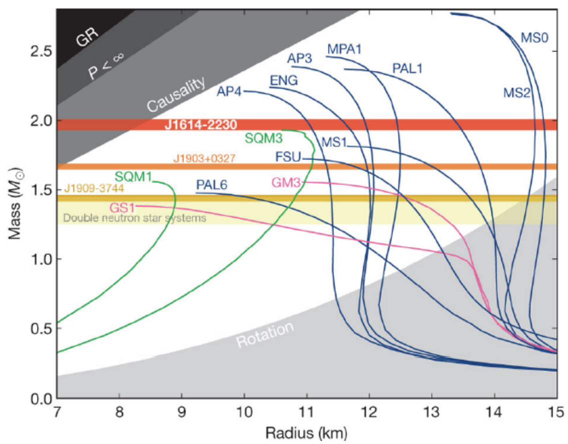


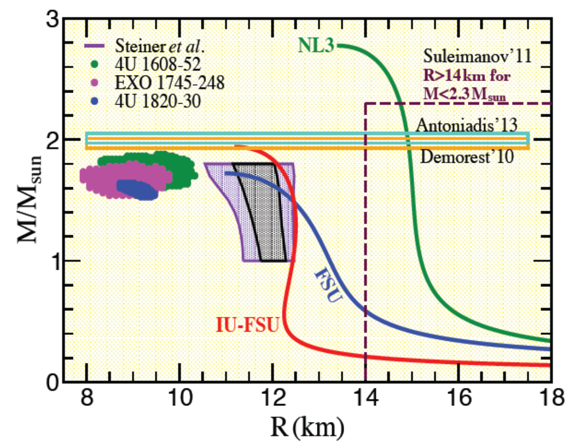
Stiffer EoS for massive neutron star

Massive neutron stars

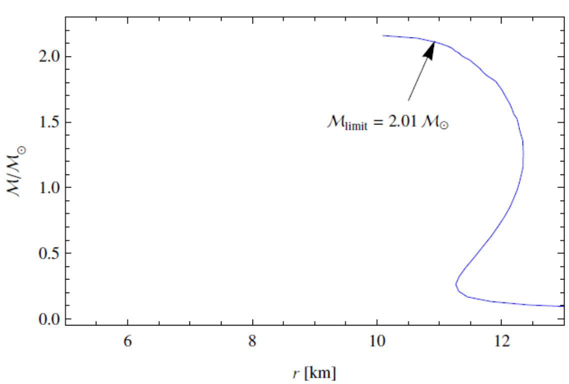
PSR J1614-2230 **(1.97 ± 0.04)** $M_\odot$ P. Demorest et al., 2010

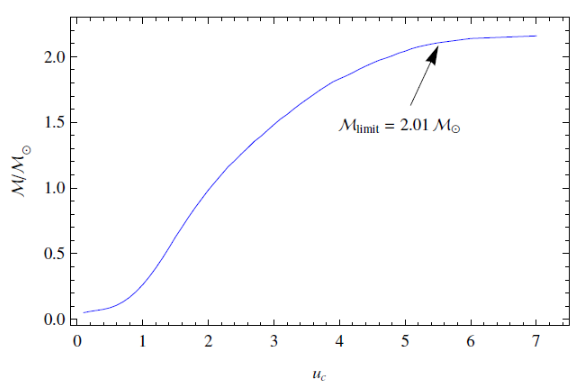
PSR J0348+0432 **(2.01 ± 0.04)** $M_\odot$ J. Antoniadis et al., 2013





[J. Piekarewicz](#) arXiv:1305.7101

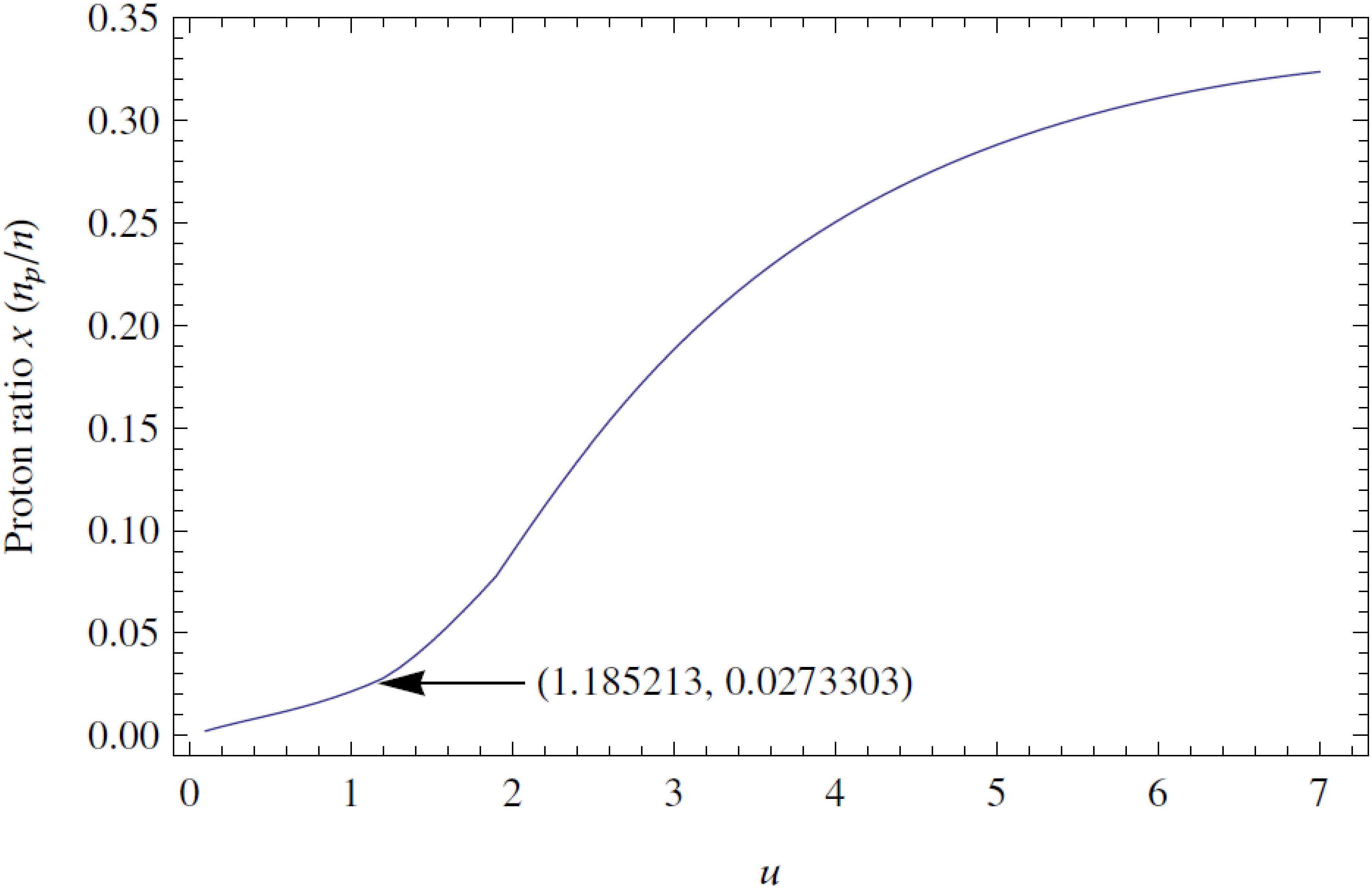




Summary

- Nuclear symmetry energy : useful indicator for new physics
- Skyrmion on the lattice(dilaton and vector mesons) → new scaling and stiffer EoS (RIB machines) → new physics ??
- New degrees of freedom in highly dense hadronic matter: strange hadrons,…., quarks, ..
- Gravitational waves from cosmic collider :advanced detectors(2015 →)


Thank you

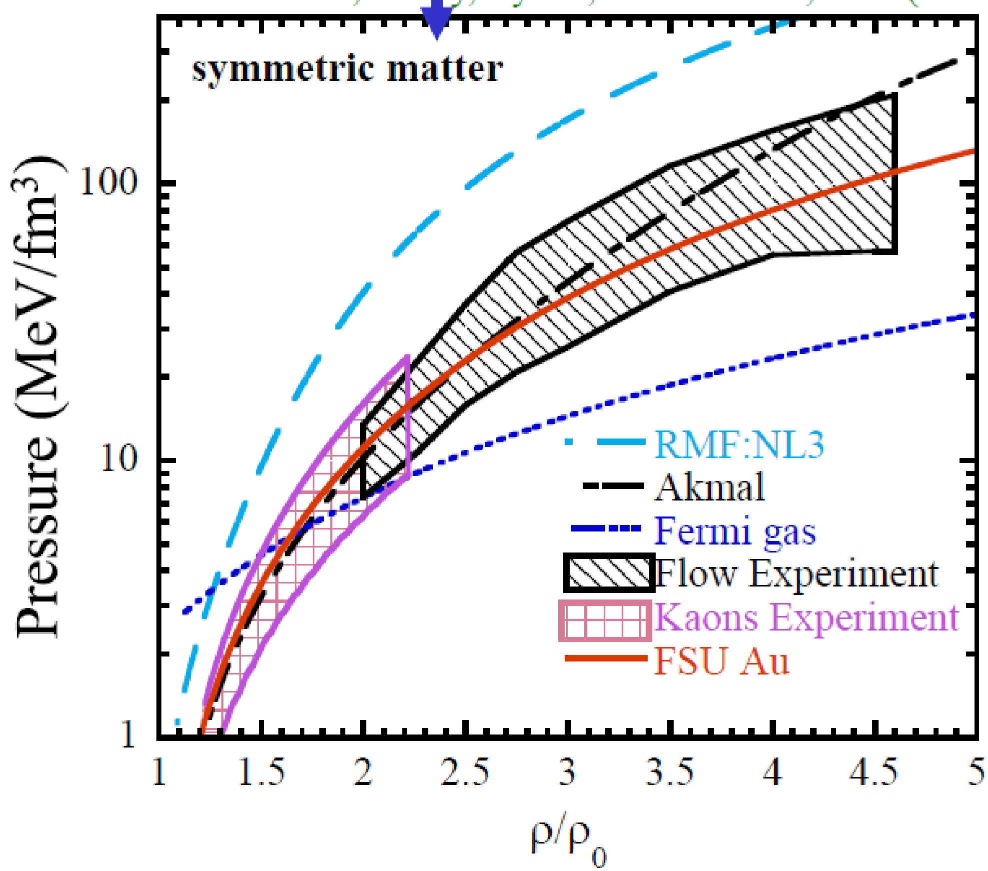


Proton ratio x (n_p/n)
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
4
5
6
7
u
(1.185213, 0.0273303)

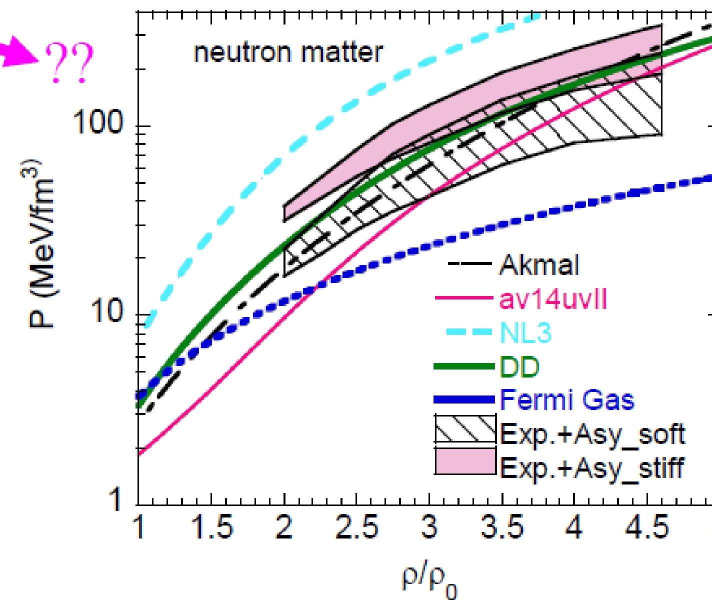
Density dependence of Symmetry Energy

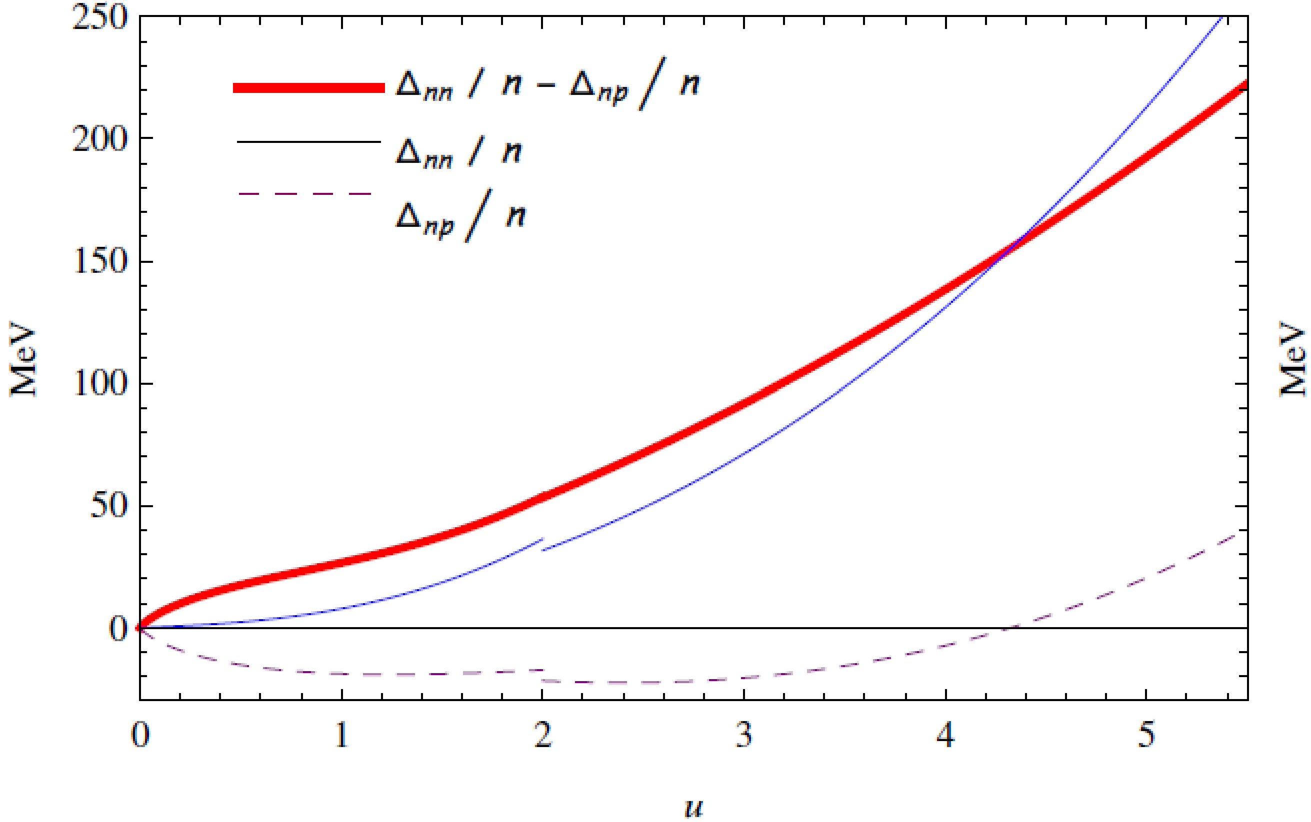
$E/A(\rho,\delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho);$ $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$

Danielewicz, Lacey, Lynch, Science 298,1592 (2002)



??





$\Delta_{nn}/n - \Delta_{np}/n$
Δ_{nn}/n
Δ_{np}/n
250
200
150
100
50
0
MeV
MeV
0
1
2
3
4
5
u