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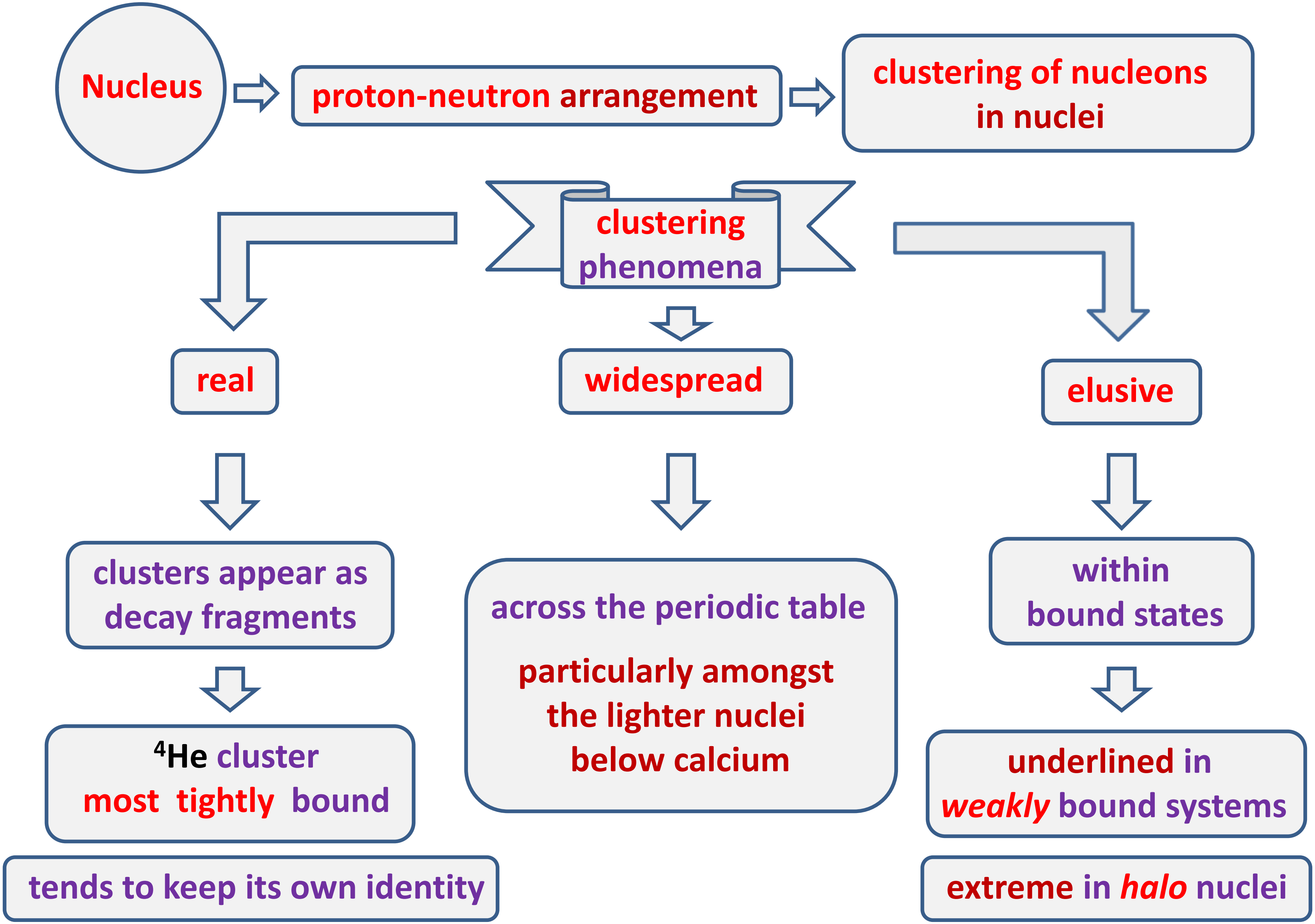
Cluster models with core excitations: Applications to radioactive nuclei.


Nucleus
proton-neutron arrangement
clustering of nucleons in nuclei
clustering phenomena
real
widespread
elusive
clusters appear as decay fragments
across the periodic table
particularly amongst the lighter nuclei below calcium
within bound states
^{4}He cluster
most tightly bound
underlined in
weakly bound systems
tends to keep its own identity
extreme in *halo* nuclei

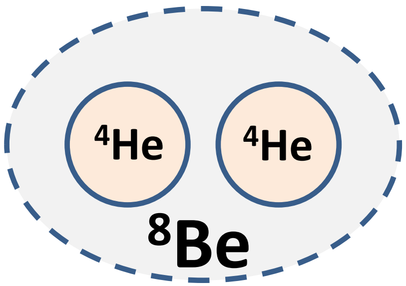
Examples:

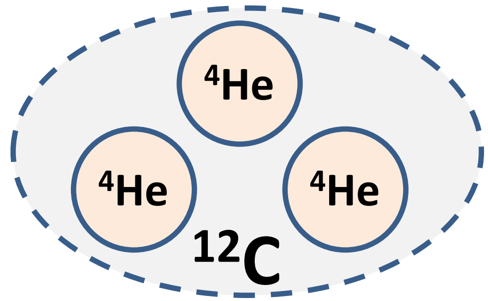
^{4}He-decay of heavy nuclei

4n nuclei: ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, …

molecular like structure composed of ^{4}He fragments nearby the threshold energy of the nuclear dissociation into these constituents

E_{8Be} = 91.84 keV
Γ_{8Be} = 5.57 eV



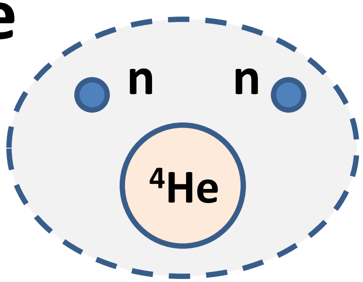


"Hoyle" state

E_{12C} = 7.654 MeV
Γ_{12C} = 8.5 eV
Q_{4He} = 7.367 MeV

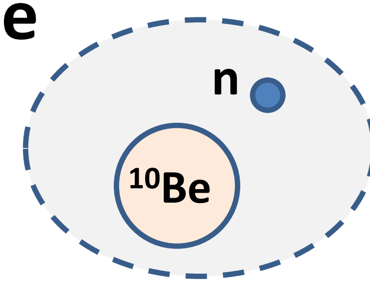
Halo nuclei

^{6}He



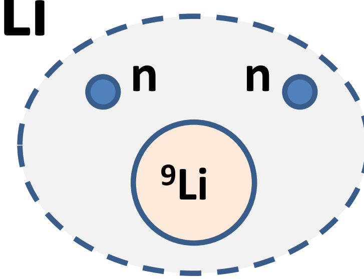
E_{6He} = -973 keV

^{11}Be



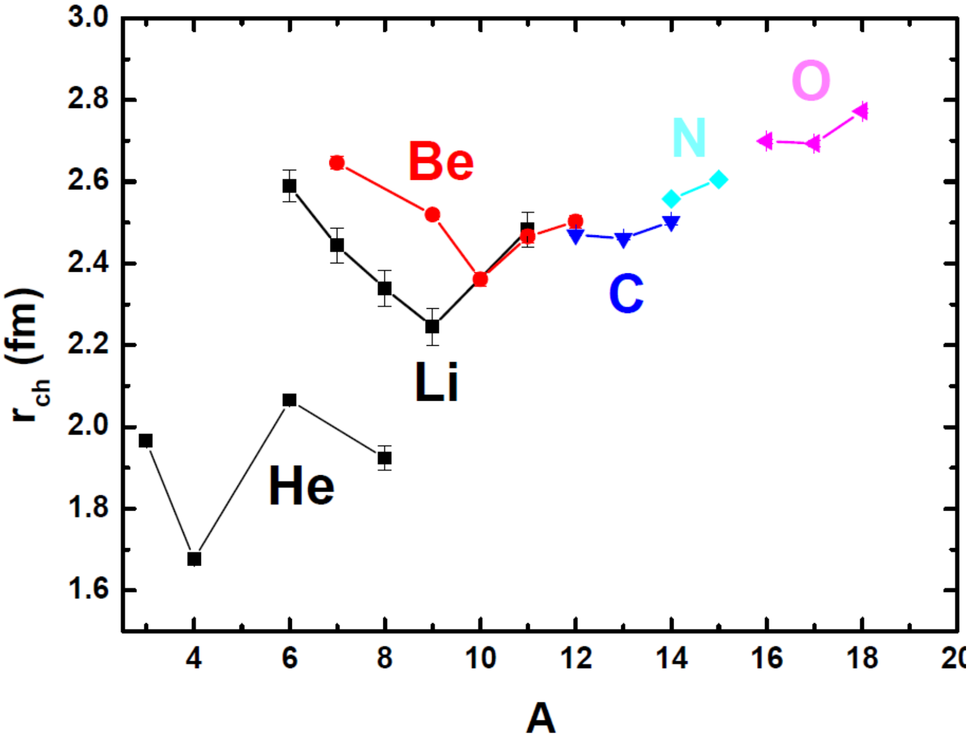
E_{11Be} = -502 keV

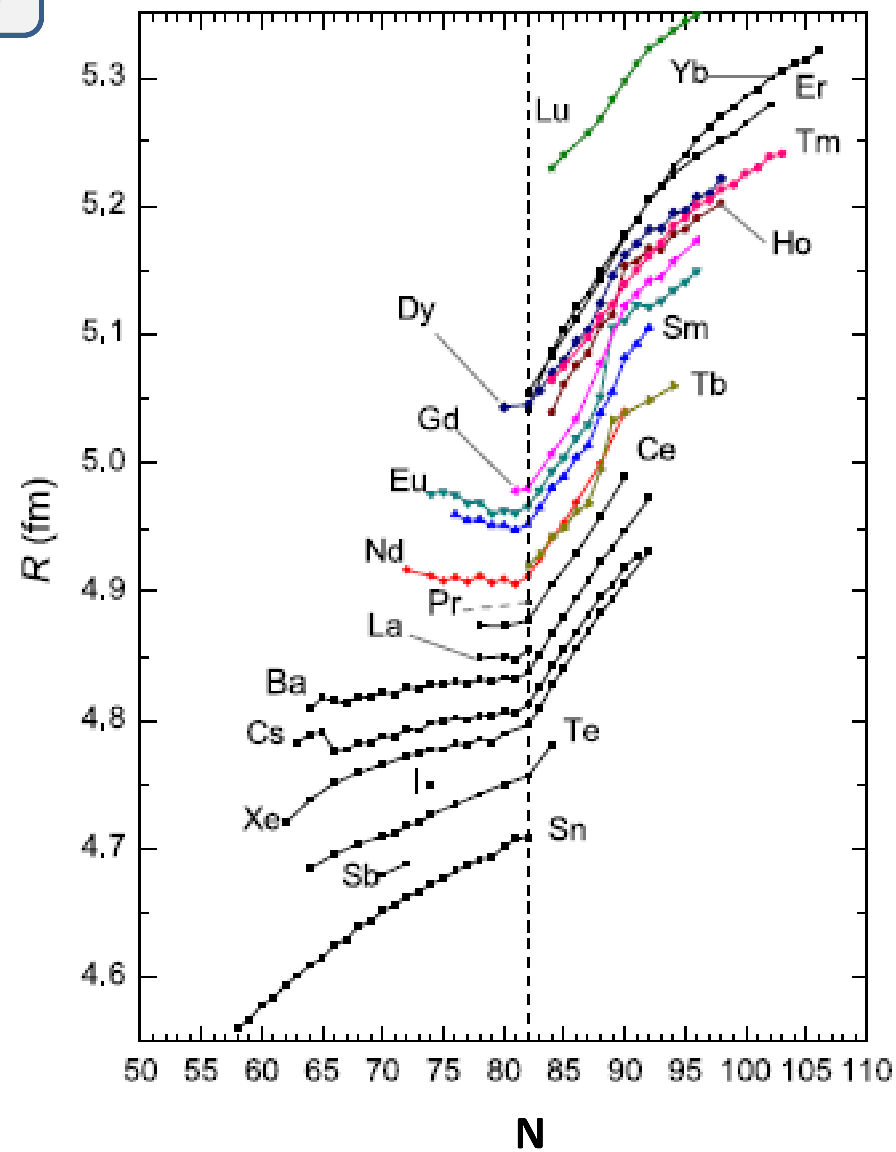
^{11}Li



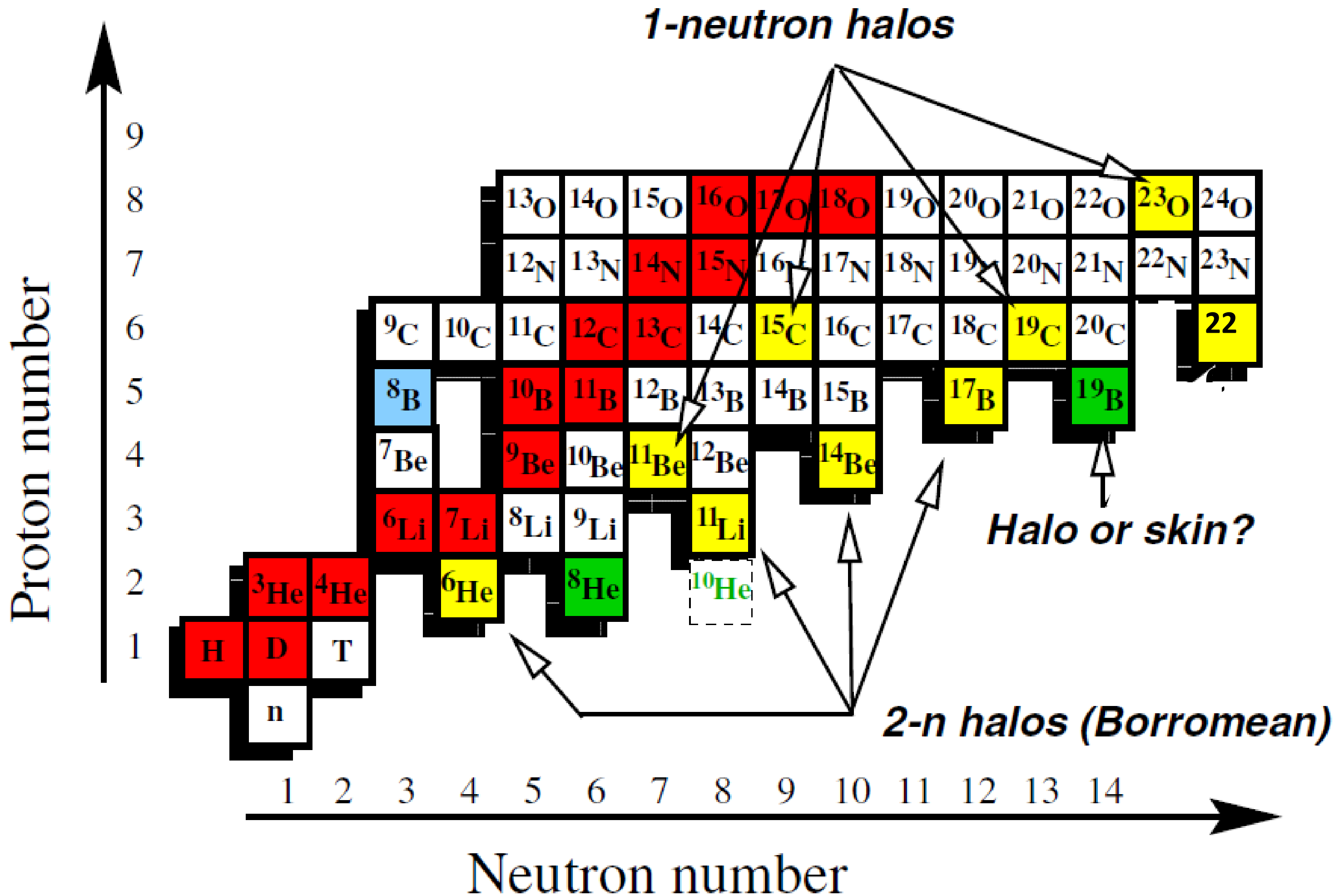
E_{11Li} = -378 keV

Charge radii





$$\langle r_{ch}^2 \rangle_A = \langle \Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A) | \frac{1}{Z} \sum_{i=1}^{Z} e_i \, (\mathbf{r}_i - \mathbf{R}_{c.m.})^2 | \Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A) \rangle$$



Unique factors for exotic nuclei

the weak binding, closeness of the particle continuum (a large diffuseness of the nuclear surface, extreme spatial dimensions for the outermost nucleons)

exotic combinations of proton and neutron numbers (prospects for completely ***new structural phenomena***)

Ab initio models

Understand the nuclear structure with realistic nuclear forces

Interactions:
NN + NNN + …

Many-Body Methods:
Green's Function Monte Carlo
No-Core Shell Model
Coupled Cluster
….

very complicated wave functions
How do they consistent with independent nucleon motion ?
(essence of the shell model)

continuum (reactions) ?

the shell models

effective interactions fitted to experimental data

Many-Body Methods:
mean-fields
(self-consistent)
+
residual interactions

shell structure evolution when nuclei are approaching driplines

Large-scale shell model calculations for medium and heavy nuclei

microscopic mean-field models and beyond

nuclear density functional theory
quasiparticle random-phase approximation
quasiparticle phonon model
…..

theoretical calculations of nuclear properties throughout the whole nuclear chart

CLUSTER MODELS

MICROSCOPIC cluster models: *exact* treatment of the antisymmetrization

Resonating Group Method (J.A. Wheeler, 1937)

Nucleons are separated into clusters and exchanged between them as if nucleons resonate between each group

$$\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A) = \mathcal{A}\{\,\Phi(\mathbf{r}_1, \cdots, \mathbf{r}_{A_C})\,\phi(\mathbf{r}_{A_C+1}, \cdots, \mathbf{r}_A)\,\psi(\mathbf{r})\,\}$$

$\mathcal{A}$ **is the antisymmetrization operator,**

$$\mathbf{r} = \frac{1}{A_C}\sum_{i=1,A_C}\mathbf{r}_i - \frac{1}{A-A_C}\sum_{i=A_C+1,A}\mathbf{r}_i$$

The variational principles is applied to the many-body hamiltonian. Equations of motion for $\psi(\mathbf{r})$ have non-local terms that represent exchanges of nucleons

Generator Coordinate Method (D.L. Hill and J.A. Wheeler, 1953)

Auxiliary parameters (generator coordinates $\alpha_1, \ldots, \alpha_N$) are introduced.

$$\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_\mathrm{A}) = \int d\alpha_1 \ldots d\alpha_N \, \Phi(\mathbf{r}_1, \ldots, \mathbf{r}_\mathrm{A}; \alpha_1, \ldots, \alpha_N) \, f(\alpha_1, \ldots, \alpha_N)$$

Slater determinant many-body wave functions

$$\Phi(\mathbf{r}_1, \ldots, \mathbf{r}_\mathrm{A}; \alpha_1, \ldots, \alpha_N)$$

are generated for parameter set $\alpha_1, \ldots, \alpha_N$

Total wave function is an integral over parameters
with the use of a weight function $f(\alpha_1, \ldots, \alpha_N)$

Variational principle leads to the Hill-Wheeler integral equation

$$\int d\alpha' \left\{ \mathrm{H}(\alpha; \alpha') - E \langle \Phi(\alpha) \mid \Phi(\alpha') \rangle \right\} f(\alpha') = 0$$

where the independent variables are generator coordinates
(not particle coordinates, all dynamical variables are already integrated out
in the integral kernels of the Hill-Wheeler equation)

In practice the radial wave function is expanded on a set of
displaced gaussian functions, centered at different locations

SEMI-MICROSCOPIC cluster models:
approximate **treatment of the antisymmetrization**

Orthogonality condition model (S. Saito, 1968)

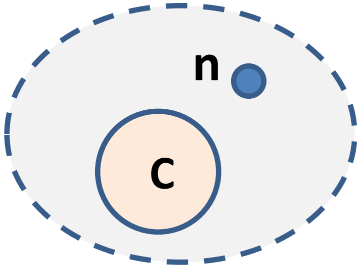
The inter-cluster wave function $\psi(\mathbf{r})$ is ORTHOGONAL to the Pauli forbidden states

The forbidden states are the null states if the complete antisymmetrization is performed. Interaction is taken place only in the (physical) space orthogonal to the forbidden states.

Effect of the complicated kernels in the RGM is well approximated by the orthogonality to the forbidden states (The exact forbidden states, however, appears only for cases with the equal oscillator parameters of shell model cluster)

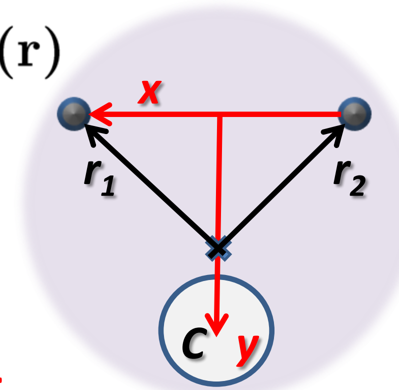
Cluster models, general formulation

wave function is factorized into a **sum of products from two parts**



Two-Body : $A = A_C + 1$

$$\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A) = \sum_i \phi_i(\mathbf{r}_1, \cdots, \mathbf{r}_{A_C})\, \psi_i(\mathbf{r})$$



Three-body : $A = A_C + 2$

$$\Psi(\mathbf{r}_1, \cdots, \mathbf{r}_A) = \sum_i \phi_i(\mathbf{r}_1, \cdots, \mathbf{r}_{A_C})\, \psi_i(\mathbf{x}, \mathbf{y})$$

The antisymmetrization operator $\mathcal{A}$ is **absent**

The sum includes core excitations :

$$H_{A_C}\, \phi_i(\mathbf{r}_1, \cdots, \mathbf{r}_{A_C}) = \epsilon_i\, \phi_i(\mathbf{r}_1, \cdots, \mathbf{r}_{A_C})$$

A coupling with excited core states involves **additional partial waxes.**
This allows to account for some emergent core degrees of freedom and get a more realistic description of nuclear properties.
(analogue to increasing the number of shells within the framework of shell-model approaches)

Total hamiltonian (two-body cluster model)

$$H_A = H_{A_C} + T_r + \sum_{i=1}^{A_C} V(r, r_i)$$

The Schrodinger equation

$$H_A \Psi(\mathrm{r}_1, \cdots, \mathrm{r}_A) = E\, \Psi(\mathrm{r}_1, \cdots, \mathrm{r}_A)$$

The wave function $\psi_i(\mathrm{r})$ is a solution of the coupled Schrodinger equations

$$(T_r + \epsilon_i - E)\, \psi_i(\mathrm{r}) + \sum_j V_{ij}(\mathrm{r})\, \psi_j(\mathrm{r}) = 0$$

matrix elements describe the core-nucleon binary interactions

$$V_{ij}(\mathrm{r}) = \langle \phi_i(\mathrm{r}_1, \cdots, \mathrm{r}_{A_C}) \mid \sum_{k=1}^{A_C} V(\mathrm{r}, \mathrm{r}_k) \mid \phi_j(\mathrm{r}_1, \cdots, \mathrm{r}_{A_C}) \rangle$$

These matrix elements define the dynamics of the system

Calculations of the bound states and continuum wave functions

The bound state wave function

$$\Psi_{JM}(\mathbf{r}_1,\cdots,\mathbf{r}_A) = \frac{1}{r}\sum_{\gamma} \chi_{\gamma}^{J}(r)\ \left[[Y_l(\hat{r})\otimes\chi_s]_j\otimes\phi_{nI}\right]_{JM}$$

$$\gamma = \{l, s, j, I, n\}$$

Set of coupled Schrodinger equations for radial wave functions

$$\left(-\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right]+\epsilon_{\gamma}-E\right)\chi_{\gamma}^{J}(r) = -\sum_{\gamma'} V_{\gamma,\gamma'}^{J}(r)\,\chi_{\gamma'}^{J}(r)$$

matrix elements

$$V_{\gamma,\gamma'}^{J}(r) = \langle\left[[Y_l(\hat{r})\otimes\chi_s]_j\otimes\phi_{nI}\right]_{JM}\mid\sum_{k=1}^{A_C}V(\mathbf{r},\mathbf{r}_k)\mid\left[[Y_{l'}(\hat{r})\otimes\chi_{s'}]_{j'}\otimes\phi_{n'I'}\right]_{JM}\rangle$$

define the dynamics of the system.

have to be specified for particular applications

The continuum wave function at the positive energy $E = \frac{\hbar^2 \mathbf{k}^2}{2\mu}$

$$\Psi_{\nu,nIM_I}{}^{(\pm)}(\mathbf{k};\mathbf{r}_1,\cdots,\mathbf{r}_A) = \sum_{ljJm_lm_jM_J} (lm_l\, s\nu \mid jm_j)\,(jm_j\, IM_I \mid JM_J)\, Y^*_{lm_l}(\hat{k})\times$$
$$\times\, \frac{1}{r}\sum_{\gamma'} \chi^J_{\gamma',\gamma}(k,\, r)\, \imath^{l'} \left[[Y_{l'}(\hat{r})\otimes\chi_s]_{j'}\otimes\phi_{n',I'}\right]_{JM_J}$$

radial wave functions in open channels at *r* → ∞

$$\chi^J_{\gamma',\gamma}(k,\, r) = \frac{\imath}{\sqrt{2\pi}}\,\frac{1}{\sqrt{k_{\gamma'}\, k_\gamma}}\,\left(H^{(-)}_{l_\gamma}(k_\gamma r)\,\delta_{\gamma',\gamma} - H^{(+)}_{l_{\gamma'}}(k_{\gamma'} r)\, S_{\gamma',\gamma}\right).$$

$$k_\gamma = \sqrt{(2\mu/\hbar^2)\ \mid E - \epsilon_\gamma \mid}$$

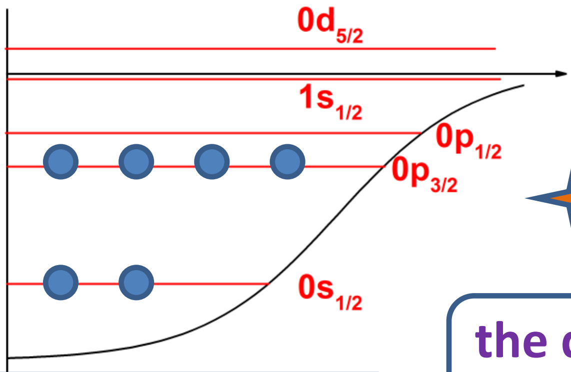
elastic and inelastic cross sections for a nucleon scattered on the core nucleus

$$\sigma_{el}(E) = \frac{\pi}{k^2}\,\frac{1}{(2s+1)(2I_i+1)}\sum_f (2J_f+1)\mid 1 - S_{i,f}\mid^2$$

$$\sigma_{in}(E) = \frac{\pi}{k^2}\,\frac{1}{(2s+1)(2I_i+1)}\sum_f (2J_f+1)\ \mid S_{i,f}\mid^2$$

The $^{11}_{4}\mathrm{Be}_7$ nucleus $\left(J^\pi = \frac{1}{2}^-, \quad E_{gr} = -502\,\mathrm{keV}\right)$

best known one-neutron halo nucleus seen as a ^{10}Be core + loosely bound neutron



the Pauli principle treatment

the deep core-neutron potential (popular approach) permits forbidden orbits

the deeply bound states are obtained as the lowest levels (forbidden orbits dominate in the structure of these states)

simply neglected

attractive features: ⇒ potentials are more universal for treating adjacent systems, similar to the ones used for calculations in usual shell models.

disadvantages ⇒ additional efforts have to be applied to exclude forbidden states from dynamics in three-body models

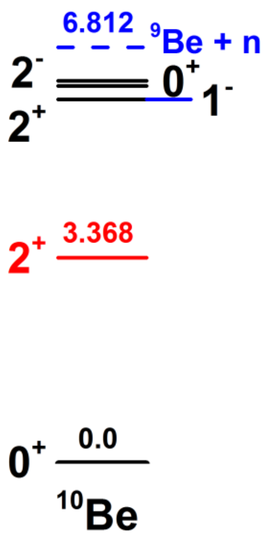
the shallow core-neutron potential does not have forbidden orbits

attractive features: ⇒ Application in three-body models does not lead to additional difficulties

disadvantages ⇓ are less universal

Description of the ^{10}Be core states

include the ground 0^+ and first excited 2^+ states



the 2^+ state is considered as a quadrupole (*I = 2*) vibration built on the *deformed axially symmetric* ground state

$$\phi_{n,IM_I} = \sqrt{\frac{2I+1}{8\pi^2}}\, D^{I*}_{M_I,0}(\omega) \mid n, I\rangle$$

The vibration operators are defined in the intrinsic coordinate system specified by the core symmetry axis $\hat{z}_C = (\theta_C, \phi_C)$ $\left(\omega = (\phi_C, \theta_C, \psi)\right)$

$$\mid n = 1, I\rangle = c^\dagger_{I0} \mid 0, 0\rangle \qquad [c_{I'0}, c^\dagger_{I0}] = \delta_{I',I}$$

The ^{10}Be core-neutron potential: non-diagonal in core states

$$V_{\Delta n} = V_I(r)\sqrt{\frac{4\pi}{2I+1}}\left(Y_I(\hat{r}) \cdot Y_I(\hat{z}_C)\right)\left(c^\dagger_{I0} + c_{I0}\right)$$

$V_I(r)$ is a surface potential (Bohr - Mottelson)

The ^{10}Be core-neutron potential: a diagonal in core states

$$V_0 = V_c(\mathrm{r}, \hat{z}_C) + V_{so}(r)\,(\mathrm{l}\cdot\mathrm{s})$$

The deformed potential $V_c(\mathrm{r}, \hat{z}_C)$ has the *axially symmetric shape* described by the surface with the quadrupole deformation β_2

$$R(\hat{z}_C) = \frac{R_0}{R_\beta}\left(1 + \beta_2\, Y_{20}\,(\hat{z}_C)\right)\;;\; R_\beta^3 = \frac{1}{4\pi}\int d\hat{z}_C\,\left(1 + \beta_2\, Y_{20}\,(\hat{z}_C)\right)^3$$

potential can be decomposed over multipoles as

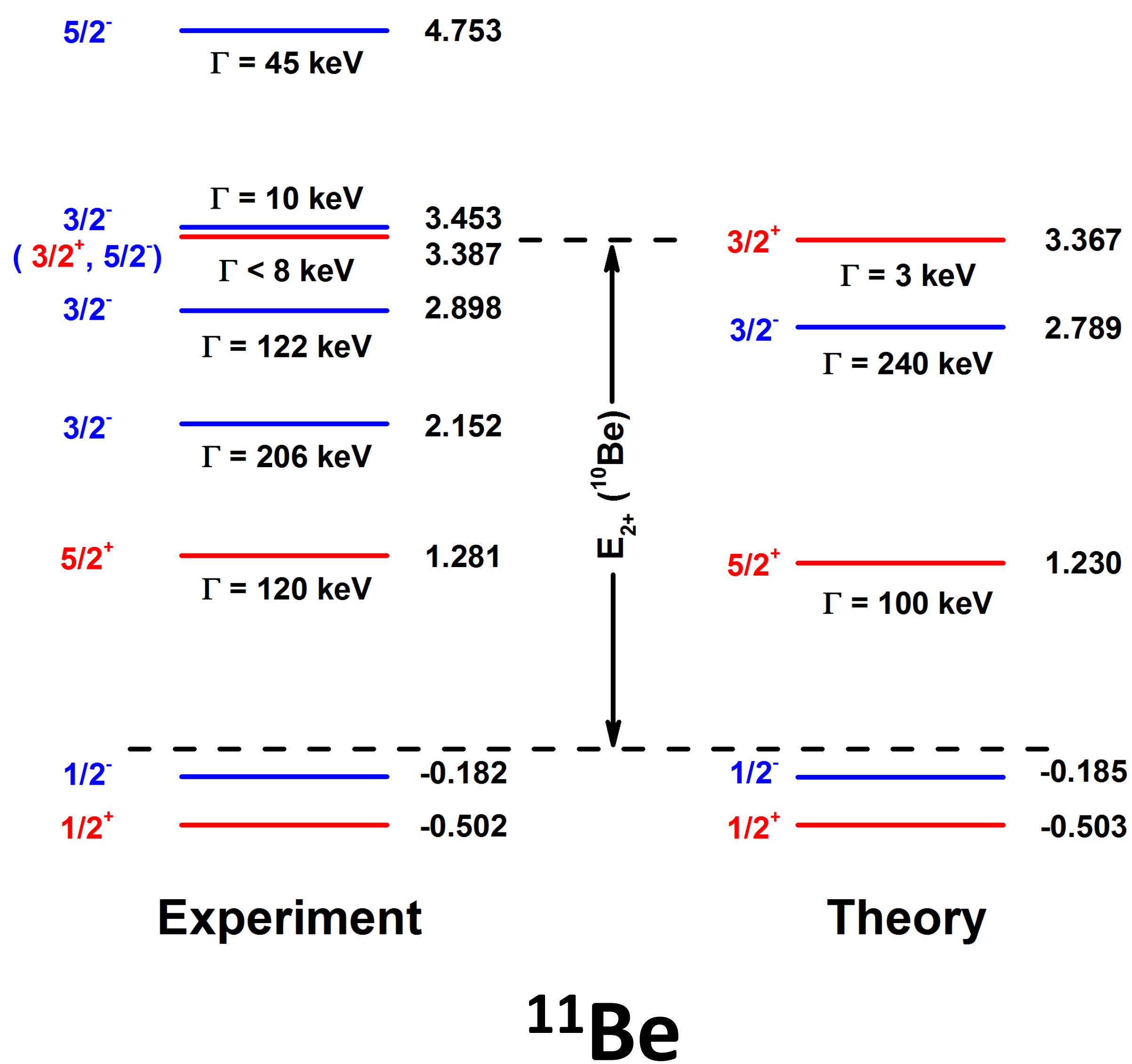
$$V_c(\mathrm{r}, \hat{z}_C) = \sum_L V_L(r)\,\frac{4\pi}{2L+1}\left(Y_L(\hat{r})\cdot Y_L(\hat{z}_C)\right)$$

$$V_L(r) = \frac{2L+1}{2}\int_{-1}^{1} d(\cos\theta)\, V_c(r, \cos\theta)\, P_L(\cos\theta)$$

Alternative description of the ^{10}Be core states

rotational model :

$$\phi_{n,IM_I} = \sqrt{\frac{2I+1}{8\pi^2}}\, D^{I*}_{M_I,0}(\omega)$$



5/2-
4.753
Γ = 45 keV
Γ = 10 keV
3/2-
3.453
(3/2+, 5/2-)
3.387
Γ < 8 keV
3/2-
2.898
Γ = 122 keV
3/2-
2.152
Γ = 206 keV
5/2+
1.281
Γ = 120 keV
1/2-
-0.182
1/2+
-0.502
E2+ (10Be)
3/2+
3.367
Γ = 3 keV
3/2-
2.789
Γ = 240 keV
5/2+
1.230
Γ = 100 keV
1/2-
-0.185
1/2+
-0.503
Experiment
Theory
11Be

The ^{11}Be 1/2$^+$ ground state properties:

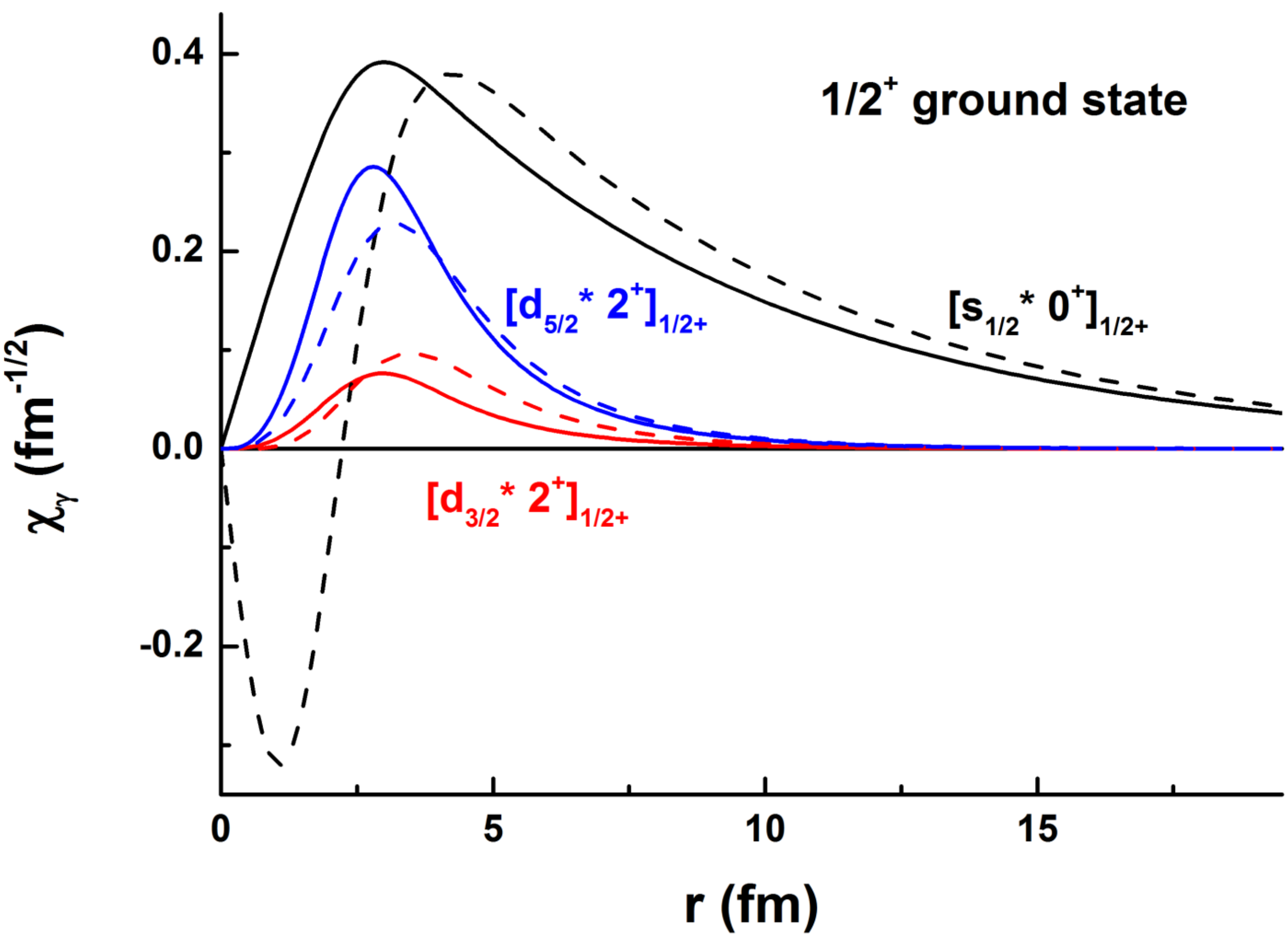
r_m - the r.m.s. matter radius

r_{C-n} - the r.m.s. distance of the halo neutron to the core c.m.

r_{ch} - the r.m.s. charge radius

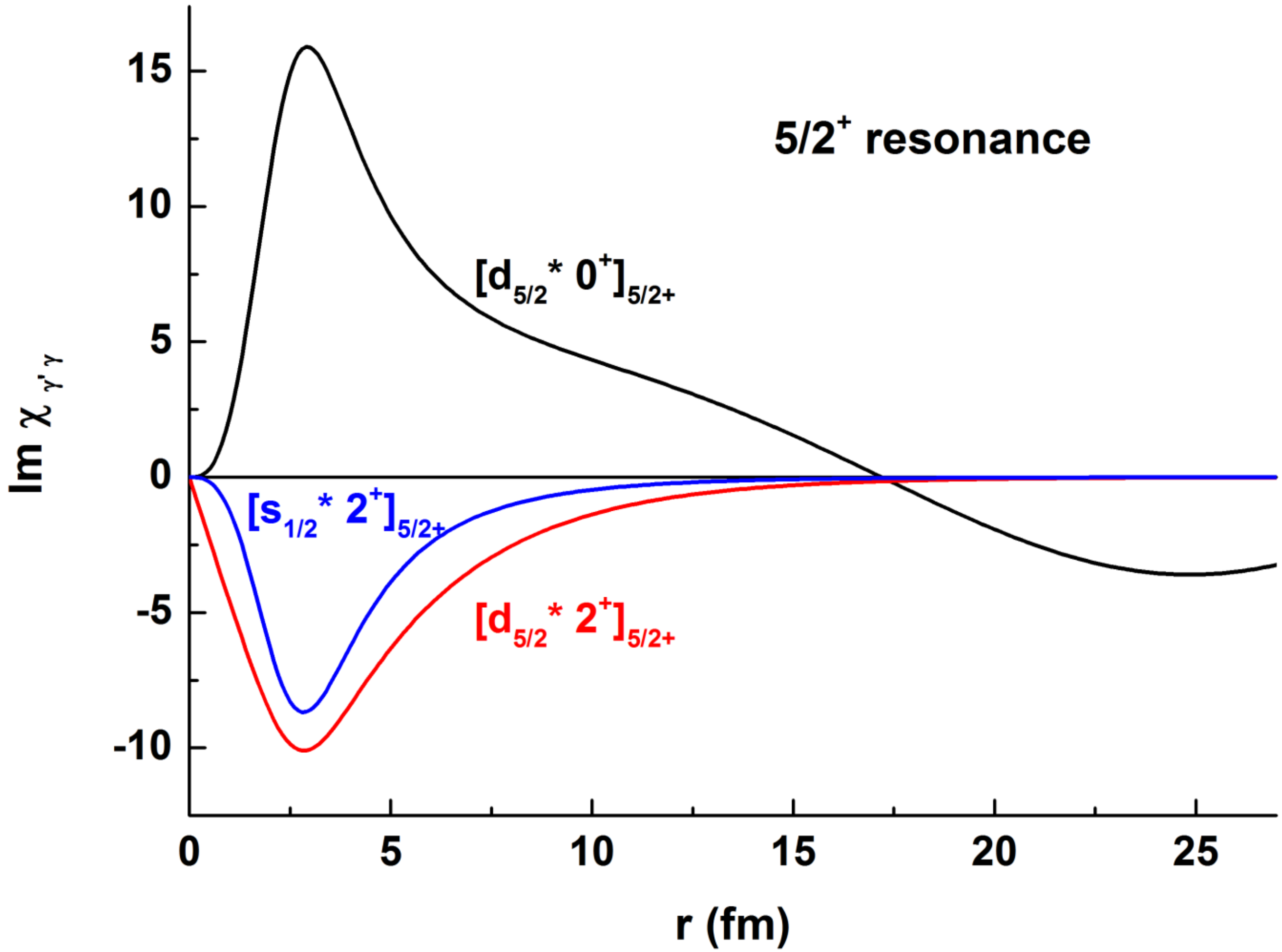
	r_m, fm	r_{C-n}, fm	r_{ch}, fm	$[s_{1/2} \otimes 0^+]$, %	$[d_{5/2} \otimes 2^+]$, %
Theory	2.76	5.84	2.42	80	18
(1)	2.93			78	20
(2)	2.91			84	13
Exp	2.73±0.05 (3)	5.77±0.17 (4)	2.466±0.016 (5)	72±4 (4)	
(6)		5.7±0.4		61±5	
(7)				84	16
(8)				71±5	

(1) F.M. Nunes *et al.*, Nucl. Phys. A 596 (1996) 271
(2) F.M. Nunes *et al.*, Nucl. Phys. A 609 (1996) 43
(3) I. Tanihata *et al.*, Phys. Lett. B 206 (1988) 592
(4) N. Fukuda *et al.*, Phys. Rev. C70 (2004) 054606
(5) A. Krieger *et al.*, Phys. Rev. Lett. 108 (2012) 142501
(6) R. Palit *et al.*, Phys. Rev. C68 (2003) 034318
(7) J.S. Winfield *et al.*, Nucl. Phys. A683 (2001) 48
(8) K.T. Schmitt *et al.*, Phys. Rev. Lett. 108 (2012) 192701

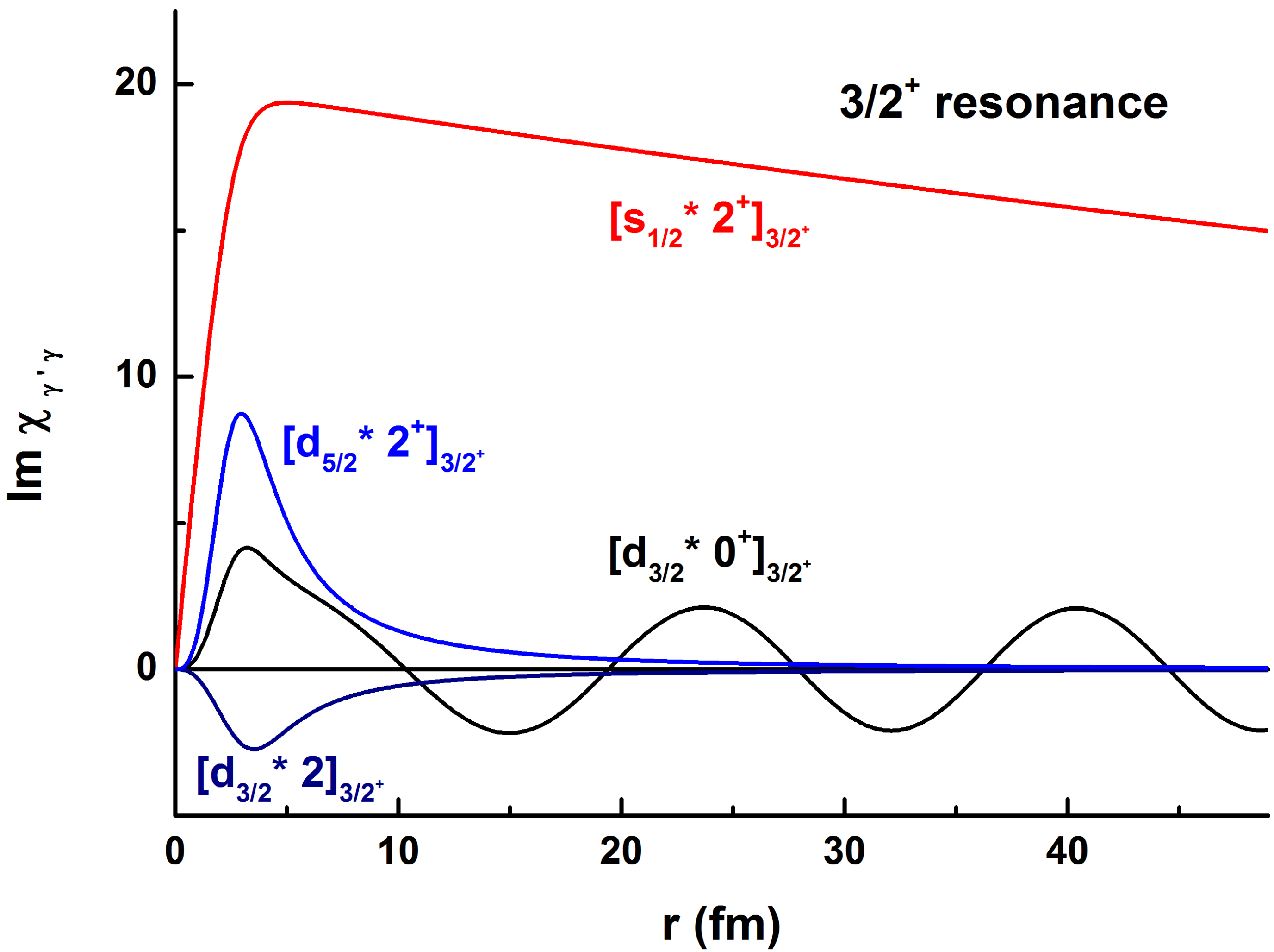


solid lines ---> "shallow" potentials

dash lines ---> " deep" potentials (F.M. Nunes *et al.,* Nucl. Phys. A 596 (1996) 271)



5/2+ resonance
[d5/2* 0+]5/2+
[s1/2* 2+]5/2+
[d5/2* 2+]5/2+
Im χ γ'γ
r (fm)
15
10
5
0
-5
-10
0
5
10
15
20
25



$$E_{3/2^+} - E_{2^+}(^{10}Be) \sim 20\,\mathrm{keV}$$

$$\Gamma < 8\,\mathrm{keV}$$

The ^{11}Be $1/2^-$ bound excited state

The description of the $1/2^-$ bound excited state has to reproduce not only the energy but simultaneously the strength for dipole transition between the bound excited and ground states.

An interesting and seldom example where we can notice the difference in applications of the models with deep and shallow potentials

	r_m, fm	r_{C-n}, fm	$[p_{1/2} \otimes 0^+]$, %	$[p_{3/2} \otimes 2^+]$, %	$B(E1, 1/2^- \to 1/2^+)(\mathrm{e^2fm^2})$
Theory	2.58	4.73	34	65	0.152
(1)	2.77	5.89	87	11	0.150
Exp (2)					0.115±0.010

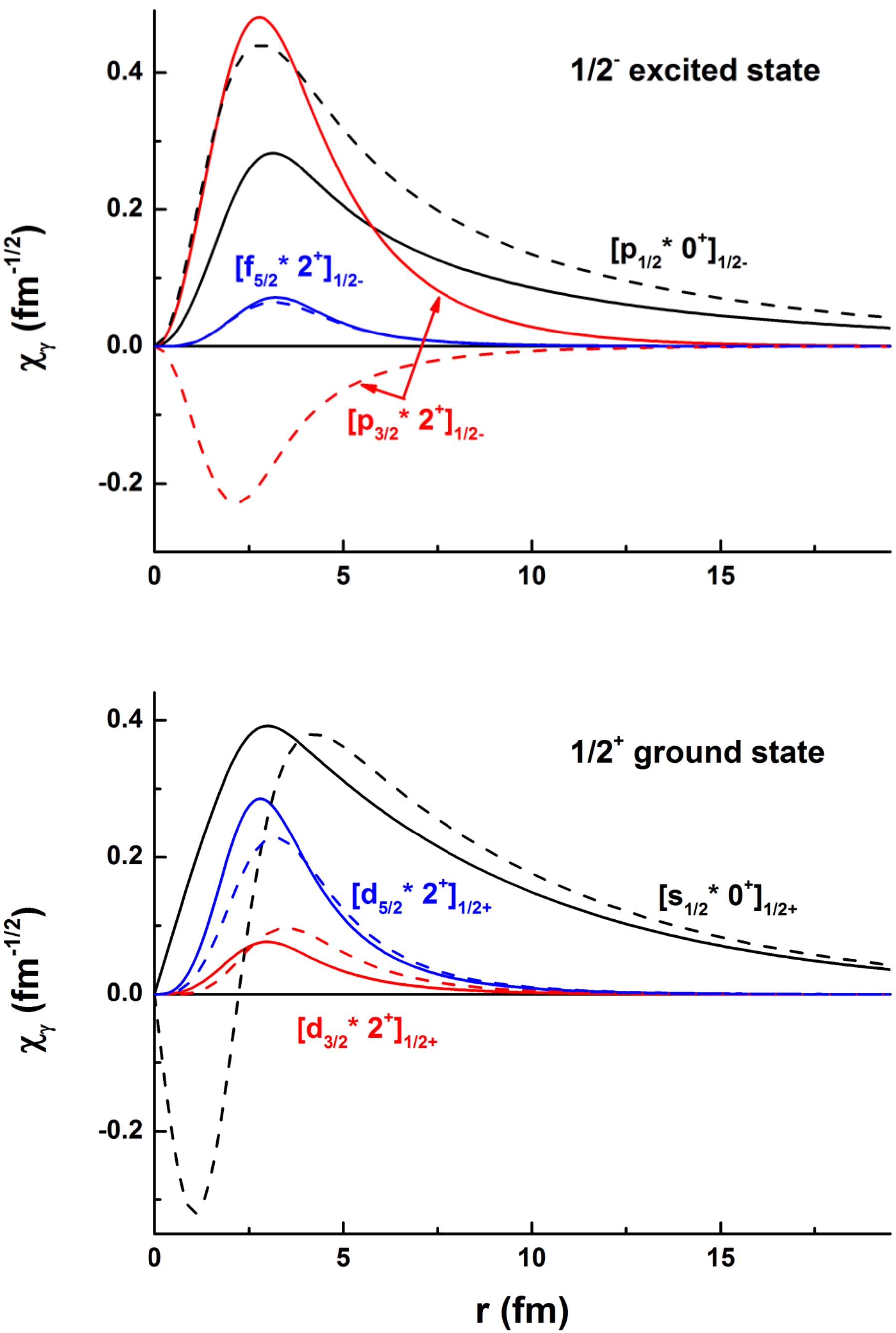
r_m - the r.m.s. matter radius

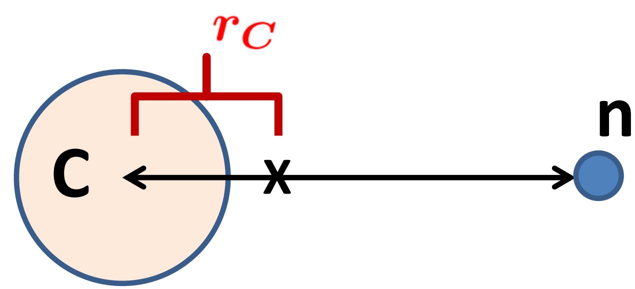
r_{C-n} - the r.m.s. distance of the halo neutron to the core c.m.

r_{ch} - the r.m.s. charge radius

(1) F.M. Nunes *et al.,* Nucl. Phys. A 596 (1996) 271

(2) D.J. Millener *et al.,* Phys. Rev. C28 (1983) 497





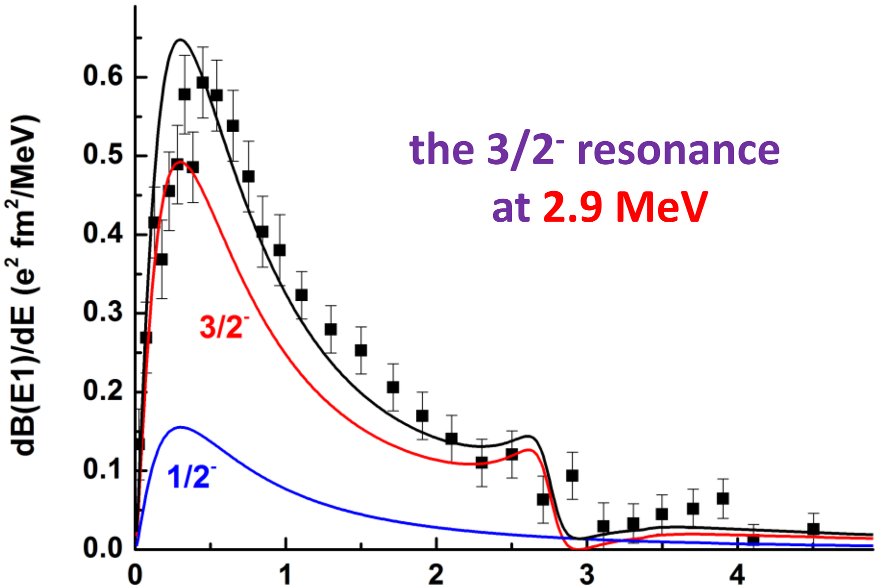
$$B(E1) = \frac{(eZ)^2}{2J_i+1} \sum_{M_i, M_f, \mu} \times \mid \langle \psi_{J_f M_f}(\mathbf{r}) \mid r_C Y_{1\mu}(\hat{\mathbf{r}}_C) \mid \psi_{J_i M_i}(\mathbf{r}) \rangle \mid^2$$

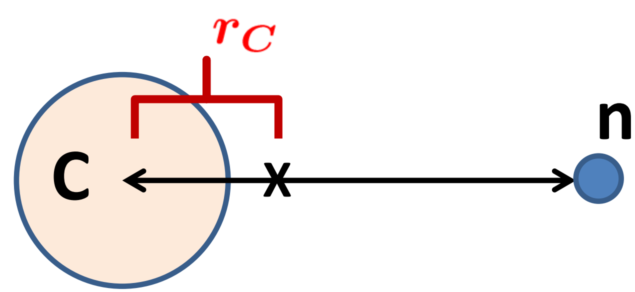
solid lines ---> “shallow” potentials

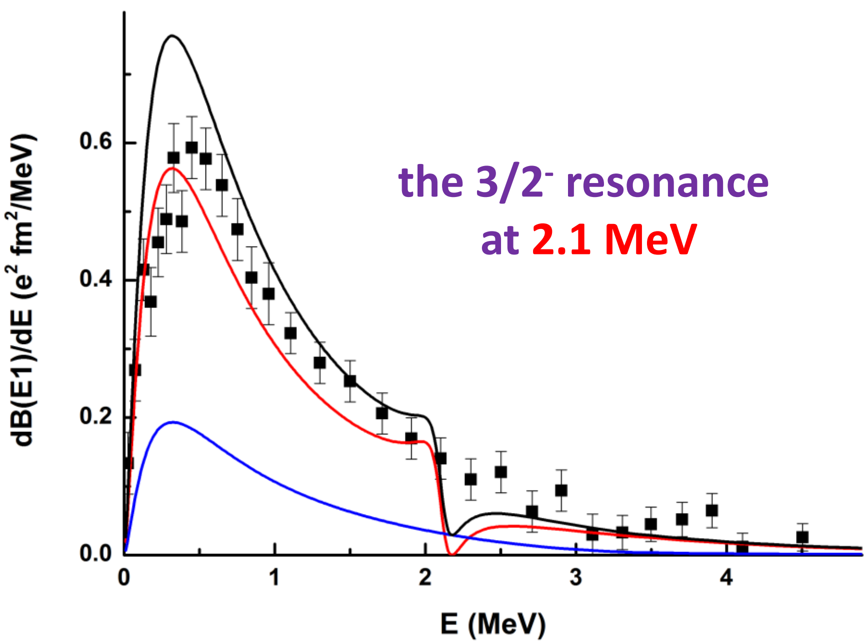
dash lines ---> “ deep” potentials

(F.M. Nunes *et al.*,
NPA 596 (1996) 271)

Dipole excitations from ground to continuum states







$$\frac{dB(E1)}{dE} = \frac{1}{2}\left(\frac{2\mu}{\hbar^2}\right)^{\frac{3}{2}} \sqrt{E}\ \frac{(eZ)^2}{2J+1} \sum_{\nu,M_I,M,\lambda}$$

$$\times \mid \langle \psi^{(-)}_{\nu,IM_I}(\mathbf{k};\mathbf{r}) \mid r_C Y_{1\lambda}(\hat{\mathbf{r}}_C) \mid \psi_{JM}(\mathbf{r}) \rangle \mid^2$$

exp. data : R. Palit *et al.*,
Phys. Rev. C68 (2003) 034318

Conclusion

Clustering is a very widespread phenomenon in light nuclei. The few-body cluster models present a natural and most transparent way to describe specific features of nuclear structure specified by the cluster degrees of freedom.

In many nuclei the cluster degrees of freedom are most important and define the basic properties of the nuclear systems. Still in many circumstances the core can not be considered as inert system and additional degrees of freedom connected to excited core states have to be taken into account. This leads to extension of few-body cluster models and increases the applicability of a cluster approach.

Two-body cluster model of the ^{11}Be nucleus with shallow core-neutron potentials describes well the experimental data concerning the bound state properties and the low-lying spectrum of continuum excitations.