Muon g-2: status and perspectives

Progress in our understanding and misunderstanding

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Introduction: What is g=2? What is g-2?

Introduction: History of Lepton Anomalies.

Theory vs Experiment

Muon g-2 (SM test)

Conclusions

Introduction

Cosmology tell us that 95% of matter is not described in text-books yet

Two search strategies:
1) High energy physics to excite heavy degrees of freedom.
No any evidence till now. We live in LHC era! (already disappointment)

2) Low energy physics to produce Rare processes in view of huge statistics.

There are some rough edges of SM.

$(g-2)_\mu$ is very famous example

$\pi_0 \rightarrow e^+e^-$ is in the list of SM test after recent experimental and theoretical progress

That's intriguing

I. History

Ein Weg zur experimentellen Prüfung der Richtungsquantelung im Magnetfeld.

Von Otto Stern in Frankfurt a. Main.

Mit zwei Abbildungen. — (Eingegangen am 26. August 1921.)

In der Quantentheorie des Magnetismus und des Zeemaneffektes wird angenommen, daß der Vektor des Impulsmomentes eines Atoms nur ganz bestimmte diskrete Winkel mit der Richtung der magnetischen Feldstärke $\mathfrak{H}$ bilden kann, derart, daß die Komponente des Impulsmomentes in Richtung von $\mathfrak{H}$ ein ganzzahliges Vielfaches von $h/2\pi$ ist[1]). Bringen wir also ein Gas aus Atomen, bei denen das

$$\mathfrak{K} = \mathfrak{m}_x \frac{\partial \mathfrak{H}}{\partial x} + \mathfrak{m}_y \frac{\partial \mathfrak{H}}{\partial y} + \mathfrak{m}_z \frac{\partial \mathfrak{H}}{\partial z}.$$

Nun führt das Atom eine gleichförmige Rotation um die Feldrichtung, d. h. um die z-Achse aus[1]), wobei $\mathfrak{m}_z$ konstant bleibt, während der Mittelwert von $\mathfrak{m}_x$ und $\mathfrak{m}_y$ über einen vollen Umlauf Null wird. Mitteln wir also bei konstantem $\frac{\partial \mathfrak{H}}{\partial x}, \frac{\partial \mathfrak{H}}{\partial y}, \frac{\partial \mathfrak{H}}{\partial z}$ über eine gegen die Umlaufdauer (die z. B. für $\mathfrak{H} = 1000$ Gauß 7.10^{-10} sec ist) große Zeit, so wird die mittlere auf das Atom wirkende Kraft:

$$\bar{\mathfrak{K}} = m_z \frac{\partial \mathfrak{H}}{\partial z}.$$

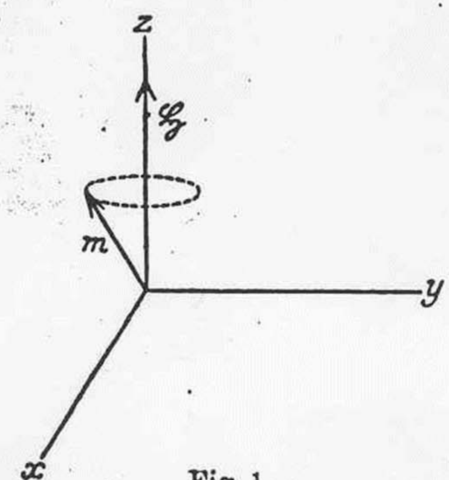


Fig. 1.

Für die auf das Atom wirkende Kraft ist also beim magnetischen Moment nur die Komponente in Richtung des Feldes selbst maßgebend, also

Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld.

Von Walther Gerlach in Frankfurt a. M. und Otto Stern in Rostock.

Mit sieben Abbildungen. (Eingegangen am 1. März 1922.)

Vor kurzem[1]) wurde in dieser Zeitschrift eine Möglichkeit angegeben, die Frage der Richtungsquantelung im Magnetfeld experimentell zu entscheiden. In einer zweiten Mitteilung[2]) wurde gezeigt, daß das normale Silberatom ein magnetisches Moment hat. Durch die Fortsetzung dieser Untersuchungen, über die wir uns im folgenden zu berichten erlauben, wurde die Richtungsquantelung im Magnetfeld als Tatsache erwiesen.

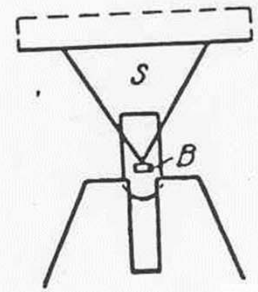
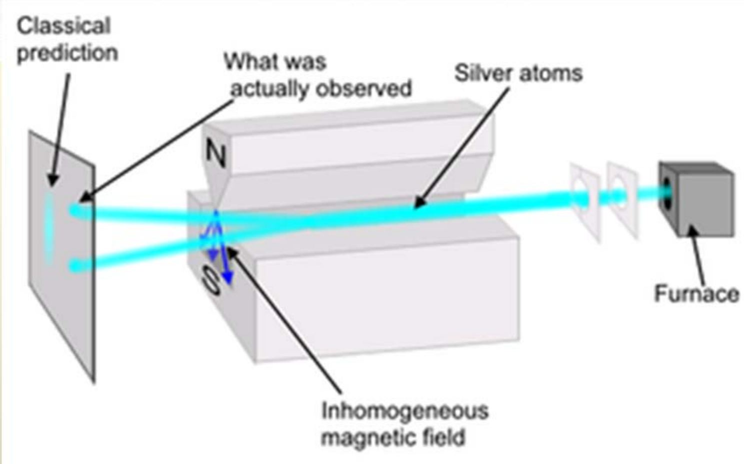


Fig. 1.



Basic of Quantum Theory – Quantum mechanics

1924. № 10.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 74.

1. *Über die Richtungsquantelung im Magnetfeld*[1]; *von Walther Gerlach und Otto Stern.*

(Hierzu Tafel III.)

Nr. der Aufnahme	Entfernung des unabgelenkten Strahles von der Schneide	Mittlere Ablenkung des abgestoßenen Strahles	
		berechnet	beobachtet
15	0,32 mm	$0{,}10_1$ mm	$0{,}10_3$ mm
14	0,21 mm	$0{,}14_6$ mm	0,15 mm

Die Genauigkeit der Messungen schätzen wir auf 10 Proz. Innerhalb dieser Fehlergrenzen zeigen also die Versuche, daß das Silberatom im Normalzustand ein Bohrsches Magneton hat.

$\Rightarrow$ $g = 2$

(in modern language)

§ 9. Ergebnis.

Die im vorstehenden mitgeteilten Versuche erbringen

1. den experimentellen Nachweis der Debye-Sommerfeldschen magnetischen Richtungsquantelung
2. die experimentelle Bestimmung des Bohrschen Magnetons.

Schließlich möchten wir dem Institutsmechanikermeister Hrn. Adolf Schmidt für seine unermüdliche und verständnisvolle Hilfe unseren aufrichtigen Dank sagen.

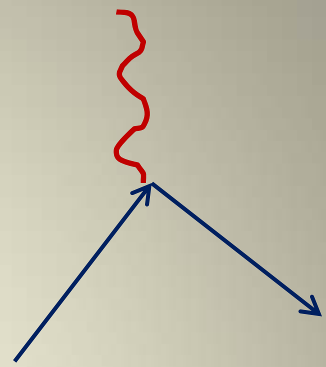
Frankfurt a. M. und Hamburg, 1923.

Dirac Equation for Spin ½ point-like Fermion

$$\left[\gamma_\mu\left(i\partial_\mu - eA_\mu\right) + m\right]\psi = 0,$$

$$\left[\left(i\gamma_\mu\partial_\mu - e\gamma_\mu A_\mu\right)^2 - m^2\right]\psi$$

$$= \left[\left(i\partial_\mu - eA_\mu\right)^2 - \frac{e}{2}\sigma_{\mu\nu}F_{\mu\nu} - m^2\right]\psi = 0,$$



where $\sigma_{\mu\nu} = \frac{i}{2}\left(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu\right)$

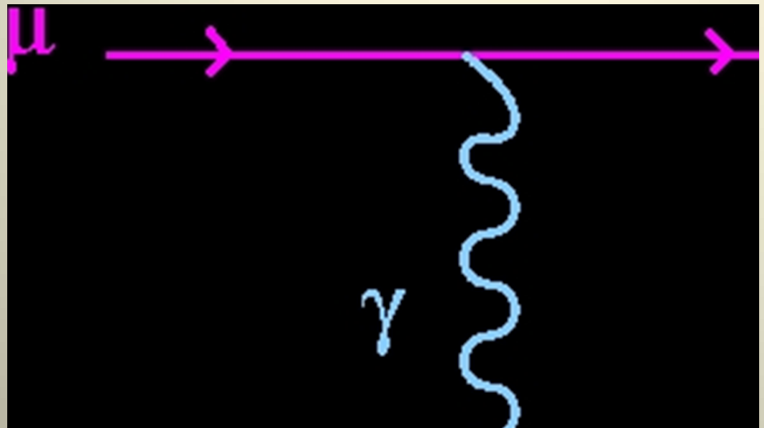
$$-\frac{e}{2}\sigma_{\mu\nu}F_{\mu\nu} = -e\left(\sigma_{0i}F_{0i} + \sigma_{ij}F_{ij}\right)$$

$$= -g\left(\frac{e}{2}\right)\left(i\gamma_0\gamma_i E_i + \sigma_i B_i\right)$$

$$\boldsymbol{g = 2}$$

Dirac Equation Predicts for point-like spin ½ charged particle: *g=2, g-2=0*

$$i\hbar\frac{\partial\psi}{\partial t} = [\frac{p^2}{2m} - \frac{e}{2m}(\vec{L}+2\vec{S})\cdot\vec{B}]\psi$$



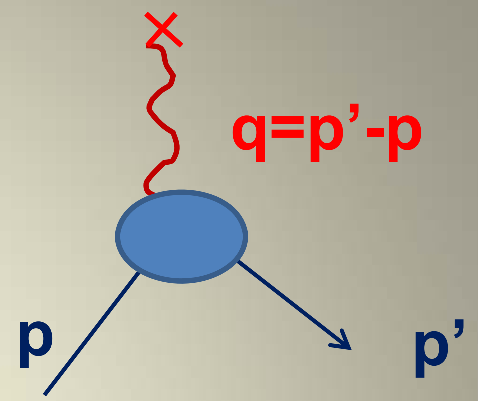
The general form of the ffγ vertex is

$$-ie\bar{u}(p')\left\{\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu}\frac{q_\nu}{2m_l}F_2(q^2) + \gamma_5\sigma_{\mu\nu}\frac{q_\nu}{2m_l}F_3(q^2)\right\}u(p)e_\mu(q)$$

- ***F_1* is the electric charge distribution $e_l = eF_1(0)$**
- ***F_2* corresponds to Anomalous Magnetic Moment (AMM) $a_l = (g_l - 2)/2 = F_2(0)$**
- ***F_3* corresponds to Anomalous Electric Dipole Moment $d_l = -e_l/(2m_l)F_3(0)$**

***$d_l = 0$* due to *T*- and *P* symmetries**

For the spin ½ fermion with structure



$$V(x) = \bar{u}(p')\Gamma_\mu(p,p')u(p)A_\mu(x)$$

$$\begin{cases} \gamma_\mu \rightarrow \Gamma_\mu(p,p') = \gamma_\mu F_1(q^2) + \dfrac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2) \\ A_\mu(x) = (0, A_i(x)) \end{cases}$$

$$\bar{u}(p')\left(\gamma_i F_1(q^2) + \frac{i\sigma^{i\nu}q_\nu}{2m}F_2(q^2)\right)u(p) \underset{q\to 0}{\approx} 2m\xi'^\dagger\left(\frac{-i}{2m}\varepsilon^{ijk}q^j\sigma^k\left[F_1(0)+F_2(0)\right]\xi\right)$$

$$V(x) = -\langle\boldsymbol{\mu}\rangle\mathbf{B}(x)$$

$$\langle\boldsymbol{\mu}\rangle = \frac{e}{m}\left[F_1(0)+F_2(0)\right]\xi'^\dagger\frac{\boldsymbol{\sigma}}{2}\xi$$

$$\boldsymbol{\mu} = g\left(\frac{e}{2m}\right)\mathbf{S},$$

$$g = 2\left[F_1(0)+F_2(0)\right] = 2 + 2F_2(0),$$

$$\mathbf{a} \equiv F_2(0) = \frac{\mathbf{g-2}}{\mathbf{2}}$$

a is not zero due to

a) Bound state effects

b) ***Radiative Corrections*** in SM

Some Definitions

A charged particle with spin S *has a magnetic moment* μ

$$L_I = \vec{\mu}_S \vec{B},$$

$$\vec{\mu}_S = g_S \left(\frac{e}{2m} \right) \vec{s},$$

$$a = \frac{g_S - 2}{2}, \quad \mu = (1 + \mathrm{a}) \frac{eh}{2m}$$

Gyromagnetic ratio

PDG

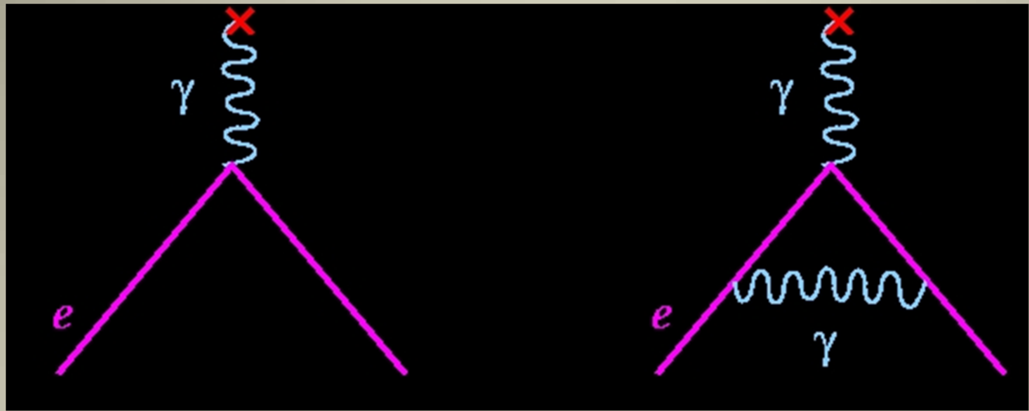
Anomaly

m_μ **=105.6583692(94) MeV,**

m_τ *= 1776.99 (29) MeV*

m_μ / m_e **= 206.768 2838(54)**

The lowest order radiative correction (



$$\Gamma_\mu = e\gamma_\mu + a_\ell \frac{ie}{2m}\sigma_{\mu\nu}q_\nu$$

$a_l = (g_l - 2)/2$

$$a = \frac{\alpha}{2\pi} = 0.001161$$

Schwinger, 1948
(Nobel prize 1965)

The Magnetic Moment of the Electron†

P. Kusch and H. M. Foley
Department of Physics, Columbia University, New York, New York

(Received April 19, 1948)

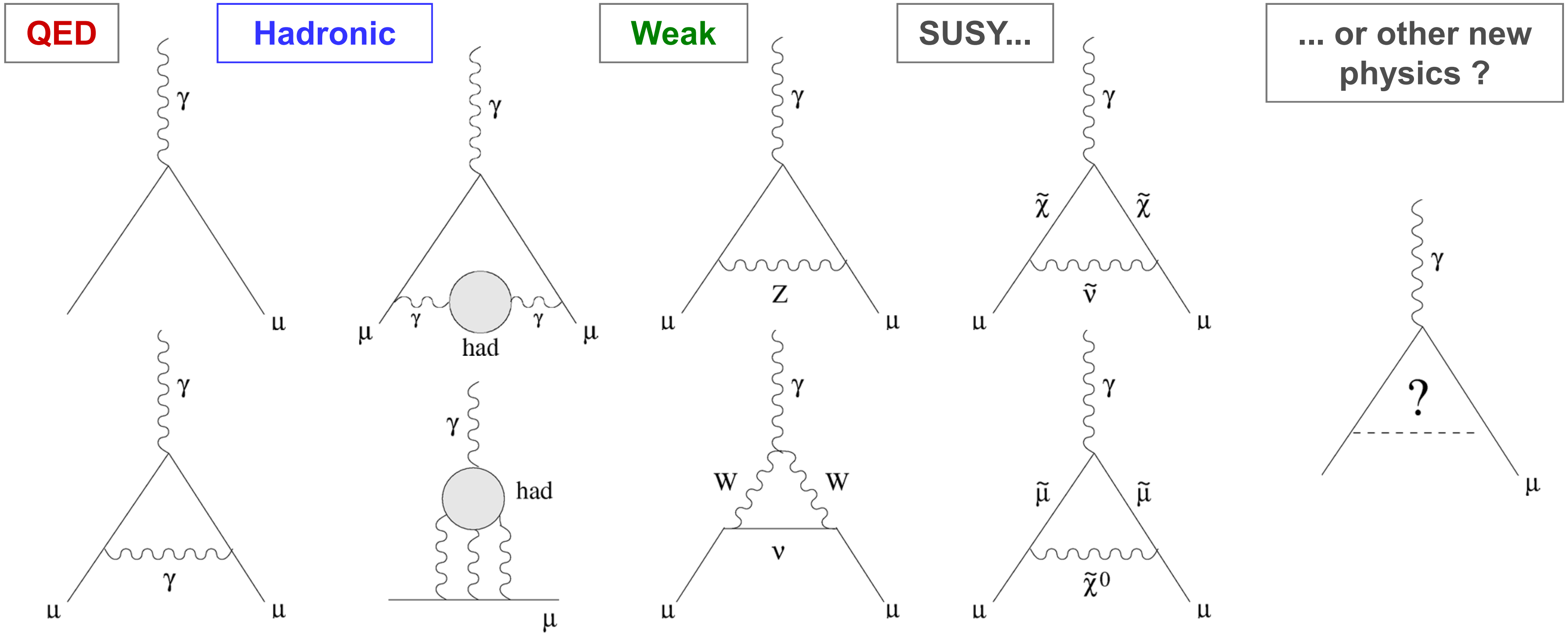
A comparison of the g_J values of Ga in the $^2P_{3/2}$ and $^2P_{\frac{1}{2}}$ states, In in the $^2P_{\frac{1}{2}}$ state, and Na in the $^2S_{\frac{1}{2}}$ state has been made by a measurement of the frequencies of lines in the *hfs* spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. Except for small residual effects, the results can be described by the statement that $g_L = 1$ and $g_S = 2(1.00119 \pm 0.00005)$. The possibility that the observed effects may be explained by perturbations is precluded by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

Theory ***meets*** *Experiment*

$$a = \frac{\alpha}{2\pi} = 0.00116$$

Basic of Quantum Field Theory – Quantum Electrodynamics

Magnetic Anomaly



Basic of Standard Model

Lepton Anomalies

- Electron anomaly is measured extremely accurately. ***QED test.***
- It is the best for determining α

Electron AMM

To measurable level a_e arises entirely from virtual electrons and photons

$$a_e^{exp} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12}\ [0.24\ \text{ppb}]$$ Harvard 2008

$$a_e^{SM} = a_e(QED) + a_e(hadron) + a_e(weak),$$

$$a_e(QED) = \sum_{n=1}^{5} C_{2n}\left(\frac{\alpha}{\pi}\right)^n + \ldots$$

The theoretical error is dominated by the uncertainty in the input value of the QED coupling $\alpha \equiv e^2/(4\pi)$

$$\alpha^{-1} = 137.035\,999\,084(51)\ [0.37\ \text{ppb}]$$

Das ist fantastisch!

QED is at the level of the best theory ever built to describe nature

Lepton Anomalies

- Electron anomaly is measured extremely accurately. ***QED test.***
- It is the best for determining α
- For a lepton L, Mass Scale contributes to a_L as $\left(m_L^2 / \Lambda^2\right)$
- Tau anomaly is difficult to measure since its fast decay

Tau anomaly

•Tau due to its highest mass is the best for searching for New Physics,

•But Tau is short living particle, so the precession method is not perspective

•The best existing limits (see S. Eidelman, M. Passera 07)

$$-0.052 < a_\tau^{Exp} < 0.013$$

are obtained at OPAL, L3 and DELPHI (LEP, CERN) from the high energy process

$$e^+e^- \to e^+e^- \tau^+\tau^- ,$$

•While the SM estimate is

$$a_\tau^{SM} = 1.17721(5)\ 10^{-3}$$

Lepton Anomalies

- Electron anomaly is measured extremely accurately. ***QED test.***
- It is the best for determining α
- For a lepton L, Mass Scale contributes to a_L as $\left(m_L^2 / \Lambda^2\right)$
- Tau anomaly is difficult to measure since its fast decay
- Muon anomaly is measured to 0.5 parts in a million (ppm) ***SM test.***
- Thus muon AMM leads to a $(m_\mu/m_e)^2 \sim 40\,000$ enhancement of the sensitivity to New Physics versus the electron AMM, the muon anomaly is sensitive to NEW physics.

SM Contributions to Muon AMM from BNL

From BNL E821 g-2 experiment (1999-2006)

$$a_\mu^{\mathrm{BNL}} = 11\,659\;208.9(6.3) \bullet 10^{-10} \quad (0.54\ \mathrm{ppm})$$

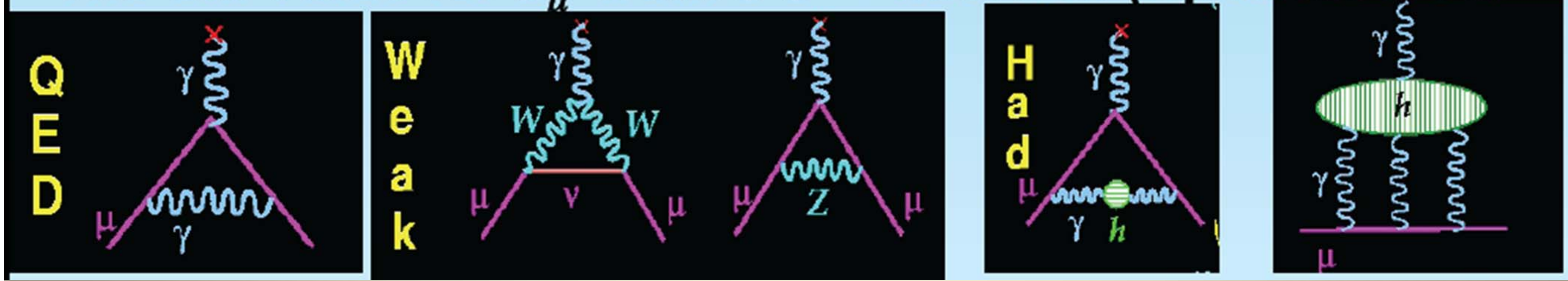
New Prop. E989 at Fermilab
0.14 ppm
KEK/JParc

From Standard Model A. Hoecker Tau2010 Update

$$a = \frac{g-2}{2}$$

$$a_\mu = \left\{ a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{Strong}} \right\}^{\mathrm{SM}} + ???$$

The SM Value for a_μ from $e^+e^- \rightarrow$ *hadrons* (Updated 9/10)



$$a_\mu^{\mathrm{SM}} = 11\,659\;180.2(4.9) \bullet 10^{-10}$$

$$\Delta a_\mu = a_\mu^{\exp} - a_\mu^{\mathrm{SM}} = 28.7(8.0) \bullet 10^{-10} \quad (3.6\sigma!)$$

$$a_\mu^{QED} = 11\,658\,471.809(0.015)\bullet 10^{-10}$$

Kinoshita&Nio 2004, 2006

plus

$$a_\mu^{EW} = 15.4(0.2)\bullet 10^{-10}$$

Czarnetski&Marciano&Vainshtein 2003

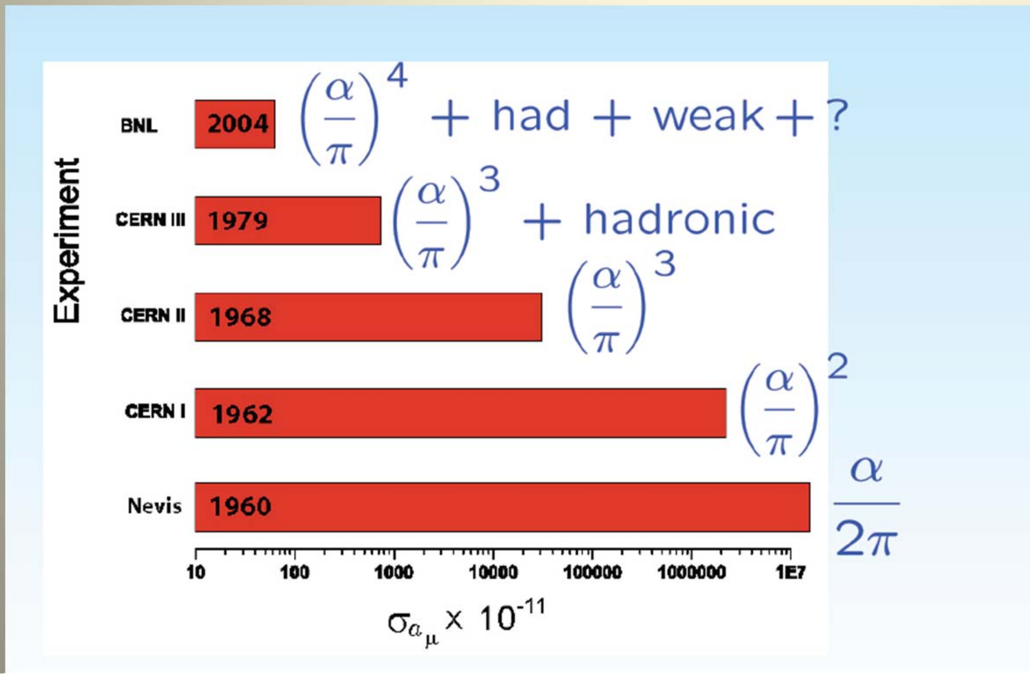
plus

the Hadronic Contribution estimated as

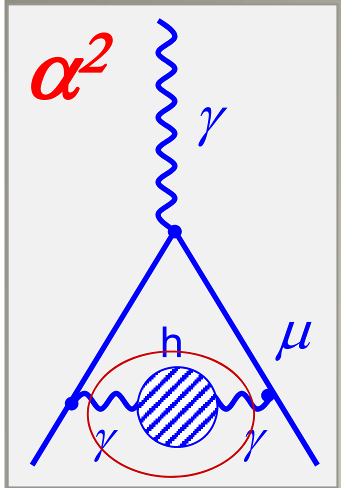
$$a_\mu^{Strong} = 693.0(4.9)\bullet 10^{-10} \quad (<1\% \text{ accuracy!})$$

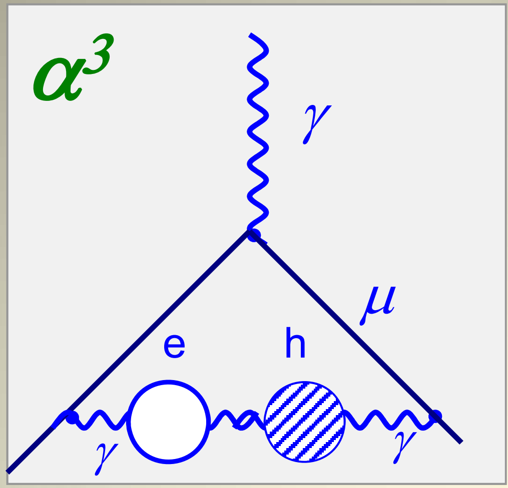
M. Davier, A. Hoecker, B. Malaescu, Z. Zhang 2010;
F. Jegerlehner, R. Szafron 2011

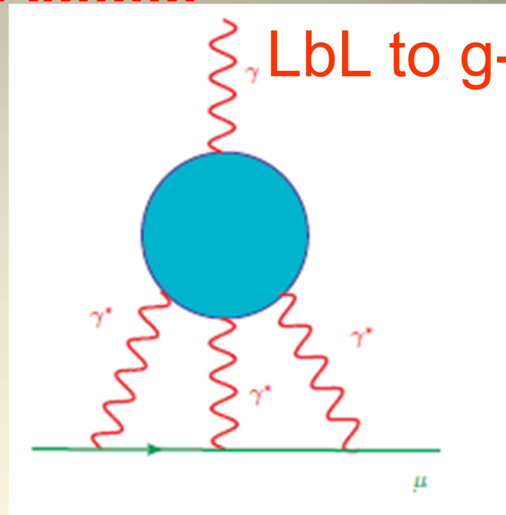
The main question how to get such accuracy from theory.



Strong contributions to Muon AMMM







$$a_\mu^{\mathrm{HVP}} = (692.3 \pm 4.2) \bullet 10^{-10}$$

Hadronic Vacuum polarization
(Davier, Hoecker, Zhang)

$$a_\mu^{\mathrm{LbL}} = (10.5 \pm 2.6) \bullet 10^{-10}$$

Hadronic Light-by-Light Scattering
(AED, A.Radzhabov, A.Zhevlakov ;
C.Fischer, T. Goecke and R.Williams)

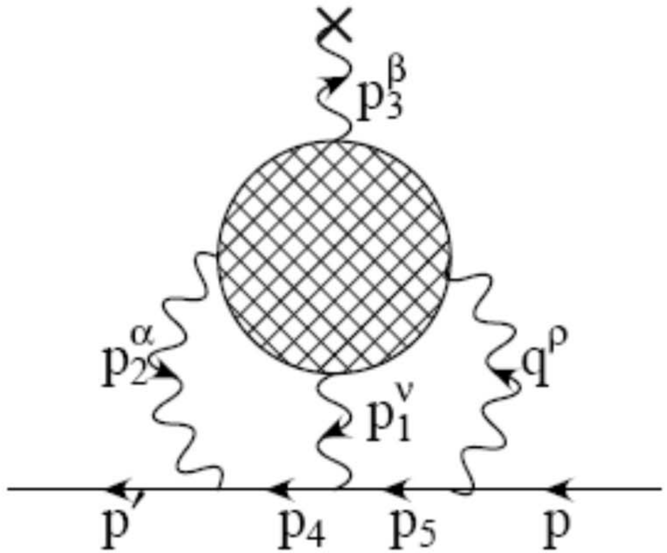
Hadronic Vacuum Polarization
contributes 99%
and half of error
Fixed by Experiment

$$a_\mu^{(2)\mathrm{hvp}} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \quad \frac{K(s)}{s} \quad R^{(0)}(s)$$

Light-by-light process
contributes 1%
and half of error

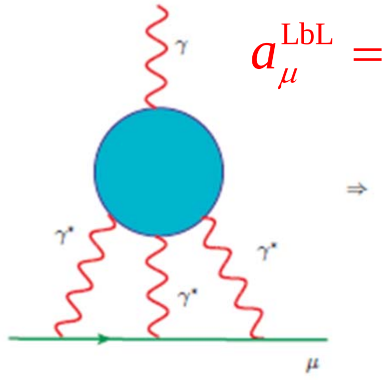
Model Dependent

Hadronic light-by-light contribution to muon $g-2$

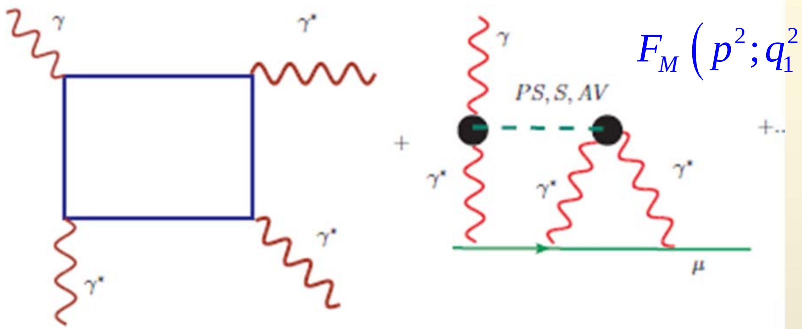


$$\mathcal{M} = |e|^7 A_\beta \int \frac{\mathrm{d}^4 p_1}{(2\pi)^4} \int \frac{\mathrm{d}^4 p_2}{(2\pi)^4} \frac{1}{q^2\, p_1^2\, p_2^2 (p_4^2 - m^2)\,(p_5^2 - m^2)}$$
$$\times \underline{\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3)}\, \bar{u}(p')\gamma_\alpha(\not{p}_4 + m)\gamma_\nu(\not{p}_5 + m)\gamma_\rho u(p)$$

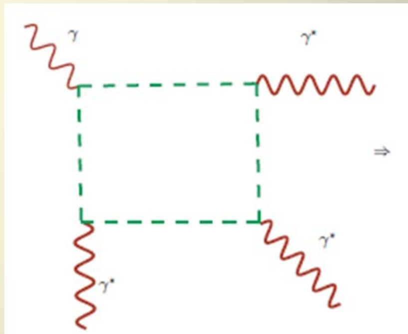
Structure of hadronic LbL contribution



$$a_\mu^{\text{LbL}} = (10.5 \pm 2.6) \bullet 10^{-10}$$



$$F_M\left(p^2; q_1^2, q_2^2\right)$$



$$a_\mu^{\text{PS,LbL}} = (9.5 \pm 1.3) \bullet 10^{-10}$$

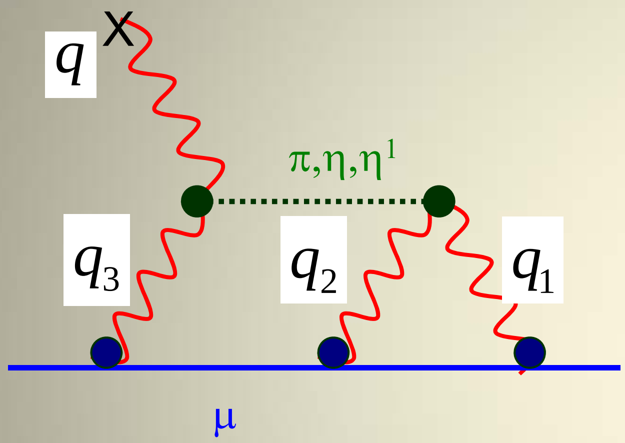
$$a_\mu^{\text{QL,LbL}} = (2.1 \pm 0.3) \bullet 10^{-10}$$

$$a_\mu^{\text{PV,LbL}} = (1.5 \pm 1.0) \bullet 10^{-10}$$

$$a_\mu^{\pi\text{L,LbL}} = -(1.9 \pm 1.9) \bullet 10^{-10}$$

$$a_\mu^{\text{S,LbL}} = -(0.7 \pm 0.7) \bullet 10^{-10}$$

Psevdoscalar meson exchange LbL contribution – Kozel (Goat) diagram



$$a_\mu^{\mathrm{LbL,PS}} = -\frac{2\alpha^3}{3\pi^2}\int_0^\infty dq_1^2 \int_0^\infty dq_2^2 \int_{-1}^{1} dt\sqrt{1-t^2}\,\frac{1}{q_3^2}\times$$

$$\times \sum_{a=\pi^0,\eta,\eta'}\Bigg[2\frac{\mathrm{F}_{a^*\gamma^*\gamma^*}\left(q_2^2;q_1^2,q_3^2\right)\mathrm{F}_{a^*\gamma^*\gamma}\left(q_2^2;q_2^2,0\right)}{q_2^2+M_a^2}I$$

$$+\frac{\mathrm{F}_{a^*\gamma^*\gamma^*}\left(q_3^2;q_1^2,q_2^2\right)\mathrm{F}_{a^*\gamma^*\gamma}\left(q_3^2;q_3^2,0\right)}{q_3^2+M_a^2}I_2\Bigg],$$

Phenomenological and QCD Constraints are used to reduce Model Dependence

Nonperturbative QCD is simulated by Nonlocal Chiral Quark model

Quark Propagator

$$\frac{\not{k}}{k^2} \Rightarrow S(k) = \frac{\not{k} + m(k^2)}{D(k^2)} \xrightarrow{k^2 \to \infty} \frac{\not{k}}{k^2}$$

Quark-Photon Vertex

$$\gamma_\mu \Rightarrow \Gamma_\mu = \gamma_\mu + \Delta\Gamma_\mu(k,k') \xrightarrow{k^2 \to \infty} \gamma_\mu,$$ where $\Delta\Gamma_\mu(k,k')$

guarantes WTI $(k' = k + q)$: $q_\mu \Gamma_\mu = S^{-1}(k') - S^{-1}(k)$

Quark-Pion vertex

$$\frac{1}{f_\pi}\gamma_5 \Rightarrow \Gamma_\pi = \frac{1}{f_\pi}\gamma_5 F(k,k') \overset{k'^2 \to \infty,\ k^2 \to \infty}{=} 0$$

The vertex F is equivalent of the light-cone pion WF

$m(k^2)$ is related to nonlocal quark condensate and thus $m(k^2) \approx M_q e^{-C(k^2)^a}$

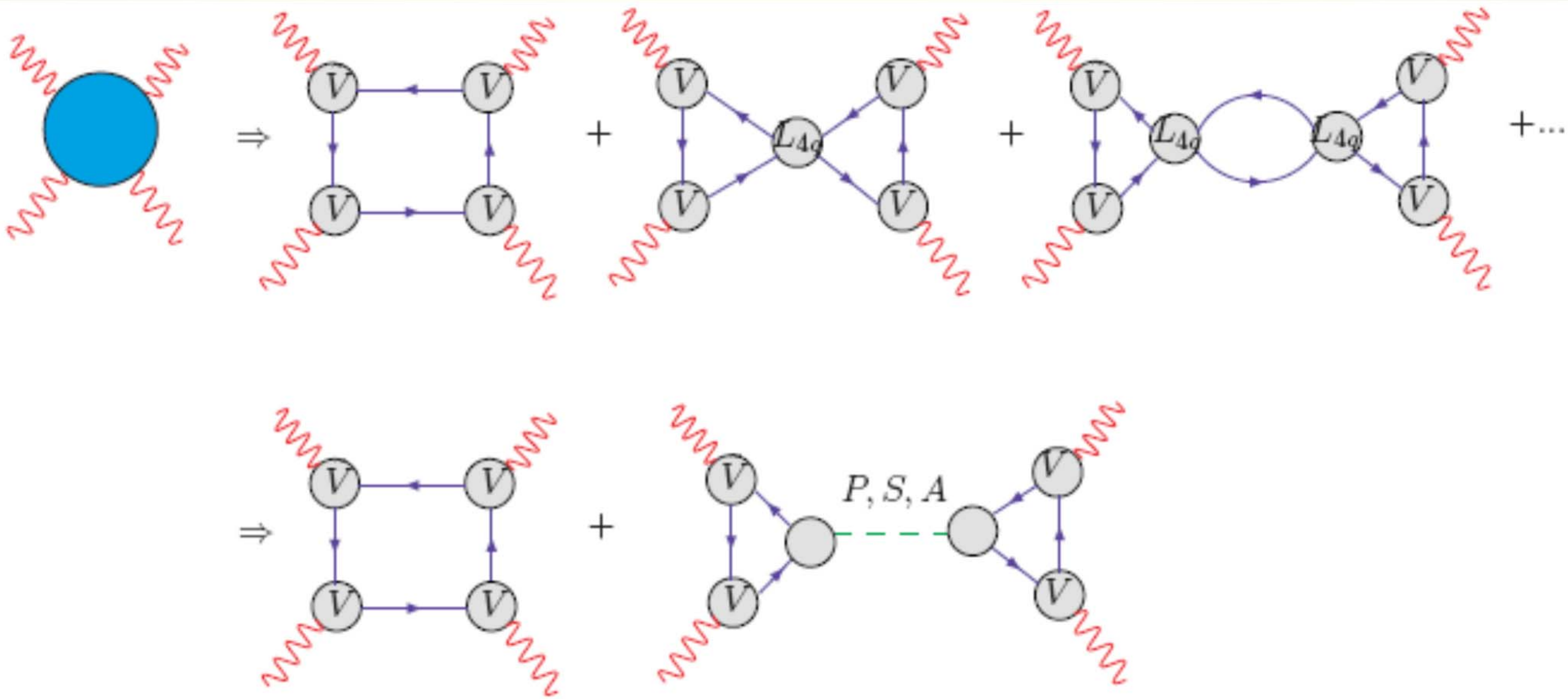
We use for the Dynamical Quark Mass the model

$$m(k^2) = M_q \exp(-2\Lambda k^2)$$

Effective Model Approach

AED, W. Broniowski PRD (2008),
AED, A. Radzhabov, A. Zhevlakov (2011)
AED, A. Radzhabov, A. Zhevlakov (2012)

$$\mathcal{L} = \bar{q}(x)(i\hat{\partial} - m_c)q(x) + \frac{G}{2}[J_S^a(x)J_S^a(x) + J_P^a(x)J_P^a(x)] - \frac{H}{4}T_{abc}[J_S^a(x)J_S^b(x)J_S^c(x) - 3J_S^a(x)J_P^b(x)J_P^c(x)], \quad (1)$$



Results on PS meson exchange LbL contribution

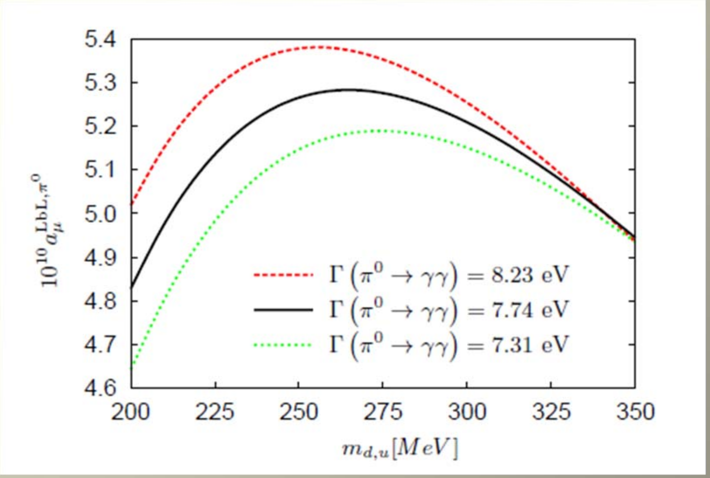
AED, AE Radzhabov, AS Zhevlakov, EPJC (2011)

Model	π^0	η	η'	$\pi^0+\eta+\eta'$
VMD [6]	5.74	1.34	1.19	8.27(0.64)
ENJL [11]	5.6			8.5(1.3)
LMD+V, VMD [7]	5.8(1.0)	1.3(0.1)	1.2(0.1)	8.3(1.2)
NJL [12]	8.18(1.65)	0.56(0.13)	0.80(0.17)	9.55(1.66)
(LMD+V)′,VMD[8]	7.97	1.8	1.8	11.6(1.0)
NχQM [13]	6.5(0.2)			
HM [16]	6.9	2.7	1.1	10.7
DIP, VMD [10]	6.54(0.25)			
DSE [15]	5.75(0.69)	1.36(0.30)	0.96(0.21)	8.07(1.20)
This work (NχQM)	5.01(0.37)	0.54	0.30	5.85

Our results are systematically lower!

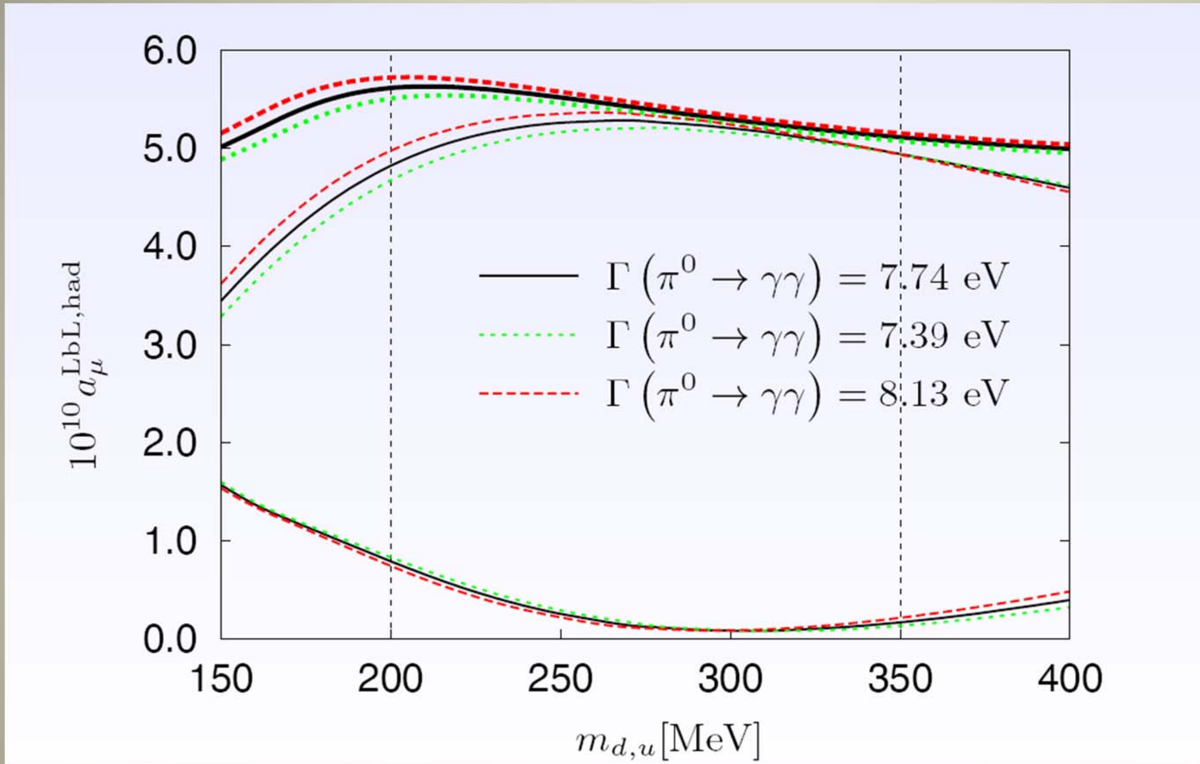
Why?

Because we use full kinematical Dependence of the photon-meson vertices!

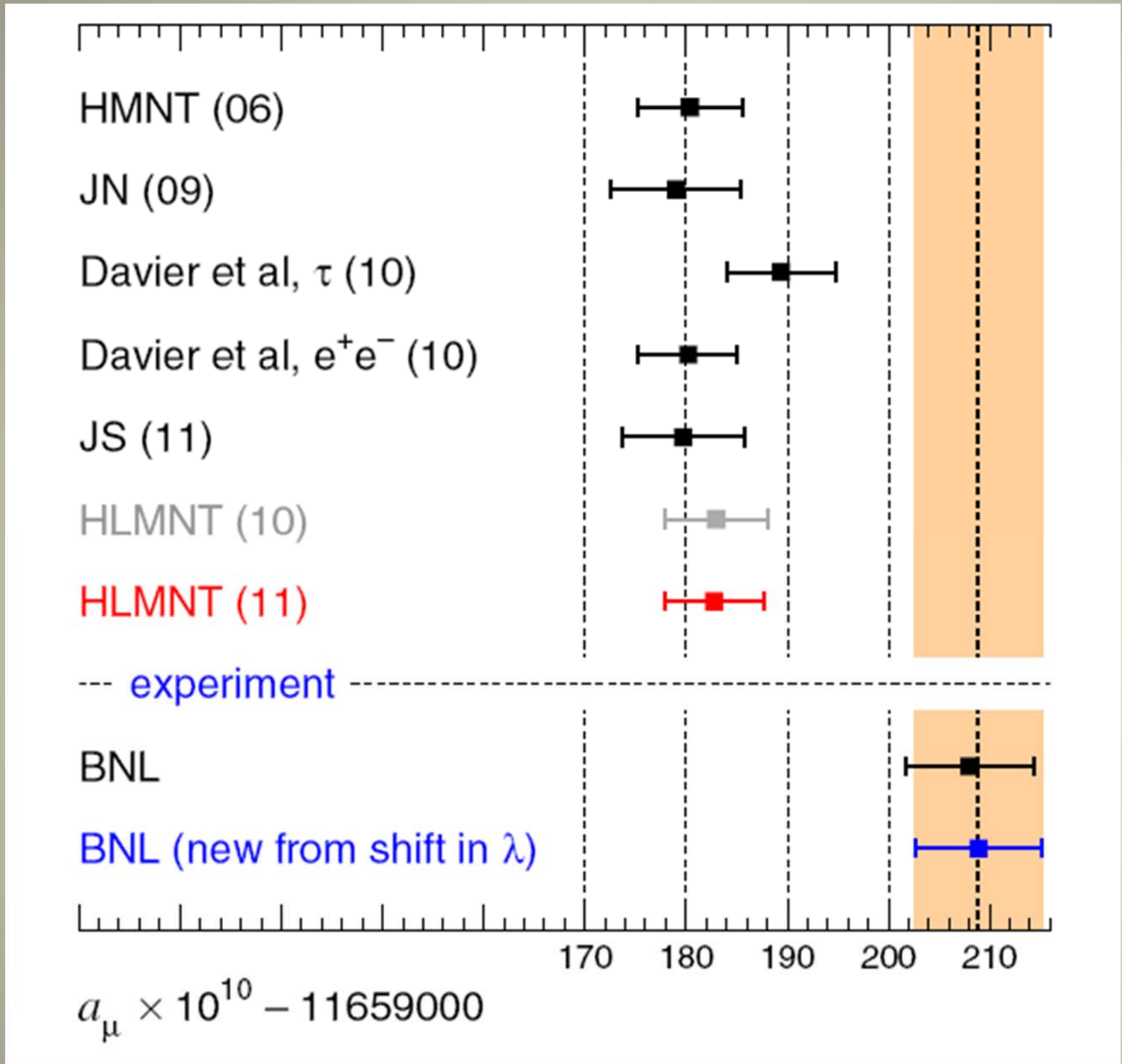


Inclusion of Scalars: Sigma, a0(980), f0(980)

AED, AE Radzhabov, AS Zhevlakov, EPJC (2012)

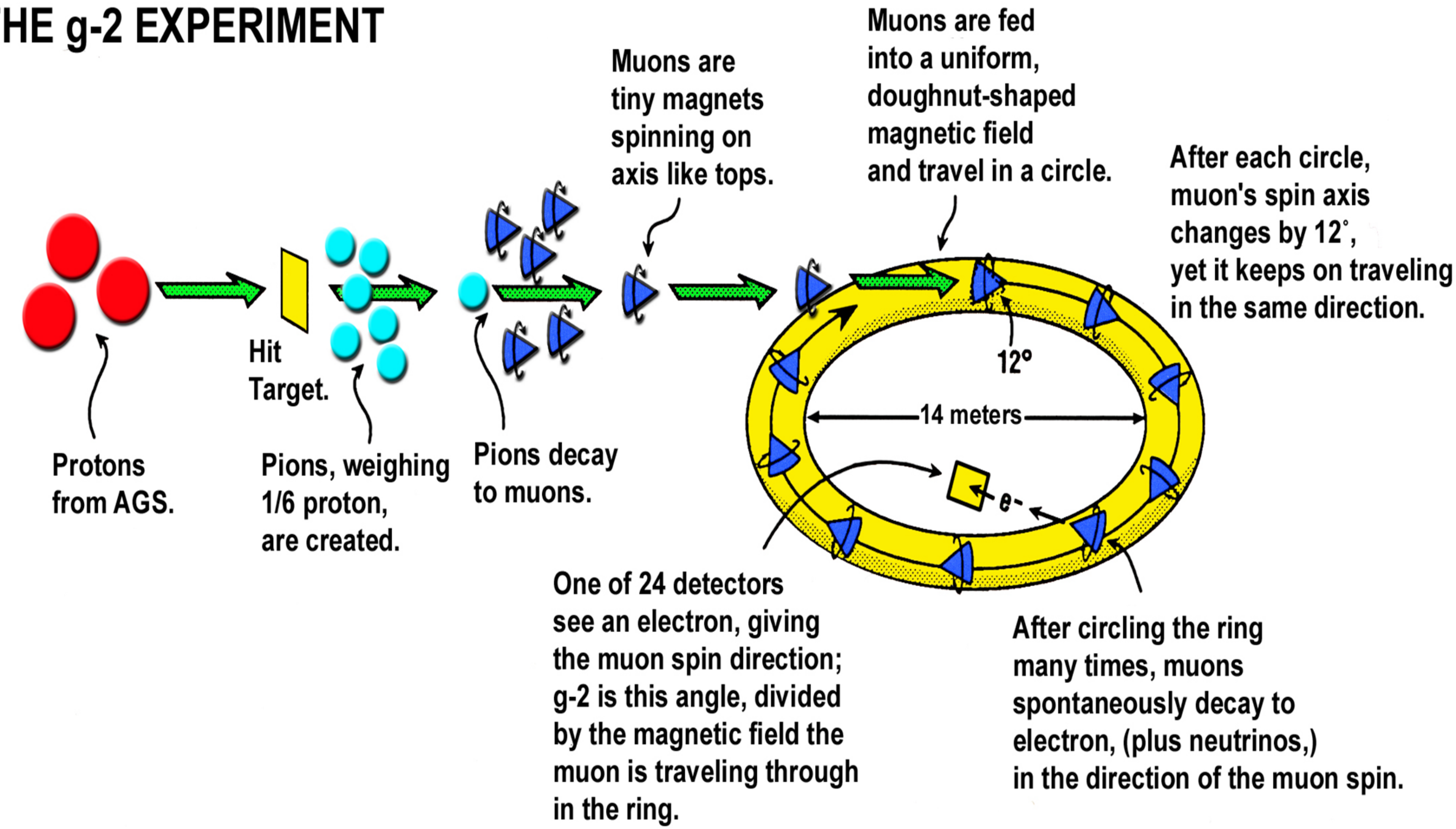


$$a_\mu^{\mathrm{PS+S,LbL}} = (6.25 \pm 0.83) \cdot 10^{-10}$$



Our results indicate that LbL is overestimated
And discrepancy may be increased by about 1 sigma

LIFE OF A MUON:
THE g-2 EXPERIMENT



Precession Method

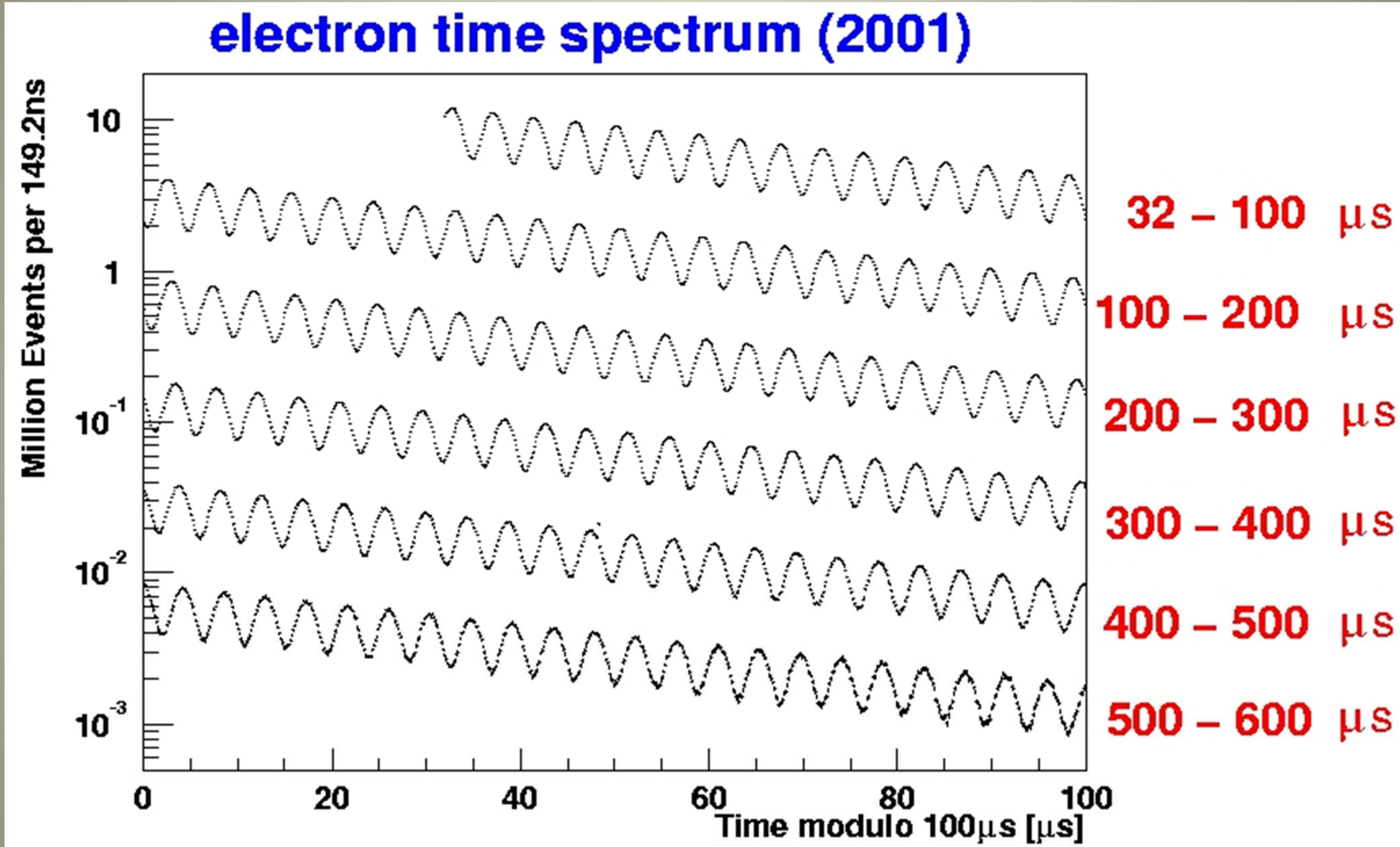
This BNL experiment is based on the fact that for $a_\mu > 0$ the spin precesses faster than the momentum vector when a muon travels transversely to a magnetic field.

The difference of the spin frequency (Larmor and Thomas) ω_S and the momentum precession (cyclotron) frequency ω_C is given by

$$\omega_a = \frac{g-2}{2}\frac{eB}{mc}$$

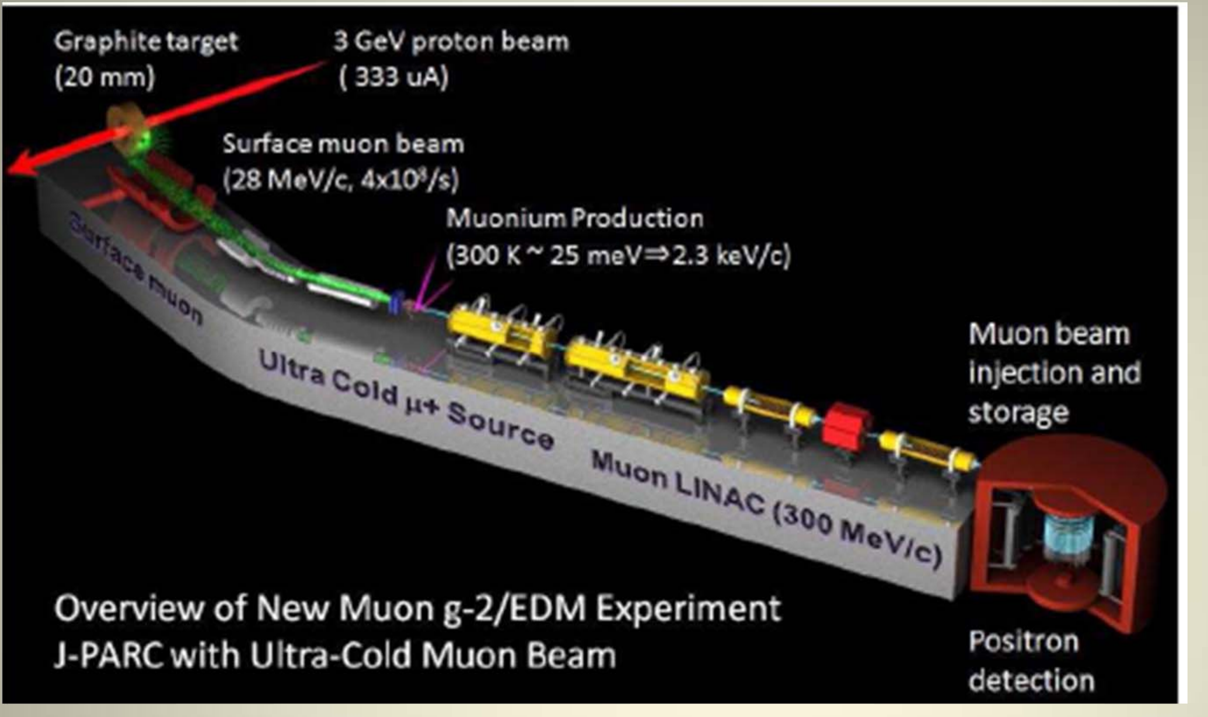
The difference frequency ω_a is the frequency with which the spin precesses relative to the momentum, and is proportional to the anomaly, rather than to g.

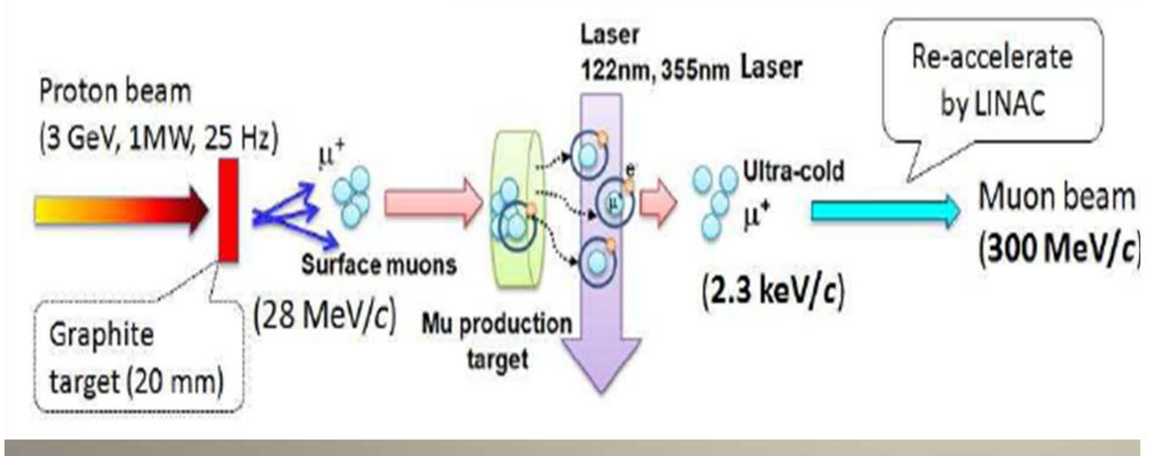
$$4 \times 10^9 \quad e^-, \; E_{e^-} \geq 1.8 \text{ GeV}$$

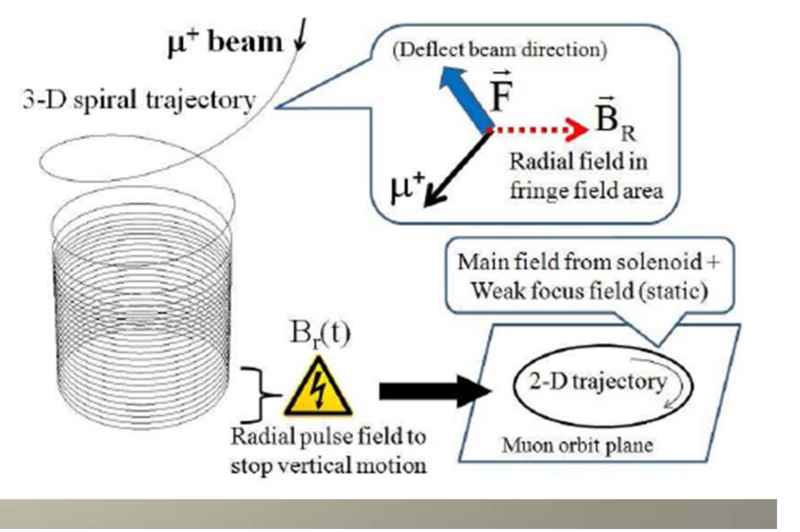


$$f(t) = N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

Precise measurment of muon g-2/EDM at JPARC







Summary

1) Study of Electron AMM provides very precise value for the QED coupling α

2) Study of Muon AMM is sensitive to effects of SM and NP

3) At present there is 3.4σ disagreement between SM and BNL experiment. New experiments at FNAL and JPARC are promising

4) New experiments at VEPP2000, KLOE2, BESS III on cross section will further diminish the error for HVP contribution

5) The account of full kinematic dependence of meson-two-photon vertex reduces the value for the LbL contribution and make agreement worse

6) The “cousin” processes to LbL is the rare decays of light PS mesons to the lepton pair are helpful for LbL and also as a test of SM

At present there is 3.3σ disagreement between SM and KTeV experiment for $\pi^0 \to e^+e^-$ This effect may be related to existence of dark matter particles with low masses 10-100 MeV

Proton Size Anomaly

$$\langle r_p^2 \rangle^{1/2} = 0.8768 \pm 0.0069 \text{ fm}$$ CODATA 2008

Lamb shift in Muonic Hydrogen (a proton orbited by a negative muon)
Nature 2010, Pohl etal (PSI)

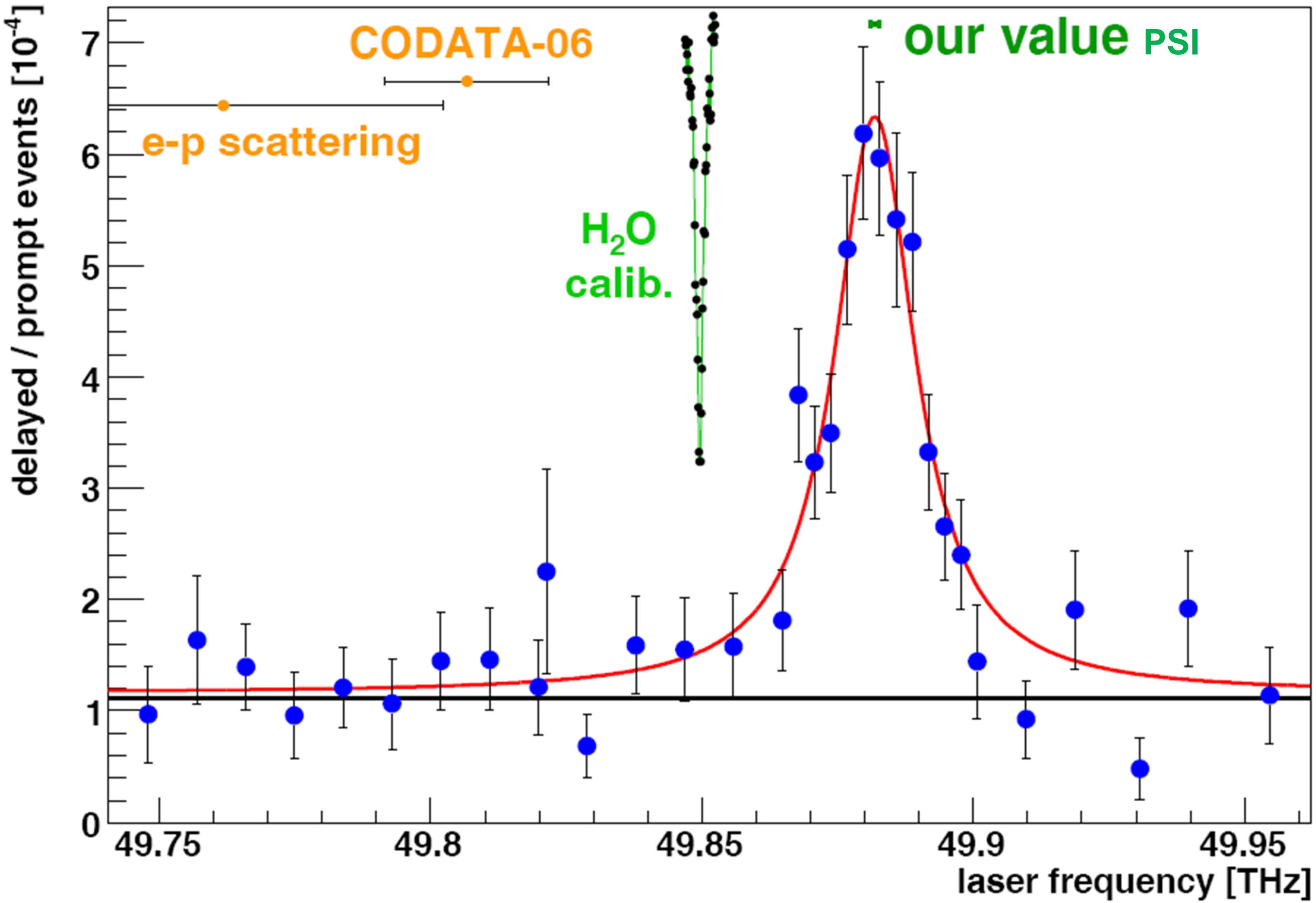
its much smaller Bohr radius compared to ordinary atomic hydrogen causes enhancement of effects related to the finite size of the proton

$$\Delta\tilde{E} \equiv E(2P_{3/2}^{F=2}) - E(2S_{1/2}^{F=1}) = 206.2949 \pm 0.0032 \text{ meV}$$

$$\Delta\tilde{E} = 209.9779(49) - 5.2262 \langle r_p^2 \rangle + 0.0347 \langle r_p^2 \rangle^{3/2}$$

$$\langle r_p^2 \rangle^{1/2} = 0.84184 \pm 0.00067 \text{ fm}$$

5 σ deviation!



CODATA-06
our value PSI
e-p scattering
H_2O calib.
delayed / prompt events [10^{-4}]
laser frequency [THz]
0
1
2
3
4
5
6
7
49.75
49.8
49.85
49.9
49.95

Rare decay $\pi0 \rightarrow$ e+e-: window to New physics?

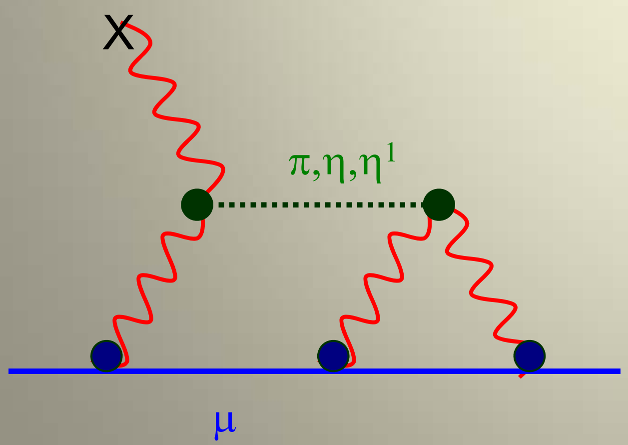
A. E. Dorokhov and M. A. Ivanov, Phys. Rev. **D75**, 114007 (2007), 0704.3498.

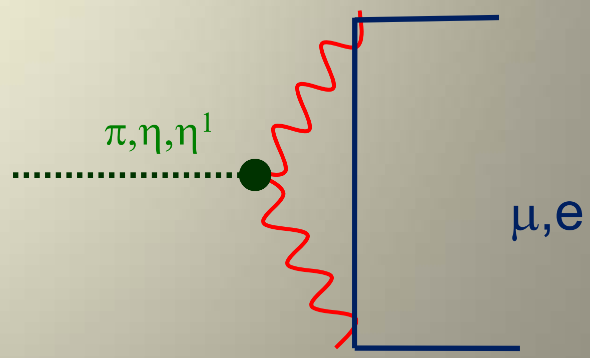
A. E. Dorokhov and M. A. Ivanov, JETP Lett. **87**, 531 (2008), 0803.4493.

A. E. Dorokhov, E. A. Kuraev, Y. M. Bystritskiy, and M. Secansky, Eur. Phys. J. **C55**, 193 (2008), 0801.2028.

A. E. Dorokhov, M. A. Ivanov, and S. G. Kovalenko, Phys. Lett. **B677**, 145 (2009), 0903.4249.

"Cousin" process to g-2





PHYSICAL REVIEW D 75, 012004 (2007)

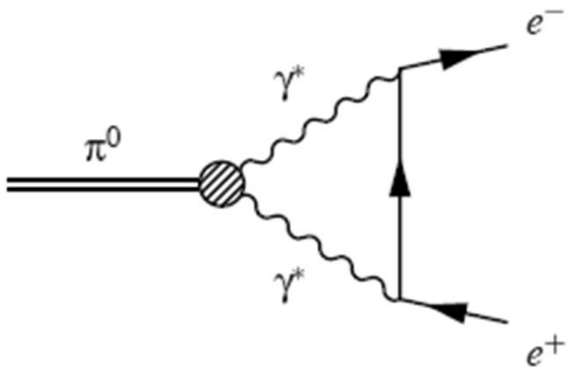
Measurement of the rare decay $\pi^0 \to e^+e^-$

(Submitted on 24 Oct 2006)

KTeV Collaboration, FERMI Lab

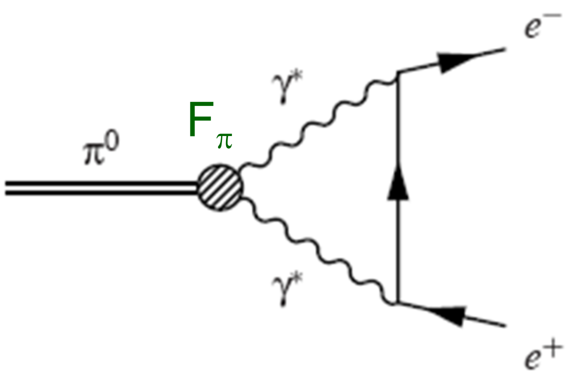
find the lowest-order rate for $\pi^0 \to e^+e^-$. We found $B^{\text{no-rad}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$, more than 7 standard deviations higher than the unitary bound. The result falls between VMD [6] and χPT predictions [8], with a significance on the difference of 2.3 and 1.5 standard deviations, respectively.

It means “no theory”, everybody happy



One of the simplest process for THEORY

Classical theory of $\pi^0 \to e^+e^-$ decay



Drell (59'), Berman,Geffen (60'), Quigg,Jackson (68')

Bergstrom,et.al. (82') Dispersion Approach

Savage, Luke, Wise (92') χPT

$$R\left(\pi^0 \to e^+e^-\right) = \frac{B\left(\pi^0 \to e^+e^-\right)}{B\left(\pi^0 \to \gamma\gamma\right)} = 2\beta\left(m_\pi^2\right)\left(\frac{\alpha}{\pi}\frac{m_e}{m_\pi}\right)^2\left[\left(\mathrm{Re}\,A\left(m_\pi^2\right)\right)^2 + \left(\mathrm{Im}\,A\left(m_\pi^2\right)\right)^2\right]$$

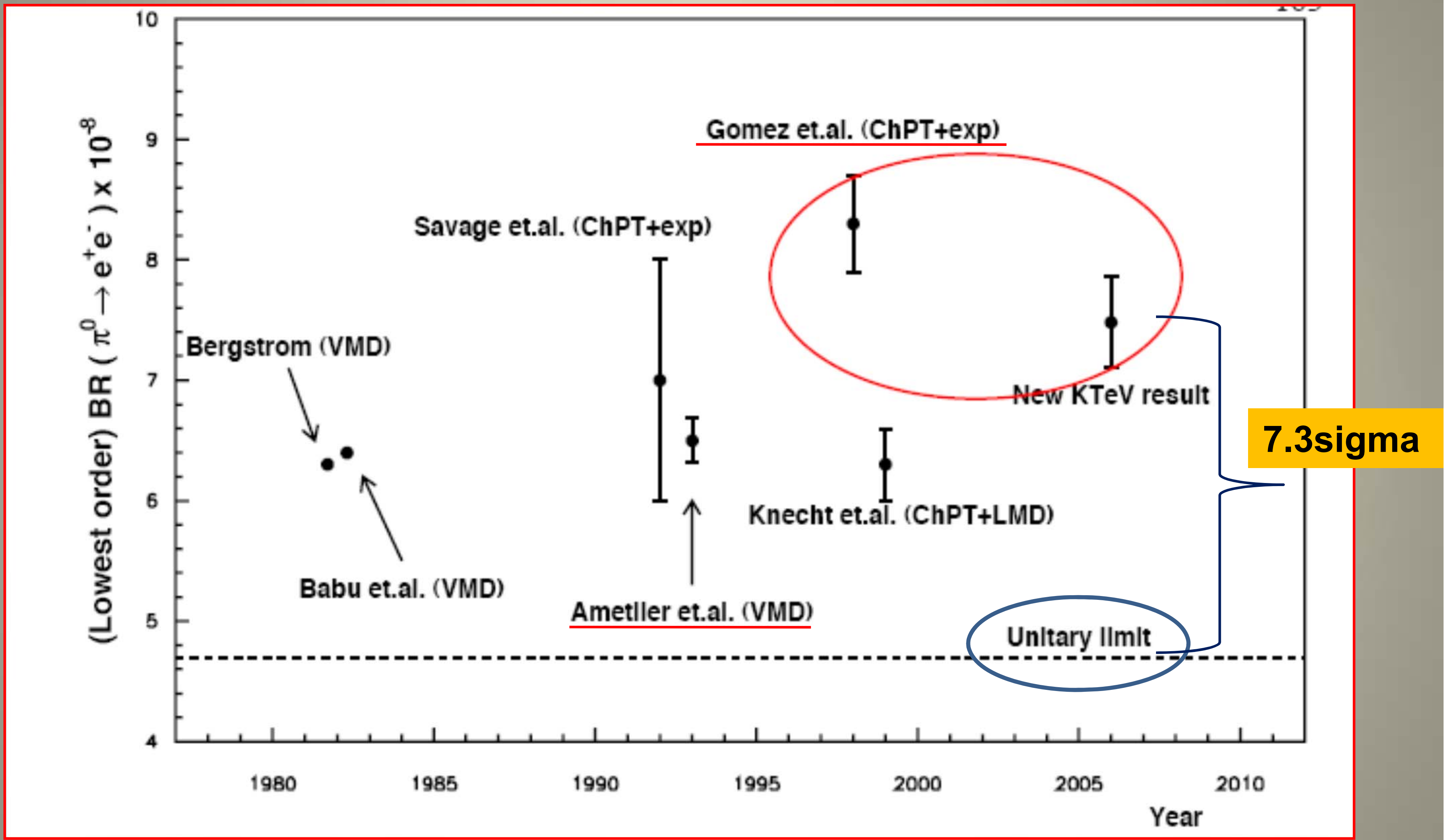
$$\beta\left(q^2\right) = \sqrt{1 - 4\frac{m_e^2}{q^2}}$$

$$A\left(q^2, p^2\right) = \frac{2i}{q^2}\int\frac{d^4k}{\pi^2}\frac{q^2k^2 - (qk)^2}{\left(k^2 + i\varepsilon\right)\left((k-q)^2 + i\varepsilon\right)\left((k-p)^2 - m_e^2 + i\varepsilon\right)}F_\pi\left(k^2,(k-q)^2\right)$$

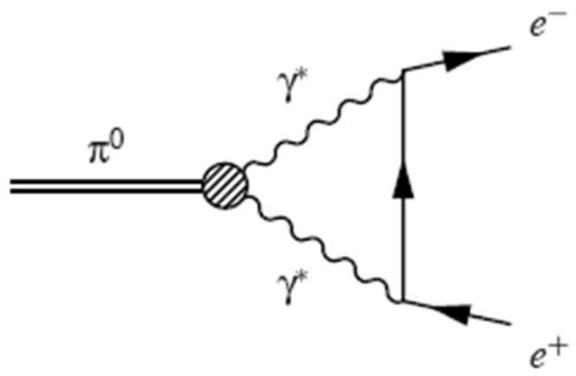
$$\mathrm{Im}\,A\left(q^2\right) = \frac{\pi}{2\beta\left(q^2\right)}\ln\left(\frac{1-\beta\left(q^2\right)}{1+\beta\left(q^2\right)}\right)$$

The Imaginary part is Model Independent;
Unitary limit (Re A =0)

$$R\left(\pi^0 \to e^+e^-\right) \geq R^{\text{unitary}}\left(\pi^0 \to e^+e^-\right) = 4.75\cdot 10^{-8}$$



Idea: Use the best model to get experimental number! It falls!!!



Transition form factor,
LbL contribution to g-2,
Test for models

$$\mathcal{A}\left(q^2\right) = \frac{2i}{q^2}\int \frac{d^4k}{\pi^2} \frac{q^2k^2 - (qk)^2}{\left(k^2 + i\varepsilon\right)\left((k-q)^2 + i\varepsilon\right)\left((k-p)^2 - m_e^2 + i\varepsilon\right)} F_\pi\left(k^2, (k-q)^2\right)$$

Well separated scales

$$m_e^2 << m_\pi^2 << \Lambda^2 \approx m_\rho^2$$

$$F_\pi^{VMD}\left(k_1^2, k_2^2\right) = \frac{1}{\left(k_1^2 + \Lambda^2\right)\left(k_2^2 + \Lambda^2\right)}$$

$$\mathrm{Re}\, A\left(m_\pi^2\right) = \ln^2\left(\frac{m_e}{m_\pi}\right) + \frac{\pi^2}{12} + 3\ln\left(\frac{m_e}{\mu}\right) + \chi_P(\mu) + O\left(\frac{m_e^2}{\Lambda^2}, \frac{m_\pi^2}{\Lambda^2}, \frac{m_e^2}{m_\pi^2}\right)$$

Model Independent because of Log,
$\chi_p(\mu)$ is the Low Energy Constant

$$\mu \approx \Lambda \approx m_\rho \ ?$$

I. Real Part of the Decay Amplitude

$$e\,A\left(m_\pi^2\right)=\ln^2\left(\frac{m_e}{m_\pi}\right)+\frac{\pi^2}{12}+3\ln\left(\frac{m_e}{\mu}\right)+\chi_P\left(\mu\right)+\mathrm{O}\left(\frac{m_e^2}{\Lambda^2},\frac{m_\pi^2}{\Lambda^2},\frac{m_e^2}{m_\pi^2}\right)$$

$$\chi_P\left(\mu\right)=-\frac{5}{4}-\frac{3}{2}\left[\int_0^{\mu^2}dt\,\frac{F_\pi\left(t,t\right)-1}{t}+\int_{\mu^2}^{\infty}dt\,\frac{F_\pi\left(t,t\right)}{t}\right]$$

$$m_e^2 << m_\pi^2 << \Lambda^2 \approx m_\rho^2$$

Similar to

$$a_\mu^{(2)\mathrm{hvp}}=\frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty}ds\,\frac{K(s)}{s}\,R^{(0)}(s)$$

Additive Factorization

The unknown Low Energy constant (LEC) is expressed as inverse moment of the Pion Transition FF at spacelike momenta !!!

A. E. Dorokhov and M. A. Ivanov, Phys. Rev. D **75** (2007) 114007

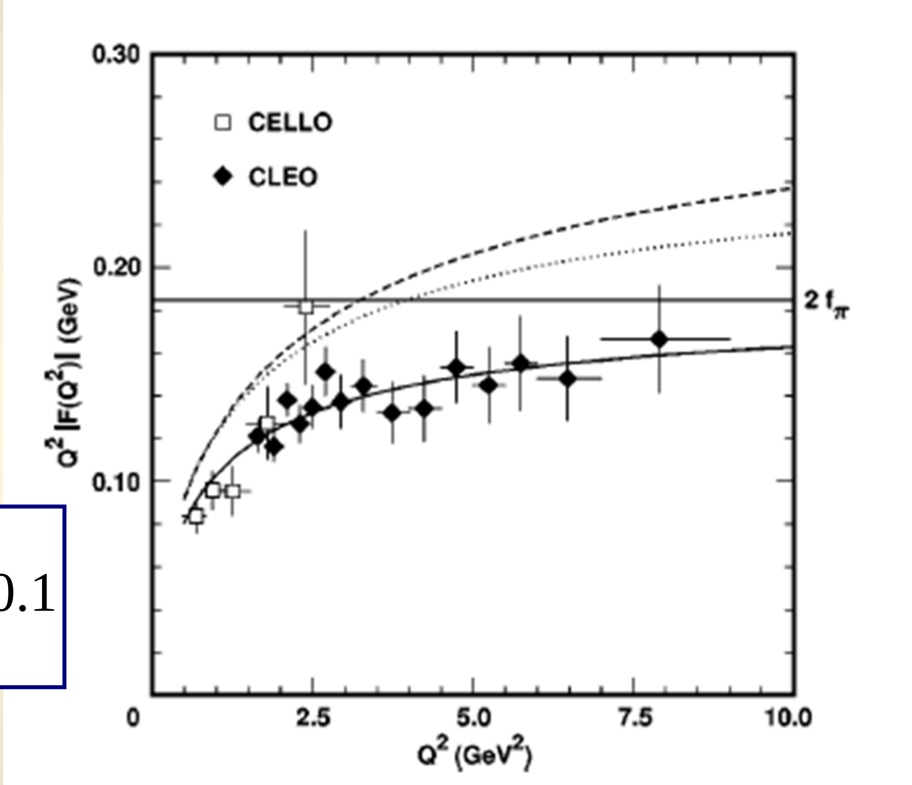
II. CLEO data on pion TFF and Lower Bound on Branching

Use inequality $F_\pi(t,t) < F_\pi(t,0)$ at spacelike $t>0$

and CLEO data (98')

$$F_\pi^{\mathrm{CLEO}}(t,0) = \frac{1}{1+t/s_0^{\mathrm{CLEO}}}$$

$$s_0^{\mathrm{CLEO}} = (776 \pm 22\ \mathrm{MeV})^2$$



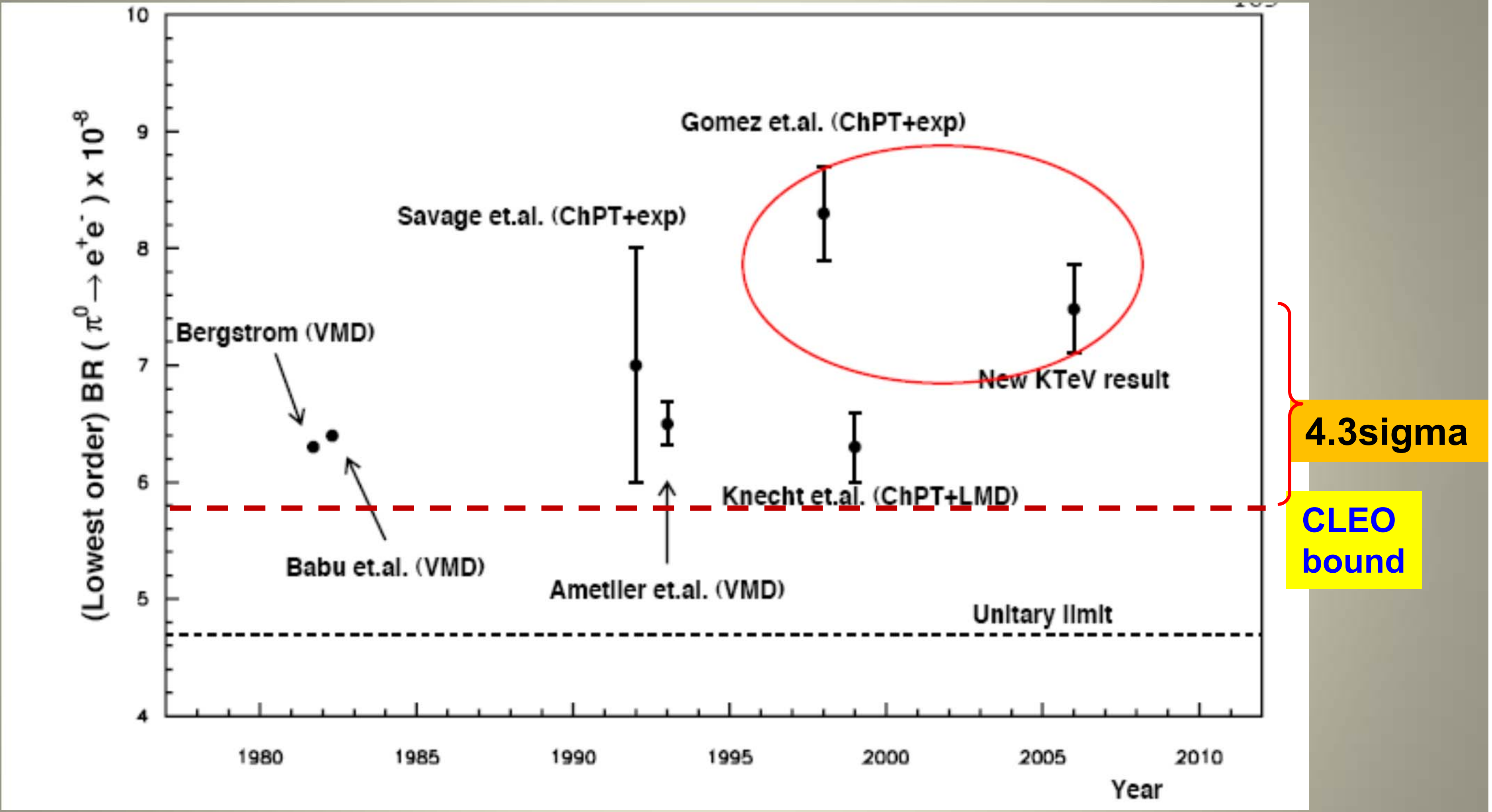
$$\mathrm{Re}\,A(q^2=0) > -\frac{3}{2}\ln\left(\frac{s_0^{\mathrm{CLEO}}}{m_e^2}\right) - \frac{5}{4} = -23.2 \pm 0.1$$

$$R(\pi^0 \to e^+e^-) \geq R^{\mathrm{CLEO}}(\pi^0 \to e^+e^-) = (5.91 \pm 0.02)\cdot 10^{-8}$$

$$R^{\mathrm{KTeV}}_{\pi\to e^+e^-} = (7.48 \pm 0.38)\cdot 10^{-8}$$

$$R(\pi^0 \to e^+e^-) \geq R^{\mathrm{unitary}}(\pi^0 \to e^+e^-) = (4.75 \pm 0.02)\cdot 10^{-8}$$

New Lower Bound



New approach immediately improves the Unitary bound for the Decay

III. $F_\pi(t,t)$ general arguments (rescaling)

Let $$F_\pi(t,t) = \frac{1}{1+t/s_1}$$ **then** $$\mathrm{Re}\,A^{\mathrm{theory}}(q^2=0) = -\frac{3}{2}\ln\left(\frac{s_1}{m_e^2}\right) - \frac{5}{4}$$

1. From $$-\frac{\partial F_\pi(t,t)}{\partial t}\bigg|_{t=0} = -2\frac{\partial F_\pi(t,0)}{\partial t}\bigg|_{t=0}$$ **one has** $$s_1 = s_0/2$$

2. From OPE QCD (Brodsky, Lepage)

$$F_\pi^{\mathrm{OPE}}(t,0)\big|_{t\to\infty} = 8\pi^2 f_\pi^2 \frac{1}{t},$$
$$F_\pi^{\mathrm{OPE}}(t,t)\big|_{t\to\infty} = \frac{8\pi^2 f_\pi^2}{3}\frac{1}{t}$$

one has $$s_1^{\mathrm{OPE}} = s_0^{\mathrm{OPE}}/3$$

F(t,0) → F(t,t) reduces to Rescaling
$s_1 = s_0 * [1/2—1/3]$

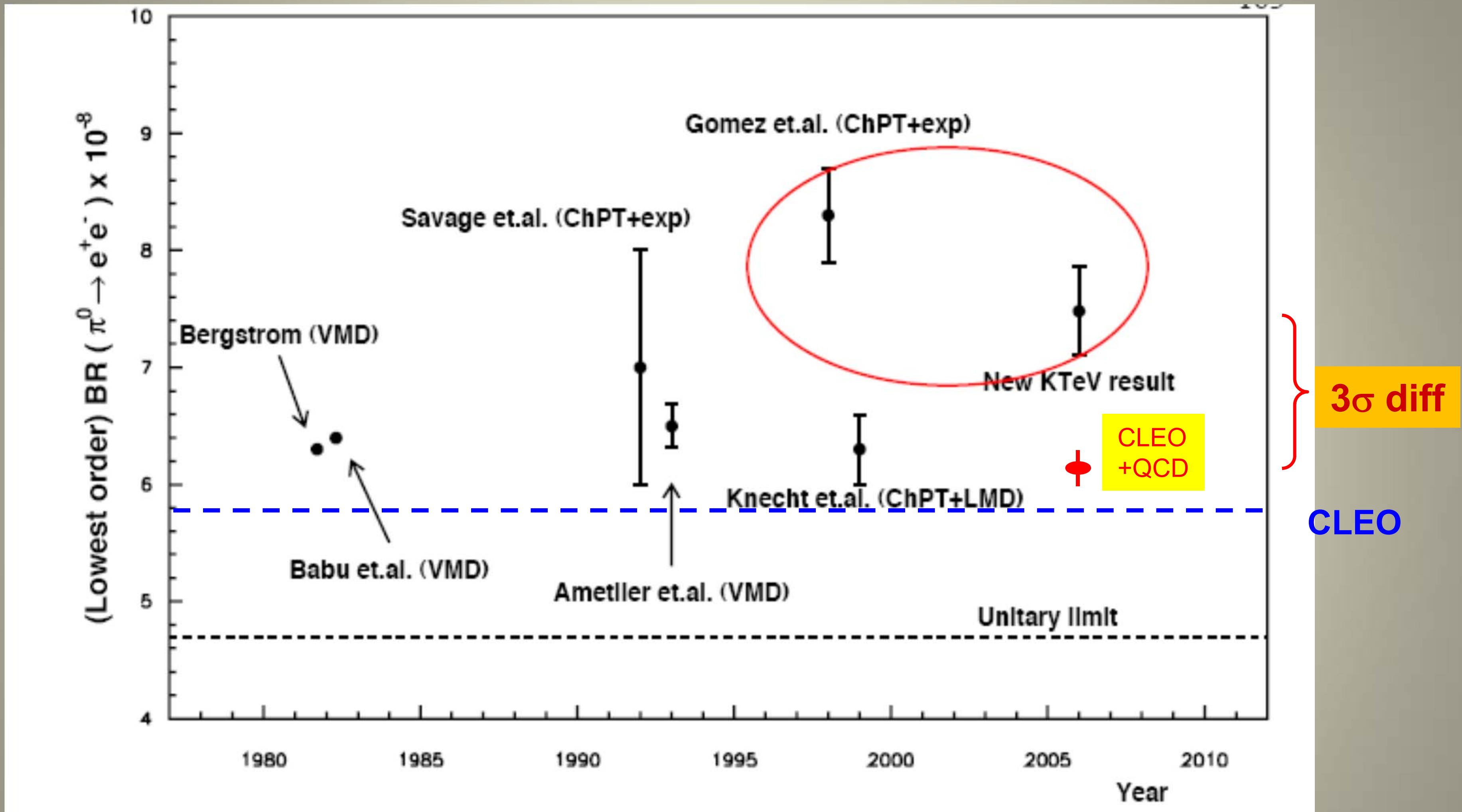
It follows $$\mathrm{Re}\,A^{\mathrm{theory}}(q^2=0) = -21.9 \pm 0.3$$

$$B^{\mathrm{KTeV}}_{\pi\to e^+e^-} = (7.48 \pm 0.38)\cdot 10^{-8}$$

$$B^{\mathrm{theory}}_{\pi\to e^+e^-} = (6.2 \pm 0.1)\cdot 10^{-8}$$

3.3σ below data!!

Theor Error is under control !!
It would required change of s_0 scale by factor more then 10!



What is next? It would be very desirable if Others will confirm KTeV result
Also, Pion transition FF need to be more accurately measured.

Possible explanations of the π^0 anomaly

1) Radiative corrections (?)

KTeV used in their analysis the results from Bergstrom 83'.

A.D.,Kuraev, Bystritsky, Secansky (EJPC 08') confirmed Numerics.

Vasko, Novotny (JHEP11)

2) Mass corrections: tiny, but visible for η and η'

A.D., M. Ivanov, S. Kovalenko (ZhETPh Lett 08' and PLB 09)

Dispersion approach and χPT are corrected by power corrections $(m_\pi/m_\rho)^n$

3) New physics

Kahn, Schmidt, Tait PRD 08' Low mass dark matter particles

Chang, Yang 08' Light CP-odd Higgs in NMSSM

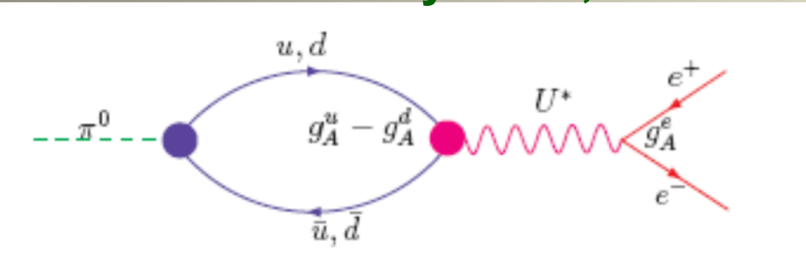
4) Experiment wrong

Waiting for new results from

KLOE, WASA@COSY, BESIII,…

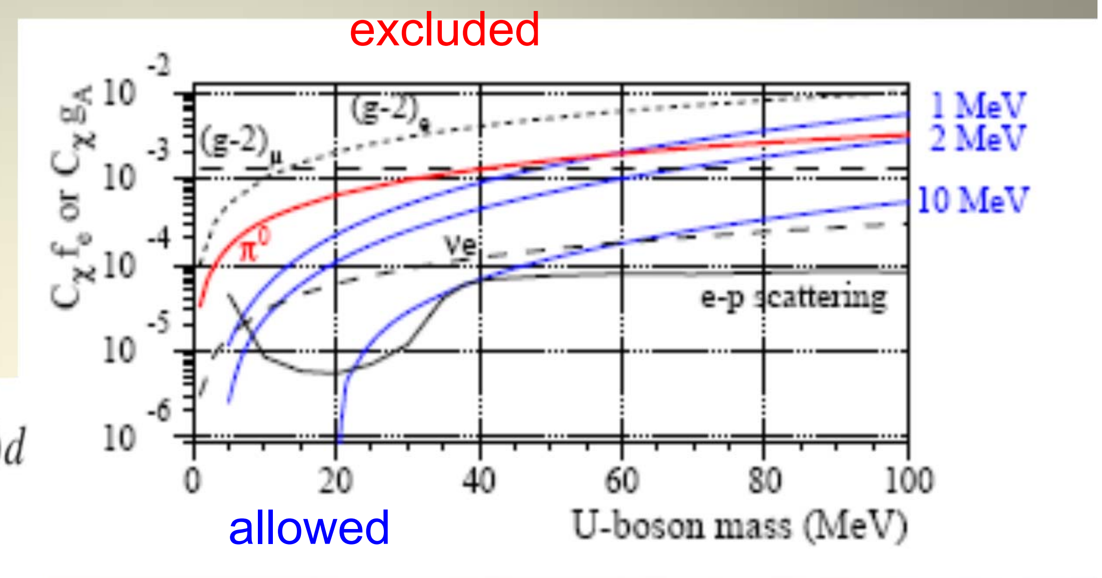
Enhancement in Rare Pion Decays from a Model of MeV Dark Matter (Boehm&Fayet)

was considered by Kahn, Schmitt and Tait (PRD 2008)



M_{U^*} ~ 10 MeV

$$\mathcal{L} \supset U_\mu\{\bar{u}\gamma^\mu(g_V^u + \gamma_5 g_A^u)u + \bar{d}\gamma^\mu(g_V^d + \gamma_5 g_A^d)d + \bar{e}\gamma^\mu(g_V^e + \gamma_5 g_A^e)e\},$$



The anomalous 511 keV γ-ray signal from Galactic Center observed by INTEGRAL/SPI (2003) is naturally explained

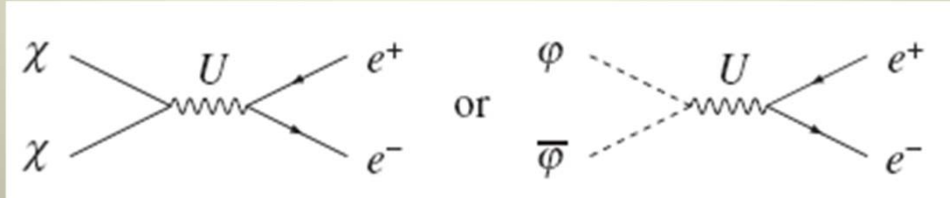


FIG. 1. Dark matter annihilations into e^+e^- pairs [12,13]. The first diagram corresponds to the pair annihilation of spin-$\frac{1}{2}$ LDM particles χ (which may be self-conjugate, or not); and the second one to the case of spin-0 particles φ.

Rare decay π0 → e+e− as a sensitive probe of light CP-odd Higgs in Next-to-Minimal SuperSymmetric Model (NMSSM)

(Qin Chang, Ya-Dong Yang, PLB, 2009)

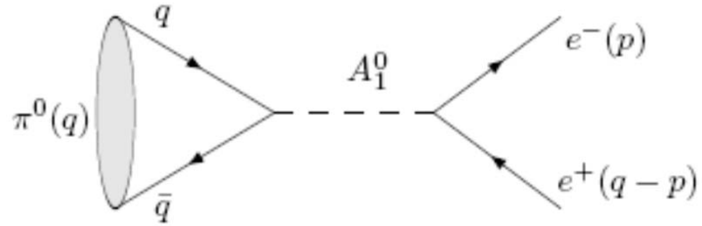
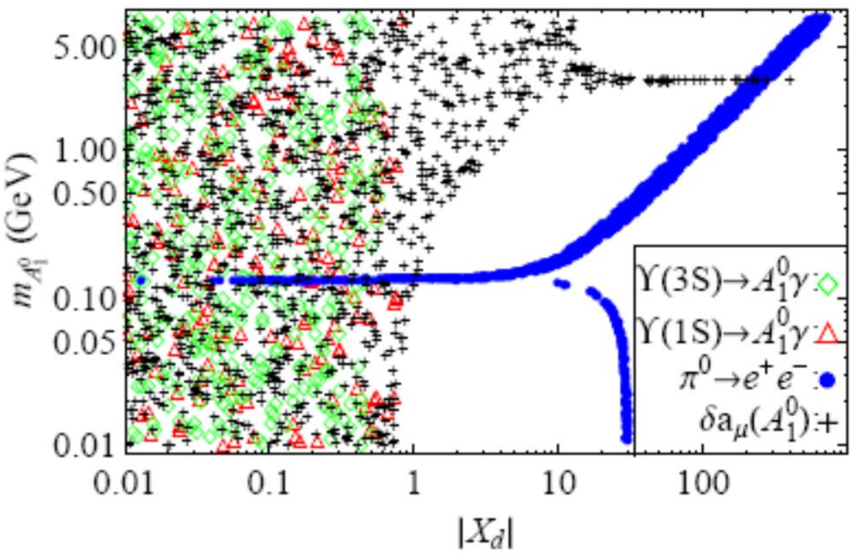


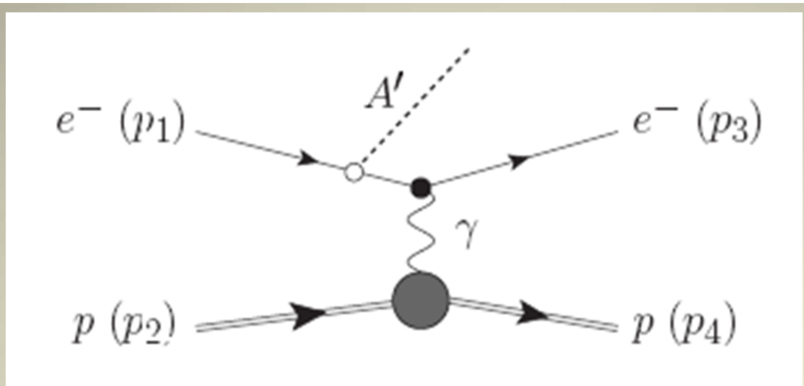
Figure 1: Relevant Feynman diagram within NMSSM.

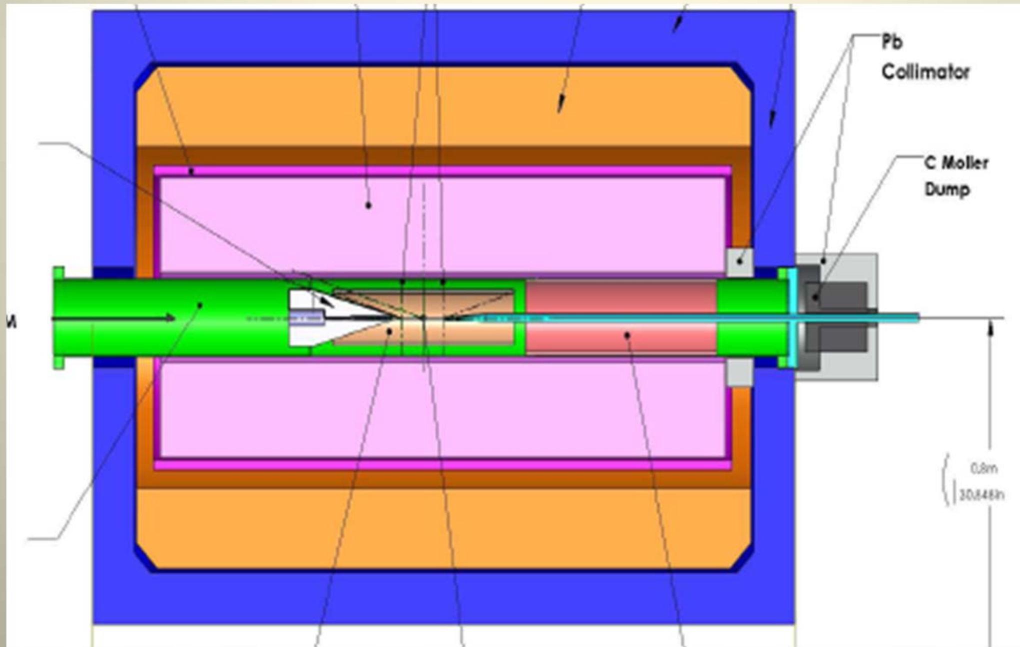


They show the combined constraints from $\Upsilon\to\gamma A^0_1$, a_μ, $\pi^0 \to$ e+e− and $\pi^0 \to\gamma\gamma$ can not be resolved simultaneously with a very light A^0_1 ($m_{A01} \simeq$ 135 MeV)

Also there is no consistency with anomaly In $\Sigma^+\to p\mu^+\mu^-$ observed by HyperCP Coll

DarkLight experiment at JLAB (project)





Other P →l+l– decays

A.D., M. Ivanov, S. Kovalenko Phys. Lett. B 677 (2009)

TABLE I: Values of the branchings $B\left(P \to l^+l^-\right)$ obtained in our approach and compared with the available experimental results.

R_0	Unitary bound	CLEO bound	CLEO+OPE	This work	Experiment	BES III (1 year)
$R_0\left(\pi^0 \to e^+e^-\right) \times 10^8$	≥ 4.69	$\geq 5.85 \pm 0.03$	6.23 ± 0.12	6.26	7.49 ± 0.38 [1]	*WASA@COSY*
$R_0\left(\eta \to \mu^+\mu^-\right) \times 10^6$	≥ 4.36	$\leq 6.23 \pm 0.12$	5.12 ± 0.27	4.64	5.8 ± 0.8 [20, 21]	0.08
$R_0\left(\eta \to e^+e^-\right) \times 10^9$	≥ 1.78	$\geq 4.33 \pm 0.02$	4.60 ± 0.09	5.24	$\leq 2.7 \cdot 10^4$ [22]	$0.7 \cdot 10^2$
$R_0\left(\eta' \to \mu^+\mu^-\right) \times 10^7$	≥ 1.35	$\leq 1.44 \pm 0.01$	1.364 ± 0.010	1.30		0.8
$R_0\left(\eta' \to e^+e^-\right) \times 10^{10}$	≥ 0.36	$\geq 1.121 \pm 0.004$	1.178 ± 0.014	1.86		$0.7 \cdot 10^3$

π->ee will be available from WASA@COSY

Mass power corrections are visible for η(η') decays

BESIII for one year will get for η ,η'->ll the limit $0.7*10^{-7}$

Very attactive **muonic decay modes**: **less rare**
and theoretically limited both from above and below

Summary

1) The processes P → l+l- as like as muon g-2 are good for test of SM.

2) Long distance physics is fixed phenomenologically. New measurements of the transition form factors at low momenta are welcome.

Radiative and mass corrections are well under control.

3) At present there is 3.3σ disagreement between SM and KTeV experiment for $\pi^0 \to e^+e^-$

KLOE, WASA@COSY, BESS III are interested in new measurements

4) If effect found persists it might be evidence for the SM extensions with low mass (10-100 MeV) particles (Dark Matter, NSSM)