

Dense Baryonic Matter in the Hidden Local Symmetry

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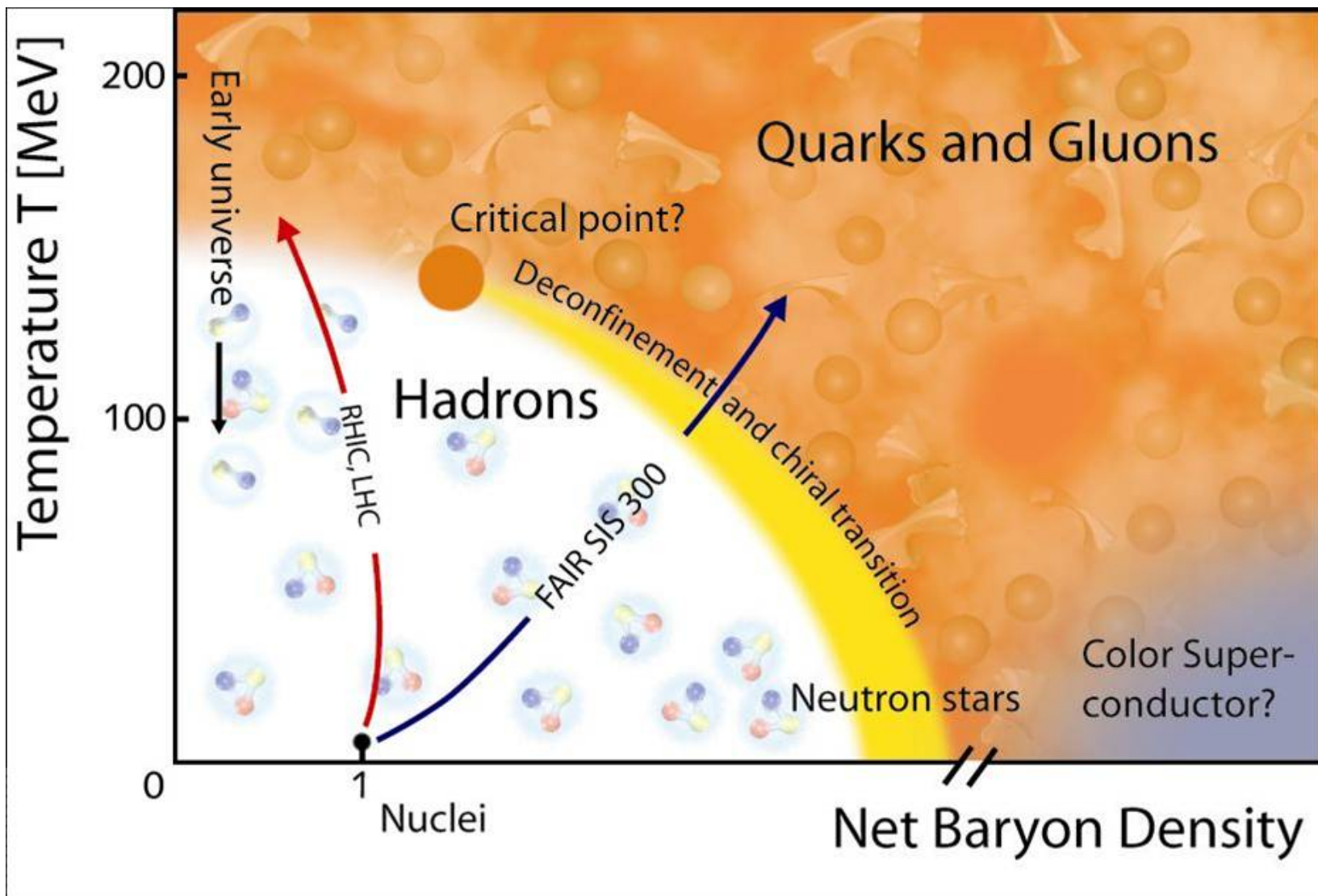
Works with

M. Rho, H. K. Lee, Y. Oh, Y.-L. Ma, G. Yang,
D.-P. Min, H.-J. Lee, J.-I. Kim, V. Vento

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- What?
- How?
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I. What are all about?



- the properties of hadrons in dense baryonic matter

II. How we do it?

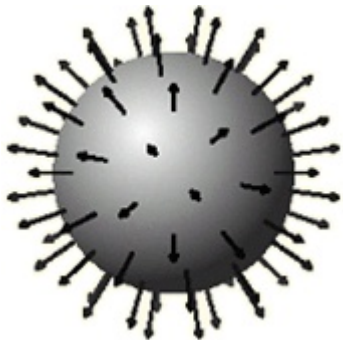
Skyrme Model

1960, T. H. R. Skyrme

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

$U(\vec{x}) = \exp(i\tau\pi/f_\pi)$: mapping from $R^3 - \{\infty\} = S^3$ to $SU(2) = S^3$

→ topological soliton



$R \sim 1 \text{ f m}$

$M \sim 1.5 \text{ GeV}$



baryon ?

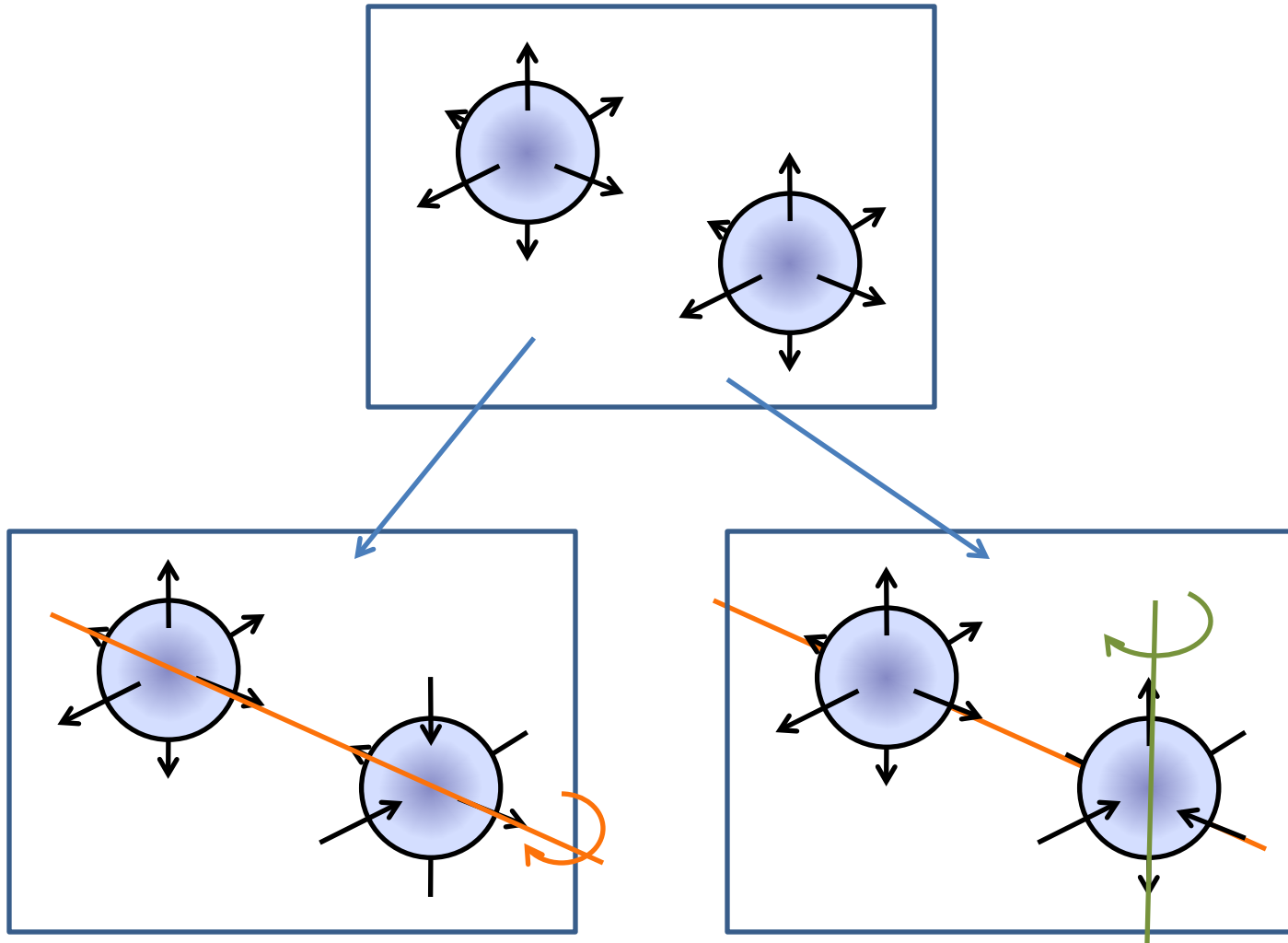
Classical object!



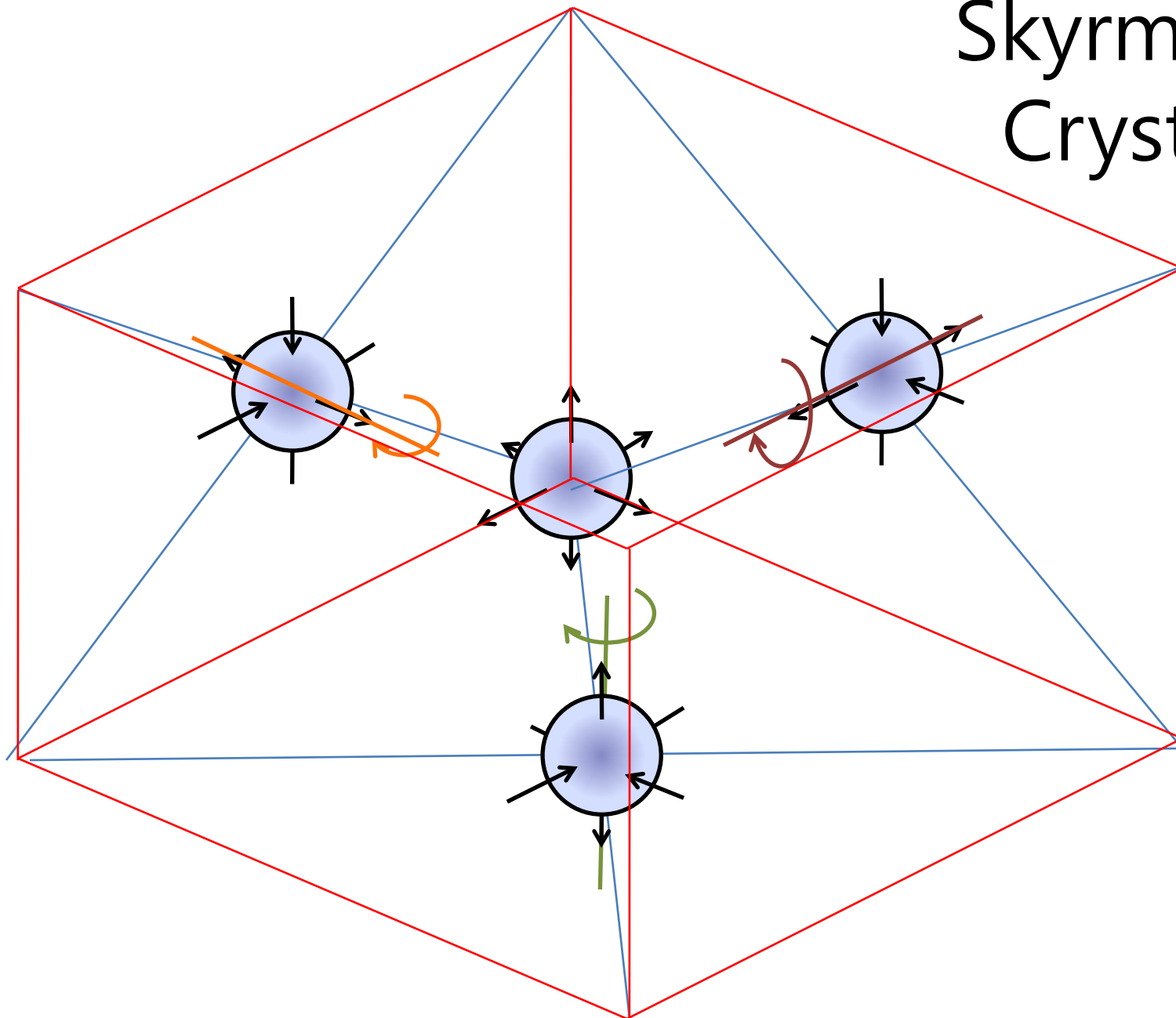
Easy to construct
many-body system

Two skyrmions

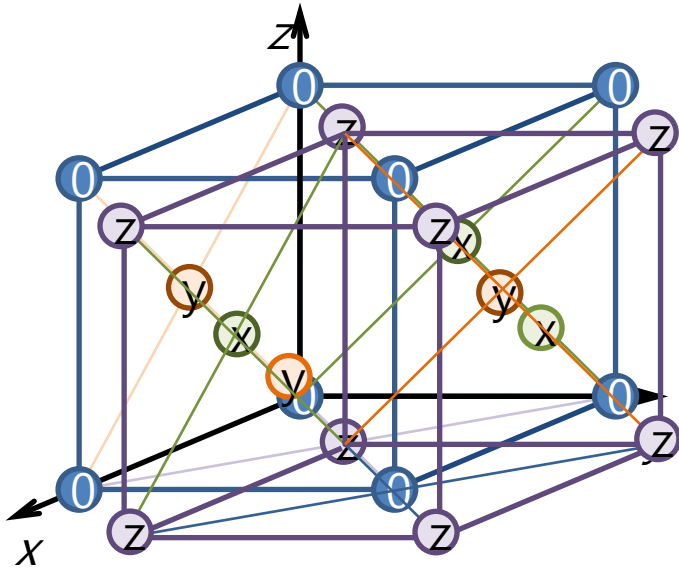
1960, T. H. R. Skyrme



Skyrmion Crystal



FCC Skymion crystal

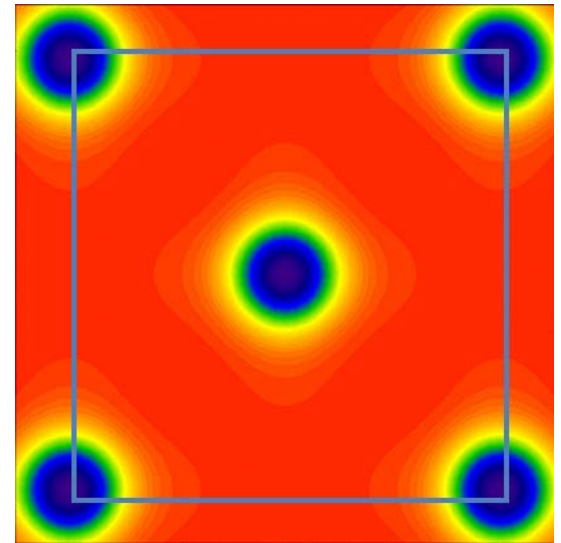
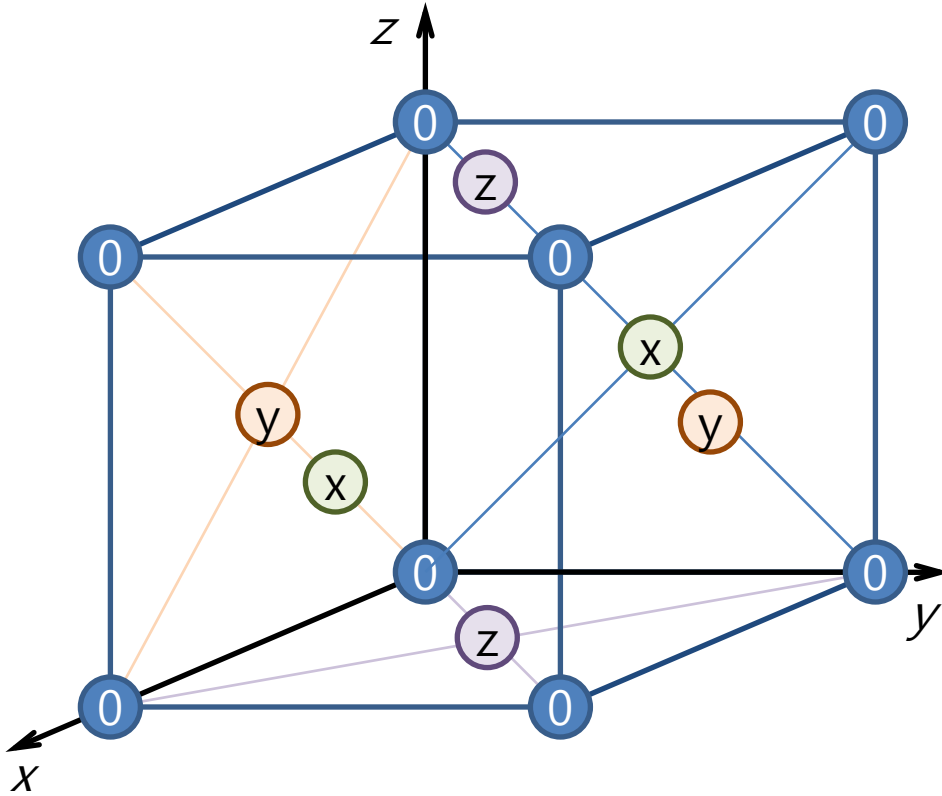


$$U(x+L/2, y+L/2, z) = \tau_z U(x, y, z) \tau_z$$

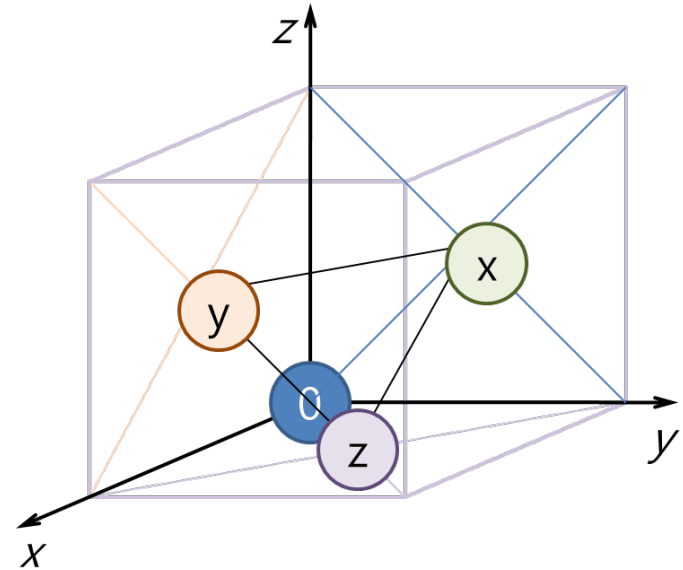
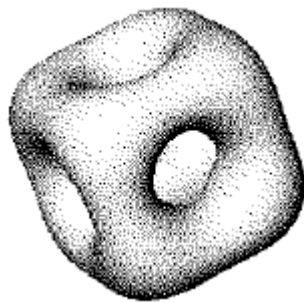
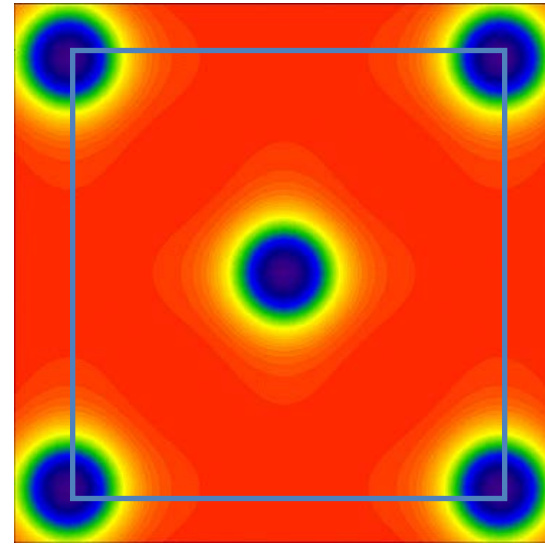
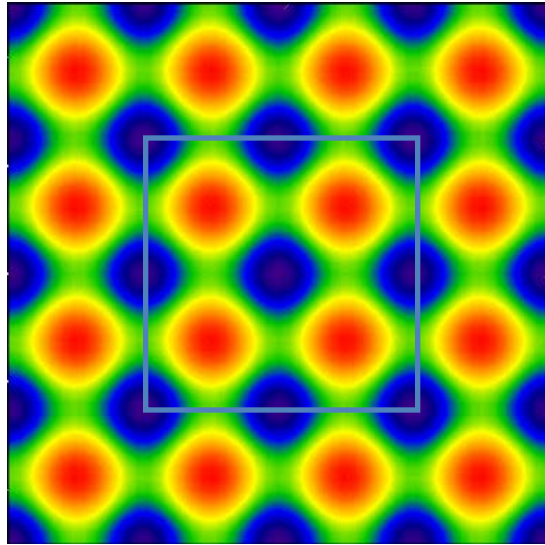
$$U(x+L/2, y, z+L/2) = \tau_y U(x, y, z) \tau_y$$

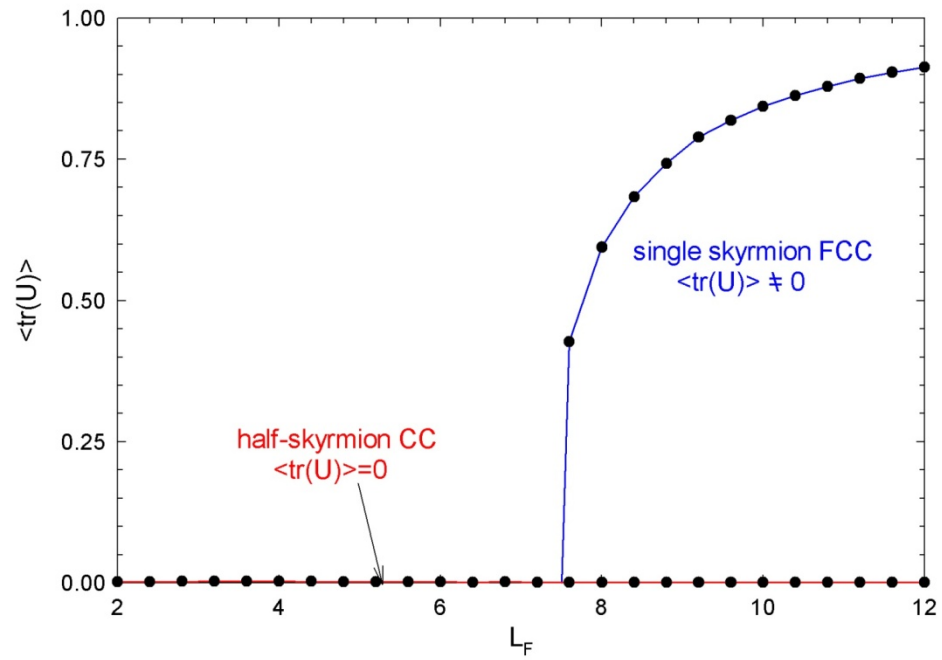
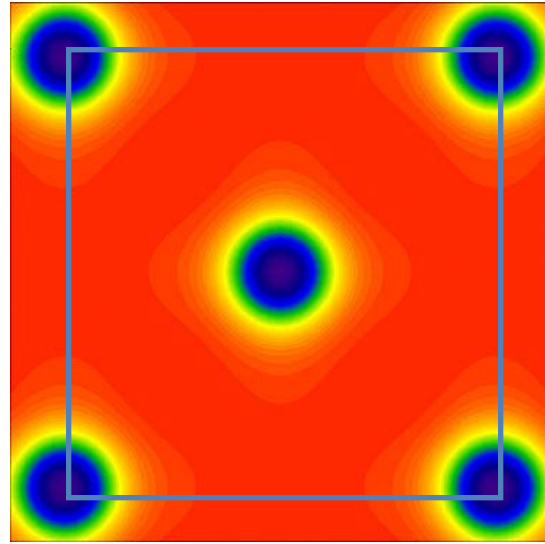
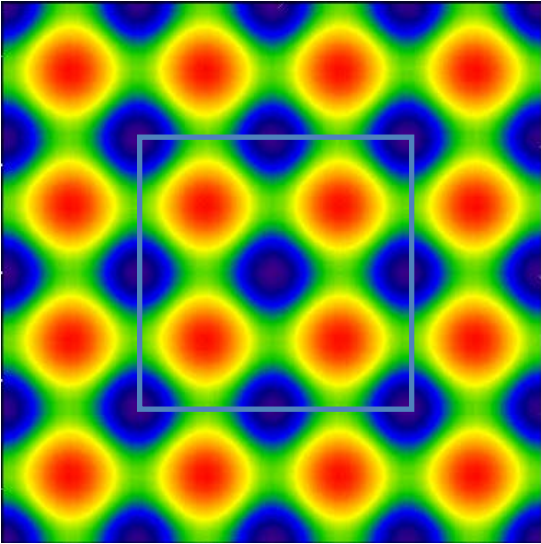
$$U(x, y+L/2, z+L/2) = \tau_x U(x, y, z) \tau_x$$

Skyrmion crystal(FCC)



Phase transition





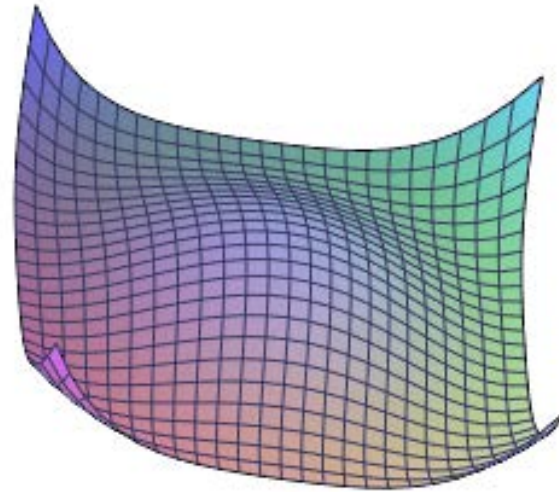
What we have found is the
classical solution
of the meson Lagrangian!




In-medium properties
of the mesons

Analogy

$$\lambda\phi^4 - \mu^2\phi^2$$



$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$


$$U_\pi = \exp(i\vec{\tau} \cdot \vec{\pi})$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \dots$$

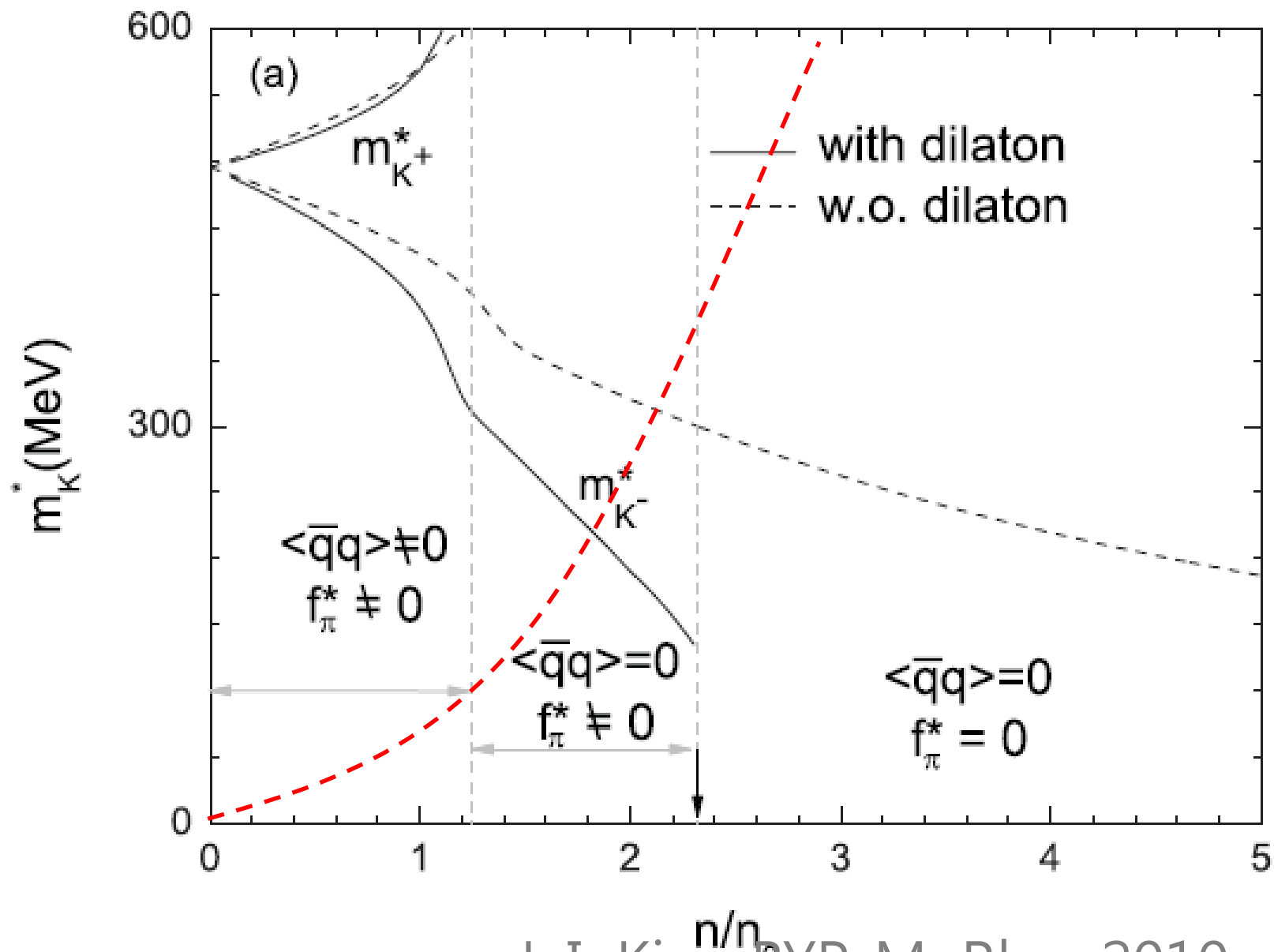
$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

$$U = \sqrt{U_\pi} U_B \sqrt{U_\pi}$$

$$\mathcal{L} = \frac{1}{2} G_{ab}(U_B) \partial_\mu \pi^a \partial^\mu \pi^b + \dots$$

$$\frac{f_\pi^*}{f_\pi} = \sqrt{\langle G_{aa}(U_B) \rangle}$$

Consequence?



J. I. Kim, B.Y.P., M. Rho, 2010

In-Medium Modification of Tensor Force

$$V_M^T(r) = S_M \frac{f_{NM}^{*2}}{4\pi} m_M^* \tau_1 \cdot \tau_2 S_{12} \\ \times \left(\left[\frac{1}{(m_{Mr}^*)^3} + \frac{1}{(m_{Mr}^*)^2} + \frac{1}{3m_{Mr}^*} \right] e^{-m_{Mr}^* r} \right)$$

III. Recent Work

Y. Ma, M. Harada, H. K. Lee, Y. Oh, B.-Y. Park,
M. Rho, to appear in Phys. Rev. D

the density $n_{1/2}$ at which the ~~skyrmions~~ Skyrmions in medium fractionize into ~~half-skyrmions~~ half-Skyrmions, bringing in a drastic change in the equation of state of dense baryonic matter. We find that the $U(1)$ field that figures in the Chern-Simons term in the ~~5-D~~ five-dimensional holographic QCD action or equivalently the ω field in the homogeneous Wess-Zumino term in the dimensionally reduced hidden local symmetry action plays a crucial role in the ~~half-skyrmion~~ half-Skyrmion phase. The importance of the ω degree of freedom may be connected to what happens in the instanton structure of elementary baryon noticed in holographic QCD. The most striking and intriguing in what is found in the model is that the pion decay constant that smoothly drops with increasing density in the ~~skyrmion~~ Skyrmion phase stops decreasing at $n_{1/2}$ and remains nearly constant in the ~~half-skyrmion~~ half-Skyrmion phase. In accordance with the large N_c consideration,

Full HLS Lagrangian up to $O(p^4)$

$$\mathcal{L}_{\text{HLS}} = \mathcal{L}_{(2)} + \mathcal{L}_{(4)} + \mathcal{L}_{\text{anom}}$$

$$\mathcal{L}_{(2)} = f_\pi^2 \text{Tr} (\hat{a}_{\perp\mu} \hat{a}_\perp^\mu) + a f_\pi^2 \text{Tr} (\hat{a}_{\parallel\mu} \hat{a}_\parallel^\mu) - \frac{1}{2g^2} \text{Tr} (V_{\mu\nu} V^{\mu\nu})$$

$$\begin{aligned} \mathcal{L}_{(4)y} = & y_1 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_\perp^\mu \hat{\alpha}_{\perp\nu} \hat{\alpha}_\perp^\nu] + y_2 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\perp\nu} \hat{\alpha}_\perp^\mu \hat{\alpha}_\perp^\nu] + y_3 \text{Tr} [\hat{\alpha}_{\parallel\mu} \hat{\alpha}_\parallel^\mu \hat{\alpha}_{\parallel\nu} \hat{\alpha}_\parallel^\nu] + y_4 \text{Tr} [\hat{\alpha}_{\parallel\mu} \hat{\alpha}_{\parallel\nu} \hat{\alpha}_\parallel^\mu \hat{\alpha}_\parallel^\nu] \\ & + y_5 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_\perp^\mu \hat{\alpha}_{\parallel\nu} \hat{\alpha}_\parallel^\nu] + y_6 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\perp\nu} \hat{\alpha}_\parallel^\mu \hat{\alpha}_\parallel^\nu] + y_7 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\perp\nu} \hat{\alpha}_\parallel^\nu \hat{\alpha}_\parallel^\mu] \\ & + y_8 \left\{ \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_\parallel^\mu \hat{\alpha}_{\perp\nu} \hat{\alpha}_\parallel^\nu] + \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\parallel\nu} \hat{\alpha}_\perp^\nu \hat{\alpha}_\parallel^\mu] \right\} + y_9 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\parallel\nu} \hat{\alpha}_\perp^\mu \hat{\alpha}_\parallel^\nu], \end{aligned}$$

$$\mathcal{L}_{(4)z} = iz_4 \text{Tr} [V_{\mu\nu} \hat{\alpha}_\perp^\mu \hat{\alpha}_\perp^\nu] + iz_5 \text{Tr} [V_{\mu\nu} \hat{\alpha}_\parallel^\mu \hat{\alpha}_\parallel^\nu].$$

$$\mathcal{L}_{\text{anom}} = \frac{N_c}{16\pi^2} [i \text{Tr} [\hat{\alpha}_L^3 \hat{\alpha}_R - \hat{\alpha}_R^3 \hat{\alpha}_L] + i \text{Tr} [\hat{\alpha}_L \hat{\alpha}_R \hat{\alpha}_L \hat{\alpha}_R] + \text{Tr} [F_V (\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L)]]$$

$$f_\pi^2 = N_c G_{\text{YM}} M_{\text{KK}}^2 \int dz K_2(z) \left[\dot{\psi}_0(z) \right]^2, \quad a f_\pi^2 = N_c G_{\text{YM}} M_{\text{KK}}^2 \lambda_1 \langle \psi_1^2 \rangle,$$

$$\frac{1}{g^2} = N_c G_{\text{YM}} \langle \psi_1^2 \rangle,$$

$$y_1 = -y_2 = -N_c G_{\text{YM}} \langle (1 + \psi_1 - \psi_0^2)^2 \rangle, \quad y_3 = -y_4 = -N_c G_{\text{YM}} \langle \psi_1^2 (1 + \psi_1)^2 \rangle,$$

$$y_5 = 2y_8 = -y_9 = -2N_c G_{\text{YM}} \langle \psi_1^2 \psi_0^2 \rangle, \quad y_6 = -y_5 - y_7,$$

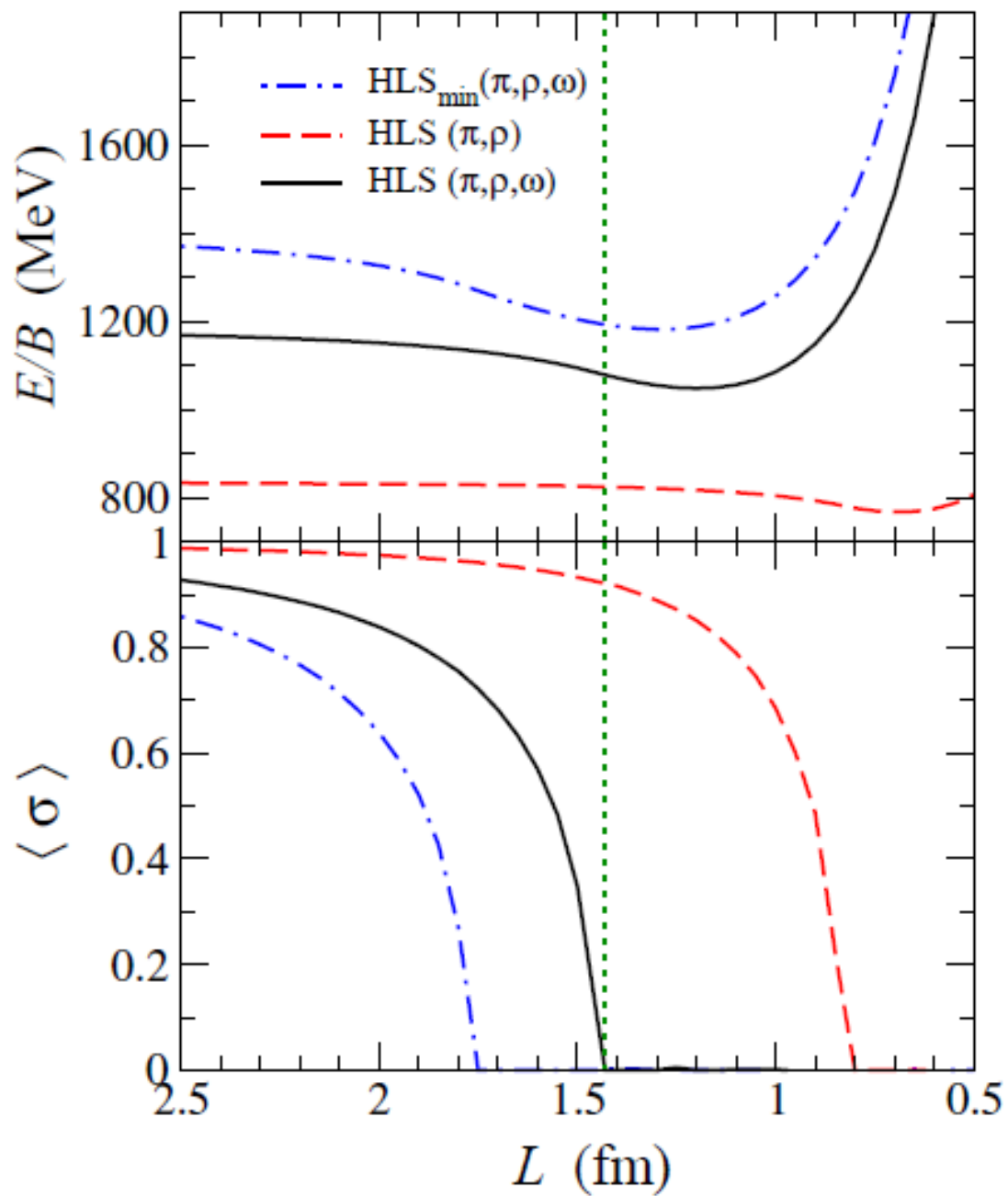
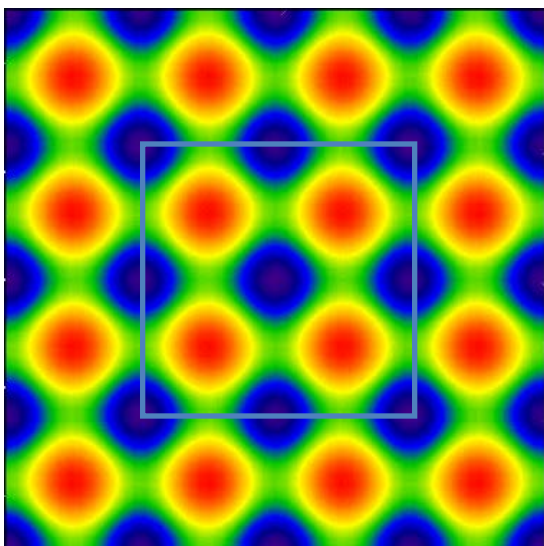
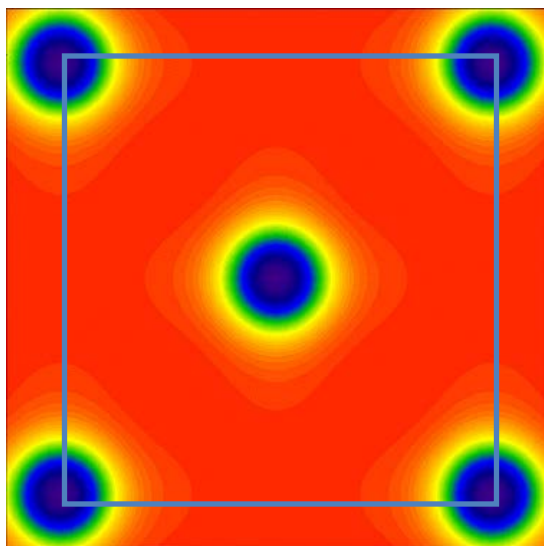
$$y_7 = 2N_c G_{\text{YM}} \langle \psi_1 (1 + \psi_1) (1 + \psi_1 - \psi_0^2) \rangle,$$

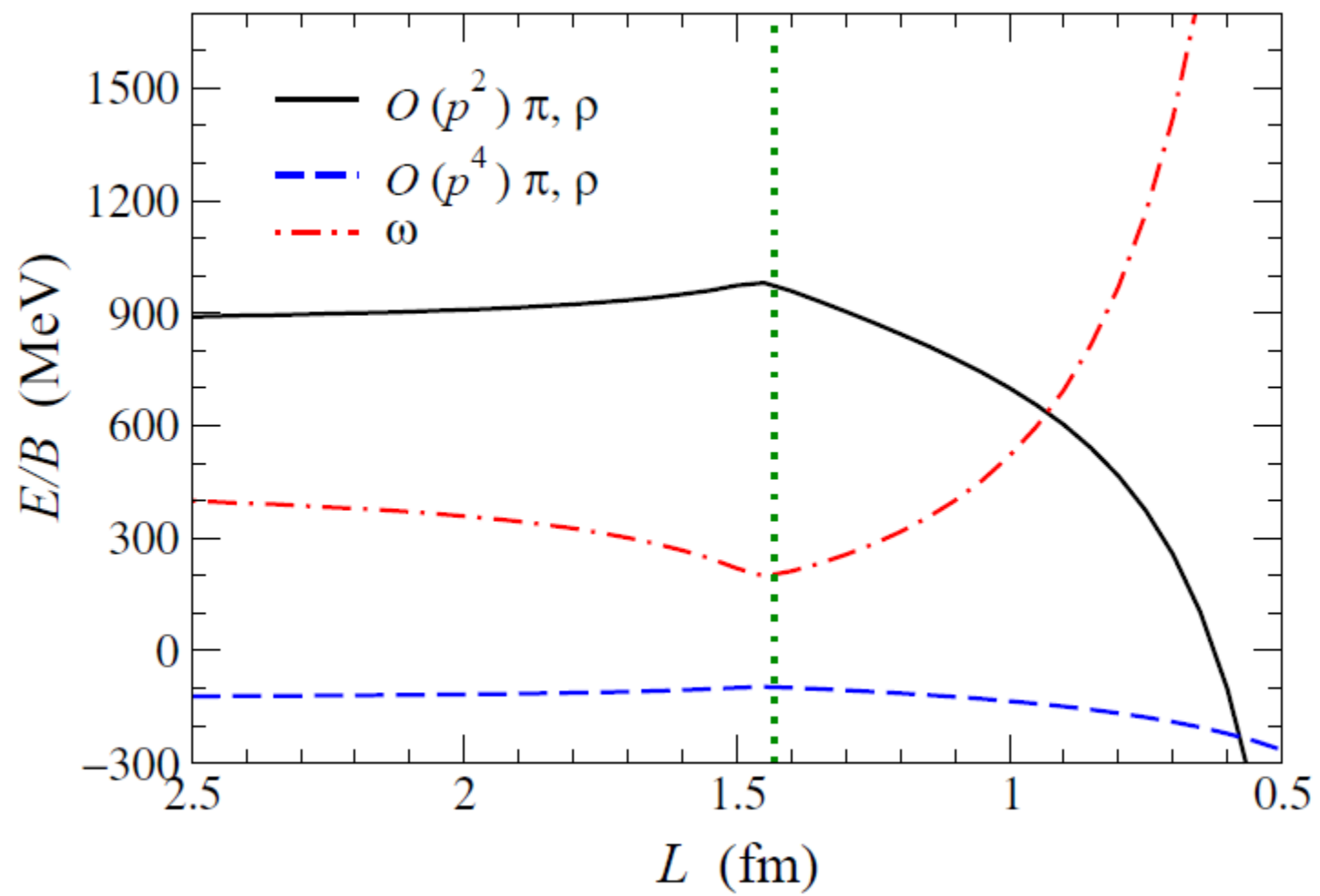
$$z_4 = 2N_c G_{\text{YM}} \langle \psi_1 (1 + \psi_1 - \psi_0^2) \rangle, \quad z_5 = -2N_c G_{\text{YM}} \langle \psi_1^2 (1 + \psi_1) \rangle,$$

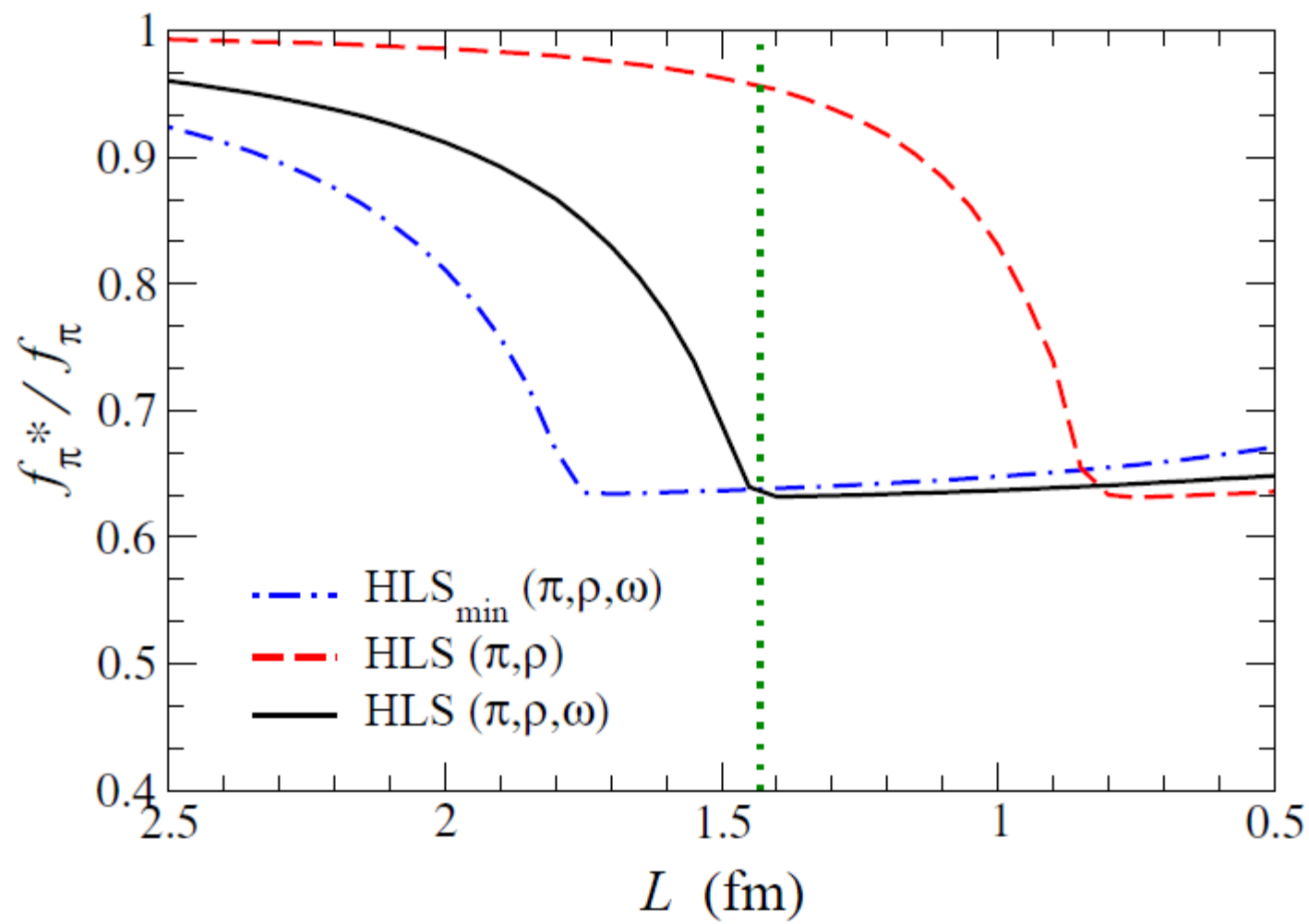
$$c_1 = \left\langle \left\langle \dot{\psi}_0 \psi_1 \left(\frac{1}{2} \psi_0^2 + \frac{1}{6} \psi_1^2 - \frac{1}{2} \right) \right\rangle \right\rangle,$$

$$c_2 = \left\langle \left\langle \dot{\psi}_0 \psi_1 \left(-\frac{1}{2} \psi_0^2 + \frac{1}{6} \psi_1^2 + \frac{1}{2} \psi_1 + \frac{1}{2} \right) \right\rangle \right\rangle$$

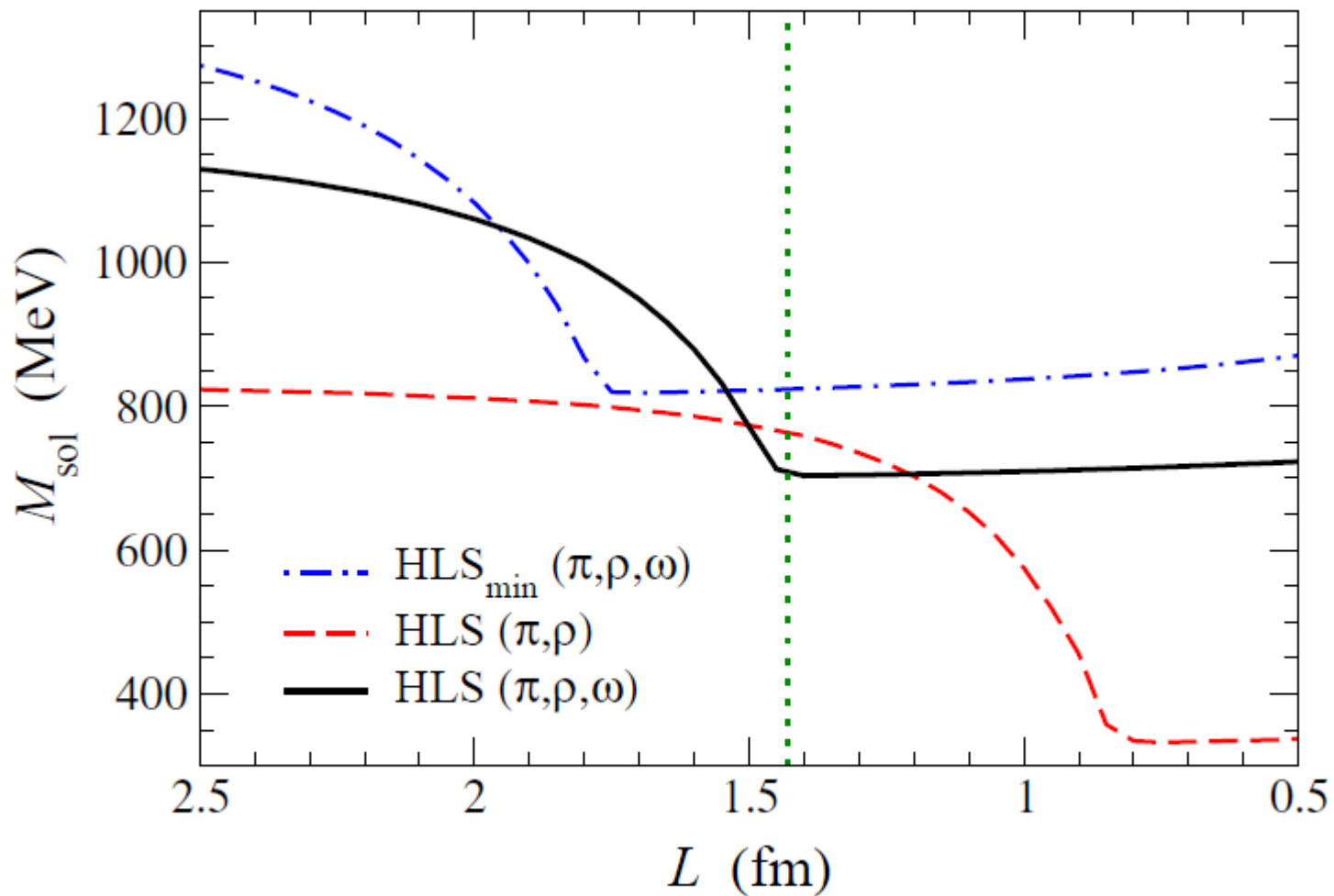
$$c_3 = \left\langle \left\langle \dot{\psi}_0 \psi_1 \left(\frac{1}{2} \psi_1 \right) \right\rangle \right\rangle,$$







f_{π}^* & m_{ρ}^*



Dilaton

Y. Ma, M. Harada, H. K. Lee, Y. Oh, B.-Y. Park,
M. Rho, [in preparation](#)

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

$$+ \frac{f_\pi^2 m_\pi^2}{4} \text{Tr}(U + U^\dagger - 2)$$

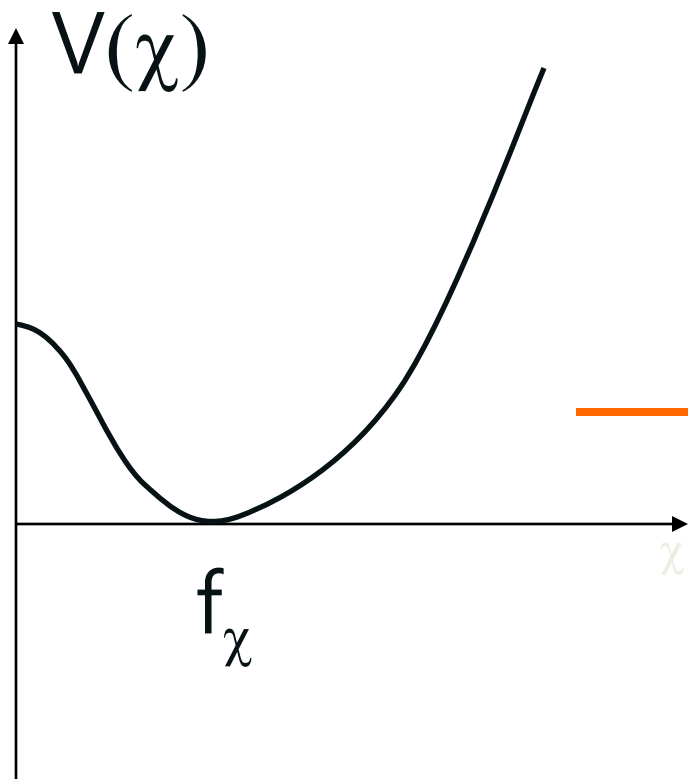
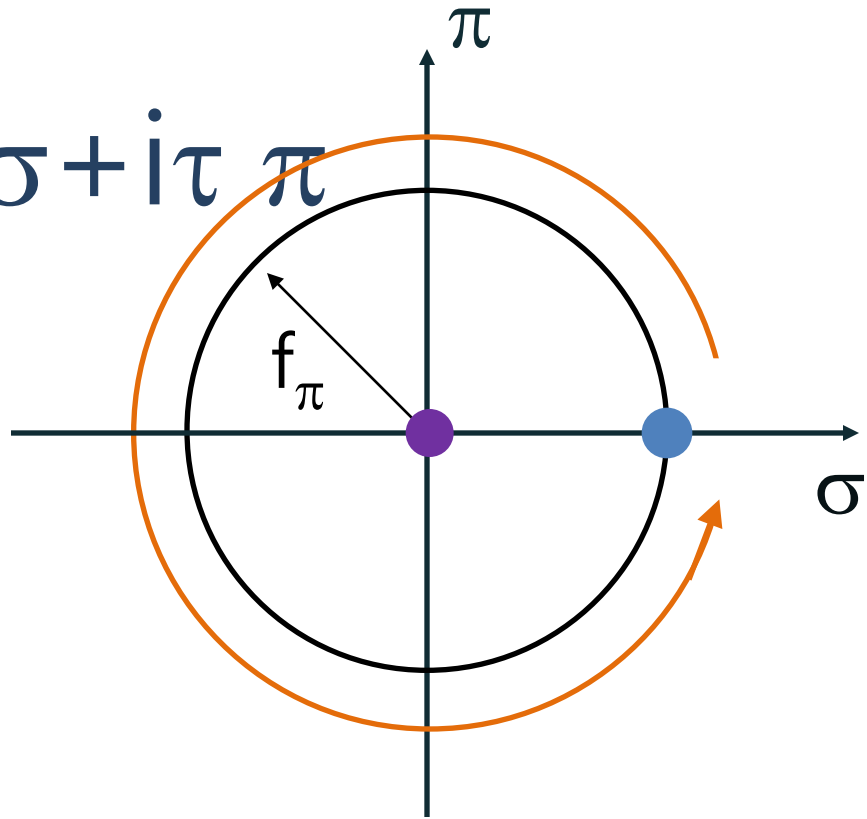
← Trace Anomaly of QCD

$$\mathcal{L} = \frac{f_\pi^2}{4} \left(\frac{\chi}{f_\chi} \right)^2 \text{Tr}(\partial_\mu U \partial^\mu U) + \frac{1}{32e^2} \text{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

$$+ \frac{f_\pi^2 m_\pi^2}{4} \left(\frac{\chi}{f_\chi} \right)^3 \text{Tr}(U^\dagger + U - 2)$$

$$+ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{4} m_\chi^2 f_\chi^2 \left((\chi / f_\chi)^4 (\ln(\chi / f_\chi) - \frac{1}{4}) + \frac{1}{4} \right)$$

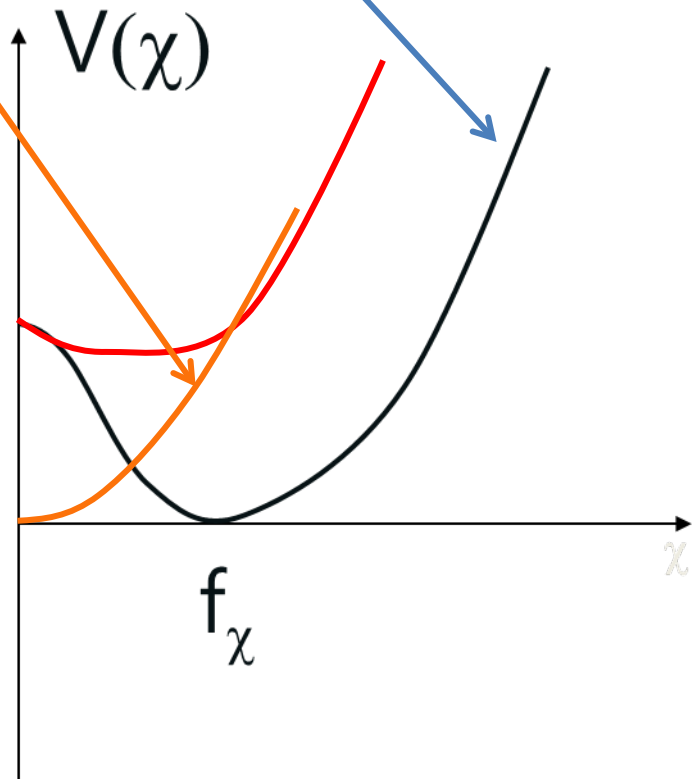
$$U \rightsquigarrow \chi U \sim \sigma + i\tau \pi$$

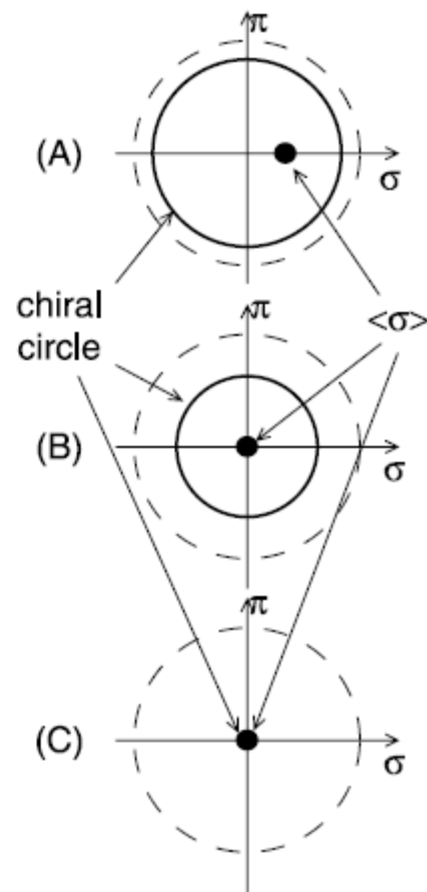
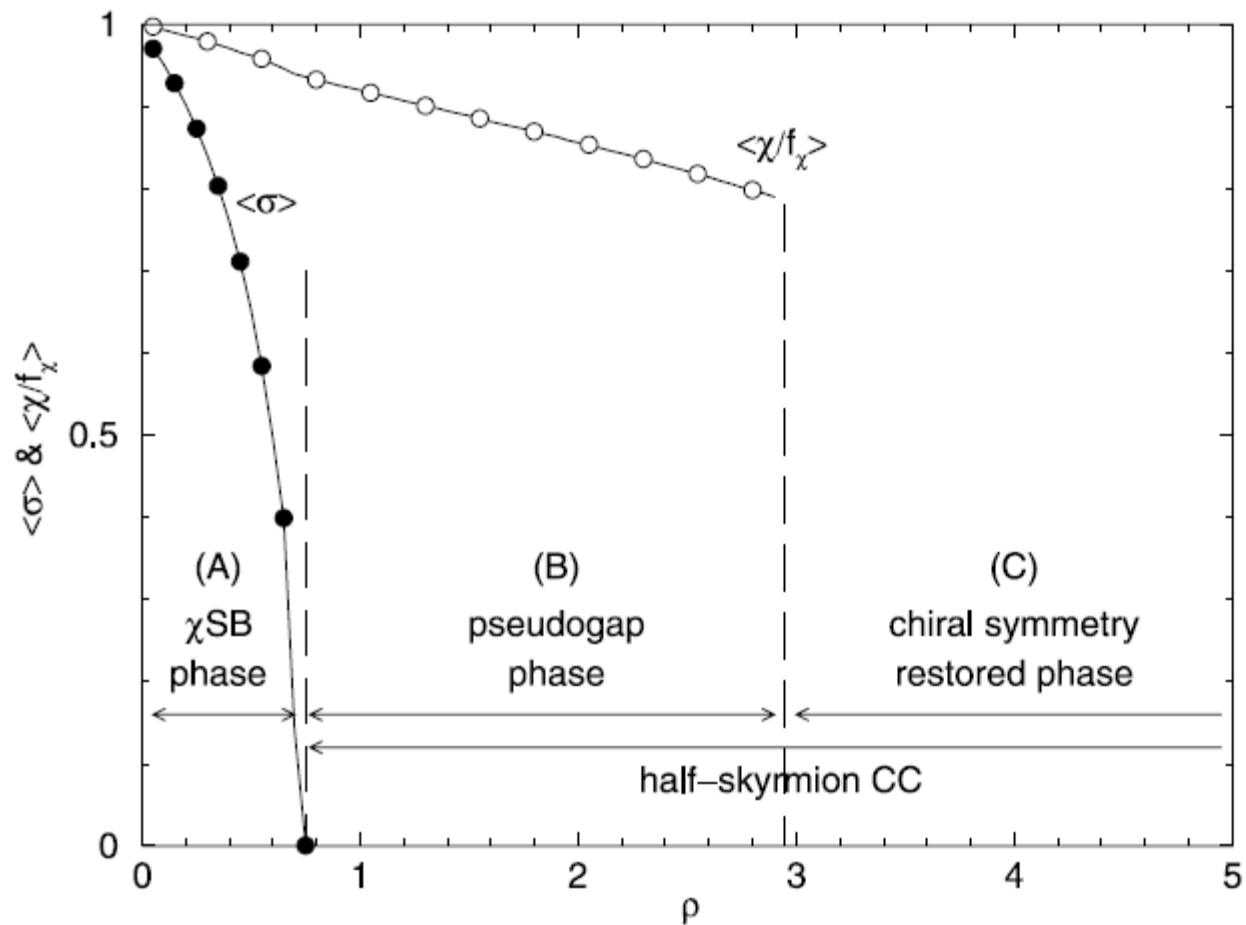


\longrightarrow Vacuum ($\rho=0$)
 $U=1$
 $\chi=f_\chi$

$$\mathcal{L} = \frac{f^2}{4} \left(\frac{\chi}{f_\chi} \right)^2 \text{Tr}(\partial_\mu U \partial^\mu U) + \frac{1}{32e^2} \text{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

$$+ \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \left(\frac{1}{4} m_\chi^2 f_\chi^2 \left(\left(\frac{\chi}{f_\chi} \right)^4 (\ln(\chi/f_\chi) - \frac{1}{4}) + \frac{1}{4} \right) \right)$$



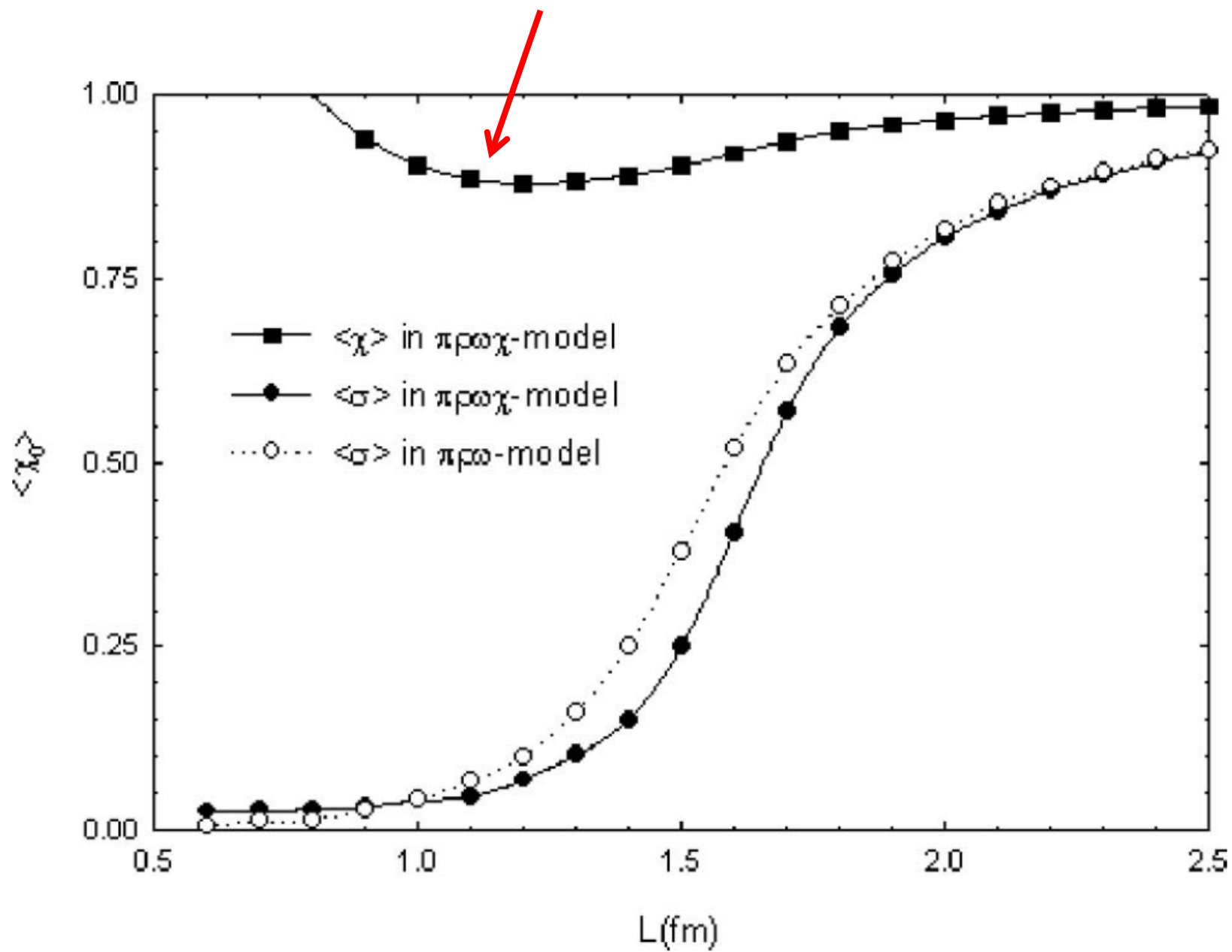


omega & dilaton

$$\mathcal{L} = \frac{f_\pi^2}{4} \left(\frac{\chi}{f_\chi} \right)^2 \text{Tr}(\partial_\mu U^\dagger \partial^\mu U). \quad \text{Minimal HLS}$$

$$\begin{aligned}
 & - \frac{f_\pi^2}{4} a \left(\frac{\chi}{f_\chi} \right)^2 \text{Tr}[\ell_\mu + r_\mu + i(g/2)(\vec{\tau} \cdot \vec{\rho}_\mu + \omega_\mu)]^2 \\
 & - \frac{1}{4} \vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{3}{2} g \omega_\mu B^\mu \\
 & + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{m_\chi^2 f_\chi^2}{4} \left[\left(\frac{\chi}{f_\chi} \right)^4 (\ln(\chi/f_\chi) - \frac{1}{4}) + \frac{1}{4} \right]
 \end{aligned}$$

KSRF relation : $m_V^2 = a f_\pi^2 g^2 \xrightarrow{\quad} m_V^*$



$$\begin{aligned}
\mathcal{L} = & \frac{f_\pi^2}{4} \left(\frac{\chi}{f_\chi} \right)^2 \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{f_\pi^2 m_\pi^2}{4} \left(\frac{\chi}{f_\chi} \right)^3 \text{Tr}(U + U^\dagger - 2) \\
& - \frac{f_\pi^2}{4} a \left(\frac{\chi}{f_\chi} \right)^2 \text{Tr}[\ell_\mu + r_\mu + i(g/2)(\vec{\tau} \cdot \vec{\rho}_\mu + \omega_\mu)]^2 \\
& - \frac{1}{4} \vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{3}{2} g \omega_\mu B^\mu + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi \\
& - \frac{m_\chi^2 f_\chi^2}{4} \left[(\chi/f_\chi)^4 \left(\ln(\chi/f_\chi) - \frac{1}{4} \right) + \frac{1}{4} \right],
\end{aligned}$$

$$(-\partial_i^2 + m_\omega^{*2})w = -\frac{3g}{2f_\pi^2} B_0$$



$$(E/B)_{\text{WZ}} = \frac{1}{4} \int_{\text{Box}} d^3x \int d^3x' \left(\frac{3g}{2} \right)^2 B_0(\vec{x}) \frac{\exp(-m_\omega^* |\vec{x} - \vec{x}'|)}{4\pi |\vec{x} - \vec{x}'|} B_0(\vec{x}')$$

$$\begin{aligned}
\mathcal{L} = & \frac{f_\pi^2}{4} \left(\frac{\chi}{f_\chi} \right)^2 \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{f_\pi^2 m_\pi^2}{4} \left(\frac{\chi}{f_\chi} \right)^3 \text{Tr}(U + U^\dagger - 2) \\
& - \frac{f_\pi^2}{4} a \left(\frac{\chi}{f_\chi} \right)^2 \text{Tr}[\ell_\mu + r_\mu + i(g/2)(\vec{\tau} \cdot \vec{\rho}_\mu + \omega_\mu)]^2 \\
& - \frac{1}{4} \vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{3}{2} g \omega_\mu B^\mu \left(\frac{\chi}{f_\chi} \right)^3 \chi \partial^\mu \chi \\
& - \frac{m_\chi^2 f_\chi^2}{4} \left[(\chi/f_\chi)^4 \left(\ln(\chi/f_\chi) - \frac{1}{4} \right) + \frac{1}{4} \right],
\end{aligned}$$

$$(-\partial_i^2 + m_\omega^{*2})w = -\frac{3g^*}{2f_\pi^2} B_0$$

$$w = -\frac{3g^*}{2f_\pi^2} \int d^3x' \frac{\exp(-m_\omega^* |\vec{x} - \vec{x}'|)}{4\pi |\vec{x} - \vec{x}'|} B_0(\vec{r}')$$

$$\begin{aligned}
\mathcal{L} = & \frac{f_\pi^2}{4} \left(\frac{\chi}{f_\chi} \right)^2 \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{f_\pi^2 m_\pi^2}{4} \left(\frac{\chi}{f_\chi} \right)^3 \text{Tr}(U + U^\dagger - 2) \\
& - \frac{f_\pi^2}{4} a \left(\frac{\chi}{f_\chi} \right)^2 \text{Tr}[\ell_\mu + r_\mu + i(g/2)(\vec{\tau} \cdot \vec{\rho}_\mu + \omega_\mu)]^2 \\
& - \frac{1}{4} \vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{3}{2} g \omega_\mu B^\mu + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi \\
& - \frac{m_\chi^2 f_\chi^2}{4} \left[\left(\chi/f_\chi \right)^4 \left(\ln(\chi/f_\chi) - \frac{1}{4} \right) + \frac{1}{4} \right],
\end{aligned}$$

U(2) sym.
Breaking?
2013

$$(-\partial_i^2 + m_\omega^2)w = -\frac{3g}{2f_\pi^2} B_0$$

$$w = -\frac{3g}{2f_\pi^2} \int d^3 x' \frac{\exp(-m_\omega |\vec{x} - \vec{x}'|)}{4\pi |\vec{x} - \vec{x}'|} B_0(\vec{r}')$$

Full HLS Lagrangian up to $O(p^4)$

$$\mathcal{L}_{\text{HLS}} = \mathcal{L}_{(2)} + \mathcal{L}_{(4)} + \mathcal{L}_{\text{anom}}$$

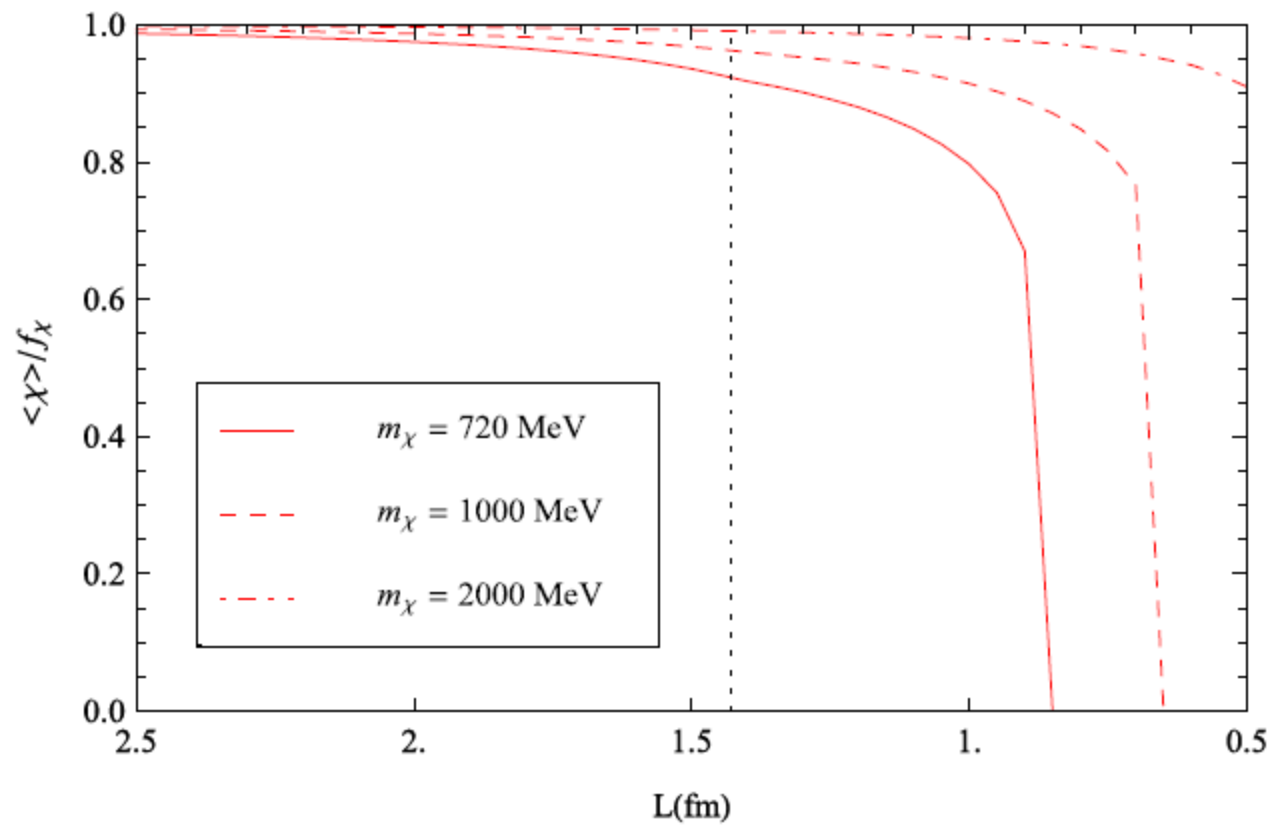
$$\mathcal{L}_{(2)} = f_\pi^2 \text{Tr} (\hat{a}_{\perp\mu} \hat{a}_{\perp}^\mu) + a f_\pi^2 \text{Tr} (\hat{a}_{\parallel\mu} \hat{a}_{\parallel}^\mu) - \frac{1}{2g^2} \text{Tr} (V_{\mu\nu} V^{\mu\nu})$$

$$\left(\frac{\chi}{f_\chi} \right)^2$$

$$\begin{aligned} \mathcal{L}_{(4)y} = & y_1 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\perp}^\mu \hat{\alpha}_{\perp\nu} \hat{\alpha}_{\perp}^\nu] + y_2 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\perp\nu} \hat{\alpha}_{\perp}^\mu \hat{\alpha}_{\perp}^\nu] + y_3 \text{Tr} [\hat{\alpha}_{\parallel\mu} \hat{\alpha}_{\parallel}^\mu \hat{\alpha}_{\parallel\nu} \hat{\alpha}_{\parallel}^\nu] + y_4 \text{Tr} [\hat{\alpha}_{\parallel\mu} \hat{\alpha}_{\parallel\nu} \hat{\alpha}_{\parallel}^\mu \hat{\alpha}_{\parallel}^\nu] \\ & + y_5 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\perp}^\mu \hat{\alpha}_{\parallel\nu} \hat{\alpha}_{\parallel}^\nu] + y_6 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\perp\nu} \hat{\alpha}_{\parallel}^\mu \hat{\alpha}_{\parallel}^\nu] + y_7 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\perp\nu} \hat{\alpha}_{\parallel}^\nu \hat{\alpha}_{\parallel}^\mu] \\ & + y_8 \left\{ \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\parallel}^\mu \hat{\alpha}_{\perp\nu} \hat{\alpha}_{\parallel}^\nu] + \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\parallel\nu} \hat{\alpha}_{\perp}^\nu \hat{\alpha}_{\parallel}^\mu] \right\} + y_9 \text{Tr} [\hat{\alpha}_{\perp\mu} \hat{\alpha}_{\parallel\nu} \hat{\alpha}_{\perp}^\mu \hat{\alpha}_{\parallel}^\nu], \end{aligned}$$

$$\mathcal{L}_{(4)z} = iz_4 \text{Tr} [V_{\mu\nu} \hat{\alpha}_{\perp}^\mu \hat{\alpha}_{\perp}^\nu] + iz_5 \text{Tr} [V_{\mu\nu} \hat{\alpha}_{\parallel}^\mu \hat{\alpha}_{\parallel}^\nu].$$

$$\mathcal{L}_{\text{anom}} = \frac{N_c}{16\pi^2} \left[i \text{Tr} [\hat{\alpha}_L^3 \hat{\alpha}_R - \hat{\alpha}_R^3 \hat{\alpha}_L] + i \text{Tr} [\hat{\alpha}_L \hat{\alpha}_R \hat{\alpha}_L \hat{\alpha}_R] + \text{Tr} [F_V (\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L)] \right]$$



IV. Summary & Discussion

Summary

We have applied the skyrmion picture to

- dense baryonic matter
- in-medium hadron properties

Skyrmion picture provides us

- simple & easy tools
- non-trivial information (to be checked by experiments)

If you need to modify the hadron properties to analyze or fit data from experiments or the observations related to dense matter, please remember



감사합니다!