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Holographic Approach to QCD

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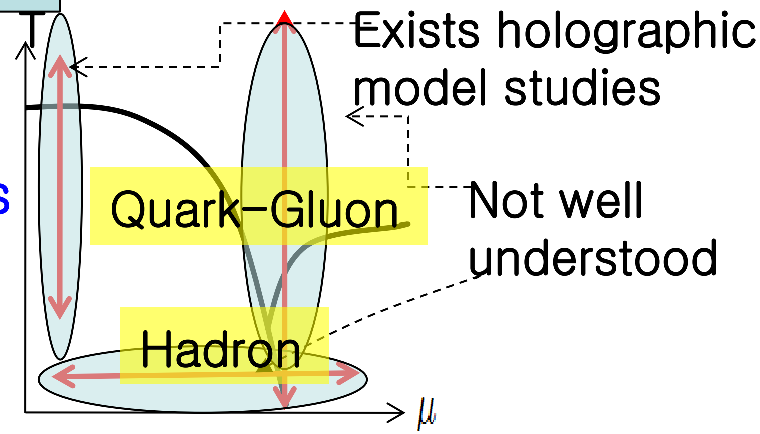
Contents

I. Introduction ：Motivation & Basics

◆ Holography idea of AdS/CFT applied to QCD is called AdS/QCD

◆ With AdS-QCD, how to explain properties such as
  confinement, χ-symm breaking,
  phases, spectra, etc. ?



◆ AdS-CFT or Holography Parameter in extra "dimension" of Energy or "radius"
3+1 dim. QFT (large Nc) with running coupling constants
<-> 4+1 dim. Effective Gravity description

Ex) 4d QFT(on"boundary"open string) <-> 5d Gravity (in"bulk",close string)

- with $\beta(g^2) = 0$ (conf. inv.) in Anti-deSitter Space

N=4 SU(Nc) SYM ⟷ Nc of D3 branes ⟷ SUGRA on AdS5 x S5

$(g_{YM}^2 \,,\, N)$ $\quad 4\pi g_s N = \dfrac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N \quad$ $(g_s \,,\, R)$

- with $\beta(g^2) \to 0$ (asym. freedom) in asymptotic AdS
- QCD ??

◆ AdS-CFT Holography : Useful tool for strongly interacting QFT such as QCD, Condensed Matter , etc.

Main idea on holography through the Dp branes

* Dp branes carry tension (energy) and charge (source for p+2 form)
  → Gravity in AdS space (dim = ((p+1)+1))

Ex) D3 brane : $ds^2 = f^{-1/2}dx_{||}^2 + f^{1/2}(dr^2 + r^2 d\Omega_5^2)$ $\quad f = 1 + \frac{4\pi g N \alpha'^2}{r^4}$

ϕ = constant (conformal symmetry)

The near horizon limit gives AdS x S5 $\quad \alpha' \to 0\ ,\quad U \equiv \frac{r}{\alpha'} = \text{fixed}$

$$ds^2 = \alpha' \left[\frac{U^2}{\sqrt{4\pi g N}} dx_{||}^2 + \sqrt{4\pi g N}\frac{dU^2}{U^2} + \sqrt{4\pi g N}\, d\Omega_5^2 \right]$$

AdS5 x S5

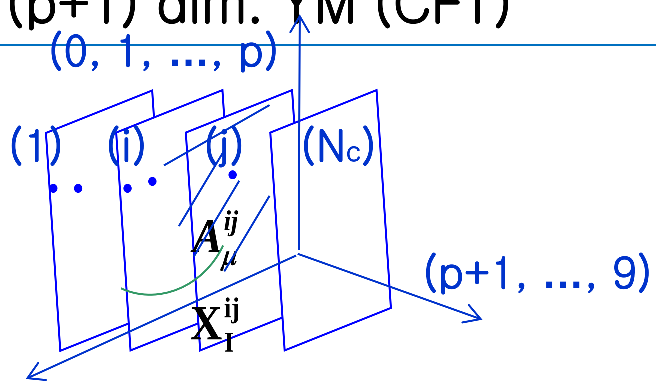
the radius of S_5
= the radius of AdS_5

$$R_{sph}^2/\alpha' = \sqrt{4\pi g N}$$

$$ds^2_{AdS(p+2)} = R^2 \left(\frac{dy^2 - dt^2 + dx_1^2 + dx_2^2 + \ldots + dx_p^2}{y^2} \right)$$

y= 1/r

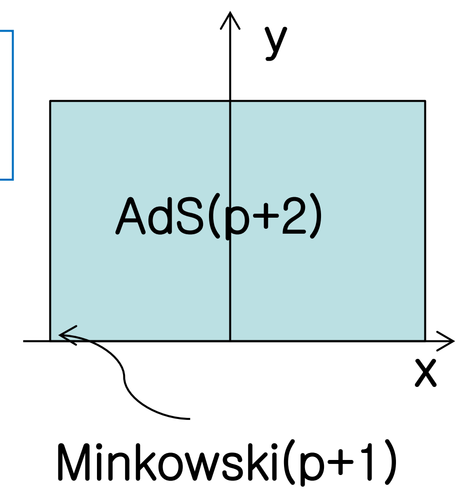
- Dp brane low E dynamics (fluctuating open strings)
  → (p+1) dim. YM (CFT)



$A_\mu, \mu = 0,1,\ldots,p$

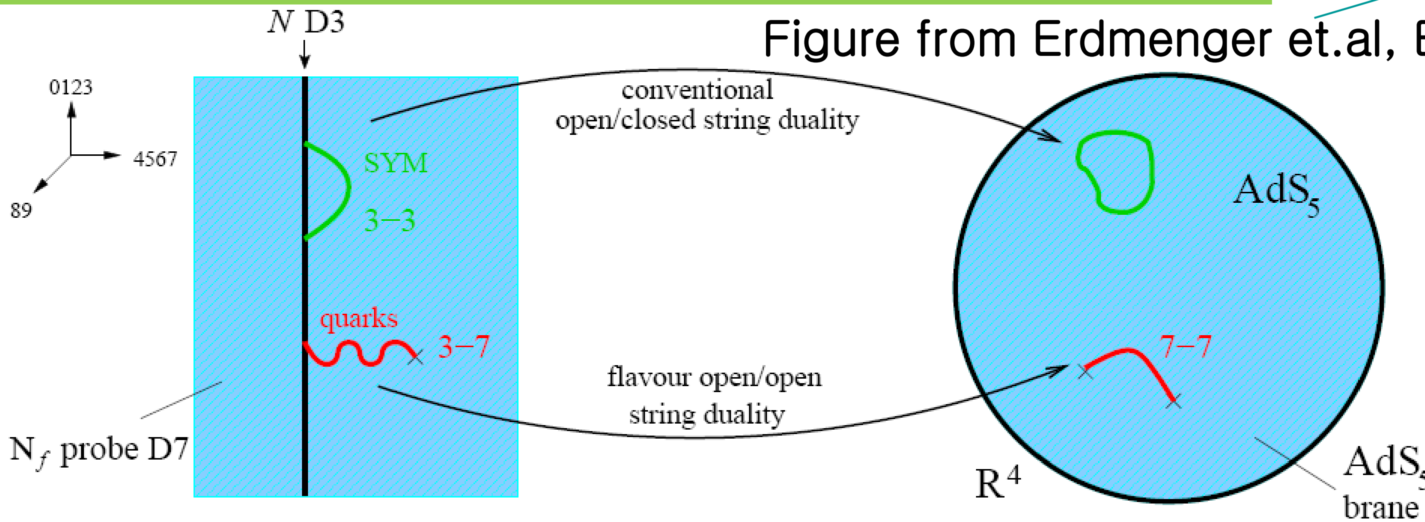
$X_I, I = p+1,\ldots,q$

$$g_s = e^{\phi} = g_{YM}^2$$



Intersecting D-Branes – Flavors, mesons, etc.

Figure from Erdmenger et.al, EPJA (2008)



7−7 open strings : Low energy dynamics for D7 branes (DBI action)

$$S_{D7} = -\mu_7 \int d^8\xi \sqrt{-\det\left(P[G]_{ab} + 2\pi\alpha' F_{ab}\right)} + \frac{(2\pi\alpha')^2}{2}\mu_7 \int P[C^{(4)}] \wedge F \wedge F$$

$$\mu_7 = [(2\pi)^7 g_s \alpha'^4]^{-1}$$

Still far from QCD !

Extension of the AdS/CFT

- the gravity theory on the asymptotically AdS space
  −> modified boundary quantum field theory (nonconf, less SUSY, etc.)

- Gravity w/ black hole background corresponds to the finite T field theory

AdS/CFT Dictionary

Witten 98; Gubser, Klebanov, Polyakov 98

Parameters (g_s , R) $4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N$ (g_{YM}^2, N)

Partition function of bulk gravity theory (semi-classial)

$$Z_{str}[\phi_0(x)] = \int_{\phi_0} \mathcal{D}\phi \exp(-S[\phi, g_{\mu\nu}])$$

$$\phi(t, \mathbf{x};\ u = \infty) = u^{\Delta-4}\phi_0(t, \mathbf{x})$$

ϕ_0 bdry value of the bulk field ϕ

Generating functional of bdry QFT for operator $\mathcal{O}$

$$Z[\phi_0(x)] = \left\langle \exp \int_{boundary} d^d x \phi_0 \mathcal{O} \right\rangle$$

$$= \int \mathcal{D}[\Phi] \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\}$$

ϕ_0 : source of the bdry op. $\mathcal{O}$

- ϕ: scalar → $S = \int d^4x du \sqrt{-g}\left(g^{ab}\partial_a\phi\partial_b\phi - m^2\phi^2\right)$ $\phi(u) \sim u^{4-\Delta}\phi_0 + u^{\Delta}\langle\mathcal{O}\rangle$
- Correlation functions by $\frac{\delta^n Z_{\rm string}}{\delta\phi_0(t_1,\mathbf{x}_1)\cdots\delta\phi_0(t_n,\mathbf{x}_n)} = \left\langle T\,\mathcal{O}(t_1,\mathbf{x}_1)\cdots\mathcal{O}(t_n,\mathbf{x}_n)\right\rangle_{\rm field\ theory}$
- 5D bulk field ϕ ←→ Operator $\mathcal{O}$
  w/ 5D mass $E(\lambda, J_1, J_2, \cdots)$ ←→ w/ Operator dimesion $\Delta(\lambda, J_1, J_2, \cdots)$
- 5D gauge symmetry ←→ Current (global symmetry)
- Radial coord. r in the bulk is proportional to the energy scale E of QFT

$\mathcal{O}$(Operator in QFT) <-> ϕ (p-form Field in 5D)

$$(\Delta - p)(\Delta + p - 4) = m_5^2$$

Δ : Conformal dimension
m_5^2 : mass (squared)

Ex)

4D: $\mathcal{O}(x)$	5D: $\phi(x, z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3
$\langle \mathrm{Tr} G^2 \rangle$ Gluon cond .	dilaton	0	4	0
$\bar{q}_L \gamma^\mu q_L$, $\bar{q}_R \gamma^\mu q_R$ baryon density	vector w/ U(1)	1	3	0

fields in gravity

- massless dilaton
- scalar field with $m^2 = -\frac{3}{R^2}$
- m=0 vector field A_μ in the $SU(N_f)$ gauge group

dual

operators of QCD

- gluon condensation $\langle \mathrm{Tr} G^2 \rangle$
- chiral condensation $\bar{q}_R q_L$
- mesons in the $SU(N_f)$ flavor group

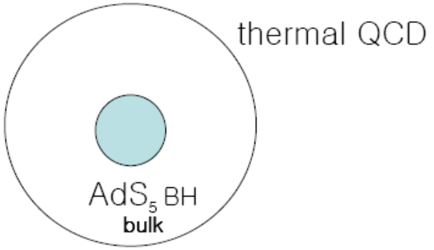
Temperatue

- Black hole gemometry
- $T = \frac{r_T}{\pi R^2}$

E. Witten (1998)

$$ds_5^2 = \frac{1}{z^2}\left(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2\right),$$

$$f^2(z) = 1 - (\frac{z}{z_T})^4 \qquad T = \frac{1}{\pi z_T}$$



Flavor degrees of freedom

- Adding probe brane
- $y(\rho) = M_q + \frac{<\bar{\psi}\psi>}{\rho^2} + \cdots \qquad (\rho >> 1)$

Chemical potential or Density

- Turning on $U(1)$ gauge field on probe brane
- $A_\mu \leftrightarrow < J^\mu > = \bar{\psi}\gamma^\mu\psi$
- $A_t = \mu + \frac{Q}{\rho^2} + \cdots \qquad (\rho >> 1)$

Source of gauge field

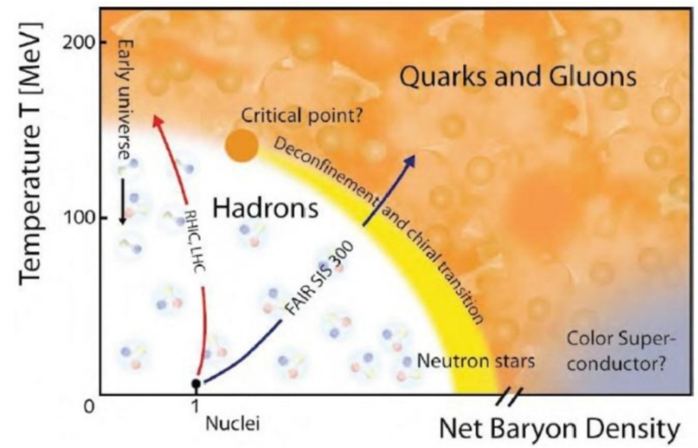
- End point of fundamental strings
- Physical object which carry $U(1)$ baryon charge
- Fundamental strings which connect probe brane and black hole
  $\longrightarrow$ Quarks
- Fundamental strings which connect probe brane and baryon vertex
  $\longrightarrow$ Baryons

II. AdS/QCD

– Holography idea of AdS/CFT applied to QCD

Goal : Using the 5 dim. dual gravity, study 4dim. QCD properties such as spectra & Phases (confining, deconfining, etc.), etc., in terms of parameters (Nc, Nf, m_q, T and μ, χ–symm., gluon condensation, etc.)

Needs the dual geometry of QCD.



Approaches :

- Top–down Approach : rooted in string theory
  Find brane config. or SUGRA solution giving the gravity dual (May put the probe brane)
  Ex) Nc of D3(D4) + M of D7(D8),
  10Dim. SUGRA solution etc.

- Bottom–up Approach:phenomenological
  Introduce fields, etc. (as needed based on AdS/CFT)
  5D setup → 4D effective Lagrangian
  * Kaluza–Klein modes
    – radial excitations of hadrons
  * Confinement in Hard/Soft Wall Model
    – by IR brane/dilaton running

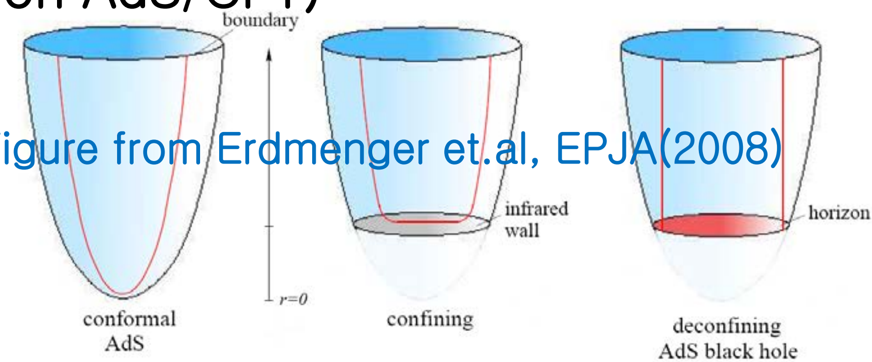
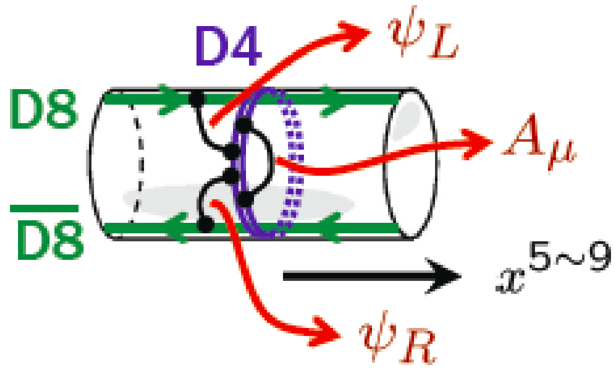


Figure from Erdmenger et.al, EPJA(2008)

Nc D4 + D8 D8 bar (Sakai-Sugimoto)

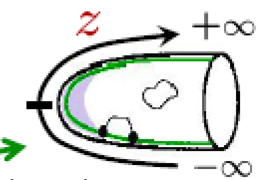
Sakai, Sugimoto hep-th/0412141, 0507073



Topology of the background :

$$\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$$

x^μ $\quad (y, z)$



D8 extended along (x,z) x S4

Effective Action on D8 (reducing S4)
for the Open Strings (Mesons)

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$K(z) = 1 + z^2$ $\qquad$ CS5-form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr}\left(\frac{1}{2} K(z)^{-1/3} F_{\mu\nu}^2 + K(z) F_{\mu z}^2\right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$\mu, \nu = 0 \sim 3$

Mode Expansion

$$A_\mu(x^\mu, z) = \sum_{n \geq 1} B_\mu^{(n)}(x^\mu) \psi_n(z)$$

$$A_z(x^\mu, z) = \sum_{n \geq 0} \varphi^{(n)}(x^\mu) \phi_n(z)$$

Some complete sets

$\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\quad \cdots$

Mesons unified
in 5dim theory

Ex) D3&D(−1) + D7 Zero T and without density

Background Metric by D3 & D−instantons

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(dt^2+d\vec{x}^2\right)+\frac{R^2}{r^2}\left(dr^2+r^2 d\Omega_5^2\right)\right],$$

$$e^{\Phi} = 1+\frac{q}{r^4}, \qquad \chi = -e^{-\Phi}+\xi_\infty .$$

(Liu & Tseytlin 9903091)

- AdSxS5 at UV
  Flat at IR (w/ dilaton singular)
- N=2 (with gluon condensation)

$$\frac{R^2}{r^2}(d\rho^2+\rho^2 d\Omega_3^2+dw_5^2+dw_6^2)$$

$$\rho^2 = w_1^2+\ldots+w_4^2$$

$$r^2 = \rho^2+w_5^2+w_6^2 = \rho^2+y^2$$

D7 Brane as a Probe

Induced metric on D7

$$ds^2_{D7} = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(dt^2+d\vec{x}^2\right)+\frac{R^2}{r^2}\left((1+y'^2)d\rho^2+\rho^2\Omega_3^2\right)\right]$$

DBI action of D7

$$S_{D7} = -\mu_7\int d^8\xi\,\sqrt{-\det\left(P[G]_{ab}+2\pi\alpha' F_{ab}\right)}+\frac{(2\pi\alpha')^2}{2}\mu_7\int P[C^{(4)}]\wedge F\wedge F$$

$$\mu_7 = [(2\pi)^7 g_s \alpha'^4]^{-1}$$

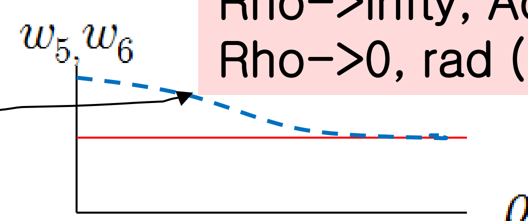
becomes (D7 wraps S3 of S5)

$$S_{D7} = -\tau_7\int dt d\rho\; e^{\Phi}\rho^3\sqrt{1+y'^2}$$

$$\tau_7 = \mu_7 V_3 \Omega_3$$

Embedding solution :

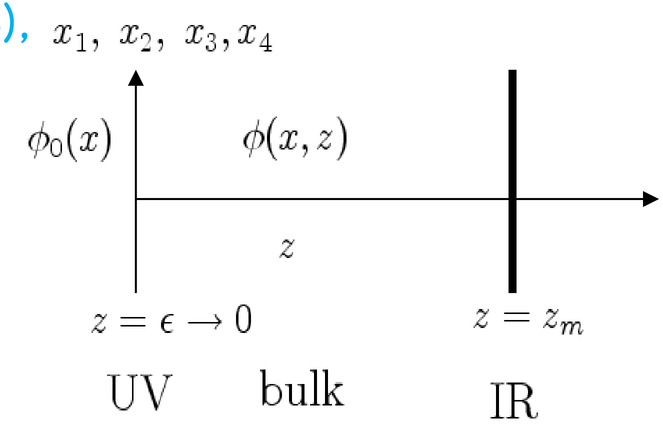
$$y(\rho) = M_q+\frac{<\bar{\psi}\psi>}{\rho^2}+\cdots \qquad (\rho >> 1)$$



Rho−>infty, AdS5xS3
Rho−>0, rad (S3)−>0

Ex) Hard wall Model

Erlich,Katz,Son,Stephanov PRL(2005),
Da Rold, Pomarol NPB(2005)



IR Brane at $z = z_m$ ⟹ Confinement

Metric – Slice of AdS metric

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu), \qquad 0 < z \leq z_m$$

5D action (Nf=2) (for χ–symm breaking)

$$S = \int d^5x\sqrt{-g}\left(-\frac{1}{2g_5^2}\mathrm{Tr}\left(L_{MN}L^{MN} + R_{MN}R^{MN}\right) + \mathrm{Tr}\left(|D_MX|^2 + m_X^2|X|^2\right)\right)$$

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3$$

$$\Sigma = \sigma 1 \qquad M = m_q 1$$

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6±0.0004 [8]	139.6*	141
m_ρ	775.8±0.5 [8]	775.8*	832
m_{a_1}	1230±40 [8]	1363	1220
f_π	92.4±0.35 [8]	92.4*	84.0
$F_\rho^{1/2}$	345±8 [15]	329	353
$F_{a_1}^{1/2}$	433±13 [6, 16]	486	440
$g_{\rho\pi\pi}$	6.03±0.07 [8]	4.48	5.29

<u>Parameters</u>

$m_q \quad \sigma \quad z_m$

$$g_5^2 = \frac{12\pi^2}{N_c}$$

(# of colors = input parameter)

AdS/CFT at finite temperature

Witten '98

(for the pure Yang-Mills theory without quark matters)

1) Low T (confining phase) : tAdS (thermal) AdS space(w/IR cutoff) no stable AdS black hole

at β(=1/Tc) ↕ QCD Phase transition ⟺ dual ⟺ ↕ Hawking-Page transition =Transition of bulk geometry

2) High T (deconfining phase) : AdS BH Schwarzschild AdS blackhole is stable

Hawking-Page phase transition

[Herzog , Phys.Rev.Lett.98:091601,2007]

The geometry is described by the following action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{g}\,(-\mathcal{R}+2\Lambda)$$

$\Lambda = -\frac{6}{R^2}$: cosmological constant

The regularized on-shell action

β' : arbitrary

1) for the tAdS, $S_{tAdS} = \frac{4R^3}{\kappa^2}\int_0^{\beta'} dt \int_\epsilon^{z_{IR}} dz \frac{1}{z^5}$

2) for the AdS BH $S_{AdSBH} = \frac{4R^3}{\kappa^2}\int_0^{\pi z_h} dt \int_\epsilon^{z_h} dz \frac{1}{z^5}$

To remove the divergence at $z=0$ introduce a UV cut-off ϵ

the period in the t-direction of tAdS β'
= the period in t-direction of AdS BH at z ϵ

⟹ $\beta' = \pi z_h \sqrt{f(\epsilon)}$

The geometry with smaller action is the stable one for given T.

$$\Delta S = \lim_{\epsilon\to 0}(S_{AdSBH} - S_{tAdS}) = \frac{\pi z_h R^3}{\kappa^2}\left(\frac{1}{z_{IR}^4} - \frac{1}{2z_h^4}\right)$$

> 0 for $T < T_c$

< 0 for $T > T_c$

III. Holographic QCD
– gluon condensation, finite density effects, etc –

towards the dual geometry of AdS/QCD

1. Gluon Condensate Background

4dim gluon condensate ↔ the dilaton in 5 dim.

Action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{G}\left(-\mathcal{R} + 2\Lambda + \frac{1}{2}\partial_M\phi\partial^M\phi\right) \qquad \Lambda = -\frac{6}{R^2} \qquad \frac{1}{\kappa^2} = \frac{4(N_c^2-1)}{\pi^2 R^3}$$

Dilaton wall solution (cf. dilaton black hole solution)

Csaki & Reece, hep-th/0608266,

$$ds^2 = \frac{R^2}{z^2}\left(\sqrt{1-c^2z^8}\,\delta_{\mu\nu}dx^\mu dx^\nu + dz^2\right),$$

$$\phi(z) = \phi_0 + \sqrt{\frac{3}{2}}\log\left(\frac{1+cz^4}{1-cz^4}\right),$$

singular at $z_c \equiv \frac{1}{c^{1/4}}$

Perturbative expansion near the boundary z → 0

$$\phi = \phi_0 + \sqrt{6}\,\frac{z^4}{z_c^4} + \mathcal{O}(z^8)\,.$$

Gluon condensate

$$\langle \mathrm{Tr}\, G^2\rangle = \frac{8\sqrt{3(N_c^2-1)}}{\pi}\frac{1}{z_c^4},$$

T-independent

General solution with metric back reaction

$$ds^2 = \frac{R^2}{z^2}\left[dz^2 + (1-f^2z^8)^{1/2}\left(\frac{1+fz^4}{1-fz^4}\right)^{a/2f}\left(d\vec{x}^2 - \left(\frac{1-fz^4}{1+fz^4}\right)^{2a/f} dt^2\right)\right]$$

$$\phi(z) = \phi_0 + \frac{c}{f}\sqrt{\frac{3}{2}}\log\left(\frac{1+fz^4}{1-fz^4}\right) \qquad f^2 = a^2 + c^2 \qquad 0 < z < f^{-1/4} := z_f$$

Kim,BHL, Park, Sin, hep-th/0702131 (JHEP 09(2007))

Note :

- For a=0, the solution reduces to the dilaton-wall solution.
- For c=0, becomes the AdS Schwarzschild black hole solution.

with T by $a = \frac{1}{4}(\pi T)^4$

- Hence, describes the finite temperature with the gluon condensation with the metric having an essential singularity at $z = f^{-1/4}$
- Thermodynamics with gluon condensation

- Gluon condensate is sensitive to the QCD deconfinement transition.
- The heavy quark potential becomes deeper as the gluon condensate value decreases.

Kim, BHL, Park, Sin, arXiv:0808.1143 (PRD80,2009).

Meson spectra in the gluon condensate background

Action

Ko, BHL, Park, JHEP 1004, (2010) (arXiv:0912.5274)

$$\Delta S = \int d^5x \sqrt{G}\, \mathrm{Tr}\left[|DX|^2 - \frac{3}{R^2}|X|^2 + \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right) \right], \qquad \frac{1}{g_5^2} = \frac{N_c}{12\pi^2 R},$$

dilaton wall solution

$$ds^2 = \frac{R^2}{z^2}\left(\sqrt{1-c^2z^8}\,\delta_{\mu\nu}dx^\mu dx^\nu + dz^2\right),$$

$$\phi(z) = \phi_0 + \sqrt{\frac{3}{2}}\log\left(\frac{1+cz^4}{1-cz^4}\right),$$

deformation of AdS corresponding to the confining phase with gluon condensation.

Eq. of motion (with axial gauge $V_z = 0,$)

$$0 = \frac{1}{\sqrt{G}}\partial_M \sqrt{G} G^{MP} G^{ij} \partial_P V_i, \qquad V_i = \int \frac{d^4k}{(2\pi)^4}\, e^{-i\omega_n t + i\vec{p}_n \vec{x}}\, V_i^{(n)}(z),$$

becomes

$$0 = \partial_z^2 V_i^{(n)} - \frac{1+3c^2z^8}{z(1-c^2z^8)}\partial_z V_i^{(n)} + \frac{m_n^2}{\sqrt{1-c^2z^8}} V_i^{(n)}$$

IR cutoff for Confined phase by

1) Hard wall at z_c or 2) braneless approach

hard wall approach

Various meson masses depending on the gluon condensation

z_c (1/GeV)	$\langle \mathrm{Tr}G^2 \rangle$ (GeV^4)	m_ρ (GeV)	m_A (GeV)	m_π (GeV)
∞	0	0.7767	1.3582	0.13961
1/0.176	0.012	0.7767	1.3583	0.13961
1/0.200	0.020	0.7767	1.3584	0.13961
1/0.250	0.049	0.7762	1.3589	0.13964
1/0.280	0.077	0.7755	1.3599	0.13970
1/0.320	0.131	0.7724	1.3612	0.13999

– As the gluon condensation increases
 mass of the vector meson decreases slightly while
 masses of the axial vector meson and pion increase very slowly

Braneless approach – singlularity identified with the IR cutoff

Boundary condition $V_i^{(1)} = 0$ at z=0, $\partial_z V_i^{(1)} = 0$ at z=zc

Fixing zc by m_ρ = 776 MeV gives zc = 1/325 MeV

Gluon condensation (for Nc = 3)

$$\langle \mathrm{Tr}\, G^2 \rangle \approx 0.139[\mathrm{GeV}^4]$$

decreases as T increases

Cf. Lattice calculation

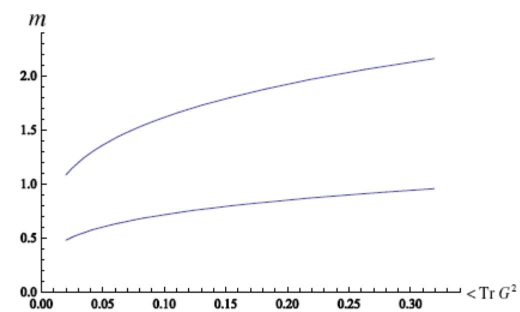
$$\langle \mathrm{Tr}\, G^2 \rangle \approx 0.012[\mathrm{GeV}^4]$$

Miller, hep-ph/0608234 (Phys. Rept, 2007)

Meson masses

z_c (1/GeV)	$\langle \mathrm{Tr} G^2 \rangle$ (GeV^4)	m_ρ (GeV)	m_A (GeV)	m_π (GeV)
1/0.200	0.020	0.4780	1.4081	0.13796
1/0.250	0.049	0.5975	1.4057	0.13808
1/0.325	0.139	0.7768	1.3574	0.14020
1/0.378	0.253	0.9035	1.2880	0.14743
1/0.400	0.319	0.9561	1.2715	0.15302

Rho-meson masses



- meson spectra well defined in spite of the singularity
- meson masses similar to those in EKSS model
  for gluon condensation larger than that of lattice calculation.
- Meson spectra significantly depend on the gluon condensate
- As the gluon condensation becomes large,
  masses of the vector meson and pion increase while
  masses of the axial vector mesons decrease

2. Dual geometry for finite chemical potential

<u>bulk</u>		<u>boundary</u>
<u>field</u> A_0	⟷	<u>dual operator</u> $\bar{\psi}\gamma_0\psi = \psi^\dagger\psi$ (quark number density)

Chamblin−Emparan−Johnson−Myers,1999
Cvetic−Gubser,1999

<u>5−dimensional action</u> dual to the gauge theory with quark matters

$$S = \int d^5x\sqrt{G}\left[\frac{1}{2\kappa^2}(-\mathcal{R}+2\Lambda)+\frac{1}{4g^2}F_{MN}F^{MN}\right]$$

Euclidean
Wick rotation $t \to -i\tau$

<u>Equations of motion</u>

1) Einstein equation $\mathcal{R}_{MN}-\frac{1}{2}G_{MN}\mathcal{R}+G_{MN}\Lambda = \frac{\kappa^2}{g^2}\left(F_{MP}F_N^P-\frac{1}{4}G_{MN}F_{PQ}F^{PQ}\right)$

2) Maxwell equation $0 = \partial_M\sqrt{-G}G^{MP}G^{NQ}F_{PQ}$

<u>Ansatz :</u>

$$\begin{cases} ds^2 = \dfrac{R^2}{z^2}\left(f(z)dt^2+d\vec{x}^2+\dfrac{1}{f(z)}dz^2\right) \\ A_0 = A(z) \quad \text{and other are zero.} \end{cases}$$

Solutions

S.-J. Sin, 2007

- most general solution, which is RNAdS BH (RN AdS black hole)

$$f(z) = 1 - mz^4 + q^2 z^6$$
$$A(z) = i\left(\mu - Qz^2\right)$$

corresponds to the deconfining phase (quark-gluon plasma)

m	black hole mass
q	black charge
μ	quark chemical potential
$Q = \sqrt{\frac{3g^2R^2}{2\kappa^2}}\, q$	quark number density

Note

1) The value of A_0 at the boundary ($z = 0$) corresponds to the quark chemical potential μ of QCD.

2) The dual operator of A_0 is denoted by Q, which is the quark (or baryon) number density operator.

3) We use $\frac{1}{2\kappa^2} = \frac{N_c^2}{8\pi^2 R^3}$ and $\frac{1}{g^2} = \frac{N_c N_f}{4\pi^2 R}$

- What is the dual geometry of the confining (or hadronic) phase ?

find non-black hole solution (BHL, Park ,Sin JHEP 0907,(2009))

$$f(z) = 1 + q^2 z^6$$
$$A(z) = i\left(\mu - Qz^2\right)$$

We call it tcAdS (thermal charged AdS space)

- baryonic chemical potential

$$\mu_B = 3\mu$$

- baryon number density

$$Q_B = Q/3$$

Note : Solutions in both phases are valid for arbitrary densities

	Hawking–Page phase transition (in dense matter)		
Phase :	hadronic phase		deconfined phase
Geometry:	tcAdS	$\leftrightarrow$	RNAdS BH

$\Downarrow$ (q = 0)

Geometry : thermal AdS $\leftrightarrow$ AdS BH

(Geometries of Hawking–Page Tr. w/o chemical potential)

For the fixed chemical potential

- dimensionless variables

$$\tilde{z}_c \equiv \frac{z_c}{z_{IR}},$$
$$\tilde{\mu}_c \equiv \mu_c z_{IR},$$
$$\tilde{T}_c \equiv T_c z_{IR},$$

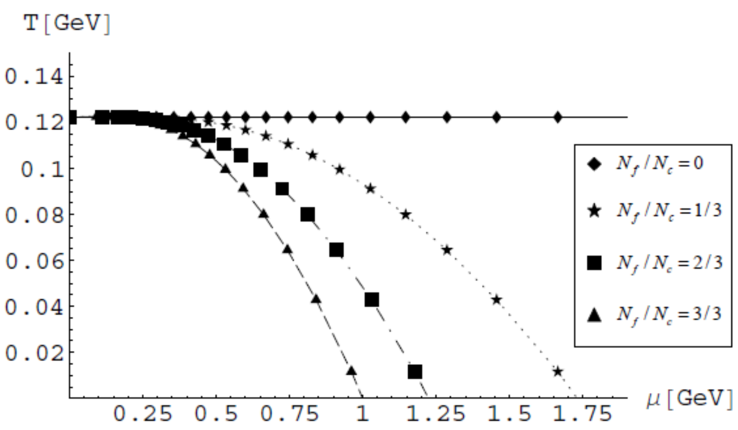
For the fixed number density

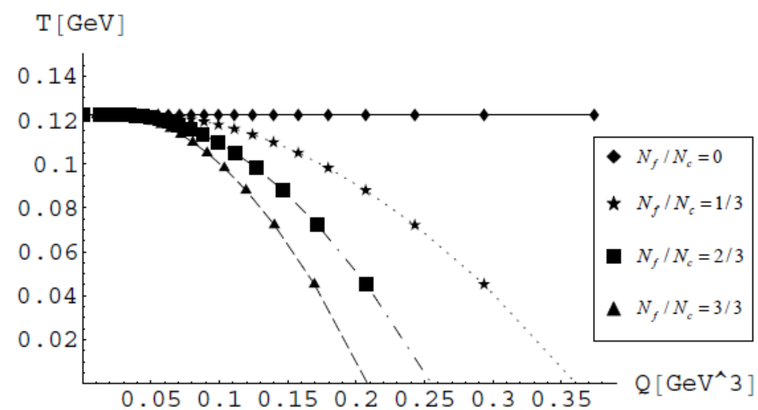
- Legendre transformation,

the Hawking–Page transition occurs at

$$\begin{cases} \tilde{\mu}_c = \sqrt{\frac{3N_c}{N_f}\frac{(1-2\tilde{z}_c^4)}{\tilde{z}_c^2(9\tilde{z}_c^2-2)}}, \\ \tilde{T}_c = \frac{1}{\pi\tilde{z}_c}\left(1-\frac{1-2\tilde{z}_c^4}{9\tilde{z}_c^2-2}\right). \end{cases}$$

$$\begin{cases} \tilde{Q}_c = \sqrt{\frac{3N_c}{2N_f}\frac{(1-2\tilde{z}_c^4)}{\tilde{z}_c^4(5\tilde{z}_c^2-2)}}, \\ \tilde{T}_c = \frac{1}{\pi\tilde{z}_c}\left[1-\frac{\tilde{z}_c^2}{2}\frac{(1-2\tilde{z}_c^4)}{(5\tilde{z}_c^2-2)}\right] \end{cases}$$





Light meson spectra in the hadronic phase

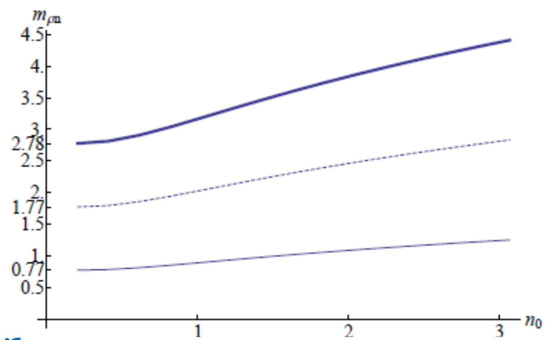
Jo, BHL, Park,Sin
JHEP 2010, arXiv:0909.3914

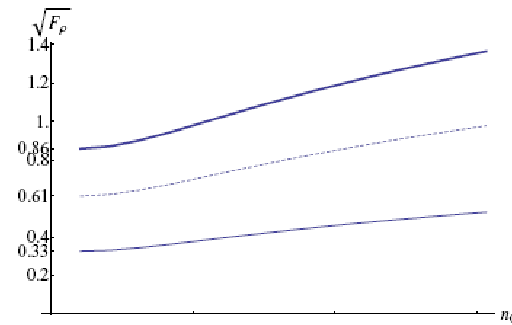
Turn on the fluctuation in bulk corresponding the meson spectra in QCD

$$\Delta S = \int d^5x\sqrt{G}\ \mathrm{Tr}\left[|DX|^2 - \frac{3}{R^2}|X|^2 + \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right] \quad M_X^2 = -3/l^2$$

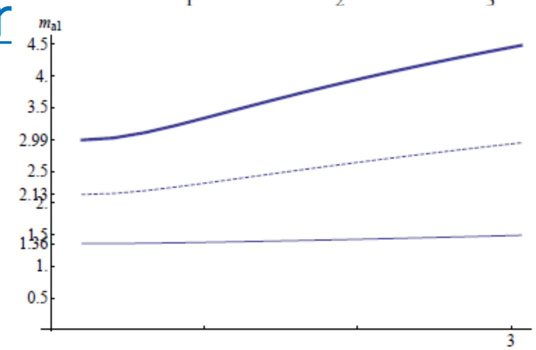
X is the dual to the quark bilinear operator <qbar q >.

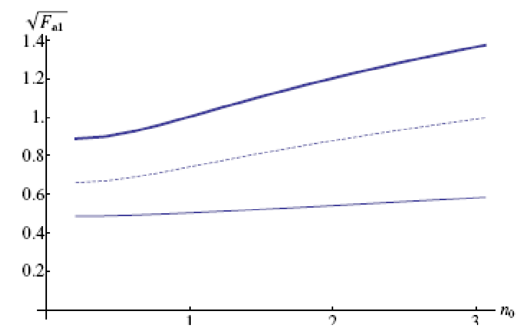
1. Vector meson



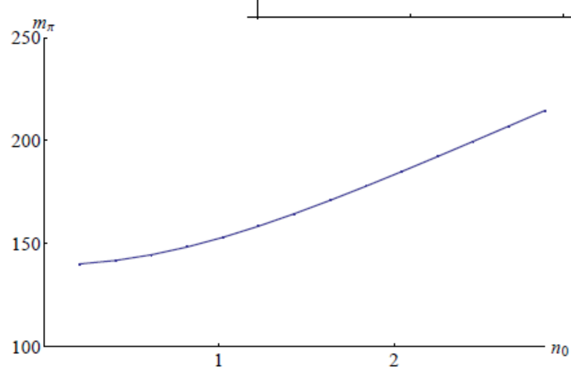


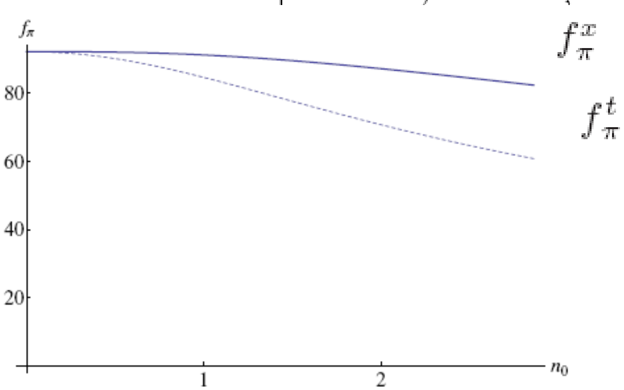
2. Axial vector meson





3. pion





4. AdS/QCD based on the D7 embedding in black D3/D-instanton geometry

B. Gwak, M. Kim, BHL, Y.Seo, S.-J. Sin, PRD

Motivation

- Alternative to the Geometrical phase Transition for in AdS/CFT ?
- Baryon Vertex (phase) and confinement at finite T (Black Hole Background) ?

Finite Temperature with Dilaton background (Solution of Type IIB SUGRA)

$$ds_{10}^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2dt^2 + d\vec{x}^2\right) + \frac{1}{f(r)^2}\frac{R^2}{r^2}dr^2 + R^3 d\Omega_5^2\right], \qquad R^4 = 4\pi g_s N_c \alpha'^2$$

$$e^{\Phi} = 1 + \frac{q}{r_T^4}\log\frac{1}{f(r)^2}, \qquad \chi = -e^{-\Phi} + \chi_0,$$

$$f(r) = \sqrt{1 - \left(\frac{r_T}{r}\right)^4},$$

Hawking Temperature

$$T = r_T/\pi R^2$$

Zero Temperature Limit : becomes near horizon geometry of D3-D(-1)

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(dt^2 + d\vec{x}^2\right) + \frac{R^2}{r^2}\left(dr^2 + r^2 d\Omega_5^2\right)\right],$$

(Liu & Tseytlin 9903091)

$$e^{\Phi} = 1 + \frac{q}{r^4}, \qquad \chi = -e^{-\Phi} + \xi_\infty.$$

- AdSxS5 at UV Flat at IR (w/ dilaton singular)
- N=2 (with gluon condensation)

Ex) Zero Temperature and without density

- AdSxS5 at UV
  Flat at IR (w/ dilaton singular)
- N=2 (with gluon condensation)

Background Metric by D3 & D−instantons

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(dt^2+d\vec{x}^2\right)+\frac{R^2}{r^2}\left(dr^2+r^2 d\Omega_5^2\right)\right],$$

$$e^{\Phi} = 1+\frac{q}{r^4}, \qquad \chi = -e^{-\Phi}+\xi_\infty .$$

(Liu & Tseytlin 9903091)

$$\frac{R^2}{r^2}(d\rho^2+\rho^2 d\Omega_3^2+dw_5^2+dw_6^2)$$

$$\rho^2 = w_1^2+\ldots+w_4^2$$

$$r^2 = \rho^2+w_5^2+w_6^2 = \rho^2+y^2$$

D7 Brane as a Probe

Induced metric on D7

$$ds_{D7}^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(dt^2+d\vec{x}^2\right)+\frac{R^2}{r^2}\left((1+y'^2)d\rho^2+\rho^2\Omega_3^2\right)\right]$$

DBI action of D7

$$S_{D7} = -\mu_7\int d^8\xi\,\sqrt{-\det\left(P[G]_{ab}+2\pi\alpha' F_{ab}\right)}+\frac{(2\pi\alpha')^2}{2}\mu_7\int P[C^{(4)}]\wedge F\wedge F$$

$$\mu_7 = [(2\pi)^7 g_s\alpha'^4]^{-1}$$

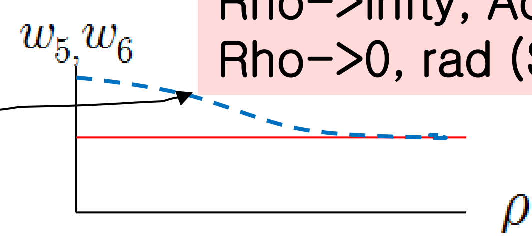
becomes (D7 wraps S3 of S5)

$$S_{D7} = -\tau_7\int dt d\rho\; e^{\Phi}\rho^3\sqrt{1+y'^2}$$

$$\tau_7 = \mu_7 V_3\Omega_3$$

Embedding solution :

$$y(\rho) = M_q+\frac{<\bar{\psi}\psi>}{\rho^2}+\cdots \qquad (\rho >> 1)$$



Rho−>infty, AdS5xS3
Rho−>0, rad (S3)−>0

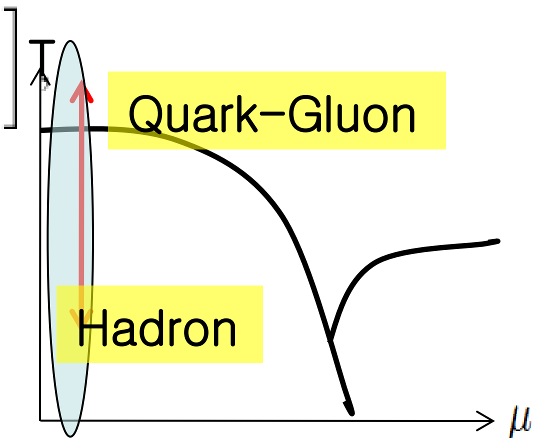
Ex) Finite Temperature and without density

Finite Temperature (Black Hole geometry) of D3/D-instanton system

$$ds_{10}^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2dt^2 + d\vec{x}^2\right) + \frac{1}{f(r)^2}\frac{R^2}{r^2}dr^2 + R^3 d\Omega_5^2\right]$$

$$e^{\Phi} = 1 + \frac{q}{r_T^4}\log\frac{1}{f(r)^2}, \qquad \chi = -e^{-\Phi} + \chi_0,$$

$$f(r) = \sqrt{1 - \left(\frac{r_T}{r}\right)^4}, \qquad T = r_T/\pi R^2.$$



Rewrite in terms of dimensionless parameter $\frac{d\xi^2}{\xi^2} = \frac{dr^2}{r^2 f^2(r)}$

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2dt^2 + d\vec{x}^2\right) + \frac{R^2}{\xi^2}\left(d\xi^2 + \xi^2 d\Omega_5^2\right)\right],$$

or

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2dt^2 + d\vec{x}^2\right) + \frac{R^2}{\xi^2}\left(d\rho^2 + \rho^2\Omega_3^2 + dy^2 + y^2 d\phi^2\right)\right].$$

where $\xi^2 = \rho^2 + y^2$

$$\left(\frac{r}{r_T}\right)^2 = \frac{1}{2}\left(\frac{\xi^2}{\xi_T^2} + \frac{\xi_T^2}{\xi^2}\right), \quad \text{and} \quad f = \left(\frac{1 - \xi_T^4/\xi^4}{1 + \xi_T^4/\xi^4}\right) \equiv \frac{\omega_-}{\omega_+}.$$

Finite Temperature and with finite density

Turn on U(1) gauge field on D7 brane

DBI action of D7

$$S_{D7} = -\tau_7 \int dt d\rho \rho^3 e^{\Phi/2} \omega_+^{3/2} \sqrt{e^{\Phi/2} \frac{\omega_-^2}{\omega_+} (1 + \dot{y}^2) - \tilde{F}^2} := \int dt d\rho \mathcal{L}_{D7},$$

$$\tau_7 = \mu_7 V_4 \Omega_3, \quad \tilde{F} = 2\pi\alpha' F_{t\rho}$$

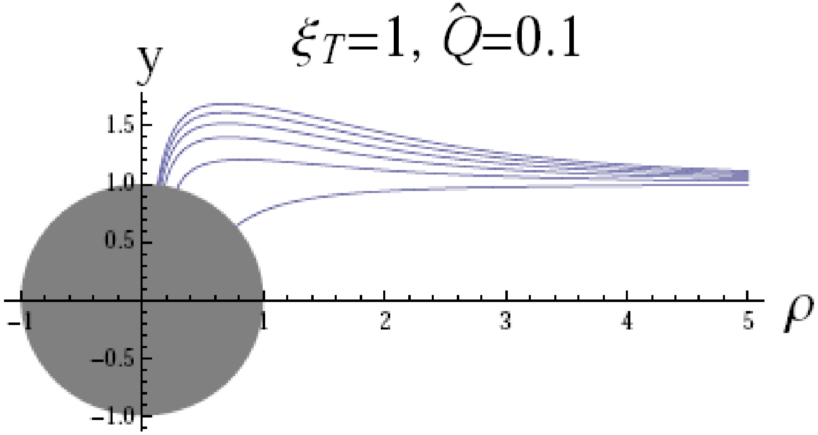
Minkowski and Black Hole embedding

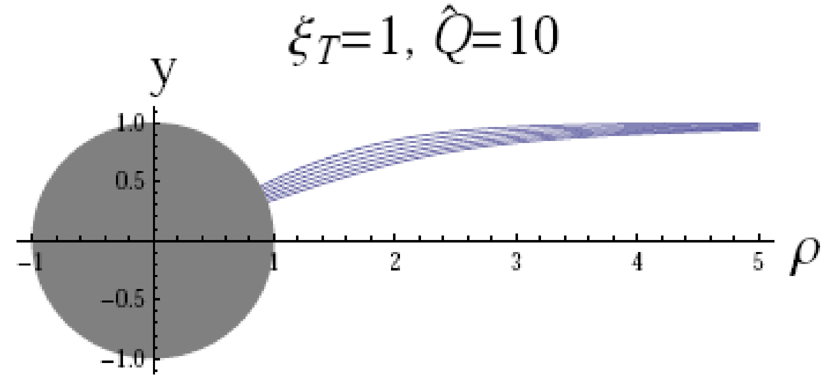
- Legendre transformation

$$\begin{aligned} \mathcal{H}_{D7} &= \tilde{F} \frac{\partial \mathcal{L}_{D7}}{\partial \tilde{F}} - \mathcal{L}_{D7} \\ &= \tau_7 \int d\rho \sqrt{e^{\Phi} \frac{\omega_-^2}{\omega_+} (1 + \dot{y}^2)} \sqrt{\frac{\tilde{Q}^2}{\tau_7^2} + \rho^6 e^{\Phi} \omega_+^3}, \end{aligned}$$

- Source of $U(1)$ gauge field on D7 brane is endpoint of fundamental strings
- There are two way to attaching fundamental strings on D7 brane

Quark Phase

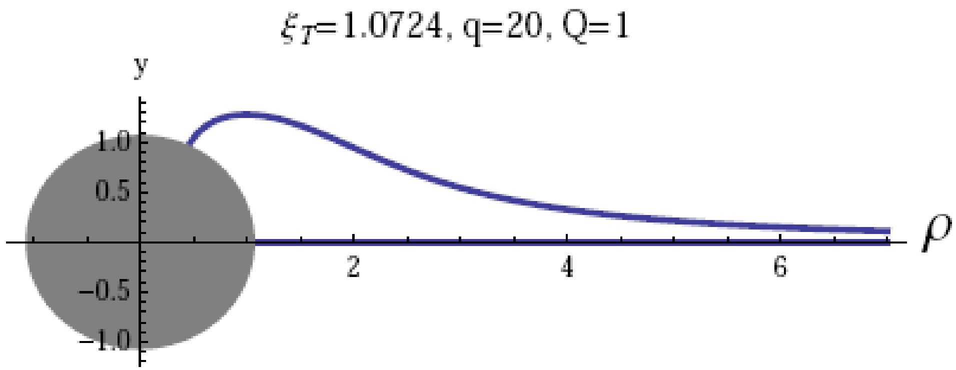


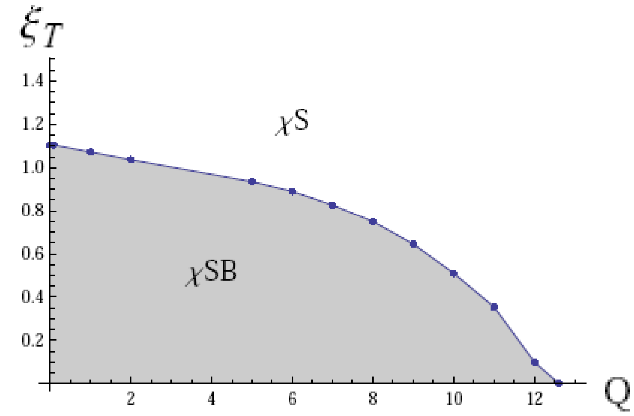


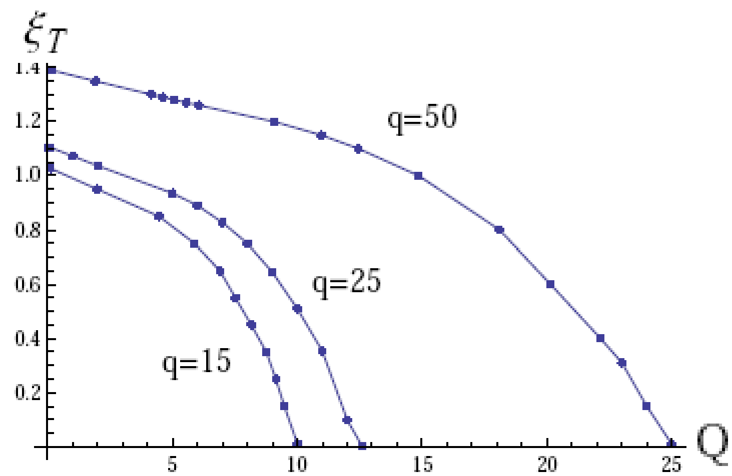
Regularity condition $\dot{y}(\rho_{min}) = \tan\theta$

- As q increases, the repulsion effect on D7 also increases.
- F1 strings connect BH horizon and probe brane
- Physical object is freely moving quark

- In $m_q \to 0$ limit, we have two phases

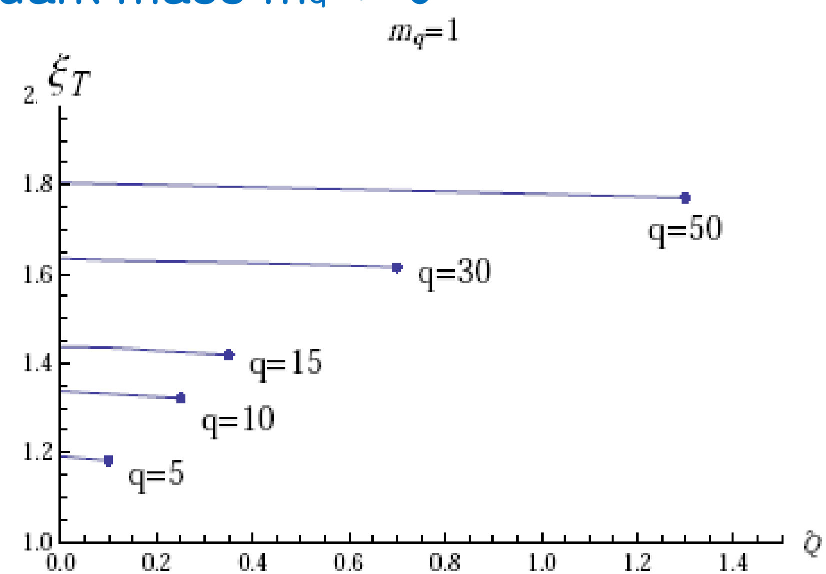






(Note : If q=0, then the trivial flat embedding is the unique solution for $m_q \to 0$.)

Finite quark mass $m_q \neq 0$



Baryon Phase

Background metric $F_{t\theta} \neq 0$

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2dt^2 + d\vec{x}^2\right) + R^2\left(\frac{d\xi^2}{\xi^2} + d\theta^2 + \sin^2\theta d\Omega_4^2\right)\right]$$

Induced metric on D5

$$ds_{D5}^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}f^2dt^2 + R^2\left(\frac{\xi'^2}{\xi^2} + 1\right)d\theta^2 + R^2\sin^2\theta d\Omega_4^2\right]$$

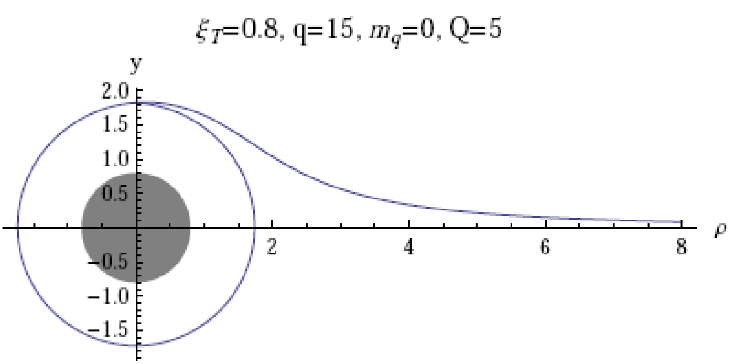
DBI action

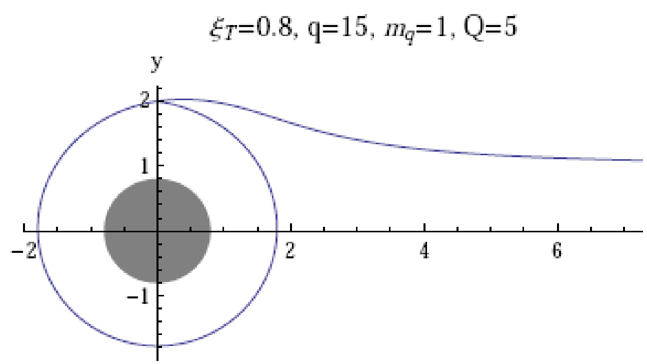
$$S_{D5} = -\mu_5 \int e^{-\Phi}\sqrt{-\det(g + 2\pi\alpha' F)} + \mu_5 \int A_{(1)} \wedge G_{(5)}$$

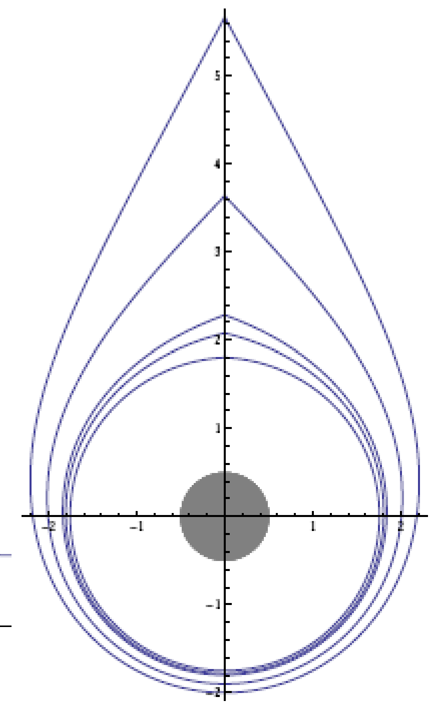
$$= \tau_5 \int dt d\theta \sin^4\theta e^{\Phi}\left[-\sqrt{e^{\Phi}\frac{\omega_-^2}{\omega_+}(\xi^2 + \xi'^2) - \tilde{F}^2} + 4\tilde{A}_t\right]$$

- F1's connect spherical D5 & probe D7
- Phys. Ob. = baryon vtx (bd state of Nc quarks)
- χ−symm. broken

D7 brane at the tip of D5 with force balance condition

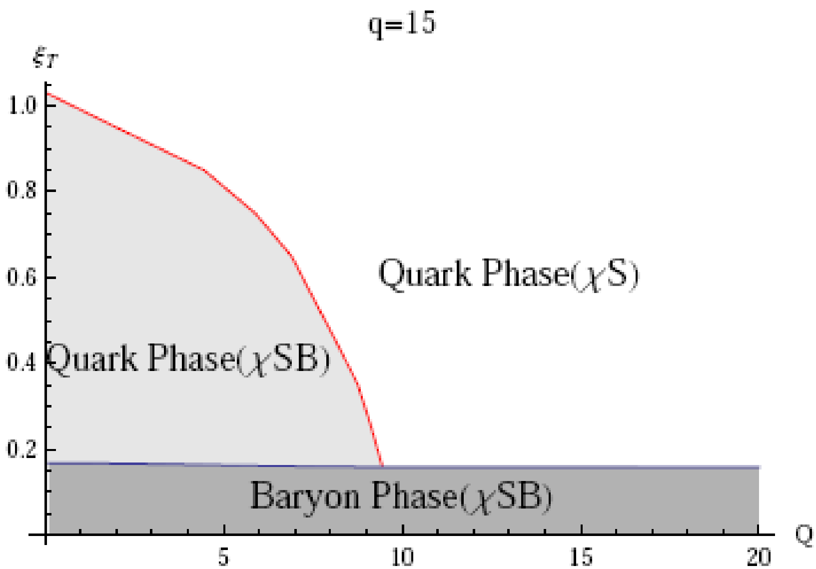


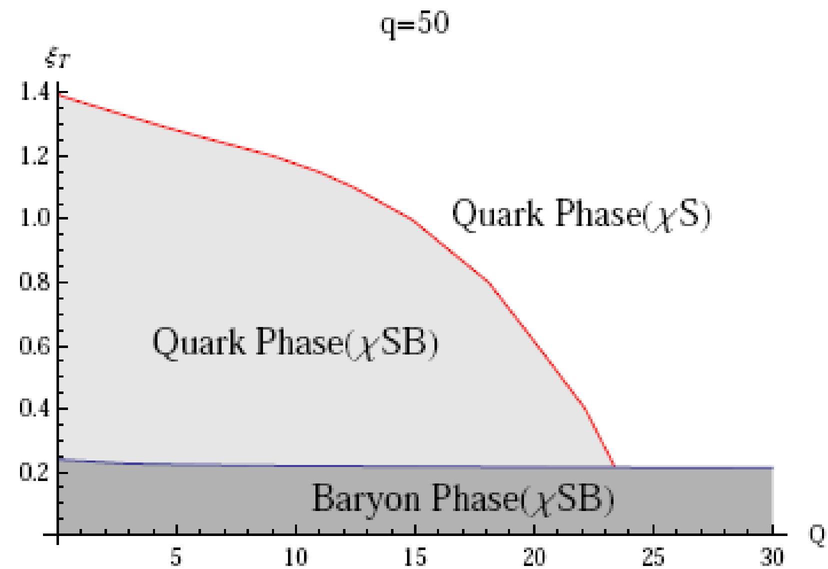




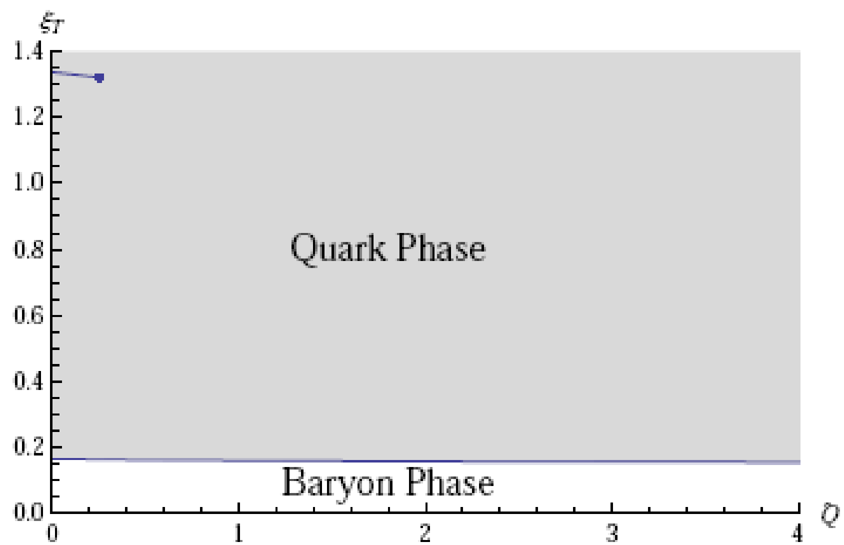
Phase Diagram

Zero quark mass





Finite quark mass



IV. Summary

- Holographic Principles ：

(d+1 dim.) (classical) Sugra ↔ (d dim.) (quantum) YM theories

- AdS/QCD – Top-down Approach & Bottom-up Approach

- QCD using Holographic dual Geometry

– w/o chemical potential –

phase ： confined phase ↔ deconfined phase transition

Geometry : thermal AdS ↔ AdS BH

Hawking-Page transition

– in dense matter – (U(1) chemical potential → baryon density)

deconfined phase by RNAdS BH ↔ hadronic phase by tcAdS

Hawking-Page phase transition

- In the hadronic phase, the quark density dependence of the light meson masses has been investigated.

IV. Summary – continued

- Holographic QCD model in D3/D-instanton background
- Two phases and phase transitions : for given T and density

  quark phase : physical objects : quarks

  baryon phase : baryon (vertex) as a physical object
- We study phase structure with and without quark mass
- We also study density dependence of chemical potential (eq. of state) and phase structure in grand canonical ensemble
- Holography Principle can be quite useful for studying the strongly interacting systems.

Thank You !