Bethe-Salpeter approach with the separable interaction for the deuteron

Serge Bondarenko, V. Burov, E. Rogochaya

Joint Institute for Nuclear Research, Dubna, Russia

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Bethe-Salpeter approach is a powerful tool to investigate reactions with the deuteron and unbound neutron-proton (np) system.

The off-shell behavior of the electromagnetic nucleon-nucleon form factors also can be studied in the reactions of elastic and inelastic electron-deuteron scattering. But firstly we need to estimate the effects of the final-state interaction.

In the report we consider influence of the electromagnetic nucleon-nucleon form factors in elastic scattering and the effects of the final-state interaction are investigated for $J = 0, 1$ partial-states of the np-pair withing the BS approach with separable kernel.

Bethe-Salpeter equation for the amplitude (deuteron)

$$\chi(p;P) = \frac{i}{4\pi^3}\, S_2(p;P) \int d^4k\, V(p,k;P)\, \chi(k;P)$$

Bethe-Salpeter equation for the nucleon-nucleon T matrix (np state)

$$T(p',p;P) = V(p',p;P) + \frac{i}{4\pi^3} \int d^4k\, V(p',k;P)\, S_2(k;P)\, T(k,p;P)$$

p', p - the relative four-momenta
P - the total four-momentum

$V(p',p;P)$ - the interaction kernel

$S_2^{-1}(k;P) = \left(\frac{1}{2}\, P\cdot\gamma + k\cdot\gamma - m\right)^{(1)} \left(\frac{1}{2}\, P\cdot\gamma - k\cdot\gamma - m\right)^{(2)}$
free two-particle Green function

Relativistic two-nucleon basis states

$|aM\rangle \equiv |\pi,\, {}^{2S+1}L^{\rho}_{J} M\rangle$

S - the total spin
L - the orbital angular momentum
J - the total angular momentum with the projection M
ρ and π - the relative-energy and spatial parity
$u_m^{\rho=\pm,\,1/2}$ - Dirac spinors ($u^{\rho=+\,1/2} \equiv u$, $u^{\rho=-\,1/2} \equiv v$)
Y_{Lm_L} - the spherical harmonics
$C^{j\ m}_{j_1 m_1 j_2 m_2}$ - Clebsch-Gordan coefficients
$U_C = i\gamma^2\gamma^0$ - the charge conjugation matrix

The spin-angular momentum functions (in c.m.s. frame):

$$\mathcal{Y}_{JM:LS\rho}(\mathbf{p})U_C =$$

$$= i^L \sum_{m_L m_S m_1 m_2 \rho_1 \rho_2} C^{S_\rho \rho}_{\frac{1}{2}\rho_1 \frac{1}{2}\rho_2} C^{JM}_{Lm_L S m_S} C^{S m_S}_{\frac{1}{2}m_1 \frac{1}{2}m_2} Y_{Lm_L}(\mathbf{p})$$

$$\times {u^{\rho_1}_{m_1}}^{(1)}(\mathbf{p}) {{u^{\rho_2}_{m_2}}^{(2)}}^{T}(-\mathbf{p})$$

Spin-angular parts

$$\mathcal{Y}_{JM:LS+}(\boldsymbol{p}) = \frac{1}{\sqrt{8\pi}}\frac{1}{4E_{\boldsymbol{p}}(E_{\boldsymbol{p}}+m)}(m+\not{p}_1)(1+\gamma_0)\mathcal{G}_{aM}(m-\not{p}_2)$$

where matrices $\mathcal{G}_{aM}$

$a = \{{}^{2S+1}L_J^+\}$	$\mathcal{G}_{aM}$
${}^1S_0^+$	$-\gamma_5$
${}^3S_1^+$	$\not{\xi}_M$
${}^1P_1^+$	$\frac{\sqrt{3}}{\vert\boldsymbol{p}\vert}(p_1\cdot\xi_M)\gamma_5$
${}^3P_0^+$	$-\frac{1}{2\vert\boldsymbol{p}\vert}(\not{p}_1-\not{p}_2)$
${}^3P_1^+$	$-\sqrt{\frac{3}{2}}\frac{1}{\vert\boldsymbol{p}\vert}\left[(p_1\cdot\xi_M)-\frac{1}{2}\not{\xi}_M(\not{p}_1-\not{p}_2)\right]\gamma_5$
${}^3D_1^+$	$\frac{1}{\sqrt{2}}\left[\not{\xi}_M+\frac{3}{2}\frac{1}{\boldsymbol{p}^2}(p_1\cdot\xi_M)(\not{p}_1-\not{p}_2)\right]$

$p_1 = (E_{\boldsymbol{p}},\boldsymbol{p})$, $p_2 = (E_{\boldsymbol{p}},-\boldsymbol{p})$ are on-mass-shell momenta, $E_{\boldsymbol{p}} = \sqrt{\boldsymbol{p}^2+m^2}$

Partial-wave decomposition

Amplitude in the c.m.s. frame

$$\chi^{1M}(k;K_{(0)}) = \sum_a \mathcal{Y}_{aM}(\mathbf{k})\ \phi_a(k_0,|\mathbf{k}|)$$

$\phi_a(k_0,|\mathbf{k}|)$ is the amplitude radial parts.

T matrix and kernel in the c.m.s. frame

$$T_{\alpha\beta,\gamma\delta}(p',p;P_{(0)}) = \sum_{JMab} (\mathcal{Y}_{aM}(-\mathbf{p}')U_C)_{\alpha\beta} \otimes (U_C\mathcal{Y}^{\dagger}_{bM}(\mathbf{p}))_{\delta\gamma}\ T_{ab}(p'_0,|\mathbf{p}'|;p_0,|\mathbf{p}|;s)$$

$$V_{\alpha\beta,\gamma\delta}(p',p;P_{(0)}) = \sum_{JMab} (\mathcal{Y}_{aM}(-\mathbf{p}')U_C)_{\alpha\beta} \otimes (U_C\mathcal{Y}^{\dagger}_{bM}(\mathbf{p}))_{\delta\gamma}\ V_{ab}(p'_0,|\mathbf{p}'|;p_0,|\mathbf{p}|;s)$$

$T_{ab}(p'_0,|\mathbf{p}'|;p_0,|\mathbf{p}|;s)$ and $V_{ab}(p'_0,|\mathbf{p}'|;p_0,|\mathbf{p}|;s)$ are the T matrix and kernel radial parts.

The partial-wave decomposed equation for amplitude

$$\phi_a(p_0,|\mathbf{p}|) = \frac{i}{4\pi^3} \sum_{bc} S_{ab}(p_0,|\mathbf{p}|;s) \int\limits_{-\infty}^{+\infty} dk_0 \int\limits_{0}^{\infty} \mathbf{k}^2 d|\mathbf{k}| \times V_{bc}(p_0,|\mathbf{p}|;k_0,|\mathbf{k}|;s)\, \phi_c(k_0,|\mathbf{k}|)$$

The partial-wave decomposed equation for T matrix

$$T_{ab}(p_0',|\mathbf{p}'|;p_0,|\mathbf{p}|;s) = V_{ab}(p_0',|\mathbf{p}'|;p_0,|\mathbf{p}|;s) + \frac{i}{4\pi^3} \sum_{cd} \int\limits_{-\infty}^{+\infty} dk_0 \int\limits_{0}^{\infty} \mathbf{k}^2 d|\mathbf{k}| \times V_{ac}(p_0',|\mathbf{p}'|;k_0,|\mathbf{k}|;s)\, S_{cd}(k_0,|\mathbf{k}|;s)\, T_{db}(k_0,|\mathbf{k}|;p_0,|\mathbf{p}|;s)$$

Separable ansatz for complex kernel

$$V_{a'a}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{m,n=1}^{N} \left[\underline{\lambda_{mn}^{r[a'a]}(s)} + i\lambda_{mn}^{r[a'a]}(s) \right] \underline{g_m^{[a']}(p_0', |\mathbf{p}'|) g_n^{[a]}(p_0, |\mathbf{p}|)}$$

underlined part $\equiv$ MYN kernels, the sum $\equiv$ MYIN kernels

$$\lambda_{mn}^{i}(s) = \theta(s - s_{th}) \left(1 - \frac{s_{th}}{s}\right) \bar{\lambda}_{mn}^{i}$$

s_{th} - the inelasticity threshold. Here MY stands for Modified Yamaguchi.

Below the only positive energy states are considered.

Solution for the BS amplitude

$$\phi_a(p_0,|\mathbf{p}|) = \frac{g_a(p_0,|\mathbf{p}|)}{(\sqrt{s}/2 - E_{\mathbf{p}} + i\epsilon)^2 - p_0^2}$$

solution for radial parts of the vertex function

$$g_a(p_0,|\mathbf{p}|) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{[a]}(p_0,|\mathbf{p}|) c_j(s)$$

with coefficients

$$c_i(s) - \sum_{k,j=1}^{N} h_{ik}(s)\lambda_{kj}(s)c_j(s) = 0$$

where functions

$$h_{ij}(s) = -\frac{i}{4\pi^3}\sum_a \int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{g_i^{[a]}(k_0,|\mathbf{k}|) g_j^{[a]}(k_0,|\mathbf{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + i\epsilon)^2 - k_0^2}$$

here $a = {}^3S_1^+, {}^3D_1^+$-states and $s = M_d^2$

Solution for the T matrix

$$T_{a'a}(p_0', |\mathbf{p}'|; p_0, |\mathbf{p}|; s) = \sum_{i,j=1}^{N} \tau_{ij}(s)\, g_i^{[a']}(p_0', |\mathbf{p}'|)\, g_j^{[a]}(p_0, |\mathbf{p}|)$$

where

$$\Big[\tau_{ij}(s)\Big]^{-1} = \Big[\lambda_{mn}^{r[a'a]}(s) + i\lambda_{mn}^{r[a'a]}(s)\Big]^{-1} + h_{ij}(s),$$

$$h_{ij}(s) = -\frac{i}{4\pi^3} \sum_a \int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{g_i^{[a]}(k_0, |\mathbf{k}|) g_j^{[a]}(k_0, |\mathbf{k}|)}{(\sqrt{s}/2 - E_{\mathbf{k}} + i\epsilon)^2 - k_0^2},$$

$g_j^{[a]}$ - the model functions, $\lambda_{ij}^{[a'a]}(s)$ - a matrix of model parameters.

Graz II covariant kernel, rank III

$$
\begin{aligned}
g_1^{(S)}(p_0,|\mathbf{p}|) &= \frac{1-\gamma_1(p_0^2-\mathbf{p}^2)}{(p_0^2-\mathbf{p}^2-\beta_{11}^2)^2}, \\
g_2^{(S)}(p_0,\mathbf{p}) &= -\frac{(p_0^2-\mathbf{p}^2)}{(p_0^2-\mathbf{p}^2-\beta_{12}^2)^2}, \\
g_3^{(D)}(p_0,|\mathbf{p}|) &= \frac{(p_0^2-\mathbf{p}^2)(1-\gamma_2(p_0^2-\mathbf{p}^2))}{(p_0^2-\mathbf{p}^2-\beta_{21}^2)(p_0^2-\mathbf{p}^2-\beta_{22}^2)^2}, \\
g_1^{(D)}(p_0,|\mathbf{p}|) &= g_2^{(D)}(p_0,|\mathbf{p}|) = g_3^{(S)}(p_0,|\mathbf{p}|) \equiv 0.
\end{aligned}
\tag{1}
$$

Table: Deuteron and low-energy scattering properties

	$p_D(\%)$	ϵ_D (MeV)	Q_D (Fm^{-2})	μ_D ($e/2m$)	$\rho_{D/S}$	r_0 (Fm)	a (Fm)
Covariant Graz II	4	2.225	0.2484	0.8279	0.02408	1.7861	5.4188
Experimental data		2.2246	0.286	0.8574	0.0263	1.759	5.424

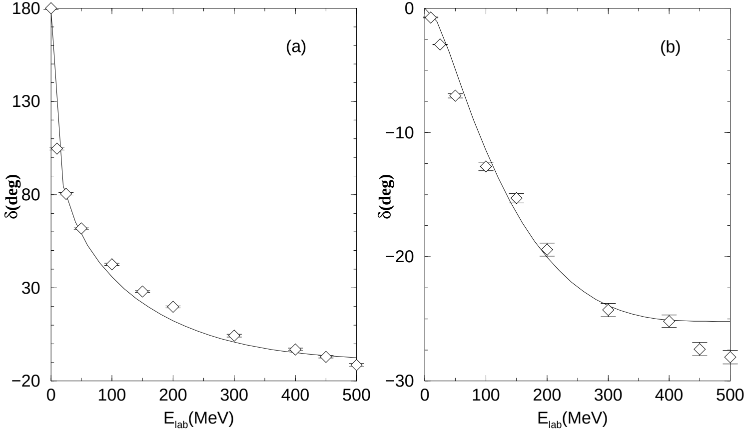


Figure: Phase shifts of the $^3S_1$ and $^3D_1$ partial states

Elastic eD scattering cross section

$$\frac{d\sigma}{d\Omega'_{\rm e}} = \left(\frac{d\sigma}{d\Omega'_{\rm e}}\right)_{\rm Mott}\left[A(q^2) + B(q^2)\tan^2\frac{\theta_{\rm e}}{2}\right],$$

$$\left(\frac{d\sigma}{d\Omega'_{\rm e}}\right)_{\rm Mott} = \frac{\alpha^2\cos^2\theta_{\rm e}/2}{4E_{\rm e}^2(1+2E_{\rm e}/M_d\sin^4\theta_{\rm e}/2)},$$

where $\theta_{\rm e}$ is the electron scattering angle, M_d is the deuteron mass, E_e is the incident electron energy.

Deuteron structure functions $A(q^2)$ and $B(q^2)$

$$A(q^2) = F_{\rm C}^2(q^2) + \frac{8}{9}\eta^2 F_{\rm Q}^2(q^2) + \frac{2}{3}\eta F_{\rm M}^2(q^2)$$

$$B(q^2) = \frac{4}{3}\eta(1+\eta)F_{\rm M}^2(q^2)$$

where $\eta = -q^2/4M_d^2 = Q^2/4M_d^2$

The tensor polarization components of the final deuteron

$$T_{20}\ \left[A + B\tan^2\frac{\theta_{\rm e}}{2}\right] = -\frac{1}{\sqrt{2}}\left[\frac{8}{3}\eta F_{\rm C}F_{\rm Q} + \frac{8}{9}\eta^2 F_{\rm Q}^2 + \frac{1}{3}\eta(1+2(1+\eta)\tan^2\frac{\theta_{\rm e}}{2})F_{\rm M}^2\right],$$

$$T_{21}\ \left[A + B\tan^2\frac{\theta_{\rm e}}{2}\right] = \frac{2}{\sqrt{3}}\eta(\eta+\eta^2\sin^2\frac{\theta_{\rm e}}{2})^{1/2}F_{\rm M}F_{\rm Q}\sec\frac{\theta_{\rm e}}{2},$$

$$T_{22}\ \left[A + B\tan^2\frac{\theta_{\rm e}}{2}\right] = -\frac{1}{2\sqrt{3}}\eta F_{\rm M}^2.$$

The electric $F_{\rm C}(q^2)$, the quadrupole $F_{\rm Q}(q^2)$ and the magnetic $F_{\rm M}(q^2)$ form factors

The normalization conditions are

$$F_{\rm C}(0) = 1, \quad F_{\rm Q}(0) = M_d^2\, Q_{\rm D}, \quad F_{\rm M}(0) = \mu_{\rm D}\frac{M_d}{m}$$

where m is the nucleon mass, $Q_{\rm D}$ and $\mu_{\rm D}$ are quadrupole and magnetic moments of the deuteron, respectively.

The deuteron current matrix element parametrization
(due to P- and T-parity conservation and gauge invariance)

$$\langle D'\mathcal{M}'|J_\mu|D\mathcal{M}\rangle = -e\xi^*_{\alpha\ \mathcal{M}'}(P')\ \xi_{\beta\ \mathcal{M}}(P)$$

$$\times\left[(P'+P)_\mu\left(g^{\alpha\beta}F_1(q^2) - \frac{q^\alpha q^\beta}{2M_d^2}F_2(q^2)\right) - (q^\alpha g^\beta_\mu - q^\beta g^\alpha_\mu)G_1(q^2)\right]$$

$\xi_{\mathcal{M}}(P)$ and $\xi^*_{\mathcal{M}'}(P')$ are the polarization 4-vectors of the initial and final deuteron, respectively.

Form factors $F_{1,2}(q^2)$, $G_1(q^2)$ are related to functions $F_{\rm C}(q^2)$, $F_{\rm Q}(q^2)$ and $F_{\rm M}(q^2)$ by the equations

$$F_{\rm C} = F_1 + \frac{2}{3}\eta\left[F_1 + (1+\eta)F_2 - G_1\right]$$

$$F_{\rm Q} = F_1 + (1+\eta)F_2 - G_1$$

$$F_{\rm M} = G_1$$

Relativistic impulse approximation (RIA)

Deuteron current matrix element

$$\langle D'\mathcal{M}'|J_\mu^{RIA}|D\mathcal{M}\rangle =$$

$$ie\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\Big\{\bar{\chi}^{1\mathcal{M}'}(P',k')\Gamma_\mu^{(\mathrm{S})}(q)\chi^{1\mathcal{M}}(P,k)(P\cdot\gamma/2-k\cdot\gamma+m)\Big\}$$

$\chi^{1\mathcal{M}}(P,k)$ - the BS amplitude of the deuteron, $P'=P+q$ and $k'=k+q/2$.

The vertex of γNN interaction

$$\Gamma_\mu^{(\mathrm{S})}(q)=\gamma_\mu F_1^{(\mathrm{S})}(q^2)-\frac{\gamma_\mu q\cdot\gamma-q\cdot\gamma\gamma_\mu}{4m}F_2^{(\mathrm{S})}(q^2)$$

is chosen to be the form factor on mass shell.

The isoscalar form factors of the nucleon

$$F_{1,2}^{(\mathrm{S})}(q^2)=(F_{1,2}^{(\mathrm{p})}(q^2)+F_{1,2}^{(\mathrm{n})}(q^2))/2$$

with normalization condition

$$F_1^{(\mathrm{S})}(0)=1/2,\quad F_2^{(\mathrm{S})}(0)=(\varkappa_p+\varkappa_n)/2$$

with $\varkappa_p=\mu_p-1$ and $\varkappa_n=\mu_n$ being anomalous parts of the proton μ_p and neutron μ_n magnetic moments, respectively.

After partial-wave decomposition and trace calculation (in LS)

$$\langle D'\mathcal{M}'|J_\mu^{RIA}|D\mathcal{M}\rangle = \mathcal{I}_{1\ \mu}^{\mathcal{M}'\mathcal{M}}(q^2)\ F_1^{(\mathrm{S})}(q^2) + \mathcal{I}_{2\ \mu}^{\mathcal{M}'\mathcal{M}}(q^2)\ F_2^{(\mathrm{S})}(q^2),$$

$$\mathcal{I}_{1,2\ \mu}^{\mathcal{M}'\mathcal{M}}(q^2) = ie\int dk_0\ |\mathbf{k}|^2\ d|\mathbf{k}|\ d(\cos\theta)$$

$$\times \sum_{a',a}\phi_{a'}(k_0',|\mathbf{k}'|)\,\phi_a(k_0,|\mathbf{k}|)\,I_{1,2\ \mathcal{M}'\mathcal{M}\ \mu}^{a'a}(k_0,|\mathbf{k}|,\cos\theta,q^2),$$

Components of the k' 4-vector

$$k_0' = (1+2\eta)k_0 - 2\sqrt{\eta}\sqrt{1+\eta}k_z - M_d\eta$$

$$k_x' = k_x, \quad k_y' = k_y$$

$$k_z' = (1+2\eta)k_z - 2\sqrt{\eta}\sqrt{1+\eta}k_0 + M_d\sqrt{\eta}\sqrt{1+\eta}$$

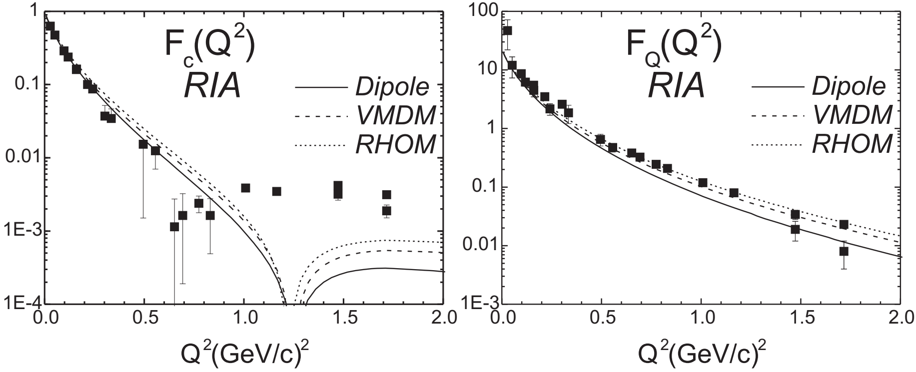
are due to Lorentz boost of the final deuteron to the LS

Nucleon form factors

In calculations the following models for nucleon form factors were investigated:

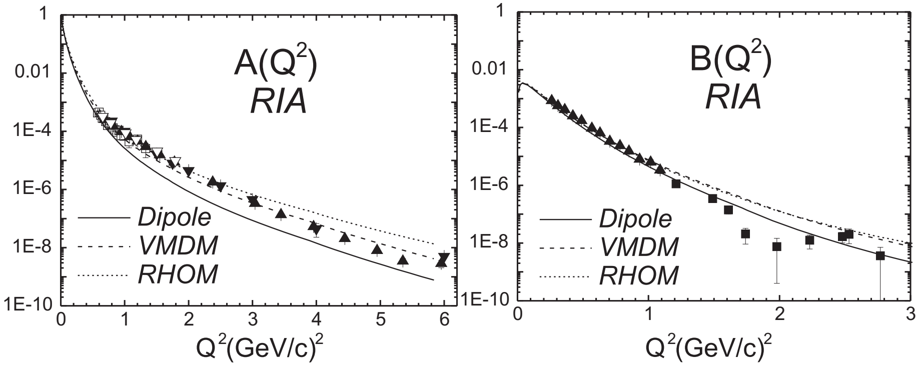
- dipole fit, $G_{\mathrm{E}}^{\mathrm{n}}(q^2) = 0$
- vector meson dominance model (VMDM), $G_{\mathrm{E}}^{\mathrm{n}}(q^2) \neq 0$
- relativistic harmonic oscillator model (RHOM), $G_{\mathrm{E}}^{\mathrm{n}}(q^2) \neq 0$

Charge $F_{\rm C}(q^2)$ and quadrupole $F_{\rm Q}(q^2)$ form factors



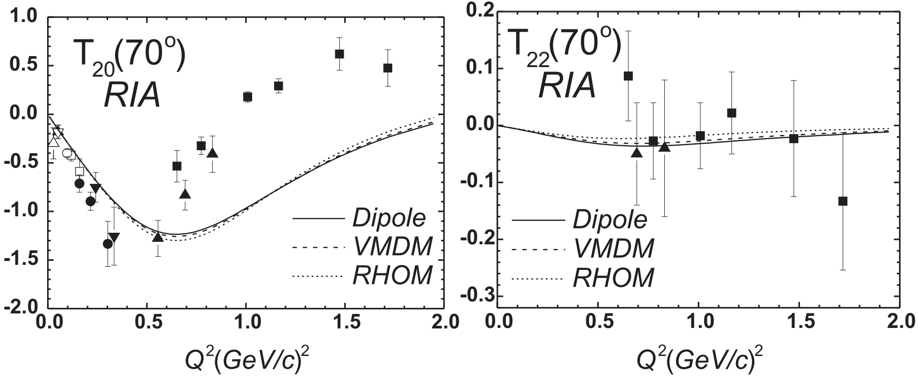
Long and short dashes represent calculations with the VMDM and RHOM nucleon form factors, respectively. The solid curve corresponds to the dipole fit.

Structure functions $A(q^2)$ and $B(q^2)$



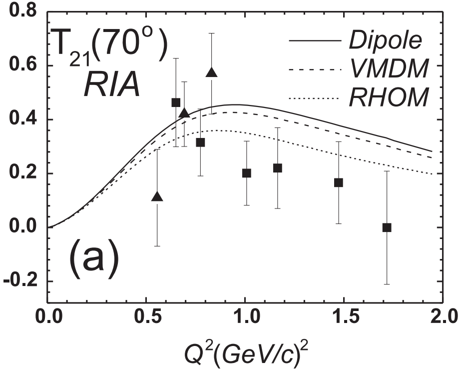
Long and short dashes represent calculations with the VMDM and RHOM nucleon form factors, respectively. The solid curve corresponds to the dipole fit.

Tensor polarization components $T_{20}(q^2)$ and $T_{22}(q^2)$ calculated at $\theta_e = 70°$.



Long and short dashes represent calculations with the VMDM and RHOM nucleon form factors, respectively. The solid curve corresponds to the dipole fit.

Tensor polarization component $T_{21}(q^2)$ calculated at $\theta_e = 70°$



Long and short dashes represent calculations with the VMDM and RHOM nucleon form factors, respectively. The solid curve corresponds to the dipole fit.

Motivation for MY(I)N kernel

Consider integral

$$\int dk_0 \int \mathbf{k}^2 d|\mathbf{k}| \frac{[g(k_0, |\mathbf{k}|)]^2}{(\sqrt{s}/2 - E_{\mathbf{k}} + i\epsilon)^2 - k_0^2}$$

Simple poles of propagators

$$k_0^{(1,2)} = \pm\sqrt{s}/2 \mp E_{\mathbf{k}} \pm i\epsilon$$

Taking into account pole and Yamaguchi-like type of $g(k_0, |\mathbf{k}|) = 1/(k_0^2 - \mathbf{k}^2 - \beta^2)$

$$(2\pi i) \int \mathbf{k}^2 d|\mathbf{k}| \frac{1}{(s/4 - \sqrt{s}E_{\mathbf{k}} + m^2 - \beta^2)^2} \frac{1}{\sqrt{s} - 2E_{\mathbf{k}} + i\epsilon}$$

Motivation for MY(I)N kernel

Consider numerator $f = s/4 - \sqrt{s}E_{\mathbf{k}} + m^2 - \beta^2$

- If $4(m-\beta)^2 < s < 4(m+\beta)^2$ then always $f < 0$ and function $1/f^n$ is integrable for any integer n and any $E_{\mathbf{k}}$.
- For bound state $s = M_d^2 = (2m - \epsilon_D)^2$. Since for $\beta_{min} = 0.2$ GeV always $\beta_{min} > \epsilon_D/2$ then function $1/f^n$ is integrable for any integer n and any $E_{\mathbf{k}}$.
- If $4(m-\beta)^2 > s > 4(m+\beta)^2$ then f can be positive and negative and $1/f^n$ is non-integrable for even n and any $E_{\mathbf{k}}$.

Critical value for $s^c = 4(m+\beta)^2$ corresponds to laboratory kinetic energy of np pair $T^c_{lab} = 4\beta + 2\beta^2/m \simeq 4\beta$. If $\beta_{min} = 0.2$ GeV then $T^{min}_{lab} = 0.8$ GeV.

The solution is to change $g(k_0, |\mathbf{k}|)$

$$g_{\rm Y}(k_0, |\mathbf{k}|) = 1/(k_0^2 - \mathbf{k}^2 - \beta^2) \to g_{\rm MY}(k_0, |\mathbf{k}|) = 1/((k_0^2 - \mathbf{k}^2 - \beta^2)^2 + \alpha^4)$$

here Y stands for Yamaguchi-like and MY - for Modified Yamaguchi

Analysis

$V(p,p') \rightarrow T_{off-mass-shell}(p,p') \rightarrow T_{on-mass-shell}(\bar{p},\bar{p}) \rightarrow (\delta,\rho,a_0,E_d...)$

δ - the phase shifts, a_0, r_0 - the low-energy parameters (the scattering length, the effective range), E_d - the deuteron binding energy.

Procedure $(J=0,1)$

calculate the kernel parameters – $\lambda_{ij}(s)$-matrix and parameter of the g-functions – to minimize the function χ^2:

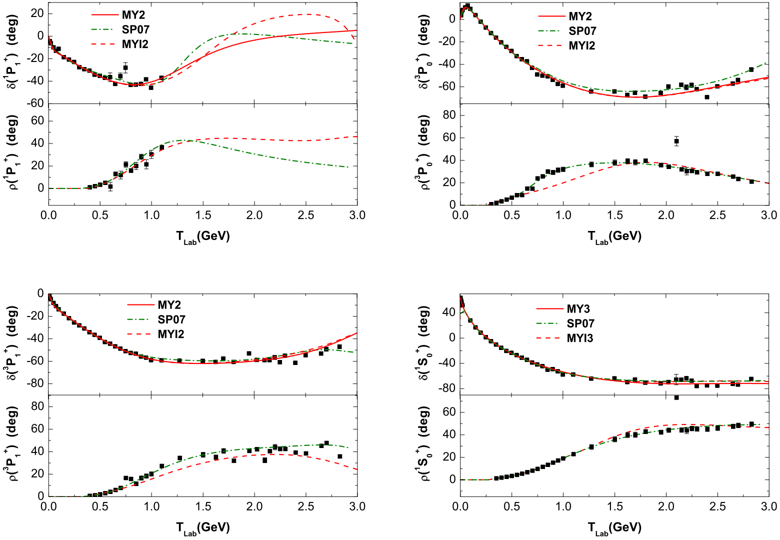
$$\chi^2 =$$
$$\sum_{i=1}^{n}(\delta^{\exp}(s_i)-\delta(s_i))^2/(\Delta\delta^{\exp}(s_i))^2 \quad \text{– for all partial-wave states}$$
$$\sum_{i=1}^{n}(\rho^{\exp}(s_i)-\rho(s_i))^2/(\Delta\rho^{\exp}(s_i))^2 \quad \text{– for all partial-wave states}$$
$$+(a_0^{\exp}-a_0)^2/(\Delta a_0^{\exp})^2 \quad \text{– for the } {}^1S_0^+ \text{ and } {}^3S_1^+ \text{ partial-wave states}$$
$$+(E_d^{\exp}-E_d)^2/(\Delta E_d^{\exp})^2 \quad \text{– for the } {}^3S_1^+\text{-}{}^3D_1^+ \text{ partial-wave states}$$
$$\{+...\}$$

Modified Yamaguchi functions

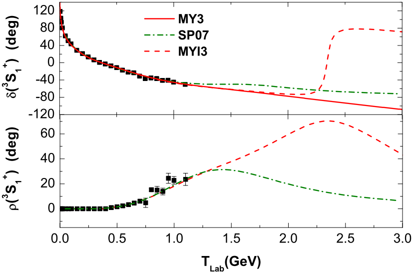
$$g_i^{[a]}(p_0,|\boldsymbol{p}|) = \frac{(p_{ci} - p_0^2 + \mathbf{p}^2)^{n_i}(p_0^2 - \mathbf{p}^2)^{m_i}}{((p_0^2 - \mathbf{p}^2 - \beta_{1i}^2)^2 + \alpha_{1i}^4)^{k_i}((p_0^2 - \mathbf{p}^2 - \beta_{1i}^2)^2 + \alpha_{1i}^4)^{l_i}}$$

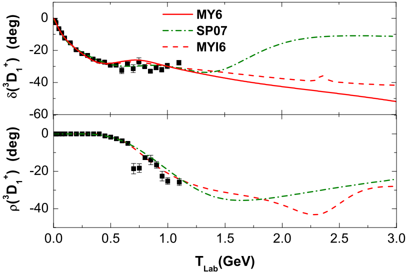
All parameters - n_i, m_i, k_i, l_i (integer), $p_{ci}, \beta_{1i}, \beta_{2i}, \alpha_{1i}, \alpha_{2i}$ (real) - depend on channel $[a]$.

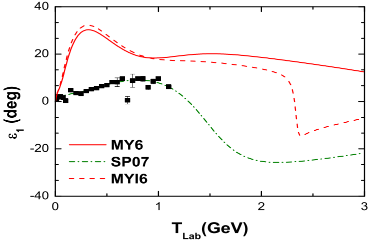
Phase shifts for ${}^1P_1^+$, ${}^3P_0^+$, ${}^3P_1^+$ and ${}^1S_0^+$ partial-wave states



Phase shifts and inelasticity parameter for triplet $^3S_1$-$^3D_1$ state







Phase shifts and inelasticity parameter for triplet $^3S_1$-$^3D_1$ state

Table: Deuteron and low-energy scattering properties

	a_{0t} (fm)	r_{0t} (fm)	p_D (%)	E_d (MeV)	$\rho_{D/S}$	μ_d $(e/2m)$
MY6	5.42	1.800	4.92	2.2246	0.0255	0.8500
Graz II	5.42	1.779	5	2.2254	0.0269	0.8512
Paris	5.43	1.770	5.77	2.2249	0.0261	0.8469
CD-Bonn	5.4196	1.751	4.85	2.2246	0.0256	0.8522
Exp.	5.424(4)	1.759(5)	4-7	2.224644(46)	0.0256(4)	0.8574

np pair wave function (amplitude)

BS amplitude of the np pair

$$\psi_{SM_S}(p,p^*;P) = \psi^{(0)}_{SM_S}(p,p^*;P) + \frac{i}{4\pi^3} S_2(p;P) \int d^4k\, V(p,k;P)\psi_{SM_S}(k,p^*;P)$$

Free term (plane-wave approximation)

$$\psi_{SM_S}(p,p^*;P) = \psi^{(0)}_{SM_S}(p,p^*;P) = (2\pi)^4 \chi_{SM_S}(p;P)\delta(p-p^*)$$

Interacting term

$$\psi^{(t)}_{SM_S}(p,p^*;P) = 4\pi i\, S_2(p;P) T(p,p^*;P)\chi_{SM_S}(p^*;P)$$

The partial-wave decomposition of the np-pair BS amplitude

$$\psi^{(t)}_{SM_S}(p,p^*;P) = 4\pi i \times \sum_{LmJMa} C^{JM}_{LmSM_S} Y^*_{Lm}(\hat{\boldsymbol{p}}^*)\mathcal{Y}_{aM}(\boldsymbol{p})\phi_{a,J:LS+}(p_0,|\boldsymbol{p}|;s), \tag{2}$$

where $p^* = (0,\boldsymbol{p}^*)$ with $|\boldsymbol{p}^*| = \sqrt{s/4-m^2}$ is the relative momentum of on-mass-shell nucleons in CM, $\hat{\boldsymbol{p}}^*$ denotes the azimuthal angle $\theta_{\boldsymbol{p}^*}$ between the $\boldsymbol{p}^*$ and $\boldsymbol{q}$ vectors and zenithal angle ϕ. Since only positive-energy partial-wave states are considered here the radial part is:

$$\phi_{a,J:LS+}(p_0,|\boldsymbol{p}|;s) = \frac{T_{a,J:LS+}(p_0,|\boldsymbol{p}|;0,|\boldsymbol{p}^*|;s)}{(\sqrt{s}/2-E_{\boldsymbol{p}}+i\epsilon)^2-p_0^2}. \tag{3}$$

Unpolarized exclusive cross-section

The 5-fold differential cross section in one-photon approximation (in laboratory system - LS)

$$\frac{d^3\sigma}{dE_e' d\Omega_e' d\Omega_p} = \frac{\sigma_{\rm Mott}}{8M_d(2\pi)^3} \frac{\boldsymbol{p}_p^2\sqrt{s}}{\sqrt{1+\eta}|\boldsymbol{p}_p| - E_p\sqrt{\eta}\cos\theta_p}$$

$$\times \left[l_{00}^0 W_{00} + l_{++}^0 (W_{++} + W_{--}) + 2l_{+-}^0 \cos 2\phi \ {\rm Re} W_{+-} - 2l_{+-}^0 \sin 2\phi \ {\rm Im} W_{+-} \right.$$

$$\left. -2l_{0+}^0 \cos\phi \ {\rm Re}(W_{0+} - W_{0-}) - 2l_{0+}^0 \sin\phi \ {\rm Im}(W_{0+} + W_{0-}) \right]$$

$\sigma_{\rm Mott} = (\alpha\cos\frac{\theta}{2}/2E_e\sin^2\frac{\theta}{2})^2$ - Mott cross section
$\alpha = e^2/(4\pi)$ - fine structure constant
M_d - mass of the deuteron, m - mass of the nucleon
$q = p_e - p_e' = (\omega, \boldsymbol{q})$ - momentum transfer
$p_e = (E_e, \boldsymbol{l})$ and $p_e' = (E_e', \boldsymbol{l}')$ - initial and final electron momenta
Ω_e' - outgoing electron solid angle
$\boldsymbol{p}_p$ - momentum of outgoing proton
$\Omega_p = (\theta_p, \phi)$ - outgoing proton solid angle
$\eta = \boldsymbol{q}^2/s$ - Lorentz boost factor

Density matrices

The virtual photon density matrix

$$l^0_{00} = \frac{Q^2}{\boldsymbol{q}^2}, \quad l^0_{0+} = \frac{Q}{|\boldsymbol{q}|\sqrt{2}}\sqrt{\frac{Q^2}{\boldsymbol{q}^2} + \tan^2\frac{\theta}{2}},$$

$$l^0_{++} = \tan^2\frac{\theta}{2} + \frac{Q^2}{2\boldsymbol{q}^2}, \quad l^0_{+-} = -\frac{Q^2}{2\boldsymbol{q}^2}$$

here $Q^2 = -q^2$

The hadron density matrix

$$W_{\lambda\lambda'} = W_{\mu\nu}\varepsilon^\mu_\lambda \varepsilon^\nu_{\lambda'}$$

λ, λ' - photon helicity components

Cartesian components

$$W_{\mu\nu} = \frac{1}{3}\sum_{s_d s_n s_p} |< np : SM_S|j_\mu|d : 1M >|^2$$

ε - photon polarization vectors, S - spin of the np pair, M_S - projection, s_d, s_n and s_p deuteron, neutron and proton momentum projections

EM current matrix element $< np : SM_S|j_\mu|d : 1M >$

Matrix element within the relativistic impulse approximation (in LS)

$$< np : SM_S|j_\mu|d : 1M >= i \sum_{n=1,2} \int \frac{d^4p^{\mathsf{CM}}}{(2\pi)^4} \times$$

$$\mathrm{Sp}\left\{\Lambda(\mathcal{L}^{-1})\bar{\psi}_{SM_S}(p^{\mathsf{CM}}, p^{*\mathsf{CM}}; P^{\mathsf{CM}})\Lambda(\mathcal{L})\Gamma_\mu^{(n)}(q) \times\right.$$

$$\left. S^{(n)}\left(\frac{K_{(0)}}{2} - (-1)^n p - \frac{q}{2}\right) \Gamma^M\left(p + (-1)^n \frac{q}{2}; K_{(0)}\right)\right\}$$

P^{CM} - total, $p^{*\mathsf{CM}}$ - relative momenta of the outgoing nucleons

p^{CM} - relative momenta in the center-of-mass system (CM)

p - relative np pair momentum in LS (p, q)

$K_{(0)} = (M_d, \mathbf{0})$ - deuteron total momentum in LS

$S^{(n)}$ - propagator of the nth nucleon

$\mathcal{L}$ - Lorenz-boost transformation along the $\boldsymbol{q}$ direction

Λ - boost operator from CM to LS:

$$\Lambda(\mathcal{L}) = \left(\frac{1+\sqrt{1+\eta}}{2}\right)^{\frac{1}{2}} \left(1 + \frac{\sqrt{\eta}\gamma_0\gamma_3}{1+\sqrt{1+\eta}}\right),$$

$\Gamma_\mu^{(n)}$ - photon-nucleon interaction vertex (on-mass-shell):

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) - \frac{1}{4m}\left(\gamma_\mu \not{q} - \not{q}\gamma_\mu\right) F_2(q^2), \tag{4}$$

ψ_{SM_S} - np pair wave function Γ^M - deuteron vertex function
are solutions of the Bethe-Salpeter equation with separable kernel

Matrix element

Plane-wave approximation

$$< np : SM_S | j_\mu | d : 1M >^{(0)} = i \sum_{n=1,2} \{ \Lambda(\mathcal{L}^{-1}) \bar{\chi}_{SM_S}(p^{*\,\mathsf{CM}}; P^{\mathsf{CM}}) \Lambda(\mathcal{L}) \Gamma_\mu^{(n)}(q)$$

$$\times S^{(n)}(\frac{K_{(0)}}{2} - (-1)^n p^* - \frac{q}{2}) \Gamma^M(p^* + (-1)^n \frac{q}{2}; K_{(0)}) \}$$

$\Lambda(\mathcal{L})$ - Lorentz-boost matrix
$\Gamma_\mu^{(n)}(q) = \gamma_\mu F_1(q) + (\gamma_\mu \hat{q} - \hat{q}\gamma_\mu) F_2(q)$ - photon-NN vertex
$F_{1,2}(q)$ - electromagnetic nucleon form-factors

Matrix element

Final-state interaction

$$< np : SM_S|j_\mu|d : 1M >^{(t)} = \frac{i}{4\pi^3} \sum_{n=1,2} \sum_{LmJM_JL'lm'} C^{JM_J}_{LmJM_J} Y_{Lm}(\hat{\boldsymbol{p}}^*)$$

$$\int_{-\infty}^{\infty} dp_0^{\mathsf{CM}} \int_0^{\infty} (\boldsymbol{p}^{\mathsf{CM}})^2 d|\boldsymbol{p}^{\mathsf{CM}}| \int_{-1}^{1} d\cos\theta^{\mathsf{CM}}_{\boldsymbol{p}} \int_0^{2\pi} d\phi$$

$$\mathrm{Sp}\left\{\Lambda(\mathcal{L}^{-1})\bar{\mathcal{Y}}_{JL'SM_J}(\boldsymbol{p}^{\mathsf{CM}})\Lambda(\mathcal{L})\Gamma^{(n)}_\mu(q)S^{(n)}\left(\frac{K_{(0)}}{2} - (-1)^n p - \frac{q}{2}\right)\right.$$

$$\left.\mathcal{Y}_{JlSm'}\left(\boldsymbol{p} + (-1)^n \frac{\boldsymbol{q}}{2}\right)\right\}$$

$$\frac{T^*_{L'L}(p_0^{\mathsf{CM}}, |\boldsymbol{p}^{\mathsf{CM}}|; 0, |\boldsymbol{p}^*|; s)}{(\sqrt{s}/2 - E_{\boldsymbol{p}} + i\epsilon)^2 - p_0^2} g_l\left(p_0 + (-1)^n \frac{\omega}{2}, \left|\boldsymbol{p} + (-1)^n \frac{\boldsymbol{q}}{2}\right|\right)$$

Calculations

- calculate trace in MAPLE, perform analytic integration over ϕ and convert expressions to FORTRAN
- analyze the poles in complex p_0 plane (poles from propagators, radial parts of deuteron vertex function and np pair amplitude can cross the Wick rotation contour and give additional contribution)
- perform (3,2,1)-fold numerical integrations in FORTRAN

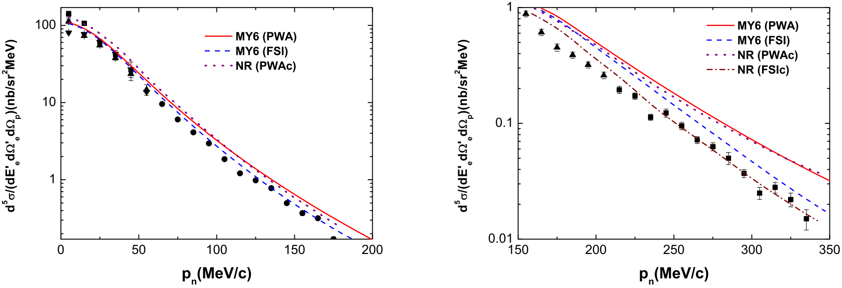
All calculations are performed with MYN kernel without taking into account the inelasticities.

Sacley data

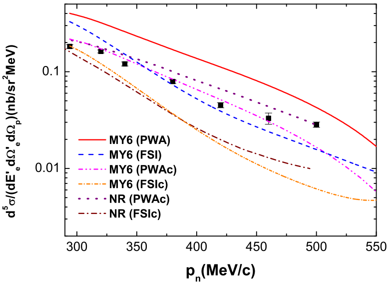
S_I, S_{II}– M. Bernheim et al., Nucl. Phys. **A365**, 349 (1981).
S_{III}– S. Turck-Chieze et al., Phys. Lett. **B142**, 145 (1984).

S.G. Bondarenko, V.V. Burov, E.P. Rogochaya. "Final-state interaction effects in electrodisintegration of the deuteron within the Bethe-Salpeter approach". JETP Lett. 94 (2012) 3.

		S_I	S_{II}	S_{III}
E_e, GeV		0.500	0.500	0.560
E'_e, GeV		0.395	0.352	0.360
θ, $^\circ$		59	44.4	25
p_n, GeV/c	min	0.005	0.165	0.294
	max	0.350	0.350	0.550
θ_n, $^\circ$	min	101.81	172.07	153.01
	max	37.78	70.23	20.81
θ_{qe}, $^\circ$	min	48.79	44.74	33.06
p_p, GeV/c	min	0.451	0.514	0.557
	max	0.276	0.403	0.306
θ_p, $^\circ$	min	0.622	2.54	13.86
	max	51.03	54.90	140.28
θ_{pe}, $^\circ$	min	49.41	47.28	46.92
	max	99.81	99.64	173.35
$\sqrt{s}$, GeV		1.929	1.993	2.057
$\sqrt{s}-2m$, GeV		0.051	0.115	0.176
Q^2, (GeV/c)^2		0.192	0.101	0.038
ω, GeV		0.105	0.148	0.200
$\|\boldsymbol{q}\|$, GeV/c		0.450	0.350	0.279



Cross section depending on recoil neutron momentum $|\boldsymbol{p}_n|$ calculated under kinematic conditions set I, II of the Sacley experiment (S_I,S_{II}). MY6 (PWA) (red solid line) - relativistic calculation in the plane-wave approximation with the MY6 potential; MY6 (FSI) (blue dashed line) - relativistic calculation including FSI effects; NR (PWAc) (violet dotted line) - nonrelativistic calculation (Shebeko *et al.*)



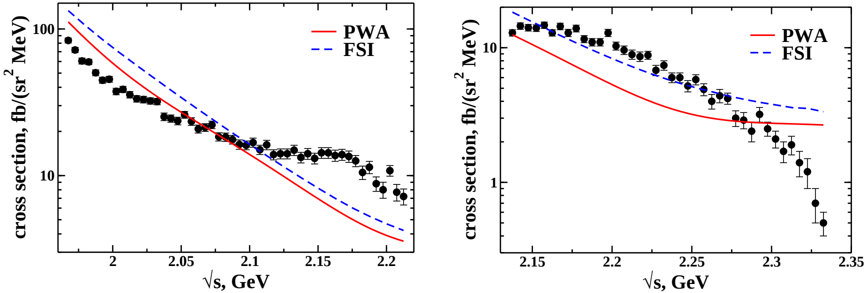
The same as in previous figures but under kinematic conditions set III of the Scale experiment (S_{III}). Two additional results are presented for comparison: MY6 (PWAc) (pink dashed-dotted-dotted line) - relativistic PWA calculation; MY6 (FSIc) (orange dashed-dotted line) - relativistic calculation with FSI effects; both obtained under current conservation condition $\omega J_0 = q_z J_z$

Bonn data

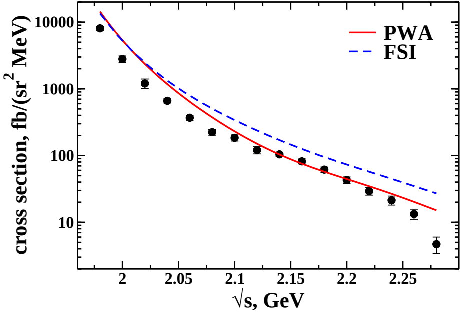
B_I, B_{II}– H. Breuker *et al.*, Nucl. Phys. **A455**, 641 (1986).
B_{III}, B_{IV}, B_V– B. Boden *et al.*, Nucl. Phys. **A549**, 471 (1992).

S.G. Bondarenko, V.V. Burov, E.P. Rogochaya. “Inelasticity of the NN-kernel for the final-state interaction in the deuteron breakup”. PoS (Baldin ISHEPP XXI), 2013.

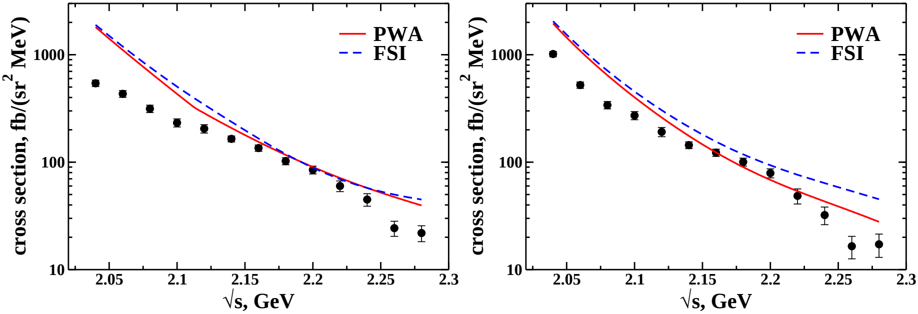
		B_I	B_{II}	B_{III}	B_{IV}	B_V
E, GeV		1.464	1.569	1.2	1.2	1.2
E', GeV	min	1.175	1.118	0.895	0.895	0.895
	max			0.800	0.800	0.800
θ, $^\circ$		21	21	20.15	20.15	20.15
p_n, GeV/c	min	0.314	0.500	0.126	0.197	0.197
	max	0.660	0.773	0.564	0.423	0.488
p_p, GeV/c	min	0.466	0.681	0.525	0.620	0.622
	max	0.664	0.791	0.834	0.929	0.889
$\sqrt{s}$, GeV	min	1.9675	2.1375	1.98	2.04	2.04
	max	2.2125	2.3325	2.28	2.28	2.28
$\sqrt{s}-2m$, GeV	min	0.090	0.260	0.101	0.161	0.161
	max	0.335	0.455	0.401	0.401	0.401
Q^2, GeV^2/c^2	min	0.257	0.255	0.154	0.145	0.145
	max	0.206	0.209	0.106	0.106	0.106
ω, GeV	min	0.162	0.348	0.148	0.210	0.210
	max	0.422	0.568	0.476	0.476	0.476
q_z, GeV/c	min	0.532	0.613	0.420	0.435	0.435
	max	0.620	0.729	0.577	0.577	0.577



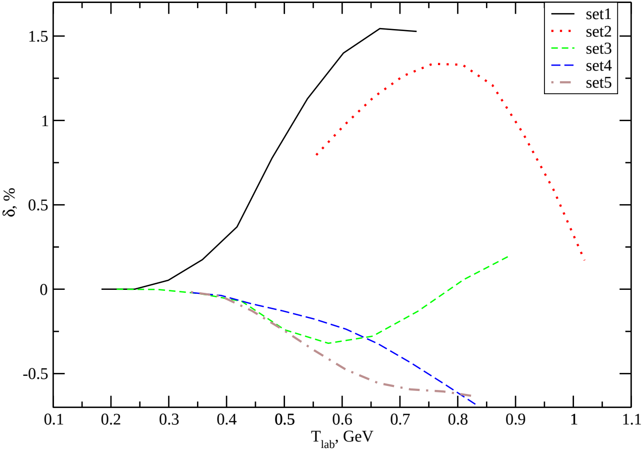
Cross section depending on $\sqrt{s}$ - invariant mass of the np-pair - calculated under kinematic conditions set I, II of the Bonn experiment (B$_I$,B$_{II}$). Solid red line - plane-way calculations, dotted black curve - final-state interaction calculations.



Cross section depending on $\sqrt{s}$ - invariant mass of the np-pair - calculated under kinematic conditions set III of the Bonn experiment (B_{III}). Solid red line - plane-way calculations, dotted black curve - final-state interaction calculations.



The same as in previous figure but under kinematic conditions set IV ans V of the Bonn experiment (B_{IV},B_V). Solid red line - plane-way calculations, dotted black curve - final-state interaction calculations.



$\delta = \frac{\mathrm{FSI without inelasticities}}{\mathrm{FSI with inelasticities}} - 1$, in % for all Bonn experimental kinematic conditions

Conclusion

- The influence of the electromagnetic nucleon-nucleon form factors in the reaction of the elastic electron-deuteron scattering is investigated in the BS formalism.
- The multirank complex separable kernels of the neutron-proton interaction for states with the total angular momentum J=0,1 are used to calculate final-state interaction effects for the deuteron electrodisintegration.
- The effects of the FSI are small at low momentum-transfer squared and energy of np-pair but become sizable at higher values of them (dozens of per cent).
- The effects of the inelasticities are relatively small (not exceed 1.5 $\%$) in the region of the laboratory kinetic energy of the np-pair from 0.2 till 1.1 GeV for unpolarized cross-section. But their contribution to the polarization characteristics should be investigated.