Top-quark condensate and spontaneous symmetry breaking in the Standard Model

Andrej Arbuzov

JINR, Dubna

In collaboration with: V.N. Pervushin, R.G. Nazmitdinov,
A.E. Pavlov, and A.F. Zakharov

[arXiv:1209.4460]

7th BLTP JINR-APCTP Joint Workshop 18th July 2013

Outline

- Motivation
- Energy scales in fundamental interactions
- Remind the standard Higgs mechanism
- Naturalness problem of SM
- Higgs boson discovery
- Condensate mechanism of symmetry breaking
- Estimate of top quark condensate
- Conclusions

Motivation (I)

- Recent discovery of a Higgs boson with mass about 126 GeV, which is consistent with SM

Motivation (I)

- Recent discovery of a Higgs boson with mass about 126 GeV, which is consistent with SM
- Still a too good description of all HEP observations within SM

Motivation (I)

- Recent discovery of a Higgs boson with mass about 126 GeV, which is consistent with SM
- Still a too good description of all HEP observations within SM
- No any experimental signal of new physics up to about 1 TeV

Motivation (I)

- Recent discovery of a Higgs boson with mass about 126 GeV, which is consistent with SM
- Still a too good description of all HEP observations within SM
- No any experimental signal of new physics up to about 1 TeV
- Intrinsic problems of the Standard Model, Standard Cosmology, and General Relativity

Motivation (I)

- Recent discovery of a Higgs boson with mass about 126 GeV, which is consistent with SM
- Still a too good description of all HEP observations within SM
- No any experimental signal of new physics up to about 1 TeV
- Intrinsic problems of the Standard Model, Standard Cosmology, and General Relativity
- Most of them are related to EXPLICIT scale invariance breaking

Motivation (I)

- Recent discovery of a Higgs boson with mass about 126 GeV, which is consistent with SM
- Still a too good description of all HEP observations within SM
- No any experimental signal of new physics up to about 1 TeV
- Intrinsic problems of the Standard Model, Standard Cosmology, and General Relativity
- Most of them are related to EXPLICIT scale invariance breaking
- We have to look for alternatives . . .

Motivation (I)

- Recent discovery of a Higgs boson with mass about 126 GeV, which is consistent with SM
- Still a too good description of all HEP observations within SM
- No any experimental signal of new physics up to about 1 TeV
- Intrinsic problems of the Standard Model, Standard Cosmology, and General Relativity
- Most of them are related to EXPLICIT scale invariance breaking
- We have to look for alternatives . . .
- Symmetry principles to be exploited

Motivation (I)

- Recent discovery of a Higgs boson with mass about 126 GeV, which is consistent with SM
- Still a too good description of all HEP observations within SM
- No any experimental signal of new physics up to about 1 TeV
- Intrinsic problems of the Standard Model, Standard Cosmology, and General Relativity
- Most of them are related to EXPLICIT scale invariance breaking
- We have to look for alternatives . . .
- Symmetry principles to be exploited
- Correspondence to SM should be preserved

Motivation (II)

Higgs boson with $m_H \approx 126$ GeV makes the SM stable up to the Planck energy scale, i.e. 10^{19} GeV. Citation:

"... where $M_{min} = 129 \pm 6$ GeV. We argue that the discovery of the SM Higgs boson in this range would be in agreement with the hypothesis of the absence of new energy scales between the Fermi and Planck scales, whereas the coincidence of M_H with M_{min} would suggest that the electroweak scale is determined by Planck physics."

New physics is not required?

F. Bezrukov, M.Y. Kalmykov, B.A. Kniehl and M. Shaposhnikov, "Higgs Boson Mass and New Physics," JHEP 1210 (2012) 140

S. Alekhin, A. Djouadi and S. Moch, "The top quark and Higgs boson masses and the stability of the electroweak vacuum," Phys. Lett. B **716** (2012) 214

Motivation (III)

Experience received from working in effective models can be further used in attempts to construct or modify a fundamental one

Below we will discuss how the mechanism of the chiral symmetry breaking can be transmitted to the SM

Remind, the Nobel Prize in Physics 2008 (one half) was awarded to Yoichiro Nambu "for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

Mechanisms of Spontaneous Symmetry Breaking (SSB) in SM and QCD are similar but have different types

Fundamental Interactions

At present to the best of our knowledge we can distinguish

6 types of fundamental interactions:

1) U(1) gauge int.
2) SU(2) gauge int.
3) SU(3) gauge int.
4) Higgs Yukawa int.
5) Higgs self coupling
6) Gravity

N.B. Textbook notion about 4 fundamental interactions is obsolete

Scale invariance breaking

The observed world is obviously not Scale Invariant (SI)

But many physical laws are SI, see e.g. Newtonian mechanics and Maxwell equations

There is only one term (the Higgs tachyon mass) in the SM Lagrangian, which explicitly breaks SI,

then we have dimensional transmutation in QCD,

and an explicitly dimensionful coupling constant in Gravity

All those make real troubles for the fundamental theory

Examples of SI breaking

1. In the Newtonian classical mechanics (w/o gravity), the laws are SI but solutions are not. The breaking happens due to the initial conditions. This is a **soft** symmetry breaking.

N.B. Dynamical symmetry breaking is a soft one (Y. Nambu)

2. In QED the SI is broken by the electron mass which enters the Lagrangian. This is an **explicit** symmetry breaking.

Due to quantum effects we have in QED also the Landau pole:

$$\alpha(Q^2) \approx \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi} \ln \frac{Q^2}{m_e^2}}, \qquad \alpha(0) \approx \frac{1}{137}, \qquad \alpha(Q_0^2) \to \infty$$

This problem is not resolved in QED, it is due to the SI breaking.

Does the Higgs boson give masses to everything around?

Does the Higgs boson really give masses to everything that we see?

NO!

Λ-term and dark matter in Cosmology?

the proton mass?

neutrino masses?

the Higgs mass itself?

We still do not understand the origin of masses

and of fundamental physical energy scales in general

Higgs boson in SM (I)

Remind the Standard Model mechanism:

$$V_{\rm Higgs}(\phi) = \frac{\lambda^2}{2}(\phi^\dagger\phi)^2 + \mu^2\phi^\dagger\phi$$

Due to spontaneous symmetry breaking (SSB) of $O(4)$ symmetry

if $\mu^2 < 0$, one component of the complex scalar doublet field $\phi = \left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right)$ acquires a non-zero vacuum expectation value

$$\langle\phi^0\rangle = v/\sqrt{2}$$

The vacuum stability condition $\lambda^2 > 0$ is always assumed

Higgs boson in SM (II)

The $O(4)$ symmetry of the Higgs field is broken spontaneously but that does not protect the Higgs mass from huge renormalizations:

$$\Delta m_H^2 \sim \Lambda^2$$

contrary to the cases of m_W and m_Z which have typical

$$\Delta m_{W,Z}^2 \sim m_{W,Z}^2 \ln \frac{\Lambda^2}{m_{W,Z}^2}$$

That is known as the naturalness or fine tuning or hierarchy problem of SM

That is because m_W and m_Z have the pure SSB origin, while m_H is related to the tachyon mass term ($\mu^2 < 0$) which breaks the conformal symmetry of SM **explicitly**

Naturalness problem

There are two general ways to solve the naturalness problem:

I. Cancel out the huge radiative corrections

— either due to some (super)symmetry

— or due to fine tuning (anthropic principle)

II. Make Λ small, i.e. $\Lambda \sim M_W$ (EW scale) with some new physics motivation

— but LHC and others do not see anything new at this scale

Higgs boson discovery (I)

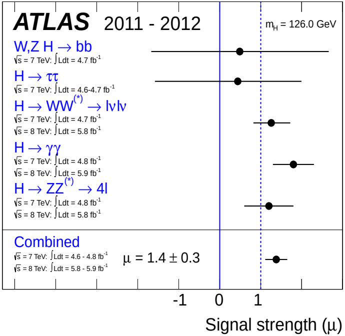
14 March 2013, ATLAS and CMS at Moriond Conf. claimed that the particle discovered at LHC “is looking more and more like a Higgs boson”

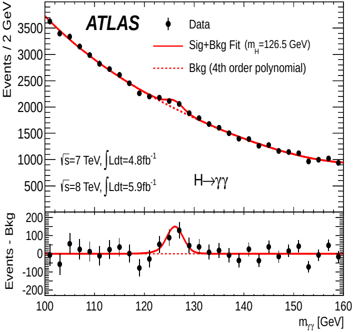
The preliminary results with the full 2012 data set are magnificent and to me it is clear that we are dealing with a Higgs boson though we still have a long way to go to know what kind of Higgs boson it is.

Experiments confirm that the discovered boson is scalar, and the observed decay rates in different channels are compatible with the ones predicted by the SM

Higgs boson discovery (II)

Example of ATLAS plots [PLB 716 (2013) 1]:





$$M_H = 126.0 \pm 0.4(stat) \pm 0.4(syst)\ \text{GeV}$$

Spontaneous conformal symmetry breaking (SCSB)

The idea: use field condensates instead of explicit fundamental terms in the Lagrangian

Remind the normal ordering in QFT (theoretical)

Remind quark and gluon condensates in QCD (phenomenological)

Remind the relation of condensates to the Casimir energy:

$$C_{\mathrm{Cas}} = 2\frac{\partial}{\partial m^2}E_{\mathrm{Cas}}, \quad E_{\mathrm{Cas}} = \frac{1}{2}\sum_k \sqrt{\mathbf{k}^2 + m^2}$$

N.B. Both the Casimir energy and condensates are finite without any new physics at a TeV scale

SCSB for Higgs (I)

The dominant terms of Higgs interactions (for $\mu \equiv 0$) are

$$L_{\rm int} = -\frac{\lambda^2}{8} h^4 - g_t h\ \bar{t} t$$

Normal ordering $\bar{t}\,t =: \bar{t}\,t : + \langle \bar{t}\,t \rangle$ gives the Higgs potential in the form

$$V_{\rm cond}(h) = \frac{\lambda^2}{8} h^4 + g_t \langle \bar{t}\,t \rangle h$$

The extremum condition $dV_{\rm cond}/dh|_{h=v} = 0$ yields

$$\frac{\lambda^2}{2} = -\frac{g_t \langle \bar{t}\,t \rangle}{v^3}$$

The Yukawa coupling g_t is known from $m_t = v \cdot g_t \simeq 173.4$ GeV

The potential takes the form

$$V_{\rm cond}(h)|_{h=v+H} = V_{\rm cond}(v) + \frac{3\lambda^2 v^2}{4} H^2 + \frac{\lambda^2 v}{2} H^3 + \frac{\lambda^2}{8} H^4$$

SCSB for Higgs (II)

So the Higgs mass is

$$m_H^2 \equiv \frac{\lambda^2}{2} 3v^2 = -\frac{3g_t \langle \bar{t}\, t \rangle}{v}$$

The question now is how to find the top quark condensate?

N.B. $\langle \bar{q}\, q \rangle < 0$

Top quark condensate

Even so that $m_t \gg \Lambda_{\rm QCD}$, the perturbative QCD doesn't give $\langle \bar{t}\, t\rangle$ since the tadpole is quadratically divergent. Formally

$$\langle \bar{t}\, t\rangle = -4N_c \int \frac{d^3p}{(2\pi)^3} \frac{m_t}{2\sqrt{p^2+m_t^2}} = -4N_c \cdot \gamma_0 \cdot m_t^3$$

Quantity γ_0 is a finite scale invariant

If the conformal symmetry is fundamental, then up to *finite radiative corrections*

$$\frac{\langle \bar{t}\, t\rangle}{m_t^3} = \frac{\langle \bar{q}\, q\rangle}{m_q^3},$$

where q is a light quark

Top quark condensate and m_H

The Gell-Mann–Oakes–Renner (1986) relation

$$m_\pi^2 \cdot F_\pi^2 = -(m_{u\ \mathrm{current}} + m_{d\ \mathrm{current}})\langle \bar{q}\, a\rangle$$

is consistent with the light quark condensate value

$$\langle \bar{q}\, q\rangle \approx -(250\ \mathrm{MeV})^3$$

found also from QCD sum rules.
Taking the constituent light quark mass $m_q \approx 330$ MeV we get the top quark condensate

$$\langle \bar{t}\, t\rangle \approx -(127\ \mathrm{GeV})^3$$

and consequently

$$m_H^2 = (130 \pm 15\ \mathrm{GeV})^2$$

Other field contributions

Assumption: γ_0 is universal

The normal ordering $HH =: HH : + < HH >$ yields in the lowest order

$$\frac{< HH >}{m_H^2} \quad = \quad \frac{1}{m_H^2} \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2\sqrt{p^2 + m_H^2}} \equiv \gamma_0 .$$

Analogously for vector fields $V_i V_j =: V_i V_j : + < VV > \delta_{ij}$,

$$< VV >= M_V^2 \cdot \gamma_0 , \quad V = W^{\pm}, Z$$

Then taking into account degrees of freedom of vector fields

$$\Delta m_H^2 = \frac{3\lambda^2}{4} < HH > + \frac{3}{8} g^2 \left(2 < WW > + \frac{< ZZ >}{\cos^2 \theta_W} \right) ,$$

$$m_H \to m_H \left[1 + 4 \frac{\Delta m_H^2}{v^2} \right]^{1/2} \approx m_H \cdot (1 + 0.02)$$

Conclusions

1. We proposed a simple modification of the SM based on the Nambu condensate mechanism. The difference from SM is only in 1.5 times lower value of the Higgs self-coupling λ which will be measured at ILC (?)

2. Here m_H and m_t are mutually related and together define EW scale

3. Our estimate of the top quark condensate is crude but the value looks natural (?)

4. The suggested mechanism automatically protects m_H from running away, since renormalization happens at the EW scale

5. The picture resembles the EW bootstrap suggested by Nambu and Bardeen at al. (1989). But their approaches were not based on the conformal symmetry. They just tried to cancel out the quadratic divergences.

6. Similar relations are used also in modern technicolor models, but the Higgs boson there is composite