Anomalies, inflow and holography

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M-theory basics:

11D theory with 32 supercharges:

$$I_{11} = \frac{1}{2\kappa_{11}^2}\left[\int d^{11}x\sqrt{-G}R_G - \frac{1}{2}G\wedge *G - \frac{1}{6}C\wedge G\wedge G\right]$$

Supersymmetric flux backgrounds ($G \neq 0$, $\mathrm{d}G = 0$):

- FR: $AdS_7 \times S^4$ max susy ($SO(5)$ symmetry)
- AdS_5 solutions
  - ◇ BBBW:
    $\left\{ \begin{array}{l} \text{(squashed) } S^4 \text{ fibered over } \Sigma_g \;-\text{ at least } U(1)^2 \text{ symmetry} \\ \text{8 or 16 supercharges} \end{array} \right.$
  - ◇ GMSW:
    $\left\{ \begin{array}{l} \mathrm{Mink}_4 \times_w \mathcal{C}(M_6) \quad \Rightarrow \quad AdS_7 \times M_6 \\ M_6 \;-\text{ compact complex } -\; S^2 \text{ bundle over Kähler } K_4 \end{array} \right.$

M-theory - strong coupling limit of type IIA strings

Higher derivative terms in string theory (α' and g_s expansion) $\sim \mathcal{R}^{3l+1}$ lift to 11D

Special CP-odd couplings : $\Rightarrow \quad C \wedge X_8$

$$\begin{aligned} X_8 &= \frac{1}{48}\left(\frac{1}{4}p_1(TM)^2 - p_2(TM)\right) \\ &= \frac{1}{(2\pi)^4}\left(-\frac{1}{768}(\text{tr}\, R^2)^2 + \frac{1}{192}\text{tr}\, R^4\right) \end{aligned}$$

- ▷ 5pt function at one-loop
  - ◇ $\hat{\chi} = \frac{1}{4!(4\pi)^2} \cdot \frac{1}{2^4}$ $\epsilon^{i_1 \cdots i_8} R_{a_1 a_2} \left(\Gamma^{a_1 a_2}\right)^{i_1 i_2} R_{a_3 a_4} \left(\Gamma^{a_3 a_4}\right)^{i_3 i_4} R_{a_5 a_6} \left(\Gamma^{a_5 a_6}\right)^{i_5 i_6} R_{a_7 a_8} \left(\Gamma^{a_7 a_8}\right)^{i_7 i_8}$
  - ◇ $\hat{\chi} = \frac{1}{16}\left(8\chi + p_1(TM)^2 - 4p_2(TM)\right)$
- ▷ Needed for **string dualities**
- ▷ Cancellations of **M5 anomalies**

M5-branes

Calssical soliton of 11D sugra

- the metric: $ds^2_{10} = e^{N_1 u(r)} \mathrm{d}s^2_6(W_{||}) + e^{N_2 u(r)} (\mathrm{d}s^2_5)_{\perp}$ (r - distance away from M5)
- the four-form : $G_4 \sim \star_{\perp} du(r)$

$$\mathrm{d}G_4 = \delta_5(r)$$

- zero-mode expansion $G_4 \to G^{(0)} + h_3 \wedge \mathrm{d}u(r) + \cdots$

$$\mathrm{d} * G_4 \sim G_4 \wedge G_4 \qquad \Rightarrow \qquad h_3 = - *_{||} h_3$$

$$\text{Theory on M5} \Leftrightarrow \begin{cases} \bullet & (2,0) \text{ tensor multiplet} \\ \bullet & (\beta^-, \psi^\alpha, x^a) \quad \alpha = 1, ..., 4; \;\; a = 1, ..., 5 \\ \bullet & SO(5) \text{ R-symmetry} \\ \bullet & ADE \text{ classification - non-Abelian M5} \end{cases}$$

Symmetries of the theory without M5 $[C \wedge G \wedge G + C \wedge [\frac{1}{4}p_1^2(TM) - p_2(TM)]]$

- shift: $C_3 \to C_3 + \mathrm{d}\Lambda$
- diffeomorphisms

With M5 $i : W_6 \hookrightarrow M_{11}$

- $\delta(\int_{M_{11}} C \wedge G \wedge G) \ \to \ \int_{W_6} i^*(\Lambda \wedge G)$
  - ⋆ M5 coupling $\int_{W_6} h_3 \wedge i^* C$
  - ⋆ $\delta h_3 = i^*(\mathrm{d}\Lambda)$ .... relative cohomology ($\mathrm{d}h = i^* G$)

- diffeomorphisms and $\delta \int C \wedge X_8$?
  - ⋆ X_8 - (made of) characteristic class(es)... $X_8 = \mathrm{d}X_7^{(0)}$ and $\delta X_7^{(0)} = \mathrm{d}X_6^{(1)}$
  - ⋆ assume trivial normal bundle ($p_i(TM)|_W = p_i(TW)$)
  - ⋆ $\delta \int C \wedge X_8 \ \to \ \int_{W_6} X_6^{(1)}$
  - ⋆ anomaly inflow

M5 ANOMALY

(2,0) tensor multiplet:

- Woldvolume chiral 2-form
  - $I_\beta = \frac{1}{5760}\big(16p_1(TW)^2 - 112p_2(TW)\big) \sim L(TW)$
- Worldvolume fermions
  - $I_D = \frac{1}{2}\widehat{A}(TW)\,\mathrm{ch}S(N)$
  - for trivial normal bundle:

$$I_D = 4 \times \frac{1}{2}\widehat{A}(TW) = \frac{1}{5760}\big(14p_1(TW)^2 - 8p_2(TW)\big)$$

- Total anomaly: $I_{\text{M5}} = \frac{1}{48}\big(\frac{1}{4}p_1(TW)^2 - p_2(TW)\big)$
- Cancelled via inflow from a bulk coupling $\sim C_3 \wedge X_8(TM)$

$$G_4\delta X_7^{(0)} \to \eta(M5)X_6^{(1)}(TM) \quad \leftrightarrow \quad \mathrm{d}^{-1}\delta\mathrm{d}^{-1}I_{(2,0)}$$

- Nontrivial normal bundle... $C \wedge G \wedge G$ is NOT ... diff invariant!

Non-trivial normal bundle

(single) M5 worldvolume :

- Chiral 2-form
  - ◇ $I_\beta = \frac{1}{5760}\left(16p_1(TW)^2 - 112p_2(TW)\right) \sim L(TW)$
- Fermions
  - ◇ $I_D = \frac{1}{2}\widehat{A}(TW)\,\mathrm{ch}S(N)$
  - ◇ $\mathrm{ch}S(N) = 4 + \frac{1}{2}p_1(N) + \frac{1}{96}p_1(N)^2 + \frac{1}{24}p_2(N) + \ldots$

$$I_D = 4 \times \frac{1}{2}\widehat{A}(TW) + \ldots = \frac{1}{5760}\left(14p_1(TW)^2 - 8p_2(TW)\right) + \ldots$$

- Total anomaly:
  $I_{\text{M5}} = \frac{1}{48}\left(\frac{1}{4}(p_1(TW)^2 + p_1(N)^2 - 2p_1(TW)p_1(N)) - p_2(TW) + p_2(N)\right)$
- Anomaly from the bulk (using $p_1(TM|_W) = p_1(TW) + p_1(N)$, ...)

$$I_{\text{bulk}} = -\frac{1}{48}\left(\frac{1}{4}(p_1(TW)^2 + p_1(N)^2 - 2p_1(TW)p_1(N)) - p_2(TW) - p_2(N)\right)$$

- The result: $I_{\text{M5}} + I_{\text{bulk}} = \frac{p_2(N)}{24}$!!!

Non-singular p-branes

Brane worldvolume W_d ($d = p + 1$) located at $y^a = 0$ ($a = 1, ..., D - d$) in M_D

$S_\epsilon(W_d)$ - S^{D-d-1} sphere bundle - boundary of tubular neighbourhood of rad ϵ, $D_\epsilon(W_d)$

- Magnetic source:

◇ $$\mathrm{d}G_{D-d-1} = 2\pi\delta(y^1)...\delta(y^{D-d})\mathrm{d}y^1 \wedge ... \wedge \mathrm{d}y^{D-y} \quad \Rightarrow \quad 2\pi\Phi_{D-d}$$

Thom class of N $\Phi_{D-d} = \begin{cases} \bullet \quad \mathrm{d}(\rho(r)e_{2n-1}/2) \qquad 2n-1 = D-d-1 \\ \bullet \quad \mathrm{d}\rho(r)e_{2n}/2 \qquad 2n = D-d-1 \end{cases}$

e_{D-d-1} - global angular form

- rank$(N) = 2n$ - sphere bundle has fibers S^{2n-1}

◇ $$\mathrm{d}e_{2n-1} = -\pi^*(\chi(N)) \qquad \chi(N) \in H^{2n}(M, \mathbb{Z})$$

- rank$(N) = 2n + 1$ - sphere bundle has fibers S^{2n}

◇ $$\mathrm{d}e_{2n} = 0 \qquad (e_{2n} = \mathrm{d}e^{(0)}_{2n-1},\ \ \delta e^{(0)}_{2n-1} = e^{(1)}_{2n-2})$$

◇ cohomology class e_{2n} : $\quad [e^2_{2n}] = \pi^*(p_n(N))$

◇ at the level of differential forms : $\quad \pi_*(e^3_{2n}) = \pi_*(e_{2n}\pi^* p_n(N)) = 2p_n(N)$

M5 ($W_6 \hookrightarrow M_{11}$) - $SO(5)$ N bundle

$\mathbf{so(5)} \cong \Lambda^2\mathbb{R}^5$ - connection on N: $\quad \Theta^{ab} = -\Theta^{ba}$

Define $\hat{y}^a = y^a/r\,; \quad (D\hat{y})^a = \mathrm{d}\hat{y}^a - \Theta^{ab}\hat{y}^b\,; \quad F^{ab} = \mathrm{d}\Theta^{ab} - \Theta^{ac}\wedge\Theta^{ca}$

$$\left\{\begin{array}{ll} \bullet & e_4(\Theta) = \frac{1}{64\pi^2}\epsilon_{a_1\ldots a_5}\left((D\hat{y})^{a_1}\ldots(D\hat{y})^{a_4} - 2F^{a_1a_2}(D\hat{y})^{a_3}(D\hat{y})^{a_4} + F^{a_1a_2}F^{a_3a_4}\right)\hat{y}^{a_5} \\ \bullet & e_3^{(0)}(\Theta) = \frac{1}{32\pi^2}\epsilon_{a_1\ldots a_5}\left(\Theta^{a_1a_2}\mathrm{d}\Theta^{a_3a_4}\hat{y}^{a_5} - \frac{1}{2}\Theta^{a_1a_2}\Theta^{a_3a_4}\mathrm{d}\hat{y}^{a_5} - 2\Theta^{a_1a_2}\mathrm{d}\hat{y}^{a_3}\mathrm{d}\hat{y}^{a_4}\hat{y}^{a_5}\right) \\ \bullet & e_2^{(1)}(\varepsilon,\,\Theta) = \frac{1}{16\pi^2}\epsilon_{a_1\ldots a_5}\varepsilon^{a_1a_2}\left(\mathrm{d}\hat{y}^{a_3}\mathrm{d}\hat{y}^{a_4}\hat{y}^{a_5} - \Theta^{a_3a_4}\mathrm{d}\hat{y}^{a_5}\right) \end{array}\right.$$

Under gauge transformations: $\delta\Theta^{ab} = (D\varepsilon)^{ab}, \quad \delta\hat{y}^a = \varepsilon^{ab}\hat{y}^b$

In the presence of M5:

- $G = \mathrm{d}C \quad \longrightarrow \quad \mathrm{d}C - 2\pi\mathrm{d}\rho\wedge e_3^{(0)}/2$
  - $\diamond \quad \delta C = -2\pi\mathrm{d}\rho\wedge e_2^{(1)}/2$
- Introduce σ_3: $\quad G_4 - 2\pi\rho e_4/2 = \mathrm{d}(C_3 - 2\pi\rho e_3^{(0)}/2) \equiv \mathrm{d}(C_3 - 2\pi\sigma_3)$
- CS couplings in presence of M5:

$$S_{\text{cs}} = \lim_{\epsilon\to 0} -\frac{1}{6(2\pi)^2}\int_{M_{11}-D_\epsilon(W_6)} (C_3 - 2\pi\sigma_3)\wedge\mathrm{d}(C_3 - 2\pi\sigma_3)\wedge\mathrm{d}(C_3 - 2\pi\sigma_3)$$

Remember $\pi_*(e_4^3) = \pi^* p_n(N)) = 2p_2(N)$

Under diffs ($SO(5)$ transformations), S_{CS} varies....

- $\delta((C_3 - 2\pi\sigma_3) = -2\pi \mathrm{d}(\rho e_2^{(1)}/2)$

$$\begin{aligned} \delta S_{\text{CS}} &= \lim_{\epsilon\to 0} \frac{1}{12\pi} \int_{M_{11}-D_\epsilon(W_6)} \mathrm{d}(\rho e_2^{(1)}/2) \wedge \mathrm{d}(C_3 - 2\pi\sigma_3) \wedge \mathrm{d}(C_3 - 2\pi\sigma_3) \\ &= -\frac{2\pi}{6} \int_{S_\epsilon(W_6)} \frac{e_4}{2} \wedge \frac{e_4}{2} \wedge \frac{e_2^{(1)}}{2} \end{aligned}$$

Anomaly cancellation!... and a key to non-Abelian $(2,0)$ theories

- $I_{\text{ M5}} + I_{\text{ bulk}} + \delta S_{\text{CS}} = 0$
- Q coincident M5 - symmetry enhancement to $SU(Q)$
  - ◇ $\mathrm{d}G_4 = 2\pi Q \mathrm{d}\rho e_4/2$
  - ◇ no new ingredients in the anomaly cancellation

⋆ $I_{\text{ M5}}(Q) = Q I_{\text{ M5}}(Q=1) + \frac{Q^3 - Q}{24} p_2(N)$

$(2,0)$ theories have ADE symmetry enhancement

- A_{Q-1}
  - ◇ remove a centre of mass (one free (2,0) multiplet) anomaly
  - ◇ $I^{(2,0)}_{A_{Q-1}} = (Q-1) I_{\text{M5}}(Q=1) + \frac{Q^3-Q}{24} p_2(N)$
- D_Q
  - ◇ $\mathbb{R}^5/\mathbb{Z}_2$ fixed points
  - ◇ $I^{(2,0)}_{D_Q} = Q I_{\text{M5}}(Q=1) + \frac{Q(2Q-1)(2Q-2)}{24} p_2(N)$
- E
  - ◇ no direct calculation (brane picture), but ... using 5D gauge CS confirm
  - ◇ $I^{(2,0)}_{A_{Q-1}} = r_G I_{\text{M5}}(Q=1) + \frac{d_G \cdot h^\vee_G}{24} p_2(N)$
  - ◇ $r_G, d_G, h^\vee_G$ - rank, dimension, dual Coxeter

CLASS $\mathcal{I}_{12}$

$$\mathcal{I}_{12} = -\frac{1}{6}\, e_4 \wedge e_4 \wedge e_4 - e_4 \wedge X_8$$

M-theory on a spacetime with non-trivial boundary:

- ◇ $S_\epsilon(W_6) \sim M_{11} - D_\epsilon(W_6):\qquad S^4 \hookrightarrow S_\epsilon(W_6) \to W_6$

Non-invariance of the action under diffeomorphisms:

- ◇ $\delta S_{\mathrm{M}} = 2\pi \int_{S_\epsilon(W_6)} \mathcal{I}_{10}^{(1)}$
  - ∗ $\mathrm{d}\mathcal{I}_{10}^{(1)} = \delta \mathcal{I}_{11}^{(0)}\,,\qquad \mathrm{d}\mathcal{I}_{11}^{(0)} = \mathcal{I}_{12}$

$$I_8^{\text{inflow}} = \int_{S^4} \mathcal{I}_{12}$$

$$I_8^{\text{inflow}} + I_8^{\text{CFT}} + I_8^{\text{decoup}} = 0$$

Four-dimensional anomalies

M5 on a Riemann surface Σ_g : $W_6 = W_4 \times \Sigma_g$

- Space M_6 (fixed by supersymmetry)
  - ◇ $S^4 \hookrightarrow M_6 \to \Sigma_{g,0}$
  - ◇ $M_6^{n=0} \hookrightarrow S_\epsilon(W_6) \to W_4$
  - ◇ $SO(5) \longrightarrow SO(2) \times SO(3)$ or $SO(2) \times SO(2)$
- $I_6^{\text{inflow}} = \int_{M_6^{n=0}} \mathcal{I}_{12}$ & $I_6^{\text{inflow}} + I_6^{\text{CFT}} + I_6^{\text{decoup}} = 0$

Riemann surface with **punctures** $\Sigma_{g,n}$:

- $M_6 = M_6^{\text{bulk}} \cup \bigcup_{\alpha=1}^{n} X_6^\alpha$
  - ◇ $S^4 \hookrightarrow M_6^{\text{bulk}} \to \Sigma_{g,n}$
  - ◇ X_6^α − geometry encoding puncture data
- $I_6^{\text{inflow}} = \int_{M_6} \mathcal{I}_{12} = I_6^{\text{inflow}}(\Sigma_{g,n}) + \sum_{\alpha=1}^{n} I_6^{\text{inflow}}(P_\alpha)$
  - ◇ $I_6^{\text{inflow}}(\Sigma_{g,n}) = \int_{M_6^{\text{bulk}}} \mathcal{I}_{12} = \int_{\Sigma_{g,n}} I_8^{\text{inflow}}$ & $I_6^{\text{inflow}}(P_\alpha) = \int_{X_6^\alpha} \mathcal{I}_{12}$

$\mathcal{N}=2$ calss $\mathcal{S}$ SCFTs

$$I_6^{\rm CFT} = (n_v - n_h)\left[\frac{1}{3}\,(c_1^r)^3 - \frac{1}{12}\,c_1^r\,p_1(TW_4)\right] - n_v\,c_1^r\,c_2^R - k_G\,c_1^r\,{\rm ch}_2(G_F)$$

- ◇ Flavor symmetry G_F
- ◇ R-symmetry $U(1)_r \times SU(2)_R$

M5 geometry: $TM_{11}|_W = TW_4 \oplus \underbrace{T\Sigma_{g,n} \oplus SO(2)}_{T^*\Sigma_{g,n}\ \text{(hyper-Kähler)}} \oplus SO(3)$

- ◇ Chern root of $SO(2)$: $n = n^{\text{4d}} + \int_{\Sigma_{g,n}} \chi(\Sigma_{g,n}) = 2c_1^r + (2(1-g)-n)$

$$\begin{cases} \bullet \quad n_v^{\rm inflow} + n_v^{\rm CFT} = n_v^{\text{free tensor}}(\Sigma_{g,0}) = \frac{1}{2}\chi(\Sigma_{g,0}) \\ \bullet \quad n_h^{\rm inflow} + n_h^{\rm CFT} = n_h^{\text{free tensor}}(\Sigma_{g,0}) = 0 \\ \bullet \quad k_{SU(k_a)}^{\rm inflow} + k_{SU(k_a)}^{\rm CFT} = 0 \end{cases}$$

S^4 reduction to AdS_7

- ◇ vacuum configuration: $G = 2\pi Q\,\mathrm{Vol}(S^4)$
- ◇ expand to include fluctuations - flux needs to be
  - ▷ invariant under $SO(5)$ gauge transformations
  - ▷ $\mathrm{d}G = 0$
  - ▷ quantisation $\int_{S^4} G/2 = 2\pi\, Q$
- ◇ $G = 2\pi Q e_4/2 +$ fluctuations in C_3 works!
- ◇ $\int_{M_{11}} C \wedge G \wedge G \quad \longrightarrow \quad \sim Q^3 \int_{AdS_7} p_2^{(0)}(A)$

- "topological sector" - extension to AdS_5 reductions

- Scalars can be included

$$64\pi^2 e_4 \longrightarrow \epsilon_{a_1\ldots a_5} \quad \Big(-\tfrac{1}{3}(D\hat{y})^{a_1}\ldots(D\hat{y})^{a_4}\frac{(T\cdot\hat{y})^{a_5}}{\hat{y}\cdot T\cdot\hat{y}} + \tfrac{4}{3}(D\hat{y})^{a_1}\ldots D\left(\frac{(T\cdot\hat{y})^{a_4}}{\hat{y}\cdot T\cdot\hat{y}}\right)\hat{y}^{a_5}$$
$$-2F^{a_1a_2}(D\hat{y})^{a_3}(D\hat{y})^{a_4}\frac{(T\cdot\hat{y})^{a_5}}{\hat{y}\cdot T\cdot\hat{y}} + F^{a_1a_2}F^{a_3a_4}\hat{y}^{a_5}\Big)$$

- ◇ consistent truncation of D=11 supergravity to D=7 fields
- ◇ $T^{ab} = (\Pi^{-1}\cdot\Pi^{-1})^{ab}$ - coset elements of $SL(5)/SO(5)$
- ◇ Deformations of GAF $\quad \Rightarrow \quad$ Consistent truncations (!?)
- ◇ Note $\quad SL(6)\times SL(2) \subset E_6 \qquad \& \qquad SL(8) -$ max subgroup of E_7

$AdS_{d+1} \times_w M^{\text{hol}}_{10-d}$

(think of M5 with $W_6 = W_d \times \Sigma_{6-d} \quad \& \quad S^4 \hookrightarrow M_{10-d} \rightarrow \Sigma_{6-d}$)

- Flux $\quad G|_{r=\epsilon} = 2\pi\, E_4$
  - ◇ E_4 invariant under symmetries of M_{10-d}
  - ◇ $\left.\begin{array}{l} \mathrm{d}E_4 = 0 \\ E_4 \ - \ \text{globally defined} \end{array}\right\} \quad \leftrightarrow \quad \int_{S^4} E_4 = N$
  - ◇ E_4 not uniquely fixed (yet)

- M_{10-d} has isometries (killing vectors k_I) and harmonic 2-forms ω_α:

$$E_4 = V_4^{\mathrm{g}} + F^X\, \omega_X^{\mathrm{g}} + F^X\, F^Y\, \sigma_{XY} + C\, p_1(TW_d) \qquad \text{for} \quad X = (I, \alpha)$$

  - ◇ Isometries
    - ⋆ Lie algebra: $\quad \pounds_I k_J \equiv \pounds_{k_I} k_J = [k_I, k_J] = f_{IJ}{}^K\, k_K$
    - ⋆ Gaugings: $\quad d\xi^m \ \rightarrow \ D\xi^m = d\xi^m + k_I^m\, A^I$
    - ⋆ Connections: $\quad F^I = dA^I - \frac{1}{2}\, f_{JK}{}^I\, A^J\, A^K$
  - ◇ $H^2(M_{10-d})$
    - ⋆ $C_3 = A^\alpha \omega_\alpha\,, \qquad \alpha = 1, ..., b^2(M_{10-d})$

$E_4 \quad \Rightarrow \quad$ equivariant class

- Action of G on $\Omega^*(M_{10-d})$

$$\begin{aligned} \alpha \quad : \quad \mathfrak{g} \quad &\rightarrow \quad \Omega^*(M_{10-d}) \\ \mathcal{X} \quad &\mapsto \quad \alpha(\mathcal{X}) \end{aligned}$$

  - ⋆ $\mathfrak{g} \ni \mathcal{X} = \mathcal{X}^I \, t_I$ ($\{t_I\}$ - a basis of $\mathfrak{g}$)
    - ◇ $\alpha(\mathcal{X}) = V_4 + \omega_I \, \mathcal{X}^I + \sigma_{IJ} \, \mathcal{X}^I \, \mathcal{X}^J$
    - ◇ $\alpha(\mathcal{X})$ - homogeneous of degree 4 ([(dif. form) $+ 2 \times$ (polynomial)])
  - ⋆ extend the Lie algebra $\mathfrak{g}$ by adding new Abelian generators t_α
    - ◇ $\mathcal{X} = \mathcal{X}^X \, t_X$
      - ∗ $\iota_\alpha = 0 \,, \qquad \pounds_\alpha = 0$ (trivial action on forms)
  - ⋆ Invariance & closure of $E_4 \quad \Leftrightarrow \quad d_{\mathrm{eq}} \alpha = 0$
    - ◇ $(d_{\mathrm{eq}} \alpha)(\mathcal{X}) = d\big(\alpha(\mathcal{X})\big) + \iota_{\mathcal{X}} \alpha(\mathcal{X})$
  - ⋆ Deformations: $\alpha(\mathcal{X}) \rightarrow \alpha(\mathcal{X}) + (d_{\mathrm{eq}} \beta)(\mathcal{X})$

- Fixing ambiguities

$$E_4^2 + 2\, X_8 = 0 \qquad \text{in cohomology of } M_{10-d}$$

(cf. $\frac{2\pi}{(2\pi \ell_p)^3} \, d * G_4 = \frac{1}{2} \, \frac{G_4}{2\pi} \, \frac{G_4}{2\pi} + X_8$)

- Chern-Simons couplings in AdS_{d+1} $\Leftrightarrow$ Holographic computation of anomalies
- Topology of M_{10-d} and E_4

$\star$ Extension to IIB (E_5 is very different from E_4!)

$\star$ Beyond "topological deformations" ...