Generalised Fayet-Iliopoulos terms in supergravity

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General comments

- Cosmological data: Expanding universe with a small positive cosmological constant.
- It is desirable to develop theoretical mechanisms to explain positivity of the cosmological constant.
- In the last few years it was appreciated by many researchers that such a mechanism is provided by spontaneously broken supergravity (de Sitter SUGRA).
- Actually the idea is not new: it goes back to the 1977 work by Deser and Zumino. However, at that time nobody was interested in a positive cosmological constant. Everyone wanted it to vanish.
- Also in 1977 Freedman provided a locally supersymmetric extension of the FI term by gauging the R-symmetry. Soon it was understood that only limited matter couplings are compatible with gauged R-symmetry.
- It has recently been realised that there exist generalised FI terms in supergravity which do not require gauged R-symmetry.

Outline

Supergravity, R-invariance and Fayet-Iliopoulos terms

- Rigid supersymmetry can be broken spontaneously using a U(1) vector multiplet with a Fayet-Iliopoulos (FI) term

  P. Fayet & J. Iliopoulos (1974)

  $$-2\xi \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, V$$

  Auxiliary field sector of the vector multiplet model with FI term: $\frac{1}{2}D^2 - \xi D \approx -\frac{1}{2}\xi^2$
- Locally supersymmetric extension of the FI term is achieved by gauging the R-symmetry.

  D. Freedman (1977)
- "...In order for a U(1) gauge theory with a FI term to be consistently coupled to supergravity, preserving gauge invariance, superpotential must be R invariant. A supersymmetric cosmological term and therefore an explicit mass-like term for the gravitino is forbidden by gauge invariance."

  R. Barbieri, S. Ferrara, D. Nanopoulos & K. Stelle (1982)

FI terms in supergravity without gauged R-symmetry

Generalised Fayet-Iliopoulos terms in supergravity, which do not require gauged R-symmetry, were proposed in:

- N. Cribiori, F. Farakos, M. Tournoy & A. Van Proeyen (22 December, 2017) [arXiv:1712.08601]

$$\mathbb{J}_{\rm FI}^{(-1)} = \int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar{\theta}\, E\, \Upsilon \mathbb{V}_{(-1)}~, \quad \mathbb{V}_{(-1)} := -4\frac{W^2\bar{W}^2\mathcal{D}W}{(\mathcal{D}^2W^2)(\bar{\mathcal{D}}^2\bar{W}^2)}$$

- SMK (15 January, 2018) [arXiv:1801.04794]

$$\mathbb{J}_{\rm FI}^{(n)} = \int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar{\theta}\, E\, \Upsilon \mathbb{V}_{(n)}~, \quad \mathbb{V}_{(n)} := -4\frac{W^2\bar{W}^2\big[(\mathcal{D}^2W^2)(\bar{\mathcal{D}}^2\bar{W}^2)\big]^n}{(\mathcal{D}W)^{4n+3}}$$

  Υ is a compensating multiplet which encodes the information about a specific off-shell supergravity.

- My work arXiv:1801.04794 was a natural extension of SMK, I. McArthur & G. Tartaglino-Mazzucchelli [arXiv:1702.02423].
- Perhaps Van Proeyen was looking for a generalised FI term of the type constructed in arXiv:1712.08601 ever since his 1983 work with Ferrara, Girardello & Kugo.

FI terms in supergravity without gauged R-symmetry

- More general FI-type terms in $\mathcal{N}=1$ SUGRA:

SMK [arXiv:1904.05201]

$$\mathbb{J}_{\rm FI}^{(\mathcal{G})}[V;\Upsilon] = \int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar\theta\, E\,\Upsilon\mathbb{V}\,\mathcal{G}\left(-\frac{\mathcal{D}^2W^2}{(\mathcal{D}W)^2}\,,\,-\frac{\bar{\mathcal{D}}^2\bar{W}^2}{(\mathcal{D}W)^2}\right)\,,$$

where

$$\mathbb{V} := -4\frac{W^2\bar{W}^2}{(\mathcal{D}W)^3}\,, \qquad W^2 := W^\alpha W_\alpha$$

and $\mathcal{G}(z,\bar z)$ is a real function of one complex variable.

$$\mathbb{J}_{\rm FI}^{(n)} \quad \Longleftrightarrow \quad \mathcal{G}(z,\bar z) = (z\bar z)^n$$

- Recent work on generalised FI terms in $\mathcal{N}=2$ SUGRA

I. Antoniadis, J. Derendinger, F. Farakos & G. Tartaglino-Mazzucchelli [arXiv:1905.09125]

is a natural extension of

SMK, I. McArthur & G. Tartaglino-Mazzucchelli [arXiv:1702.02423].

FI terms in supergravity without gauged R-symmetry

- Standard FI term

$$J_{\mathrm{FI}} = \frac{1}{2} D$$

- Special generalised FI term (CTVP construction)

$$\mathbb{J}_{\mathrm{FI}}^{(-1)} = \frac{1}{2} D + O(\psi) \ ,$$

  where ψ denotes the photino/Goldstino.

- Most general FI-type term

$$\mathbb{J}_{\mathrm{FI}}^{(\mathcal{G})} = \frac{1}{2} D + O(\psi, F) \ ,$$

  with F the Maxwell field.

Gravity as conformal gravity coupled to a compensator

Einstein-Hilbert action with a cosmological term

$$S_{\mathrm{EH}} = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\, e\, R - \frac{\Lambda}{\kappa^2}\int \mathrm{d}^4x\, e$$

Weyl-invariant reformulation S. Deser (1970), B. Zumino (1970)

$$S = \frac{1}{2}\int \mathrm{d}^4x\, e\left(\nabla^a\varphi\nabla_a\varphi + \frac{1}{6}R\varphi^2 - \lambda\varphi^4\right) ,$$

where φ is a nowhere vanishing conformal compensator (φ^{-1} exists).
Weyl transformations

$$\begin{aligned}\delta\nabla_a &= \sigma\nabla_a + (\nabla^b\sigma)M_{ba} , \qquad \delta\varphi = \sigma\varphi , \\ \nabla_a &:= e_a{}^m\partial_m + \frac{1}{2}\omega_a{}^{bc}M_{bc} , \qquad [\nabla_a, \nabla_b] = \frac{1}{2}R_{ab}{}^{cd}M_{cd}\end{aligned}$$

Weyl invariance is part of the gauge freedom of conformal gravity.
In the case of Weyl-invariant formulation for Einstein's gravity, imposing Weyl gauge $\varphi = \frac{\sqrt{6}}{\kappa} = \mathrm{const}$ takes us back to the original action.

Off-shell formulations for supergravity: a review

- Pure 4D $\mathcal{N}=1$ supergravity can be realised as conformal supergravity coupled to a compensating supermultiplet.

  M. Kaku & P. Townsend (1978)
  W. Siegel (1978), W. Siegel & J. Gates (1979)
  T. Kugo & S. Uehara (1983)

- Different off-shell formulations for supergravity correspond to different compensators.

  W. Siegel & J. Gates (1979)
  S. Ferrara, L. Girardello, T. Kugo & A. Van Proeyen (1983)

- The simplest way to describe $\mathcal{N}=1$ conformal supergravity in superspace is to make use of the geometry proposed by

  R. Grimm, J. Wess & B. Zumino (1978)

  This superspace geometry was used in the very first published work on the old minimal formulation for $\mathcal{N}=1$ supergravity:

  J. Wess and B. Zumino, Phys. Lett. B **74**, 51 (1978)

  Old minimal supergravity was developed independently by

  K. Stelle & P. West, Phys. Lett. B **74**, 330 (1978)
  S. Ferrara & P. van Nieuwenhuizen, Phys. Lett. B **74**, 333 (1978)

Grimm-Wess-Zumino superspace geometry

Superspace covariant derivatives

$$\mathcal{D}_A := (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot\alpha}) = E_A{}^M \partial_M + \Omega_A{}^{\beta\gamma} M_{\beta\gamma} + \bar\Omega_A{}^{\dot\beta\dot\gamma} \bar M_{\dot\beta\dot\gamma} .$$

Graded commutation relations

$$\begin{aligned}
&\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} = -2\mathrm{i}\mathcal{D}_{\alpha\dot\alpha} , \\
\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} &= -4\bar R M_{\alpha\beta} , \qquad \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = 4R\bar M_{\dot\alpha\dot\beta} , \\
\Big[\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot\beta}\Big] &= \mathrm{i}\varepsilon_{\alpha\beta}\Big(\bar R\,\bar{\mathcal{D}}_{\dot\beta} + G^\gamma{}_{\dot\beta}\mathcal{D}_\gamma - (\mathcal{D}^\gamma G^\delta{}_{\dot\beta})M_{\gamma\delta} + 2\bar W_{\dot\beta}{}^{\dot\gamma\dot\delta}\bar M_{\dot\gamma\dot\delta}\Big) \\
&\quad +\mathrm{i}(\bar{\mathcal{D}}_{\dot\beta}\bar R)M_{\alpha\beta} .
\end{aligned}$$

Torsion superfields R, $G_{\alpha\dot\alpha} = \bar G_{\alpha\dot\alpha}$ and $W_{\alpha\beta\gamma}$ obey the Bianchi identities:

$$\bar{\mathcal{D}}_{\dot\alpha} R = 0 , \qquad \bar{\mathcal{D}}_{\dot\alpha} W_{\alpha\beta\gamma} = 0 , \qquad \bar{\mathcal{D}}^{\dot\alpha} G_{\alpha\dot\alpha} = \mathcal{D}_\alpha R$$

R, $G_{\alpha\dot\alpha}$ and $W_{\alpha\beta\gamma}$ are supergravity analogues of the scalar curvature, traceless Ricci tensor and self-dual Weyl tensor, respectively.

Super-Weyl transformations

P. Howe & R. Tucker (1978)

$$
\begin{aligned}
\delta_\sigma \mathcal{D}_\alpha &= (\bar\sigma - \frac{1}{2}\sigma)\mathcal{D}_\alpha + (\mathcal{D}^\beta\sigma)\, M_{\alpha\beta} \ , \\
\delta_\sigma \bar{\mathcal{D}}_{\dot\alpha} &= (\sigma - \frac{1}{2}\bar\sigma)\bar{\mathcal{D}}_{\dot\alpha} + (\bar{\mathcal{D}}^{\dot\beta}\bar\sigma)\bar{M}_{\dot\alpha\dot\beta} \ , \\
\delta_\sigma \mathcal{D}_{\alpha\dot\alpha} &= \frac{1}{2}(\sigma + \bar\sigma)\mathcal{D}_{\alpha\dot\alpha} + \frac{\mathrm{i}}{2}(\bar{\mathcal{D}}_{\dot\alpha}\bar\sigma)\mathcal{D}_\alpha + \frac{\mathrm{i}}{2}(\mathcal{D}_\alpha\sigma)\bar{\mathcal{D}}_{\dot\alpha} \\
&\quad + (\mathcal{D}^\beta{}_{\dot\alpha}\sigma)M_{\alpha\beta} + (\mathcal{D}_\alpha{}^{\dot\beta}\bar\sigma)\bar{M}_{\dot\alpha\dot\beta} \ ,
\end{aligned}
$$

where σ is an arbitrary covariantly chiral scalar superfield, $\bar{\mathcal{D}}_{\dot\alpha}\sigma = 0$.
The torsion tensors transform as follows:

$$
\begin{aligned}
\delta_\sigma R &= 2\sigma R + \frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\bar\sigma \ , \\
\delta_\sigma G_{\alpha\dot\alpha} &= \frac{1}{2}(\sigma + \bar\sigma)G_{\alpha\dot\alpha} + \mathrm{i}\mathcal{D}_{\alpha\dot\alpha}(\sigma - \bar\sigma) \ , \\
\delta_\sigma W_{\alpha\beta\gamma} &= \frac{3}{2}\sigma W_{\alpha\beta\gamma} \ .
\end{aligned}
$$

Off-shell formulations for supergravity: a review

- Old minimal supergravity
  Its conformal compensator is a chiral scalar superfield S_0, $\bar{\mathcal{D}}_{\dot\alpha} S_0 = 0$, with the super-Weyl transformation

$$\delta_\sigma S_0 = \sigma S_0$$

  Pure supergravity action

$$S_{\rm OMSG} = -\frac{3}{\kappa^2}\int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta\, E\, \bar S_0 S_0 + \left\{ \frac{\mu}{\kappa^2} \int \mathrm{d}^4x\mathrm{d}^2\theta\, \mathcal{E}\, S_0^3 + \mathrm{c.c.} \right\} ,$$

  where $E^{-1} = \mathrm{Ber}(E_A{}^M)$ and $\mathcal{E}$ is the chiral density.
- New minimal supergravity
  Its conformal compensator is a real linear superfield, $\bar{\mathbb{L}} - \mathbb{L} = (\bar{\mathcal{D}}^2 - 4R)\mathbb{L} = 0$, with the super-Weyl transformation

$$\delta_\sigma \mathbb{L} = (\sigma + \bar\sigma)\mathbb{L}$$

  Pure supergravity action (no cosmological terms is allowed)

$$S_{\rm NMSG} = \frac{3}{\kappa^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\, \mathbb{L} \ln \frac{\mathbb{L}}{|S_0|^2}$$

Fayet-Iliopoulos terms in off-shell supergravity

Fayet-Iliopoulos terms in new minimal supergravity

Consider new minimal supergravity coupled to a nonlinear σ-model and a U(1) vector multiplet with FI term. The complete action is

$$S = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\,\mathbb{L}\left\{\frac{3}{\kappa^2}\ln\frac{\mathbb{L}}{|S_0|^2} + K(\phi^i,\bar{\phi}^{\bar{i}})\right\} + S[V] ,$$

where $S[V]$ denotes the vector multiplet action,

$$S[V] = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\left\{\frac{1}{8}V\mathcal{D}^\alpha(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V - 2\xi\mathbb{L}V\right\} .$$

All matter multiplets ϕ^i are assumed to be neutral under the super-Weyl transformations, $\delta_\sigma\phi^i = 0$. (Simplest example)
The action is invariant under Kähler transformations

$$K(\phi,\bar{\phi}) \to K(\phi,\bar{\phi}) + F(\phi) + \bar{F}(\bar{\phi}) ,$$

with $F(\phi)$ an arbitrary holomorphic function.
It is also invariant under the gauge transformations of V,

$$\delta_\lambda V = \lambda + \bar{\lambda} , \qquad \bar{\mathcal{D}}_{\dot{\alpha}}\lambda = 0 .$$

Fayet-Iliopoulos terms in old minimal supergravity

Applying a superfield Legendre transformation to the theory considered above, we end up with a dual formulation which describes old minimal supergravity coupled to a nonlinear σ-model and a U(1) vector multiplet with FI term. The resulting action is

$$
\begin{aligned}
S = & -\frac{3}{\kappa^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\, \bar{S}_0 \exp\left(\frac{2}{3}\xi\kappa^2 V\right) S_0 \exp\left(-\frac{\kappa^2}{3}K(\phi,\bar{\phi})\right) \\
& + \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\, \frac{1}{8} V\mathcal{D}^\alpha(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V \ .
\end{aligned}
$$

Kähler invariance:

$$
K(\phi,\bar{\phi}) \to K(\phi,\bar{\phi}) + F(\phi) + \bar{F}(\bar{\phi}) \ , \qquad S_0 \to \mathrm{e}^{\frac{\kappa^2}{3}F(\phi)} S_0
$$

Gauge invariance

$$
V \to V + \lambda + \bar{\lambda} \ , \qquad S_0 \to \mathrm{e}^{-\frac{2}{3}\xi\kappa^2\lambda} S_0
$$

Fayet-Iliopoulos term in old minimal supergravity

"...The new minimal auxiliary field formulation is equivalent to the restricted class of old minimal formulation, namely the one with R symmetry. This symmetry is a necessary and sufficient condition for the Fayet-Iliopoulos term to be introduced."

S. Ferrara, L. Girardello, T. Kugo & A. Van Proeyen (1983)

Nilpotent real scalar supermultiplet

Nilpotent real scalar supermultiplet

$\mathcal{N} = 1$ Goldstino supermultiplet model proposed in

SMK, I. McArthur & G. Tartaglino-Mazzucchelli [arXiv:1702.02423]

is described in terms of a real scalar superfield V with the properties: (i) it is super-Weyl invariant, $\delta_\sigma V = 0$; and (ii) it is constrained by

$$V^2 = 0 \ , \qquad V \mathcal{D}_A \mathcal{D}_B V = 0 \ , \qquad V \mathcal{D}_A \mathcal{D}_B \mathcal{D}_C V = 0 \ .$$

In order for V to describe a Goldstino supermultiplet, the real descendant $\mathcal{D}W := \mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot\alpha} \bar{W}^{\dot\alpha}$ has to be nowhere vanishing, with

$$W_\alpha := -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V \ , \qquad \bar{\mathcal{D}}_{\dot\beta} W_\alpha = 0 \ .$$

Dynamics is governed by the super-Weyl invariant action

$$S[V] = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta \, E \left\{ \frac{1}{8} V \mathcal{D}^\alpha (\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V - 2\xi \mathbb{L} V \right\} \ .$$

Note: $S[V]$ also describes a massless vector multiplet with FI term provided V is unconstrained. In this case $S[V]$ is gauge invariant.

Nilpotent real scalar supermultiplet

- Super-Weyl transformation laws:

$$\delta_\sigma V = 0 \ , \qquad \delta_\sigma W_\alpha = \frac{3}{2}\sigma W_\alpha \ , \qquad \delta_\sigma(\mathcal{D}W) = (\sigma + \bar{\sigma})\mathcal{D}W \ .$$

- Constraints $V^2 = 0 \ , \quad V\mathcal{D}_A\mathcal{D}_B V = 0 \ , \quad V\mathcal{D}_A\mathcal{D}_B\mathcal{D}_C V = 0$ imply

$$V = -4\frac{W^2\bar{W}^2}{(\mathcal{D}W)^3} \ , \qquad W^2 := W^\alpha W_\alpha \ .$$

- Important by-product:
  Consider a massless vector multiplet realised in terms of a real unconstrained prepotential V and gauge-invariant field strength $W_\alpha = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V$ such that $\mathcal{D}W \neq 0$.
  Then the following composite

$$\mathbb{V} = -4\frac{W^2\bar{W}^2}{(\mathcal{D}W)^3} = \bar{\mathbb{V}}$$

  is well defined and super-Weyl invariant, $\quad \delta_\sigma \mathbb{V} = 0$.

Generalised FI terms in supergravity

Generalised FI terms in supergravity

We couple (conformal) supergravity to a massless vector multiplet such that SUSY is in a spontaneously broken phase. The corresponding real prepotential V has the properties:

- It is defined modulo gauge transformations

$$\delta_\lambda V = \lambda + \bar{\lambda} , \qquad \bar{\mathcal{D}}_{\dot{\alpha}} \lambda = 0$$

- It is super-Weyl inert, $\delta_\sigma V = 0$.
- The top component of V is nowhere vanishing,

$$\mathcal{D}W := \mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \neq 0 , \qquad W_\alpha := -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V$$

Example: vector multiplet model with FI term in new minimal supergravity with action

$$S[V] = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E \left\{ \frac{1}{8} V \mathcal{D}^\alpha (\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha V - 2\xi \mathbb{L} V \right\} .$$

Equation of motion for V: $\mathcal{D}W = -2\xi\mathbb{L} \neq 0$.

Generalised FI terms in supergravity

Since $\mathcal{D}W$ is nowhere vanishing, we can introduce real scalar composite

$$\mathbb{V} := -4\frac{W^2\bar{W}^2}{(\mathcal{D}W)^3} , \qquad W^2 := W^\alpha W_\alpha .$$

The properties of $\mathbb{V}$ are as follows:

- $\mathbb{V}$ is gauge invariant, $\delta_\lambda \mathbb{V} = 0$;
- $\mathbb{V}$ is super-Weyl invariant, $\delta_\sigma \mathbb{V} = 0$;
- $\mathbb{V}$ obeys the nilpotency conditions

$$\mathbb{V}\,\mathbb{V} = 0 , \quad \mathbb{V}\mathcal{D}_A\mathcal{D}_B\mathbb{V} = 0 , \quad \mathbb{V}\mathcal{D}_A\mathcal{D}_B\mathcal{D}_C\mathbb{V} = 0$$

  and, therefore, $\mathbb{V}$ may be interpreted to be a Goldstino superfield.

$\mathbb{V}$ can be used to obtain a super-Weyl invariant functional

$$\mathbb{J}^{(0)}_{\rm FI}[V;\Upsilon] = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\,\Upsilon\mathbb{V} , \qquad \delta_\sigma\Upsilon = (\sigma+\bar{\sigma})\Upsilon$$

Υ may be identified with a compensator: (i) $\Upsilon = \bar{S}_0 S_0$ in pure OMSG; and (ii) $\Upsilon = \mathbb{L}$ in NMSG. More general choices are possible.

Generalised FI terms in supergravity

- More general FI-like terms:

SMK [arXiv:1904.05201]

$$\mathbb{J}_{\rm FI}^{(\mathcal{G})}[V;\Upsilon] = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\,\Upsilon\mathbb{V}\,\mathcal{G}\left(-\frac{\mathcal{D}^2W^2}{(\mathcal{D}W)^2}\,,\,-\frac{\bar{\mathcal{D}}^2\bar{W}^2}{(\mathcal{D}W)^2}\right)\ ,$$

where $\mathcal{G}(z,\bar z)$ is a real function of one complex variable.

- In old minimal supergravity coupled to chiral matter

$$S_{\rm SG} = -3\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\,\bar S_0\,\mathrm{e}^{-\frac{1}{3}K(\phi,\bar\phi)}S_0 + \left\{\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\,S_0^3W(\phi) + \mathrm{c.c.}\right\}$$

it is necessary to choose

$$\Upsilon = \bar S_0\mathrm{e}^{-\frac{1}{3}K(\phi,\bar\phi)}S_0$$

in order to preserve Kähler invariance of the complete action.

I. Antoniadis, A. Chatrabhuti, H. Isono and R. Knoops [arXiv:1805.00852]

Supergravity models with generalised FI terms

Complete supergravity-matter action

SMK [arXiv:1904.05201]

$$S = S_{\rm SG} + S[V] - 2\xi \mathbb{J}^{(\mathcal{G})}_{\rm FI}[V;\Upsilon]$$

$S_{\rm SG}$ describes supergravity coupled to other matter supermultiplets.
For instance, old minimal supergravity with chiral matter is described by

$$S_{\rm SG} = -3\int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar\theta\, E\,\bar S_0\,{\rm e}^{-\frac{1}{3}K(\phi,\bar\phi)}S_0 + \left\{\int {\rm d}^4x{\rm d}^2\theta\,\mathcal{E}\,S_0^3 W(\phi) + {\rm c.c.}\right\}$$

$S[V]$ is a superconformal action of the form

$$S[V] = \frac{1}{2}\int {\rm d}^4x{\rm d}^2\theta\,\mathcal{E}\,W^2 + \int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar\theta\,E\,\frac{W^2\,\bar W^2}{(\mathcal{D}W)^2}\,\mathcal{H}\left(-\frac{\mathcal{D}^2W^2}{(\mathcal{D}W)^2}, -\frac{\bar{\mathcal{D}}^2\bar W^2}{(\mathcal{D}W)^2}\right) ,$$

where $\mathcal{H}(z,\bar z)$ is a real function of one complex variable.

Supergravity models with generalised FI terms

Complete supergravity-matter action

$$S = S_{\rm SG} + S[V] - 2\xi \mathbb{J}^{(\mathcal{G})}_{\rm FI}[V;\Upsilon]$$

is highly nonlinear. However its functional form drastically simplifies provided the ordinary gauge field contained in V is chosen to be a flat connection. In such a case the gauge freedom allows us to make V a nilpotent superfield obeying the constraints

$$V\,V = 0~, \qquad V\mathcal{D}_A\mathcal{D}_B V = 0~, \qquad V\mathcal{D}_A\mathcal{D}_B\mathcal{D}_C V = 0~.$$

Then it may be shown that

$$S[V] - 2f\mathbb{J}^{(n)}_{\rm FI}[V;\Upsilon] = \frac{h}{2}\int {\rm d}^4x{\rm d}^2\theta\,\mathcal{E}\,W^2 - 2\xi g\int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar\theta\,E\,\Upsilon V$$

where $\quad h := 1 + \frac{1}{2}\mathcal{H}(1,1) > 0~, \qquad g := \mathcal{G}(1,1) \neq 0$.
Modulo an overall factor, this is the Goldstino multiplet action given in
SMK, I. McArthur & G. Tartaglino-Mazzucchelli [arXiv:1702.02423]

Component analysis

It is of interest to work out the bosonic sector of the model in the vector multiplet sector.

- Component fields of the vector multiplet

$$W_\alpha| = \psi_\alpha \ , \quad -\frac{1}{2}\mathcal{D}^\alpha W_\alpha| = D \ , \quad \mathcal{D}_{(\alpha} W_{\beta)}| = 2\mathrm{i}\hat{F}_{\alpha\beta} \ = \mathrm{i}(\sigma^{ab})_{\alpha\beta}\hat{F}_{ab}$$

  Bar-projection $U|$ means switching off the Grassmann variables $\theta, \bar{\theta}$.

- U(1) field strength

$$\begin{aligned}\hat{F}_{ab} \ &= \ F_{ab} - \frac{1}{2}(\Psi_a\sigma_b\bar{\psi} + \psi\sigma_b\bar{\Psi}_a) + \frac{1}{2}(\Psi_b\sigma_a\bar{\psi} + \psi\sigma_a\bar{\Psi}_b) \ , \\ F_{ab} \ &= \ \nabla_a V_b - \nabla_b V_a - \mathcal{T}_{ab}{}^c V_c \ ,\end{aligned}$$

  with $V_a = e_a{}^m(x)\, V_m(x)$ the gauge one-form, and $\Psi_a{}^\beta$ the gravitino.

- ∇_a denotes spacetime covariant derivative with torsion.

Component analysis

∇_a denotes spacetime covariant derivative with torsion

$$[\nabla_a, \nabla_b] = \mathcal{T}_{ab}{}^c \nabla_c + \frac{1}{2} \mathcal{R}_{abcd} M^{cd} \ ,$$

$$\mathcal{T}_{abc} = -\frac{\mathrm{i}}{2}(\Psi_a \sigma_c \bar{\Psi}_b - \Psi_b \sigma_c \bar{\Psi}_a) \ .$$

where $\mathcal{R}_{abcd}$ is the curvature tensor and $\mathcal{T}_{abc}$ is the torsion tensor.
Component expressions:

$$-\frac{1}{4}\mathcal{D}^2 W^2| = D^2 - 2F^2 + \text{fermionic terms} \ , \qquad F^2 := F^{\alpha\beta} F_{\alpha\beta}$$

Component analysis

- Direct calculations give the component bosonic Lagrangian

$$\begin{aligned}\mathcal{L}(F_{ab}, D) &= -\frac{1}{2}(F^2 + \bar{F}^2) \\ &\quad + \frac{1}{2}D^2 \left\{1 + \frac{1}{2}\mathcal{H}\left(1 - \frac{2F^2}{D^2}\,, 1 - \frac{2\bar{F}^2}{D^2}\right)\left|1 - \frac{2F^2}{D^2}\right|^2\right\} \\ &\quad - \xi D\, \mathcal{G}\left(1 - \frac{2F^2}{D^2}\,, 1 - \frac{2\bar{F}^2}{D^2}\right)\left|1 - \frac{2F^2}{D^2}\right|^2 \Upsilon| \,.\end{aligned}$$

- In order for the supergravity action in to give the correct Einstein-Hilbert Lagrangian, one has to impose the super-Weyl gauge $\Upsilon| = 1$.
- Choice $\mathcal{G}_{\rm CFTV}(z, \bar{z}) = (z\bar{z})^{-1}$ is somewhat special since the last term becomes linear in D and independent of the field strength. N. Cribiori, F. Farakos, M. Tournoy & A. Van Proeyen [1712.08601]
- However, our general model describes spontaneously broken local supersymmetry for any $\mathcal{G}$, and thus there is nothing unique in choice $\mathcal{G}_{\rm CFTV}(z, \bar{z}) = (z\bar{z})^{-1}$ from the conceptual point of view.

Generalised FI terms as quantum corrections

- No loop corrections to the standard FI term in SYM theories
  W. Fischler, H-P. Nilles, J. Polchinski, S. Raby & L. Susskind (1981)
  M. Grisaru & W. Siegel (1982)
- It appears that generalised FI terms can be generated quantum mechanically.
  SMK & I. McArthur (work in progress)

Some recent developments

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Some recent developments

Generalised FI terms in $\mathcal{N} = 2$ SUGRA

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