Higher order vertices in HS theory

S. Didenko
(based on work with O.A Gelfond, A.V. Korybut and M. A. Vasiliev)

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Plan

AdS/CFT in HS

- Weak-weak duality which does not require supersymmetry (Sundborg, Klebanov, Polyakov, Leigh, Petkou, Sezgin, Sundell)

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- Giombi and Yin tests from equations of motion: substantial piece of evidence that many of $3pt$ functions match.
- Generic structure of $3pt$-correlators (Maldacena, Zhiboedov)

$$\langle JJJ \rangle = \cos^2\phi \langle JJJ \rangle_b + \sin^2\phi \langle JJJ \rangle_f + \frac{1}{2}\sin(2\phi)\langle JJJ \rangle_o$$

Locality

$$S = \int \phi\Box\phi + \overbrace{D..D\phi D..D\phi D..D\phi}^{\text{local}} + \overbrace{D..D\phi D..D\phi D..D\phi D..D\phi}^{\text{(non-)local?}}$$

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- Quartic order indicates possible non-localities from holographic reconstruction Bekaert, Erdmenger, Ponomarev, Sleight; Sleight, Taronna

(Non)locality

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$$dω(Y|x) + ω ∗ ω = Υ(ω, ω, C) + Υ(ω, ω, C, C) + \dots ,$$
$$dC(Y|x) + [ω, C]_∗ = Υ(ω, C, C) + Υ(ω, C, C, C) + \dots .$$

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$$d\omega(Y|x) + \omega * \omega = \Upsilon(\omega, \omega, C) + \Upsilon(\omega, \omega, C, C) + \dots ,$$
$$dC(Y|x) + [\omega, C]_* = \Upsilon(\omega, C, C) + \Upsilon(\omega, C, C, C) + \dots .$$

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Perturbative vertices on AdS background Ω

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General solution is given by Vasiliev equations

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$$C_{\alpha_1\ldots\alpha_n\ldots}C^{\alpha_1\ldots\alpha_n\ldots}\,, \qquad n<\infty \qquad \text{local}$$

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$$f(y)*g(y)=f(y)e^{i\epsilon^{\alpha\beta}\overleftarrow{\partial}_\alpha\overrightarrow{\partial}_\beta}g(y)\,,$$

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- How come cubic vertex is local?

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$$D_\Omega \omega = (e \wedge e)^{\alpha\beta} \partial_\alpha \partial_\beta C(0) .$$

Goals and summary

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- Develop perturbation theory for HS equations that is consistent with locality.

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- Calculate explicitly some lower and higher order interaction vertices.

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- Vertices $\Upsilon(\omega, \omega, C)$ and $\Upsilon(\omega, \omega, C, C)$ are found to have ultra-local form.
- The meaning of parameter β is revealed.

Vasiliev equations

Vasiliev equations in $d = 4$

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\begin{aligned}
&\mathrm{d}_x W + W * W = 0\,,\\
&\mathrm{d}_x S + [W, S]_* = 0\,,\\
&\mathrm{d}_x B + [W, B]_* = 0\,,\\
&S * S = -i\theta_\alpha\theta^\alpha + i\eta B * \gamma + c.c.\,,\\
&[S, B]_* = 0\,, \quad \gamma = e^{iz_\alpha y^\alpha}\theta^\beta\theta_\beta
\end{aligned}
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(f * g)(z, y) = \frac{1}{(2\pi)^2}\int dudvf(z+u; y+u)g(z-v; y+v)e^{iu_\alpha v^\alpha}
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$$
W(Z, Y|x) = \omega(Y|x) + \dots\,, \quad B(Z, Y|x) = C(Y|x) + \dots
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Perturbation theory

Vacuum

$$S^0 = z_\alpha \theta^\alpha\,, \quad B^0 = 0\,, \quad W^0 = \omega(Y|x)$$

$$\mathrm{d}_x \omega + \omega * \omega = 0\,.$$

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$$[S^0, f] \sim d_z f$$

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Conventional homotopy:

$$\Delta_0 J = z^\alpha \frac{\partial}{\partial \theta^\alpha} \int_0^1 \frac{dt}{t} J(tz, y; t\theta)$$

Shifted homotopy

Alternative way of writing solution

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$$\Delta_{q,\beta} J = \int e^{iu_\alpha v^\alpha}\int_0^1 \frac{\mathrm{d}\tau}{\tau}(z+q-v)^\alpha \frac{\partial}{\partial\theta^\alpha} J(\tau z - (1-\tau)(q-v); y+\beta u; \tau\theta)$$

Homotopy properties

- Resolution of identity

$$\{d_z,\Delta_{q,\beta}\}=1-h_{q,\beta}\,,\quad h_{q,\beta}J(z,y;\theta)=\int e^{iu_\alpha v^\alpha}J(-q+v;y+\beta u;0)$$

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- anti-commutativity

$$\Delta_{p,\beta_1}\Delta_{q,\beta_2} = -\Delta_{q,\beta_2}\Delta_{p,\beta_1}\,, \qquad h_{p,\beta_1}\Delta_{q,\beta_2} = -h_{q,\beta_2}\Delta_{p,\beta_1}$$

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- triangle identity

$$h_{d,\beta_1}\Delta_{c,\beta_2}\Delta_{b,\beta_3} + h_{d,\beta_1}\Delta_{b,\beta_3}\Delta_{a,\beta_4} = h_{c,\beta_2}\Delta_{b,\beta_3}\Delta_{a,\beta_4} + h_{d,\beta_1}\Delta_{c,\beta_2}\Delta_{a,\beta_4}$$

Homotopy properties

- Scaling property

$$h_{(1-\beta)a,0}\Delta_{(1-\beta)b,0}\Delta_{(1-\beta)c,0}f(z,y) = (1-\beta)^2 h_{a,0}\Delta_{b,0}\Delta_{c,0}f((1-\beta)z,y)$$

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- Star exchange

$$\Delta_{\hat{q},\,\beta}(f(y)*J(z;y;\theta)) = f(y)*\Delta_{\hat{q}+\hat{p},\,\beta}J(z;y;\theta)\,,\ \hat{q} = (1-\beta)q\,,\ \hat{p} = (1-\beta)p$$

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$$p_\alpha f(y)\equiv f(y)p_\alpha:=-i\frac{\partial}{\partial y^\alpha}f(y)$$

Locality theorem

- Structure theorem (O.A. Gelfond). There is the *even* one-form sector of HS equations (W and S) that is respected by $\Delta_{s_e,\beta}$ – homotopies and the *odd* zero-form one (B) respected by $\Delta_{s_o,\beta}$

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- Pfaffian locality theorem (Gelfond+Vasiliev). The degree of non-locality of zero-form HS vertices gets reduced on the homotopies prescribed by the structure theorem.

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$$\det P_{ij} = 0\,, \quad \exp P_{ij}\partial^i_\alpha \partial^{j\alpha} \quad \Rightarrow \quad \text{measure of non-locality}$$

Strategy

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- Keeping fixed arbitrary β. Locality is reached in the limit $\beta \to -\infty$

$\Upsilon(\omega, \omega, C)$ – vertex

$$d_z S_1 = -\frac{\eta}{2} C * \gamma \quad \Rightarrow \quad S_1 = -\frac{\eta}{2} \Delta_{0,\beta}(C * \gamma) = -\frac{\eta}{2} C * \Delta_p \gamma \,,$$

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$$W_1 = -\frac{\eta}{4i} \left(C * \omega * \Delta_{p+t} \Delta_{p+2t} \gamma - \omega * C * \Delta_{p+t} \Delta_p \gamma\right) \,,$$

$$p_\alpha C(Y) := i \frac{\partial}{\partial y^\alpha} C(Y) \qquad t_\alpha \omega(Y) := i \frac{\partial}{\partial y^\alpha} \omega(Y)$$

$\Upsilon(\omega,\omega,C)$ – vertex

$$\mathrm{d}_x\omega+\omega*\omega=\frac{\eta}{4i}(\omega*\omega*C*X_{\omega\omega C}+C*\omega*\omega*X_{C\omega\omega}+\omega*C*\omega*X_{\omega C\omega}),$$

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where

$$X_{\omega\omega C} = h_{p+t_1+t_2}\Delta_p\Delta_{p+t_2}\gamma\,,$$
$$X_{C\omega\omega} = h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+2t_1+2t_2}\gamma\,,$$
$$X_{\omega C\omega} = -h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+t_2}\gamma - h_{p+t_1+2t_2}\Delta_{p+2t_2}\Delta_{p+t_2}\gamma\,.$$

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$$\mathrm{d}_x\omega+\omega*\omega=\frac{\eta}{4i}(\omega*\omega*C*X_{\omega\omega C}+C*\omega*\omega*X_{C\omega\omega}+\omega*C*\omega*X_{\omega C\omega})\,,$$

where

$$X_{\omega\omega C}=h_{p+t_1+t_2}\Delta_p\Delta_{p+t_2}\gamma\,,$$
$$X_{C\omega\omega}=h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+2t_1+2t_2}\gamma\,,$$
$$X_{\omega C\omega}=-h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+t_2}\gamma-h_{p+t_1+2t_2}\Delta_{p+2t_2}\Delta_{p+t_2}\gamma\,.$$

explicitly

$$\Upsilon_{\omega\omega C}=\frac{\eta}{2i}\int_{[0,1]^3}d^3\tau\delta(1-\tau_1-\tau_2-\tau_3)e^{i(1-\tau_3)\partial_1^\alpha\partial_{2\alpha}}$$
$$\partial^\alpha\omega((1-\tau_1)y)\partial_\alpha\omega(\tau_2 y)\,C(-i\tau_1\partial_1-i(1-\tau_2)\partial_2)\,,$$

$\Upsilon(\omega, \omega, C)$ – vertex

$$\mathrm{d}_x\omega+\omega*\omega = \frac{\eta}{4i}(\omega*\omega*C*X_{\omega\omega C}+C*\omega*\omega*X_{C\omega\omega}+\omega*C*\omega*X_{\omega C\omega})\,,$$

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$$X_{\omega C\omega} = -h_{p+t_1+t_2}\Delta_{p+t_1+2t_2}\Delta_{p+t_2}\gamma - h_{p+t_1+2t_2}\Delta_{p+2t_2}\Delta_{p+t_2}\gamma\,.$$

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$$\partial^\alpha\omega((1-\tau_1)y)\partial_\alpha\omega(\tau_2 y)\, C(-i\tau_1\partial_1 - i(1-\tau_2)\partial_2)\,,$$

No y – dependence in $C(y)$ $\Rightarrow$ ultra local form

$\Upsilon(\omega, C, C)$ – vertex

Solving for $B(z, y)$ to the second order

$$B_2 := B_2^q = \frac{\eta}{4i}\Delta_{(1-\beta)q}(C*C*(\Delta_{p_2}-\Delta_{p_1+2p_2})\gamma)\,, \qquad q = v_1 p_1 + v_2 p_2$$

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Locality requirement

$$v_2 - v_1 = 1$$

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$$X_{\omega CC} = h_{p_2}\Delta_{p_1+2p_2}\Delta_{p_1+2p_2+t}\gamma\,,$$
$$X_{CC\omega} = h_{p_2+2t}\Delta_{p_2+t}\Delta_{p_1+2p_2+2t}\gamma\,,$$
$$X_{C\omega C} = (h_{p_1+2p_2+2t} - h_{p_2})\Delta_{p_2+t}\Delta_{p_1+2p_2+t}\gamma$$

$\Upsilon(\omega, C, C)$ – vertex

Explicitly,

$$\Upsilon_{\omega CC} = \frac{\eta}{2i} \int_{[0,1]^3} d^3\tau \delta(1-\tau_1-\tau_2-\tau_3)(\partial_1^\alpha + \partial_2^\alpha)\partial_\alpha^\omega$$
$$\omega((1-\tau_3)y)\, C(\tau_1 y - i(1-\tau_2)\partial^\omega)\, C(-(1-\tau_1)y + i\tau_2\partial^\omega)\,,$$

The result is perfectly local in accordance with PLT

Generalities

Extracting $\Upsilon(\omega, \omega, C, C)$ from equations of motion

$$d_x\omega+\omega*\omega = -\{\omega, W_1\}_* -(d_x W_1 + d_x W_2 + W_1 * W_1 + \{\omega, W_2\}_*)+O(C^3)$$

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$$\Upsilon^{\eta\eta}(\omega, \omega, C, C) = -h_{0,\,\beta}(W_1 * W_1 + \{\omega, W_2(\beta)\})\Big|_{\eta\eta}$$

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$$\Upsilon^{\eta\eta}(\omega, \omega, C, C) = \Upsilon^{\eta\eta}_{\omega\omega CC} + \Upsilon^{\eta\eta}_{\omega C\omega C} + \Upsilon^{\eta\eta}_{C\omega\omega C} + \Upsilon^{\eta\eta}_{C\omega C\omega} + \Upsilon^{\eta\eta}_{CC\omega\omega} + \Upsilon^{\eta\eta}_{\omega CC\omega}$$

$$\Upsilon^{\eta\eta}_{C\omega\omega C}(\beta) = -h_{0,\,\beta}(W_1 * W_1)\Big|_{C\omega\omega C}$$

calculation

$$h_{0,\beta}(W_1 * W_1) \sim \int_0^1 d\tau_{1,2} \frac{\beta^2(1-\tau_1)(1-\tau_2)}{(1-\beta(\tau_1\circ\tau_2))^4} e^{\frac{i}{1-\beta(\tau_1\circ\tau_2)}A^{\alpha}(y+\beta B)_{\alpha}+iC}$$

$$\tau_{\circ} \equiv \tau_1 \circ \tau_2 = \tau_1(1-\tau_2)+\tau_2(1-\tau_1)$$

calculation

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$$\lim_{\beta\to-\infty} h_{0,\beta}(W_1 * W_1) \quad \text{exists and is ultra-local!}$$

$$I(\beta) = \int_{[0,1]^2} d\tau_{1,2} \frac{\beta^2(1-\tau_1)(1-\tau_2)}{(1-\beta\tau_\circ)^4} \exp\Big(-\frac{i\beta}{\xi}\tau_1(1-\tau_2)A - \frac{i\beta}{\xi}\tau_2(1-\tau_1)B + \frac{i}{\xi}C\Big)$$

calculation

$$h_{0,\beta}(W_1 * W_1) \sim \int_0^1 d\tau_{1,2} \frac{\beta^2(1-\tau_1)(1-\tau_2)}{(1-\beta(\tau_1\circ\tau_2))^4} e^{\frac{i}{1-\beta(\tau_1\circ\tau_2)}A^\alpha(y+\beta B)_\alpha + iC}$$

$$\tau_\circ \equiv \tau_1\circ\tau_2 = \tau_1(1-\tau_2)+\tau_2(1-\tau_1)$$

$$\lim_{\beta\to-\infty} h_{0,\beta}(W_1 * W_1)$$ exists and is ultra-local!

$$I(\beta) = \int_{[0,1]^2} d\tau_{1,2} \frac{\beta^2(1-\tau_1)(1-\tau_2)}{(1-\beta\tau_\circ)^4} \exp\left(-\frac{i\beta}{\xi}\tau_1(1-\tau_2)A - \frac{i\beta}{\xi}\tau_2(1-\tau_1)B + \frac{i}{\xi}C\right)$$

$$I(-\infty) = \int d^3_\Delta\tau\, \tau_3 \exp\left(i\tau_1 A + i\tau_2 B + i\tau_3 C\right)$$

Result

$$\Upsilon^{\eta\eta}_{C\omega\omega C}=\frac{\eta^2}{4}\int_{[0,1]^2}d\sigma d\sigma'\,\sigma\sigma'\int d^3_\Delta\tau\,(t_{1\alpha}t_2^\alpha)^2$$
$$\tau_3\,\exp\Big[-i(\tau_2\sigma+\tau_1\sigma'+\tau_3\sigma\sigma')t_{1\alpha}t_2^\alpha\Big]C(-\tau_1\sigma' t_2-(1-\sigma(1-\tau_1))t_1)$$
$$\omega(-\tau_3\sigma y)\omega(-\tau_3\sigma' y)C(\tau_2\sigma t_1+(1-\sigma'(1-\tau_2))t_2)\,,$$

$$t_{1,2}=-i\partial_{\omega_{1,2}}$$

Properties

- Ultra-locality of (anti)holomorphic vertices $\Upsilon(\omega,\omega,C(0),C(0))$
- Vanish on the gravitational background

$$\Upsilon = 0 \quad \text{for} \quad \omega = \omega_{s\leq 2}$$

What is β?

- Introduce the following reordering

$$O_\beta f(z,y) = \int \frac{dudv}{(2\pi)^2} f(z+v, y+\beta u)\exp(iu_\alpha v^\alpha)$$

$$\Delta_{q,\beta} = O_\beta^{-1}\Delta_{q,0}O_\beta$$

(For $\beta = 1$ – De Filippi, Iazeollа, Sundell)

- β – star-product

$$f\star_\beta g = \int f(z+u', y+u)g(z-(1-\beta)v-v', y+v+(1-\beta)v')e^{iu_\alpha v^\alpha + iu'_\alpha v'^\alpha}$$

Conclusion

- A β – class of homotopies is introduced for solving Vasiliev equations.
- Part of the quartic and quintic vertices $\Upsilon(\omega, \omega, C)$ and $\Upsilon(\omega, \omega, C, C)$ were explicitly calculated and shown to have ultra local form in the limit $\beta \to -\infty$. In addition it was shown that lower order vertices $\Upsilon(\omega, \omega, C)$ and $\Upsilon(\omega, C, C)$ are β -independent
- Holomorphic part of $\Upsilon(\omega, \omega, C, C)$ vanishes on any gravitational background.
- Interpretation of β as the reordering parameter is proposed.