On partially massless supergravity

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01.08.2017

Outlook

Massive gravity

- Now we have models of massive gravity (bigravity) with usual massless ones supplemented with some smart potential.
- Natural question: do these models admit supersymmetric extensions?
- There are many examples of spontaneously broken supergravities:

$$s = 2, \ m = 0 \quad \Leftrightarrow \quad s = 3/2, \ m \neq 0$$

- It easy to understand that the reverse situation is impossible:

$$s = 2, \ m \neq 0 \quad \Leftrightarrow \quad s = 3/2, \ m = 0$$

- Thus the most ”simple” possibility

$$s = 2, \ m \neq 0 \quad \Leftrightarrow s = 3/2, \ m \neq 0$$

Massive spin-2 in de Sitter space

- In the dS_4 space unitarity puts restriction on the spin-2 mass:

$$m^2 > 2\Lambda$$

- In the forbidden region $m^2 < 2\Lambda$ helicity 0 component becomes ghost.
- At the boundary

$$m^2 = 2\Lambda$$

  helicity 0 components decouples leaving us this the particle with four degrees of freedom (± 2, ± 1) — partially massless spin-2.
- Complete non-linear interacting theory for such partially massless case has never been constructed, only cubic vertex is known.
- In $d > 4$ such vertex requires terms with up four derivatives, but in $d = 4$ there exists vertex with just two.

Lagrangian formulation

- Frame-like gauge invariant description requires one-forms $\Omega^{[ab]}$, f^a, A and zero-form $B^{[ab]}$
- Free Lagrangian in dS_4:

$$\begin{aligned} \mathcal{L}_0 &= \frac{1}{2}\hat{E}_{ab}\Omega^a{}_c\Omega^{bc} - \frac{1}{2}\hat{E}_{abc}\Omega^{ab}Df^c + \frac{1}{2}B_{ab}B^{ab} - \hat{E}_{ab}B^{ab}DA \\ &\quad + m_0\hat{E}_{ab}\Omega^{ab}A + m_0\hat{e}_a B^{ab} f_b \end{aligned}$$

- It is invariant under the following gauge transformations

$$\begin{aligned} \delta\Omega^{ab} &= D\eta^{ab}, \qquad \delta f^a = D\xi^a - e_b\eta^{ab} + m_0 e^a\xi \\ \delta B^{ab} &= -m_0\eta^{ab}, \qquad \delta A = D\xi + \frac{m_0}{2}e_a\xi^a \end{aligned}$$

Here

$$m_0{}^2 = 2\Lambda$$

Gauge invariant objects

- Four gauge invariant objects (two and one-forms):

$$
\begin{aligned}
\mathcal{R}^{ab} &= D\Omega^{ab} + \frac{m_0}{2} E^a{}_b B^{ab} \\
\mathcal{T}^a &= Df^a - e_b\Omega^{ab} + m_0 e^a A \\
\mathcal{B}^{ab} &= DB^{ab} + m_0\Omega^{ab} \\
\mathcal{A} &= DA - \frac{1}{2} E_{ab} B^{ab} + \frac{m_0}{2} e_a f^a
\end{aligned}
$$

- ”on-shell”

$$
\mathcal{T}^a \approx 0, \qquad \mathcal{A} \approx 0
$$

- Lagrangian in terms of these objects:

$$
\mathcal{L} = a_1 \hat{E}_{abcd} \mathcal{R}^{ab} \mathcal{R}^{cd} + a_2 \hat{E}_{ab} \mathcal{B}^a{}_c \mathcal{B}^{bc} + a_3 \hat{E}_{abc} \mathcal{B}^{ab} \mathcal{T}^c
$$

Lagrangian formulation

- Spin-3/2 one-form Φ and spin-1/2 zero-form ϕ
- Lagrangian in dS_4:

$$\begin{aligned} \mathcal{L}_0 &= -\frac{i}{2}\hat{E}_{abc}\bar{\Phi}\Gamma^{abc}D\Phi + \frac{i}{2}\hat{e}_a\bar{\phi}\gamma^a D\phi \\ &\quad -\frac{3M}{2}\hat{E}_{ab}\bar{\Phi}\Gamma^{ab}\Phi + 3im\hat{e}_a\bar{\Phi}\gamma^a\phi - M\bar{\phi}\phi \end{aligned}$$

- It is invariant under the following gauge transformations:

$$\delta_0\Phi = D\zeta + \frac{iM}{2}e_a\gamma^a\zeta, \qquad \delta_0\phi = 3m\zeta$$

Here

$$M^2 = m^2 - \Lambda \quad \Rightarrow \quad m^2 > \Lambda$$

Gauge invariant objects

- Pair of the gauge invariant objects:

$$\begin{aligned} \mathcal{F} &= D\Phi + \frac{iM}{2} e_a \gamma^a \Phi + \frac{m}{12} E_{ab} \Gamma^{ab} \phi \\ \mathcal{C} &= D\phi - 3m\Phi + \frac{iM}{2} e_a \gamma^a \phi \end{aligned}$$

- and their conjugated ones:

$$\begin{aligned} \bar{\mathcal{F}} &= D\bar{\Phi} - \frac{iM}{2} e_a \bar{\Phi} \gamma^a - \frac{m}{12} E_{ab} \bar{\phi} \Gamma^{ab} \\ \bar{\mathcal{C}} &= D\bar{\phi} - 3m\bar{\Phi} - \frac{iM}{2} e_a \bar{\phi} \gamma^a \end{aligned}$$

- Lagrangian in terms of these objects:

$$\mathcal{L}_0 = b_1 \hat{E}_{abcd} \bar{\mathcal{F}} \Gamma^{abcd} \mathcal{F} + i b_2 \hat{E}_{abc} \bar{\mathcal{F}} \Gamma^{abc} \mathcal{C} + b_3 \hat{E}_{ab} \bar{\mathcal{C}} \Gamma^{ab} \mathcal{C}$$

Massless case

- Deformation: we consider the most general quadratic corrections for all gauge invariant objects and appropriate linear corrections to the gauge transformations

$$\begin{aligned} \mathcal{R} &\Rightarrow \hat{\mathcal{R}} = \mathcal{R} + \Phi\Phi \\ \delta\Phi &\Rightarrow \hat{\delta}\Phi = \delta\Phi + \Phi\xi \end{aligned}$$

- Require that the deformed objects transform covariantly:

$$\delta\hat{\mathcal{R}} \sim \mathcal{R}\xi$$

- Interacting Lagrangian

$$\mathcal{L}_{int} \sim \hat{\mathcal{R}}\hat{\mathcal{R}} + \mathcal{R}\mathcal{R}\Phi$$

- All non-trivial cubic vertices for the massless higher spin fields in AdS_d, $d \geq 4$ can be reproduced (Vasiliev 2011)

Types of vertices

- Trivially gauge invariant vertices

$$\mathcal{L}_1 \sim \mathcal{R}\mathcal{R}\mathcal{R} \quad \Rightarrow \quad \delta\Phi = 0, \quad [\delta_1, \delta_2]\Phi = 0$$

- Abelian vertices:

$$\delta\Phi \sim \mathcal{R}\xi$$

$$[\delta_1, \delta_2]\Phi = 0$$

- Non-abelian vertices

$$\delta\Phi \sim \Phi\xi \ (+\mathcal{R}\xi)$$

$$[\delta_1, \delta_2]\Phi \neq 0$$

Massive (partially massless) case

- In the gauge invariant formulation for massive (partially massless) fields we have a full set of gauge invariant objects and the free Lagrangians can be rewritten using them. This allow us to apply the same procedure for the construction of cubic vertices.
- Now we have not only one-forms (gauge fields) but also zero-forms (Stueckelberg fields)
- In-particular we face a lot of possible field redefinitions
- The possibility to reproduce all non-trivial vertices containing massless and massive (partially massless) fields is still an open question, but at least we have some working examples.
- One of examples — partially massless spin-2:
  - in $d > 4$ four derivatives vertex is abelian
  - in $d = 4$ two derivatives vertex is non-abelian

Deformation procedure

- Supertransformations for the bosonic components (up to the terms that produce vanishing on-shell contributions or can be removed by field redefinitions):

$$\delta\Omega^{ab} = Mc_0\bar{\Phi}\Gamma^{ab}\zeta, \qquad \delta f^a = ic_0\bar{\Phi}\gamma^a\zeta$$
$$\delta B^{ab} = \frac{Mm_0}{3m}c_0\bar{\phi}\Gamma^{ab}\zeta, \qquad \delta A = \frac{m_0}{6m}c_0\bar{\phi}\gamma^a\zeta$$

- Supertransformations for the fermionic components:

$$\delta\Phi = \frac{Md_0}{4}\Gamma^{ab}\Omega^{ab}\zeta + id_0\gamma^a f^a\zeta, \qquad \delta\phi = d_0\Gamma^{ab}B^{ab}\zeta$$

- Note that at this stage parameters c_0 and d_0 are independent
- Recall that

$$m_2{}^2 = 2\Lambda, \qquad m_{3/2}{}^2 > \Lambda$$

Interacting Lagrangian

- Interacting Lagrangian looks like:

$$\begin{aligned}\mathcal{L} &= a_1 \hat{E}_{abcd} \hat{\mathcal{R}}^{ab} \hat{\mathcal{R}}^{cd} + a_2 \hat{E}_{ab} \hat{\mathcal{B}}^a{}_c \hat{\mathcal{B}}^{bc} + a_3 \hat{E}_{abc} \hat{\mathcal{B}}^{ab} \hat{\mathcal{T}}^c \\ &+ b_1 \hat{E}_{abcd} \hat{\bar{\mathcal{F}}} \Gamma^{abcd} \hat{\mathcal{F}} + i b_2 \hat{E}_{abc} \hat{\bar{\mathcal{F}}} \Gamma^{abc} \hat{\mathcal{C}} + b_3 \hat{E}_{ab} \hat{\bar{\mathcal{C}}} \Gamma^{ab} \hat{\mathcal{C}} \\ &+ i b_4 \hat{E}_{abcd} \bar{\mathcal{F}} \Gamma^{abc} \mathcal{C} f^d + b_5 \hat{E}_{abc} \bar{\mathcal{C}} \Gamma^{ab} \mathcal{C} f^c + b_6 \hat{E}_{abcd} \bar{\mathcal{F}} \Gamma^{abcd} \mathcal{C} A\end{aligned}$$

- It contains quadratic, cubic and even quartic terms.
- There are two independent solutions — abelian and non-abelian ones.
- For the non-abelian one we obtain the relation on two coupling constants (as expected):

$$2 a_1 c_0 = 3 b_1 d_0$$

Discussion

- Thus the status of partially massless supergravity is the same as of partially massless gravity itself:
  - we have constructed interactions in the first non-trivial approximation
  - it does not guarantee that the complete interacting theory exists but at lest we do not have one more ”no-go” theorem
- Next step — to investigate general case of massive supergravity and in-particular to see what happens in the partially massless limit
- Supersymmetric extensions for the massive bigravities is also worth to investigate.