On some phases in brane matter

A. A. Zheltukhin

[a] Kharkov Inst. of Physics and Technology, Kharkov, 61108, Ukraine
[b] Physics Department, Stockholm University, AlbaNova, Nordita , KTH Royal Inst. of Technology and Stockholm University
SE 106 91 Stockholm, Sweden

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Dirac branes, Cartan world-sheet multiplets and R squared gravity

The Dirac action for p-branes in $\mathbf{R}^{1,D-1}$ is built from the determinant of the induced metric $g_{\mu\nu} = \partial_\mu \mathbf{x} \partial_\nu \mathbf{x}$

$$S = T_p \int d^{p+1}\xi \sqrt{|det(\partial_\mu \mathbf{x} \partial_\nu \mathbf{x})|} \tag{1}$$

That gives the nonlinear wave eqn. on hyper w-s Σ_{p+1} swept by $\mathbf{x}(\xi^\mu)$

$$\Box^{(p+1)}\mathbf{x} := \nabla^\mu \nabla_\mu \mathbf{x} = \frac{1}{\sqrt{|g|}} \partial_\mu (\sqrt{|g|} g^{\mu\nu} \partial_\nu \mathbf{x}). \tag{2}$$

Projection of (2) on orts $\mathbf{n}_a(\xi) \perp$ to Σ_{p+1} results in the minimality conds.

$$g^{\mu\nu} l^a_{\mu\nu} \equiv Sp(l^a) = 0, \ \ (a = p+1, p+2, ..., D-p-1), \tag{3}$$

where $l_{\mu\nu}{}^a$ is the second fundamental form of Σ_{p+1}

$$l_{\mu\nu}{}^a := \mathbf{n}^a \partial_{\mu\nu} \mathbf{x} \equiv \mathbf{n}^a \nabla_\mu \partial_\nu \mathbf{x}, \tag{4}$$

The orthonormal vectors $\mathbf{n}_A(\mathbf{x})$ at $\mathbf{x}$ form a moving frame in $\mathbf{R}^{1,D-1}$

$$\mathbf{n}_A(\mathbf{x}) \mathbf{n}_B(\mathbf{x}) = \eta_{AB}, \quad A, B = (0, 1, .., D-1), \tag{5}$$
$$d\mathbf{x} = \omega^A(d\xi)\mathbf{n}_A, \quad d\mathbf{n}_A = -\omega_A{}^B(d\xi)\mathbf{n}_B.$$

We will use ω^A and ${\omega_A}^B$ $\mathbf{x}$ as new variables instead of the D-hedron $(\mathbf{x}, \mathbf{n}_A)$. The integrability conditions results in Maurer-Cartan eqs.

$$d \wedge \omega_A + {\omega_A}^B \wedge \omega_B = 0, \tag{6}$$

$$d \wedge {\omega_A}^B + {\omega_A}^C \wedge {\omega_C}^B = 0 \quad \rightarrow \quad {F_A}^B = 0. \tag{7}$$

The presence of Σ_{p+1} breaks the Poincare symmetry of $\mathbf{R}^{1,D-1}$
$ISO(1, D-1) \rightarrow ISO(1, p-1) \times SO(D-p-1)$
and $\mathbf{n}_A$ split into two subsets: $\mathbf{n}_A = (\mathbf{n}_i, \mathbf{n}_a)$,
where $\mathbf{n}_i,\ (i, k = 0, 1, ..., p)$ are tangent and $\mathbf{n}_a$ orthogonal to Σ_{p+1}.
The remaining symmetry of the rotations is $SO(1, p) \times SO(D-p-1)$ describes the tangent Lorentz rotations in Σ_{p+1} accompanied with rotations in the $(D-p-1)$-dim. subspace normal to Σ_{p+1}.
Then the $(p+1)(D-p-1)$ N-G bosons are effectively described by the massless Cartan multiplets of the right gauge group $SO(1, D-1)$

$${\omega_A}^B(d\xi) = \begin{pmatrix} {A_i}^k(d\xi) & {W_i}^b(d\xi) \\ {W_a}^k(d\xi) & {B_a}^b(d\xi) \end{pmatrix}. \tag{8}$$

The diagonal submatrices ${A_{\mu i}}^k d\xi^\mu$ and ${B_{\mu a}}^b d\xi^\mu$ describe the gauge fields in the fundamental rep-s of $SO(1,p)$ and $SO(D-p-1)$ subgroups.
${W_{\mu i}}^b d\xi^\mu$ describes a charged vector multiplet in the bi-fundamental rep. of $SO(1,p)\times SO(D-p-1)$ with the covariant derivative

$$(D_\mu W_\nu)_i{}^a = \partial_\mu {W_{\nu i}}^a + {A_{\mu i}}^k {W_{\nu k}}^a + {B_\mu}^a{}_b {W_{\nu i}}^b. \tag{9}$$

The invariant forms ω^A for the global translations of $\mathbf{R}^{1,D-1}$ are $\delta^A_m dx^m$.
Projections of $d\mathbf{x}$ on $\mathbf{n}_A(\xi)$ give the forms referred to a moving frame on Σ_{p+1}

$$\omega^A = d\mathbf{x}(\xi)\mathbf{n}^A(\xi) \equiv dx^m {n_m}^A(\xi). \tag{10}$$

PDE's (10) represent N-G translation modes $x^m(\xi)$ through $\omega^A_m(\xi)$ if integrab. conds. (6) are satisfied. In view of orhogonality $\mathbf{n}_a(\xi)d\mathbf{x}(\xi) = 0$ we have

$$\omega^a(d\xi) = 0 \quad \rightarrow \quad d\mathbf{x}(\xi) = \omega^i(d)\mathbf{n}_i(\xi). \tag{11}$$

Then $ds^2 = d\mathbf{x}^2$, including the induced metric $g_{\mu\nu}$ of Σ_{p+1}, takes the form

$$ds^2 = \omega_i\omega^i = \omega^i_\mu \omega_{i\nu} d\xi^\mu d\xi^\nu \equiv g_{\mu\nu}(\xi) d\xi^\mu d\xi^\nu. \tag{12}$$

This shows that $\omega^i_\lambda(\xi)$ is the vielbein of Σ_{p+1}

$$g_{\mu\nu} := \omega^i_\mu \eta_{ik} \omega^k_\nu, \;\; \omega^i_\mu \omega^\mu_k = \delta^i_k. \tag{13}$$

The solution of M-C Eqs. (6) yields the tetrade postulate

$$D^{\|}_{[\mu}\omega^i_{\nu]} \equiv \partial_{[\mu}\omega^i_{\nu]} + A_{[\mu}{}^i{}_k\omega_{\nu]}{}^k = 0, \tag{14}$$

which expresses $A_\mu{}^i{}_k$ through ω^i_μ together with the constraints

$$\omega^i_{[\mu}W_{\nu]ia} = 0 \tag{15}$$

having the general solution

$$W_{\mu i}{}^a = -l_{\mu\nu}{}^a\omega^\nu_i, \tag{16}$$

where $l^a_{\mu\nu} = l^a_{\nu\mu}$ is the second fundamental form of Σ_{p+1}.
Then M-C Eqs. (7) are transformed into the Gauss-Ricci-Codazzi eqs.

$$R_{\mu\nu}{}^\gamma{}_\lambda = l_{[\mu}{}^{\gamma a}l_{\nu]\lambda a}, \tag{17}$$
$$H_{\mu\nu}{}^{ab} = l_{[\mu}{}^{\gamma a}l_{\nu]\gamma}{}^b, \tag{18}$$
$$\nabla^\perp_{[\rho}l_{\mu]\nu}{}^a = 0. \tag{19}$$

These eqs. and Eqs. (3) yield a complete set of data describing fundamental branes in terms of the Cartan multiplets of the gauge group $SO(D-p-1)$.

The $SO(D-p-1)$ and diff invariant action of p-branes sweeping a minimal hyper world-sheet Σ^{min}_{p+1} which is consistent with the G-R-C eqs. (17-19) is

$$S_{Dir} = \frac{1}{k_p^2}\int d^{p+1}\xi\,\sqrt{|g|}\,\{-\frac{1}{4}Sp(H_{\mu\nu}H^{\nu\mu})$$

$$+\frac{1}{2}\nabla^{\perp}_{\mu}l_{\nu\rho a}\nabla^{\perp\{\mu}l^{\nu\}\rho a} - \nabla^{\perp}_{\mu}l^{\mu}_{\rho a}\nabla^{\perp}_{\nu}l^{\nu\rho a} + V_{Dir}(l)\}. \tag{20}$$

The invariant potential $V_{Dir}(l)$ encodes self interaction of the N-G multiplet $l^{a}_{\mu\nu}$ in the gravitational *background* $g_{\mu\nu}(\xi^{\rho})$ and has the form

$$V_{Dir} = -\frac{1}{2}Sp(l_a l_b)Sp(l^a l^b) + Sp(l_a l_b l^a l^b) - Sp(l_a l^a l_b l^b) + \mathrm{c_p}, \tag{21}$$

where $\mathrm{c_p}$ is an integration constant.
To derive V_{Dir} we used the Bianchi identities

$$[\nabla^{\perp}_{\gamma},\,\nabla^{\perp}_{\nu}]l^{\mu\rho a} = R_{\gamma\nu}{}^{\mu}{}_{\lambda}l^{\lambda\rho a} + R_{\gamma\nu}{}^{\rho}{}_{\lambda}l^{\mu\lambda a} + H_{\gamma\nu}{}^{a}{}_{b}l^{\mu\rho b} \tag{22}$$

for the metric and Y-M covariant derivative

$$\nabla^{\perp}_{\mu}l_{\nu\rho}{}^{a} := \partial_{\mu}l_{\nu\rho}{}^{a} - \Gamma^{\lambda}_{\mu\nu}l_{\lambda\rho}{}^{a} - \Gamma^{\lambda}_{\mu\rho}l_{\nu\lambda}{}^{a} + B^{ab}_{\mu}l_{\nu\rho b}. \tag{23}$$

The Euler-Lagrange PDEs have a unique solution describing p-branes provided that the Ricci-Codazzi eqs.(18-19) were chosen as the *Cauchy initial data*.

The latter turned out to be invariants of the evolution prescribed by S_{Dir}.

The Gauss eqs. (17) are treated as the evolution PDEs for $g_{\mu\nu}$. They are consistent with the used variational principle since they have selected V_{Dir}.

Then the EOM become equivalent to the identities

$$\nabla^{\perp\mu}\mathcal{H}^{ab}_{\mu\nu} = 0, \quad \nabla^{\perp\mu}\nabla^{\perp}_{[\mu}l^{a}_{\nu]\rho} = 0 \tag{24}$$

produced by the covariant differentiation of the Ricci-Codazzi eqs.
They can be equivalently written in the form of the generalized Maxwell-Y-M and Newton eqs. in the gravit. field defined by Gauss eqs. (17)

$$\nabla^{\perp}_{\nu}H^{\nu\mu}_{ab} = j^{\mu}_{ab}, \quad j^{\mu}_{ab} = Sp(l_{[a}\nabla^{\perp\mu}l_{b]}), \quad \nabla^{\perp}_{\mu}j^{\mu}_{ab} = 0, \tag{25}$$

$$\nabla^{\perp}_{\mu}\nabla^{\perp\mu}l^{\nu\rho a} = \frac{1}{2}\frac{\partial V_{Dir}}{\partial l_{\nu\rho a}} \equiv (2l_b l^a l^b - l^a l_b l^b - l_b l^b l^a)^{\nu\rho} - l^{\nu\rho}_{b}Sp(l^b l^a). \tag{26}$$

We conclude that S_{Dir} (20) with the chosen potential V_{Dir} (21) reformulates the Dirac p-brane dynamics it terms of the Cartan multiplets.

The potential term V_{Dir} can be represented in the form

$$V_{Dir} = -\frac{1}{4}R_{\mu\nu\gamma\lambda}R^{\mu\nu\gamma\lambda} - \frac{1}{2}R_{\mu\nu}R^{\mu\nu} + \frac{1}{4}H_{\mu\nu ab}H^{\nu\mu ab} + \mathrm{c_p}. \tag{27}$$

Eq. (27) was derived using the G-R eqs.(17-18) and (3) resulting in

$$\frac{1}{2}R_{\mu\nu\gamma\lambda}R^{\mu\nu\gamma\lambda} = Sp(l_a l_b)Sp(l^a l^b) - Sp(l_a l_b l^a l^b), \tag{28}$$

$$\frac{1}{2}H^{ab}_{\mu\nu}H^{\mu\nu}_{ab} = Sp(l_a l_b l^a l^b) - Sp(l_a l^a l_b l^b), \quad Spl^a = 0, \tag{29}$$

which were combined with the quadratic expressions for the Ricci tensor $R_{\mu\nu}$ and the scalar curvature R of the *minimal* hyper w-s Σ^{min}_{p+1}

$$R_{\mu\nu} = -(l^a l_a)_{\mu\nu}, \quad R = -Sp(l^a l_a). \tag{30}$$

The potential (27) contains the curvature squared terms considered in f(R) gravity. In the codimention 1, i.e. when $D = p + 2$, $B^{ab}_{\mu} \equiv 0$ since $a = b = p + 1$ and

S_{Dir} (20) is reduced to the action

$$S_{D=p+2} = \frac{1}{k_p^2}\int d^{p+1}\xi\,\sqrt{|g|}\Big(\frac{1}{2}\nabla_\mu l_{\nu\rho\perp}\nabla^\mu l^{\nu\rho\perp} - \nabla_\mu l^\mu_{\rho\perp}\nabla_\nu l^{\nu\rho\perp} - \frac{1}{2}[Sp(l_\perp l^\perp)]^2 + \mathrm{c_p}\Big), \quad (31)$$

where $(p+1)$ is denoted as $\perp$ and the metric covariant derivative as ∇_μ is

$$\nabla_\mu l^\perp_{\nu\rho} := \partial_\mu l^\perp_{\nu\rho} - \Gamma^\lambda_{\mu\nu} l^\perp_{\lambda\rho} - \Gamma^\lambda_{\mu\rho} l^\perp_{\nu\lambda}, \qquad l^\perp_{\lambda\rho} = -l_{\lambda\rho\perp} \equiv -l_{\lambda\rho}. \quad (32)$$

Using Eqs. (27-30) we observe that V_{Dir} in $S_{D=p+2}$ can be rewritten as

$$\frac{1}{2}[Sp(l_\perp l^\perp)]^2 = \frac{1}{2}R^2. \quad (33)$$

Eqs. (25-26) are reduced to the equations

$$\Box l^\perp_{\nu\rho} = l^\perp_{\nu\rho} Sp\,(l^{\perp 2}) \equiv R\, l^\perp_{\nu\rho}, \quad Sp\, l^\perp = 0, \quad (34)$$

where $\Box \equiv \nabla_\mu\nabla^\mu$ is the D'Alembert-Beltrami operator for a tensor field on Σ^{min}_{p+1}.

We built the Dirac brane mapping into the R^2 model formulated in terms of the interacting Y-M and N-G fields on its w-s Σ^{min}_{p+1} which we call as the brane matter.

Possible phases of brane matter

To discuss physics of the model we restore its coupling constants using $\hbar = c = 1$ and the coordinate dimensions: $[x^m] = [\xi^\mu] = [L] \rightarrow [g_{\mu\nu}] = 1$; $[l^a_{\mu\nu}] = [B^{ab}_\mu] = [\nabla^\perp_\nu] = [L^{-1}]$.

This defines the dimensions of the constants T_p, k_p and $\mathrm{c_p}$

$$[T_p] = [L^{-d_p}], \;\; [k_p] = [L^{\frac{d_p-4}{2}}], \;\; [\mathrm{c_p}] = [L^{-4}], \;\; d_p := p+1. \tag{35}$$

Then we obtain

$$[k_p] = [T_p]^{\frac{4-d_p}{2d_p}} \tag{36}$$

Transition to the fields with the canonical dimension $[L^{\frac{2-d_p}{2}}]$

$$\tilde{l}^a_{\mu\nu} := k_p^{-1} l^a_{\mu\nu}, \qquad \tilde{B}^{ab}_\mu := k_p^{-1} B^{ab}_\mu \tag{37}$$

shows that k_p coincides with the gauge coupling of $SO(D-p-1)$

$$\begin{aligned} \nabla^\perp_\mu \tilde{l}^a_{\mu\nu} = k_p^{-1}\nabla^\perp_\mu l_{\nu\rho}{}^a \equiv \partial_\mu \tilde{l}_{\nu\rho}{}^a - \Gamma^\lambda_{\mu\nu}\tilde{l}_{\lambda\rho}{}^a - \Gamma^\lambda_{\mu\rho}\tilde{l}_{\nu\lambda}{}^a + k_p \tilde{B}^{ab}_\mu \tilde{l}_{\nu\rho b}, \\ \tilde{H}_{\mu\nu}{}^{ab} = k_p^{-1} H_{\mu\nu}{}^{ab} \equiv (\partial_{[\mu}\tilde{B}_{\nu]} + k_p \tilde{B}_{[\mu}\tilde{B}_{\nu]})^{ab} \end{aligned} \tag{38}$$

for $D \geq p+3$ when $\tilde{B}^{ab}_\nu \neq 0$.

In this case k_p squared is equal to the interaction coupling λ_p in $V_{Dir}(\tilde{l})$

$$\lambda_p = k_p^2 \tag{39}$$

In terms of the canonical fields the action (20) takes the form

$$S_{Dir} = \int d^{p+1}\xi \sqrt{|g|}\{-\frac{1}{4}Sp(\tilde{H}_{\mu\nu}\tilde{H}^{\nu\mu}) + \frac{1}{2}\nabla^{\perp}_{\mu}\tilde{l}_{\nu\rho a}\nabla^{\perp\{\mu}\tilde{l}^{\nu\}\rho a} - \nabla^{\perp}_{\mu}\tilde{l}^{\mu}_{\rho a}\nabla^{\perp}_{\nu}\tilde{l}^{\nu\rho a} \tag{40}$$
$$+ k_p^2\,[-\frac{1}{2}Sp(\tilde{l}_a\tilde{l}_b)Sp(\tilde{l}^a\tilde{l}^b) + Sp(\tilde{l}_a\tilde{l}_b\tilde{l}^a\tilde{l}^b) - Sp(\tilde{l}_a\tilde{l}^a\tilde{l}_b\tilde{l}^b)] + \Lambda_p\},$$

where $\Lambda_p := \mathrm{c_p}/k_p^2$ is a cosmological constant of dimension $[\Lambda_p] = [T_p] = [L^{-d_p}]$.
If $\mathrm{c_p} = 0 \rightarrow \Lambda_p = 0$ and S_{Dir} has only one coupling constant k_p which must depends on T_p. In view of (36) one can conjecture that

$$k_p \sim T_p^{\frac{4-d_p}{2d_p}} \equiv T_p^{\frac{3-p}{2(p+1)}} \qquad \lambda_p \sim T_p^{\frac{3-p}{p+1}}. \tag{41}$$

Note the absence of the H-E term and its gravitational constant G_p^{Newt} in S_{Dir} (20). This constant is independent of k_p and has the dimension $[G_p^{Newt}] = [L^{d_p-2}]$ equal to $[k_p^{2\frac{d_p-2}{d_p-4}}] = [T_p^{\frac{1-p}{1+p}}]$ if $p \neq 3$, because k_3 is dimensionless.

By analogy with condenced matter physics one can treat $T_p = 0$ and α_p

$$\alpha_p := \frac{3-p}{2(p+1)}, \qquad (p = 1, 2, .., D-1) \tag{42}$$

as the critical temperature and exponent depending on p. The limit $T_p \to 0$ corresponds to superplanckian energies when M_{Planck} becomes negligible.

The power law (41) implies three different regimes defined by the sign of α_p.

They correspond to phases of the fields representing the brane matter in (40) and correlating with p.

So, we observe the *decrease* of the constants in cases when $p < 3$ corresponding to strings and membranes when T_p *decreases*

$$k_1 \sim T_1^{\frac{1}{2}}, \;\; \lambda_1 \sim T_1; \qquad k_2 \sim T_2^{\frac{1}{6}}, \;\; \lambda_2 \sim T_2^{\frac{1}{3}}$$

and their *increase* for the phases corresponding to p-branes with $p > 3$

$$k_4 \sim T_4^{-\frac{1}{10}}, \;\; \lambda_4 \sim T_4^{-\frac{1}{5}}; \;\; k_5 \sim T_5^{-\frac{1}{6}}, \;\; \lambda_5 \sim T_1^{-\frac{1}{3}};$$

including the limiting case $k_\infty \sim T_\infty^{-\frac{1}{2}}, \;\; \lambda_\infty \sim T_\infty^{-1}$ when $D, p \to \infty$.

The dependence of $R_{\mu\nu}{}^{\gamma}{}_{\lambda}$ and $\tilde{H}_{\mu\nu}{}^{ab}$ on k_p follows from the G-R-C eqs. (17-19) rewritten in terms of the canonical fields

$$R_{\mu\nu}{}^{\gamma}{}_{\lambda} = k_p^2 \tilde{l}_{[\mu}{}^{\gamma a} \tilde{l}_{\nu]\lambda a} \tag{43}$$

$$\tilde{H}_{\mu\nu}{}^{ab} = k_p \tilde{l}_{[\mu}{}^{\gamma a} \tilde{l}_{\nu]\gamma}{}^{b}, \tag{44}$$

$$\nabla^{\perp}_{[\mu} \tilde{l}_{\nu]\rho a} = 0. \tag{45}$$

These eqs. show that in superplanckian region the string and membrane curvatures goes to zero that corresponds to their inflation.

On the contrary, the curvature of branes with $p > 3$ go to infinity that could be treated as their collapse if rotations do not compensate their large gravity force.

Thus, at high energies the regimes of confinement ($k_{3,4,\dots} \to \infty$) or asymptotic freedom ($k_1, k_2 \to 0$) in the field model (40) correspond to brane matter phases describing its collapse or inflation dependening on the value of p.

In the low-energy limit, corresponding to $T_p \to \infty$, the curvatures go to infinity for $p = 1, 2$ that could be treated as the collapse of non-rotating strings and membranes contrary to an inflation of the rotating branes with $p > 3$.

Finally, the field regime when $\alpha_p = 0$ corresponding $p = 3$. In this exclusive case the coupling constant k_p in (40) is dimensionless and independent of T_p
Since in our analysis the constant Λ_p with the dimension $[L^{-4}]$ vanishes the action (40) becomes invaiant under the global Weyl transformations

$$g'_{\mu\nu}(\xi) = \rho g_{\mu\nu}(\xi), \quad \tilde{l}'_{\mu\nu}(\xi) = \rho^{1/2}\tilde{l}_{\mu\nu}(\xi), \quad \tilde{B}'^{ab}_{\mu}(\xi) = B^{ab}_{\mu}(\xi). \tag{46}$$

Here we arrive at the well-studied conformal invariant theory of gravity.
There exist many papers devoted to investigations of conformal gravity carried out by outstanding scientists including Weyl and Dirac. See e.g. the recent book V. Pervushin, A. Pavlov, "Principles of Quantum Universe", LAP LAMBERT Academic Publishing, Saarbrucken, 2013 and refs. there including also intersting results of the BLTP scientists in scale invariant gravity.

The simplest example of fundamental branes studied in the talk shows that the correspondence between branes and field theories on their hyper world-sheets may shed light on the problem of unification of the Standard Model and gravity.

Summary

1. The correspondence between fundamental *p*-branes in D-dim. Minkowski space and their world-sheet gauge multiplets is studied.
It is shown that the interaction potential of the Nambu-Goldstone w-s multiplet encodes curvature squared terms in the action of R squared gravity.

2. Based on dimensional analysis of the w-s action with zero cosmological constant, we propose the power law $k_p \sim T_p^{\frac{3-p}{2(p+1)}}$ for the coupling constant as the function of the *p*-brane tension T_p. This points to possible existence of brane matter phases interpreted as asymptotic freedom and confinement phases. Their connection with collapse and inflation of *p*-branes is discussed.

THANK YOU FOR YOUR ATTENTION!