

Construction of inflationary multifield models with non-minimal coupling

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based on

**M.N. Dubinin, E.Yu. Petrova, E.O. Pozdeeva,
M.V. Sumin, and S.Yu. Vernov,
arxiv:1705.09624**

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Models with scalar fields are very useful to describe the observable evolution of the Universe as the dynamics of the spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) background with

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Generic quantum corrections to the action of the scalar field minimally coupled to gravity include the term, proportional to $\phi^2 R$.

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The confirmed discovery of the Higgs boson at the Large Hadron Collider (CERN) has initiated an intense research activity with the aim to understand the cosmological implications of this fundamental scalar field. A fundamental step towards the unification of physics at all energy scales could be the description the inflation using particle physics models.

There are models of inflation, where the role of the inflaton is played by the Higgs field nonminimally coupled to gravity.

INFLATION MOTIVATED BY THE QFT

In my talk I consider the multifield inflationary scenarios connected with the quantum field theory.

A number of advantages of simplified SUSY GUTs in comparison with nonsupersymmetric GUTs such as naturally longer period of exponential expansion and better stability of the effective Higgs potential with respect to radiative corrections due to cancelation of loop diagrams have been noted quite long ago:

J. Ellis, D. V. Nanopoulos, K. A. Olive and K. Tamvakis, *Primordial supersymmetric inflation*, *Nucl. Phys.* **B221** (1983) 524.

B. A. Ovrut and P. J. Steinhardt, *Supersymmetric inflation, baryon asymmetry and the gravitino problem*, *Phys. Lett.* **B147** (1984) 263.

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There are a lot of papers that describe inflation using supersymmetry:

G. R. Dvali, *Phys. Lett.* **B387** (1996) 471 [hep-ph/9605445].

K. Enqvist and A. Mazumdar, *Phys. Rept.* **380** (2003) 99, [hep-ph/0209244].

L. E. Ibanez, F. Marchesano and I. Valenzuela, *J. High Energy Phys.* **1501** (2015) 128 [arXiv:1411.5380 [hep-th]].

The main idea is to use the Standard Model all the way to the Planck scale, and to introduce neither supersymmetry, nor any extension of the Standard Model gauge group:

C. D. Froggatt and H. B. Nielsen, *Standard model criticality prediction: Top mass 173 ± 5 GeV and Higgs mass 135 ± 9 GeV*, Phys. Lett. B **368** (1996) 96 [arXiv:hep-ph/9511371].

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The first inflationary model with the SM Higgs boson, proposed by: J. L. Cervantes-Cota and H. Dehnen, *Induced gravity inflation in the standard model of particle physics*, Nucl. Phys. **B442** (1995) 391 [astro-ph/9505069],
is not realistic.

THE HIGGS-DRIVEN INFLATION

There are models of inflation, where the role of the inflaton is played by the Higgs field nonminimally coupled to gravity. (F.L. Bezrukov and M. Shaposhnikov, *Phys. Lett. B* **659** (2008) 703–706, arXiv:0710.3755). They add $W^{(0)}(\phi)$ to the standard GR term and get the following action:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{M_{PL}^2}{2} + \xi \phi^2 \right) R - \frac{1}{2} (\partial\phi)^2 - \lambda (\phi^2 - \phi_0^2)^2 \right].$$

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This model have been actively studied

A.O. Barvinsky, A.Y. Kamenshchik, and A.A. Starobinsky, *J. Cosmol. Astropart. Phys.* **0811** (2008) 021 (arXiv:0809.2104);

F. Bezrukov, D. Gorbunov and M. Shaposhnikov, *J. Cosmol. Astropart. Phys.* **0906** (2009) 029, arXiv:0812.3622;

A. De Simone, M.P. Hertzberg and F. Wilczek, *Phys. Lett. B* **678** (2009) 1 (arXiv:0812.4946);

A.O. Barvinsky, A.Y. Kamenshchik, C. Kiefer, A.A. Starobinsky, and C.F. Steinwachs, *J. Cosmol. Astropart. Phys.* **0912** (2009) 003 (arXiv:0904.1698);

J. Garcia-Bellido, D.G. Figueroa, and J. Rubio, *Phys. Rev. D* **79** (2009) 063531 (arXiv:0812.4624);

F. Bezrukov, *Class. Quant. Grav.* **30** (2013) 214001 (arXiv:1307.0708)

OBSERVATION DATA

There are a few main parameters that can be obtained by the observation data

Planck 2015 results. XX. Constraints on inflation, *Astron.Astrophys.* **594** (2016) A20 [arXiv:1502.02114]:

- The joint BICEP2/Keck Array and Planck constraint on tensor-to-scalar power ratio r : $r < 0.11$ (95% c.l.).

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$$n_s = 0.968 \pm 0.006.$$

- This gives an upper bound on the Hubble parameter during inflation:

$$H < 3.6 \times 10^{-5} M_{Pl}, \quad (95\%CL).$$

Similar bound has been calculated in the GUT motivated inflationary model by A.O. Barvinsky and A.Yu. Kamenshchik, *Phys. Lett. B* **332** (1994) 270 [gr-qc/9404062].

- The Planck2015 data analysis confirms the prediction of the Starobinsky R^2 inflationary model and the Bezrukov–Shaposhnikov Higgs-driven inflation.

Two Higgs doublets of the MSSM

The considering cosmological model will initially formulate in terms of two Higgs doublets of the MSSM:

$$\Phi_1 = \begin{pmatrix} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix}, \quad (1)$$

$$\Phi_2 = \begin{pmatrix} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{pmatrix}, \quad (2)$$

where $\omega_{1,2}^+$ are complex scalar fields, $\eta_{1,2}$ and $\chi_{1,2}$ are real fields, the vacuum expectation values $v_1/\sqrt{2}$ and $v_2/\sqrt{2}$ are usually redefined in $(v, \tan \beta)$ parametrization:

$$v = \sqrt{v_1^2 + v_2^2}, \quad \tan(\beta) = \frac{v_2}{v_1}.$$

$v/\sqrt{2}$ is the vacuum expectation value of the neutral Higgs field.

The MSSM-inspired Cosmological Model with Non-minimal Interaction

- We start with the non-minimal coupling model :

$$S = \int d^4x \sqrt{-g} [f(\Phi_1, \Phi_2) R - \delta^{ab} g^{\mu\nu} \partial_\mu \Phi_a^\dagger \partial_\nu \Phi_b - V(\Phi_1, \Phi_2)], \quad (3)$$

where g is the determinant of metric tensor $g_{\mu\nu}$, and R is the scalar curvature.

- In the single-field Higgs-driven inflation the function f has been chosen as a sum of the Hilbert–Einstein term and the induced gravity term. We choose the function f in an analogous form:

$$f(\Phi_1, \Phi_2) = \frac{M_{Pl}^2}{2} + \xi_1 \Phi_1^\dagger \Phi_1 + \xi_2 \Phi_2^\dagger \Phi_2 \quad (4)$$

where ξ_1 and ξ_2 are positive dimensionless constants.

The MSSM Effective Potential

- The MSSM effective potential:

$$V(\Phi_1, \Phi_2) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - [\mu_{12}^2(\Phi_1^\dagger\Phi_2) + h.c.] + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + h.c. \right].$$

has been proposed by P. Fayet, Nucl. Phys. **D90**, 104 (1975);
K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, Prog. Theor. Phys. **68**, 927 (1982)

Model in Terms of Scalar Fields

- Two Higgs doublets of the MSSM can be parameterized using the $SU(2)$ states

$$\Phi_1 = \left(\begin{array}{c} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{array} \right), \quad \Phi_2 = \left(\begin{array}{c} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{array} \right), \quad (5)$$

- The $SU(2)$ eigenstates (ω_a^\pm, η_a and $\chi_a, a = 1, 2$) are expressed through mass eigenstates of the Higgs bosons h, H_0, A and H^\pm and the Goldstone bosons G^0, G^\pm by means of two orthogonal rotations¹

$$\left(\begin{array}{c} \eta_1 \\ \eta_2 \end{array} \right) = \mathcal{O}_\alpha \left(\begin{array}{c} H_0 \\ h \end{array} \right), \quad \left(\begin{array}{c} \chi_1 \\ \chi_2 \end{array} \right) = \mathcal{O}_\beta \left(\begin{array}{c} G^0 \\ A \end{array} \right), \quad \left(\begin{array}{c} \omega_1^\pm \\ \omega_2^\pm \end{array} \right) = \mathcal{O}_\beta \left(\begin{array}{c} G^\pm \\ H^\pm \end{array} \right),$$

where the rotation matrix

$$\mathcal{O}_X = \left(\begin{array}{cc} \cos X & -\sin X \\ \sin X & \cos X \end{array} \right), \quad X = \alpha, \beta.$$

¹E. Akhmetzyanova, M. Dolgoplov, and M. Dubinin, Phys. Rev. D **71** (2005) 075008 [arXiv:hep-ph/0405264].

- We consider MSSM effective potential at the tree level
- The dimensionless factors λ_i ($i=1,\dots,7$) including to the potential can be expressed, using the $SU(2)$ and $U(1)$ gauge couplings g_2 and g_1 ²:

$$\lambda_{1,2}^{\text{tree}}(M_{SUSY}) = \frac{g_1^2 + g_2^2}{8}, \quad \lambda_3^{\text{tree}}(M_{SUSY}) = \frac{g_2^2 - g_1^2}{4},$$

$$\lambda_4^{\text{tree}}(M_{SUSY}) = -\frac{g_2^2}{2}, \quad \lambda_{5,6,7}^{\text{tree}}(M_{SUSY}) = 0.$$

- The boson masses are $m_Z = 91.2\text{GeV}$ and $m_W = 80.4\text{GeV}$. Using $v = 246\text{Gev}$, from

$$m_Z = \frac{v}{2} \sqrt{g_1^2 + g_2^2}, \quad m_W = \frac{v}{2} g_2$$

we fix $g_1 = 0.36$, $g_2 = 0.65$.

- Values of all λ_i^{tree} are given!

²K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. **68** (1982) 927; R. A. Flores and M. Sher, Ann. Phys. (N.Y.) **148** (1983) 95. ▶ ◀ ≡ ≡ ≡ ↺ ↻

The dimension-two parameters μ_1^2, μ_2^2 and μ_{12}^2 are fixed using the minimization conditions:

$$\mu_1^2 = -m_A^2 \sin^2(\beta) + \frac{m_Z^2}{2} \cos(2\beta),$$

$$\mu_2^2 = -m_A^2 \cos^2(\beta) - \frac{m_Z^2}{2} \cos(2\beta),$$

$$\mu_{12}^2 = m_A^2 \sin(\beta) \cos(\beta).$$

Masses of the CP-even scalars h and H_0 (m_h and m_{H_0}), the charged scalar mass (m_{H^\pm}) can be expressed through the CP-odd scalar mass m_A , Z -boson mass m_Z and mixing angles β and α :

$$m_h^2 = m_Z^2 \sin^2(\alpha + \beta) + m_A^2 \cos^2(\alpha - \beta),$$

$$m_{H_0}^2 = m_Z^2 \cos^2(\alpha + \beta) + m_A^2 \sin^2(\alpha - \beta),$$

$$m_{H^\pm}^2 = m_A^2 + m_W^2,$$

$$\tan(2\alpha) = \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2} \tan(2\beta).$$

The potential includes two free parameters β and m_A .

From the MSSM we also get $m_A \ll M_{Pl}$.

There are some restriction on $\tan(\beta)$ from the LHC.

Reformulation in Terms of Scalar Fields

- We choose the mixing angles $\beta = \pi/2$ and $\alpha = 0$ in the unitary gauge $G^0 = G^\pm = 0$ and get
- The following isodoublet convolutions

$$\Phi_1^\dagger \Phi_1 = \frac{1}{2}(\Omega_\pm^2 + \Omega_0^2), \quad \Phi_2^\dagger \Phi_2 = \frac{h_v^2}{2},$$

$$\Phi_1^\dagger \Phi_2 = \frac{h_v}{2}(H_0 + iA), \quad \Phi_2^\dagger \Phi_1 = \frac{h_v}{2}(H_0 - iA),$$

where $h_v = h + v$, $\Omega_0^2 = H_0^2 + A^2$, and $\Omega_\pm^2 = 2H^+H^-$.

- The kinetic terms of the canonical form

$$\partial_\mu \Phi_1^\dagger \partial^\mu \Phi_1 = \partial_\mu H^- \partial^\mu H^+ + \frac{1}{2}(\partial A)^2 + \frac{1}{2}(\partial H_0)^2, \quad \partial_\mu \Phi_2^\dagger \partial^\mu \Phi_2 = \frac{1}{2}(\partial h)^2.$$

- The initial action of the MSSM-inspired model can be written as

$$S_J = \int d^4x \sqrt{-\tilde{g}} \left[f(\phi^I) \tilde{R} - \frac{1}{2} \delta_{IJ} \tilde{g}^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - V(\phi^I) \right], \quad (6)$$

where tilde denominates the metric tensor and curvature in the Jordan frame.

- In our case $V(\phi^I)$ and $f(\phi^I)$ depend on five real scalar fields

$$\phi^1 = \frac{H^+ + H^-}{\sqrt{2}}, \quad \phi^2 = \frac{H^+ - H^-}{\sqrt{2}i}, \quad \phi^3 = A, \quad \phi^4 = H_0, \quad \phi^5 = h_\nu.$$

- The non-minimal interaction can be presented in the form

$$f(\Phi_1, \Phi_2) = \frac{M_{Pl}^2}{2} + \frac{\xi_1}{2} (\Omega_{\pm}^2 + \Omega_0^2) + \frac{\xi_2}{2} h_v^2$$

- The potential has the following form

$$V(h_v, \Omega_0, \Omega_{\pm}) = -m_1^2 h_v^2 + m_2^2 (\Omega_0^2 + \Omega_{\pm}^2) + \nu_1 (h_v^4 + \Omega_0^4 + \Omega_{\pm}^4) - \\ - 2\nu_1 h_v^2 \Omega_0^2 + 2\nu_2 h_v^2 \Omega_{\pm}^2 + 2\nu_1 \Omega_0^2 \Omega_{\pm}^2,$$

where

$$\Omega_{\pm}^2 = (\phi^1)^2 + (\phi^2)^2, \quad \Omega_0^2 = (\phi^3)^2 + (\phi^4)^2 \\ m_1^2 = \frac{m_Z^2}{4}, \quad m_2^2 = \frac{m_A^2}{2} + \frac{m_Z^2}{4}, \quad \nu_1 = \frac{g_1^2 + g_2^2}{32}, \quad \nu_2 = \frac{g_2^2 - g_1^2}{32}.$$

Conformal Transformations of Multifield Models

- Generic action which is dependent on N scalar fields ϕ^I , $I = 1, \dots, N$ with the standard kinetic term and nonminimal coupling to gravity

$$S_J = \int d^4x \sqrt{-\tilde{g}} \left[f(\phi^I) \tilde{R} - \frac{1}{2} \delta_{IJ} \tilde{g}^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - V(\phi^I) \right]. \quad (7)$$

tilde denotes the metric tensor and curvature in the Jordan frame.

- This action can be transformed to the following action in the Einstein frame ³

$$S_E = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{2} \mathcal{G}_{IJ} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J - W \right], \quad (8)$$

where

$$\mathcal{G}_{IJ} = \frac{M_{Pl}^2}{2f(\phi^K)} \left[\delta_{IJ} + \frac{3f_{,I} f_{,J}}{f(\phi^K)} \right], \quad W = M_{Pl}^4 \frac{V}{4f^2}, \quad M_{Pl} \equiv \frac{1}{\sqrt{8\pi G}}.$$

- Metric tensors in the Jordan and the Einstein frames are related by the equation

$$g_{\mu\nu} = \frac{2}{M_{Pl}^2} f(\phi^I(x)) \tilde{g}_{\mu\nu}(x).$$

³R.N. Greenwood, D.I. Kaiser, E.I. Sfakianakis, Phys. Rev. D **87** (2013) 064021

Equations of Motion in the FLRW Metric

- Let us consider a spatially flat FLRW universe with metric interval

$$ds^2 = - dt^2 + a^2(t) (dx_1^2 + dx_2^2 + dx_3^2),$$

where $a(t)$ is the scale factor.

- Varying the action S_E , we get

$$\begin{cases} H^2 = \frac{1}{3M_{Pl}^2} \left(\frac{\dot{\sigma}^2}{2} + W \right), \\ \dot{H} = -\frac{1}{2M_{Pl}^2} \dot{\sigma}^2, \end{cases} \quad (9)$$

where the Hubble parameter $H = \dot{a}/a$,

$$\dot{\sigma}^2 = \mathcal{G}_{IJ} \dot{\phi}^I \dot{\phi}^J,$$

and dots mean the cosmic time derivatives.

- Field equations have the following form ⁴

$$\ddot{\phi}^I + 3H\dot{\phi}^I + \Gamma^I_{JK}\dot{\phi}^J\dot{\phi}^K + \mathcal{G}^{IK}W'_{,K} = 0, \quad (10)$$

where Γ^I_{JK} is the Christoffel symbol for the field-space manifold, calculated in terms of \mathcal{G}_{IJ} ,

$$W'_{,K} = \partial W / \partial \phi^K.$$

Hereafter, primes denote derivatives with respect to the fields.

⁴D.I. Kaiser, E.A. Mazenc, E.I. Sfakianakis, *Phys. Rev. D* **87** (2013) 064004 [arXiv:1210.7487].

- During inflation the Hubble parameter is positive and the scalar factor is a monotonically increasing function. To describe the evolution of scalar fields during inflation we use the number of e-foldings $N_e = \ln(a/a_e)$, as a new measure of time.

Using $d/dt = H d/dN_e$ one can write eqs. (9) and (10) in the form

$$H^2 = \frac{2W}{6M_{Pl}^2 - (\sigma')^2}, \quad (11)$$

$$\frac{d \ln H}{dN_e} = -\frac{1}{2M_{Pl}^2} (\sigma')^2, \quad (12)$$

$$\frac{d\phi^I}{dN_e} = \psi^I, \quad (13)$$

$$\frac{d\psi^I}{dN_e} = -\left(3 + \frac{d \ln H}{dN_e}\right) \psi^I - \Gamma^I_{JK} \psi^J \psi^K - \frac{1}{H^2} G^{IK} W'_{,K}, \quad (14)$$

- In order to calculate the observables, spectral index n_s and tensor-to-scalar ratio r , slow-roll parameters are introduced analogously to the single-field inflation

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta_{\sigma\sigma} = M_{Pl}^2 \frac{\mathcal{M}_{\sigma\sigma}}{W}, \quad (15)$$

where

$$\mathcal{M}_{\sigma\sigma} \equiv \hat{\sigma}^K \hat{\sigma}^J (\mathcal{D}_K \mathcal{D}_J W),$$

$\sigma^I = \dot{\phi}^I / \dot{\sigma}$ is the unit vector in the field space and \mathcal{D} denotes a covariant derivative with respect to the field-space metric, $\mathcal{D}_I \phi^J = \partial_I \phi^J + \Gamma_{IK}^J \phi^K$.

Parameters of an inflationary model

- The spectral index n_s and tensor-to-scalar ratio r at the time when a characteristic scale (50–65 e-foldings before the end of inflation) can be calculated using the single-field expressions valid to lowest order in slow-roll parameters ⁵

$$n_s = 1 - 6\epsilon + 2\eta_{\sigma\sigma}, \quad r = 16\epsilon. \quad (16)$$

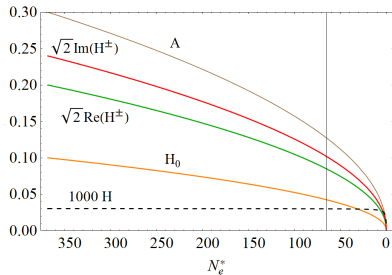
- The testing of inflationary scenarios can be realized using inflationary parameters obtained from Planck mission data: [spectral index of curvature perturbations \$n_s = 0.968 \pm 0.006\$](#) and the upper bound on the tensor-to-scalar ratio $r < 0.11$

⁵D. I. Kaiser, E. A. Mazenc and E. I. Sfakianakis, *Phys. Rev. D* **87** (2013) 064004 [1210.7487 [astro-ph.CO]]; K. A. Malik and D. Wands, *Phys.Rep.* **475** (2009) 1 [arXiv:0809.4944]

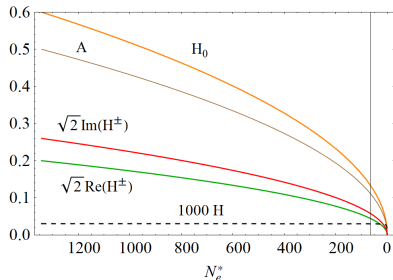
Numerical solutions of the equations of motion

Scenario	ξ_1	ξ_2	ϕ_0^1	ϕ_0^2	ϕ_0^3	ϕ_0^4	ϕ_0^5
A_1	2500	any	0.2	0.24	0.3	0.1	0
A_2	2500	any	2×10^{-3}	0	0.45	0.2	0
A_3	2500	any	0.2	0.26	0.5	0.6	0
A_4	40	any	0.8	0.9	0.5	0.7	0
B_1	1100	500	0.3	0.2	0	0	0.1
B_2	1100	500	0.4	0.6	0	0	0.3

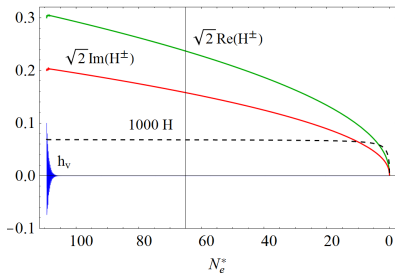
Table: Initial conditions (in units of M_{Pl}) for trajectories with successful inflationary scenarios, CP-odd Higgs boson mass $m_A = 200$ GeV.



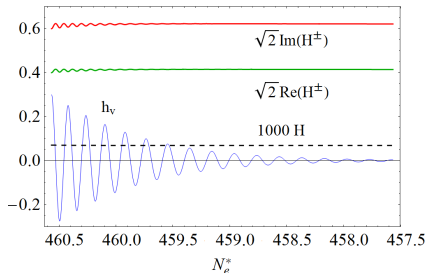
(a)



(b)



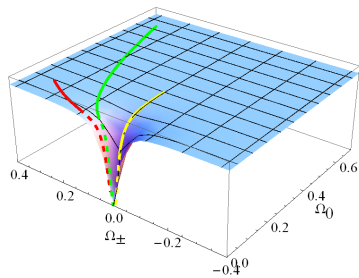
(c)



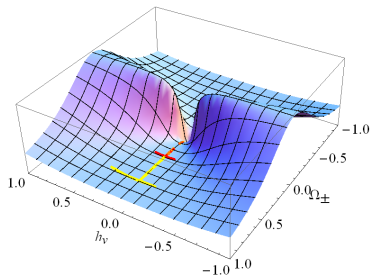
(d)

Figure: Evolution for the scenarios A_1 (a), A_3 (b), B_1 (c) and B_2 (d).

- A) the field system rolls slowly down to the potential minimum
- B) all nonzero fields demonstrate rapidly damped oscillations going to zero h_V for the number of e-foldings before the end of inflation $N_e^* \gg 65$.



(a)



(b)

Figure: Parametric plots of the fields' evolution superimposed on the Einstein-frame potential for A parameter sets, plot (a), and B parameter sets, plot (b), see the parameter sets in table 1. The trajectories shown here have the initial condition (in units of M_{Pl}): (a) $\phi_0^5=0$ and $\phi_0^1=0.2$, $\phi_0^2=0.24$, $\phi_0^3=0.3$, $\phi_0^4=0.1$ (red line); $\phi_0^1 = 2 \times 10^{-3}$, $\phi_0^2=0$, $\phi_0^3=0.45$, $\phi_0^4=0.2$ (yellow line); $\phi_0^1=0.2$, $\phi_0^2=0.26$, $\phi_0^3=0.5$, $\phi_0^4=0.6$ (green line); (b) $\phi_0^3 = \phi_0^4 = 0$ and $\phi_0^1=0.3$, $\phi_0^2=0.2$, $\phi_0^5=0.1$ (red line); $\phi_0^1=0.4$, $\phi_0^2=0.6$, $\phi_0^5=0.3$ (yellow line). The dashed lines correspond to the inflationary stage when $0 \leq N_e^* \leq 65$.

Scenario	ϕ_{in}^1	ϕ_{in}^2	ϕ_{in}^3	ϕ_{in}^4
A_1	0.0849	0.1019	0.1274	0.0425
A_2	0.0008	0	0.1725	0.0767
A_3	0.0446	0.0579	0.111	0.134
A_4	0.7984	0.8982	0.4990	0.6986

Table: Initial conditions (fields in units of M_{Pl}) at $N_e^* = 65$ in the scenarios of type A.

Scenario	ϕ_{in}^1	ϕ_{in}^2	ϕ_{in}^5
B_1	0.2367	0.1578	$1.2 \cdot 10^{-8}$
B_2	0.1578	0.2367	$-9.9 \cdot 10^{-21}$

Table: Initial conditions (fields in units of M_{Pl}) at $N_e^* = 65$ in the scenarios of type B.

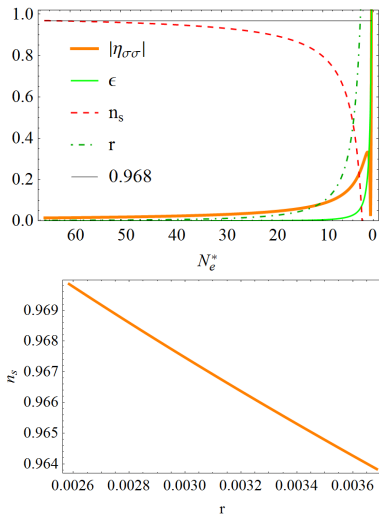


Figure: Evolution of the slow-roll parameters ϵ (light green line), $|\eta_{\sigma\sigma}|$ (dashed orange line), scalar-to-vector ratio r (green line) and spectral index n_s (red line) as a number of e-foldings before the end of inflation N_e^* ; (n_s, r) contour at $65 \leq N_e^* \leq 55$.

Scenario	$H [10^{-5}]$	r	n_s
A_1	2.99983	0.00266259	0.969398
A_2	2.99983	0.00266259	0.969398
A_3	2.99983	0.00266255	0.969399
A_4	187.444	0.00174899	0.969258
B_1	6.81778	0.00266322	0.969396
B_2	6.81778	0.00266325	0.969396

Table: The Hubble parameter H , tensor-to-scalar ratio r and spectral index n_s for successful inflationary scenarios at $N_e^* = 65$, $m_A=200$ GeV.

- For the Hubble parameter $H \sim 10^{-5} M_{Pl}$ the values of n_s and r coincide up to five and three digits, correspondingly. Such "attractor behavior" when over a wide range of initial conditions the system evolves along the same trajectory in the course of inflation is known for single-field models, but it is not an obvious observation, generally speaking, for multifield models.

- It has been shown in a large number of analyses ⁶ that there are several classes of the single-field inflationary models such that within a given class all models predict the same values of observable parameters n_s and r in the leading $1/N_e$ approximation. These classes are known as *cosmological attractors*.
- Similar analysis of two-field inflationary models has been made in ⁷.

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⁷D.I. Kaiser and E.I. Sfakianakis, *Phys. Rev. Lett.* **112** (2014) 011302 [1304.0363];
R. Kallosh and A. Linde, *J. Cosmol. Astropart. Phys.* **1312** (2013) 006 [1309.2015].

The Strong Coupling Approximation

- The idea of a cosmological attractor is based on an observation that the kinetic term in Jordan frame practically does not affect the slow-roll parameters if the "strong coupling regime" is respected during inflation. In the case of multifield models the field system is in the SC regime if the following inequality is respected:

$$\delta_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J \ll \frac{3}{f(\phi^K)} f_{,I} f_{,J} \partial_\mu \phi^I \partial_\nu \phi^J. \quad (17)$$

- In the SC approximation the initial action can be written as

$$S_E = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{g^{\mu\nu}}{2} \partial_\mu \Theta \partial_\nu \Theta - \frac{M_{Pl}^4 V}{4f^2} \right].$$

- The role of inflaton in the strong coupling approximation is performed by the "effective field"

$$\Theta = \sqrt{\frac{3}{2}} M_{Pl} \ln \left(\frac{f}{f_0} \right), \quad (18)$$

where f_0 is a positive constant with the same dimension as f .

- In terms of which the action S_E includes the standard kinetic term of Θ and does not include kinetic terms of any other scalar fields which can be interpreted as model parameters.
- This circumstance allows one to calculate the inflationary parameters in the SC approximation using the single-field model. If we adjust Θ in such a way that $\Theta = 0$ corresponds to $\Omega_0 = 0$ and $\Omega_\pm = 0$, then $f_0 = M_{Pl}^2/2$.

- In the scenario *A* we set $\phi^5 = h_v = 0$ during inflation, while in the scenario *B* one can observe that inflation starts when

$$h_v^2 \ll \sum_{l=1}^4 (\phi^l)^2.$$

- So in both scenarios we neglect h_v and write the potential in the form

$$V_{sc} = m_2^2 (\Omega_0^2 + \Omega_{\pm}^2) + \nu_1 (\Omega_0^2 + \Omega_{\pm}^2)^2.$$

- The function f is approximated by

$$f_{sc} = \frac{M_{Pl}^2}{2} + \frac{\xi_1}{2} (\Omega_{\pm}^2 + \Omega_0^2)$$

- Using approximated expressions for potential and non-minimal coupling function we can rewrite
- the Jordan frame potential in the form

$$V_{sc} = \frac{m_2^2}{\xi_1} (2f - M_{Pl}^2) + \frac{\nu_1}{\xi_1^2} (2f - M_{Pl}^2)^2, \quad (19)$$

- the Einstein frame potential can be written as follows

$$W_{sc} = \frac{M_{Pl}^4 (M_{Pl}^2 - 2f_{sc}) [(M_{Pl}^2 - 2f_{sc})\nu_1 - m_2^2\xi_1]}{4f_{sc}^2\xi_1^2}. \quad (20)$$

Using $m_2^2 \xi_1 \ll M_{Pl}^2 \nu_1$, from (20) we get

$$W_{sc} \simeq \frac{M_{Pl}^4 \nu_1}{\xi_1^2} \left(\frac{M_{Pl}^2}{2f_{sc}} - 1 \right)^2 = \frac{M_{Pl}^4 \nu_1}{\xi_1^2} \left(1 - \frac{M_{Pl}^2}{2f_0} e^{-\sqrt{6}\Theta/(3M_{Pl})} \right)^2.$$

The slow-roll parameters are

$$\epsilon = \frac{M_{Pl}^2}{2} \left(\frac{W'_\Theta}{W} \right)^2 = \frac{4}{3} \left(e^{\sqrt{6}\Theta/(3M_{Pl})} - 1 \right)^{-2},$$

$$\eta = M_{Pl}^2 \frac{W''_\Theta}{W} = \frac{4 \left(e^{\sqrt{6}\Theta/(3M_{Pl})} - 2 \right)}{3 \left(e^{\sqrt{6}\Theta/(3M_{Pl})} - 1 \right)^2}.$$

With these analytic expressions for the slow-roll parameters in the SC approximation the inflationary parameters can be easily calculated.

It is convenient to express the inflationary parameters as a functions of f_{sc}

$$n_s = 1 - \frac{8M_{Pl}^2 (M_{Pl}^2 + 2f_{sc})}{3(M_{Pl}^2 - 2f_{sc})^2}, \quad r = \frac{64M_{Pl}^4}{3(M_{Pl}^2 - 2f_{sc})^2}. \quad (21)$$

- The values of inflationary parameters r and n_s , calculated Eq. (21), are close to the parameter values that have been found numerically.

Scenario	f_{in}/M_{Pl}^2	r	n_s
A_1	43.35	0.0029	0.9682
A_2	44.83	0.0027	0.9692
A_3	44.94	0.0027	0.9693
A_4	44.13	0.0028	0.9687
B_1	45.12	0.0027	0.9695
B_2	45.02	0.0027	0.9694

Table: The inflationary parameters in the strong coupling approximation calculated at $h_v = 0$, $m_Z = 0$ and $m_A = 0$.

Conclusions

- 1 We analyze the inflationary scenarios which could be induced by the two-Higgs-doublet potential of the Minimal Supersymmetric Standard Model (MSSM), where five scalar fields have non-minimal couplings to gravity.
- 2 Observables following from such MSSM-inspired multifield inflation are calculated and a number of consistent inflationary scenarios are constructed.
- 3 Cosmological evolution with different initial conditions for the multifield system leads to consequences fully compatible with observational data on the spectral index and the tensor-to-scalar ratio.
- 4 It is demonstrated that the strong coupling approximation is precise enough to describe such inflationary scenarios.

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Thank for your attention