AdS$_2$ Holography

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Based on works with

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JT model

a.k.a Barbashov-Nesterenko-Chervyakov, Almheiri-Polchinski...
In the 2nd order formalism the action reads

$$\Gamma = -\frac{1}{2\kappa^2}\int_{\mathcal{M}} d^2x\sqrt{g}\,X\,(R+2),$$

where X is the dilaton. Classical solutions are (Euclidean) AdS_2 geometries. By introducing an independent connection one-form ω and two auxiliary fields X^a the action may be transformed into the 1st order form

$$I_{\mathrm{JT}} = -\frac{k}{2\pi}\int\left(X^a(\mathrm{d}e_a + \epsilon_a{}^b\omega\wedge e_b) + X\mathrm{d}\omega + \tfrac{1}{2}X\epsilon^{ab}e_a\wedge e_b\right)$$

e_a is the zweibein, ϵ is the Levi-Civita tensor.

Let $L_0, L_\pm$ be the $\mathfrak{sl}(2)$ generators

$$[L_I, L_J] = (I - J)L_{I+J} \qquad I, J = +1, -1, 0$$

and let $\mathbf{X} = X^I L_I$, $A = A_I L^I$. Then the 1st order JT action may be written as

$$I = \frac{k}{\pi} \int_{\mathcal{M}} \mathrm{tr}(\mathbf{X}(\mathrm{d}A + A \wedge A))$$

which is just a 2D BF action.
All classical solutions of the JT model (and, in fact, of any 2D dilaton gravity) can be subdivided into two categories:

- "Constant dilaton solutions": the dilaton X is a constant defined through the action, $X^{\pm} = 0$. These are isolated points in the space of solutions.
- "Linear dilaton solutions": the dilaton X varies (and thus may be taken as on of the coordinates). These solutions have much more freedom.

These two families of the solutions behave very differently from the holographic prospective.

The idea: The JT model has no local physical degrees of freedom, meaning that all degrees of freedom are pure gauge. In the presence of boundaries, some gauge symmetries are broken. Thus some degrees of freedom become physical (=asymptotic states). They transform under some symmetry group (former gauge transformations) that is called the asymptotic symmetry group.

To implement this idea one has to find a set of the asymptotic conditions and a boundary action that (i) ensure a consistent variation principle with a finite on-shell action, (ii) give an interesting asymptotic symmetry. All at the same moment. Hopeless.

To make life more simple ....

... there are boundary charges. They are

- computed from function derivatives of the bulk constraints.
- On the physical asymptotic states they have to be finite and non-constant.

(Boundary action may be postponed)
In the JT gravity this method yields that the asymptotic states are defined by the **leading terms in the asymptotic expansion** of X^I (in contrast to higher dimensional gravities where the leading terms do not fluctuate).
Here comes our first result: constant dilaton backgrounds give a trivial holography.

Asymptotic conditions

Let φ be the coordinate on the asymptotic boundary, and $\rho \to \infty$ be the asymptotic region. Let us partially fix the gauge to $A_\rho = 0$ and impose the condition that the rest of the field tend to some constant values

$$A_\varphi|_{\partial\mathcal{M}} \equiv a(\varphi) = L_+\mathcal{L}^+(\varphi) + L_0\mathcal{L}^0(\varphi) + L_-\mathcal{L}^-(\varphi)$$
$$\mathbf{X}|_{\partial\mathcal{M}} \equiv x(\varphi) = L_+\mathcal{X}^+(\varphi) + L_0\mathcal{X}^0(\varphi) + L_-\mathcal{X}^-(\varphi)$$

Then the AdS_2 asymptotic conditions are obtained by a transformation

$$A = b(\rho)^{-1}\left(\mathrm{d} + a(\varphi)\mathrm{d}\varphi\right) b(\rho)$$
$$\mathbf{X} = b(\rho)^{-1}\, x(\varphi)\, b(\rho)$$

A convenient choice is

$$b = e^{\rho L_0}$$

The asymptotic symmetry consists of (former) gauge transformations that move the asymptotic conditions onto themselves. It can be analyzed in two possible ways:
- by making the full canonical analysis (hard) or
- by analyzing the representations of the symmetry (easier).
There are the following choices for the asymptotic conditions:

(i) All $\mathcal{X}^I$ and $\mathcal{L}^I$ are allowed to fluctuate. The symmetry algebra is the current algebra $\widehat{\mathfrak{sl}(2)}_0$.

(ii) Restrict $\mathcal{L}_0 = 0$. The resulting symmetry is warped conformal algebra (Virasoro and a $\widehat{u(1)}$ current algebra).

(iii) Restrict in addition $\mathcal{L}^+ = 1/2$. The resulting algebra is Virasoro (the only case studied so far in the literature).

(iv) Take $\mathcal{L}^+ = 1$ and $\mathcal{L}^- = 0$ but allow $\mathcal{L}^0$ to fluctuate. This gives the $\hat{u}_k(1)$ algebra.

which make the variational problem in the 2nd order formalism consistent reads:

$$\Gamma = -\frac{1}{2\kappa^2}\int_{\mathcal{M}} d^2x\sqrt{g}\,X\,(R+2) - \frac{1}{\kappa^2}\int_{\partial\mathcal{M}} dx\,\sqrt{\gamma}\,X\,K + \frac{1}{\kappa^2}\int_{\partial\mathcal{M}} dx\,\sqrt{\gamma}\,\left(\sqrt{X^2+c_0} + \frac{1}{2\,X}\,\gamma^{\mu\nu}\,\partial_\mu X\partial_\nu X\right)\ ,$$

In the 1st order formalism it is even uglier.

Holographic duals in 1D

By using the boundary action, one can figure out the holographic duals. They depend on the boundary conditions.
In the case (iii) - Virasoro asymptotic symmetry - depending on a particular parametrization of the states, one gets:

- The de Alfaro-Fibini-Furlan conformal quantum mechanics.
- The finite-temperature Schwarzian action, which is a limiting case of the Sachdev-Ye-Kitaev (SYK) model (cf. Maldacena-Stanford-Yang).

Other results

Entropy: a consistency check. The computations of black hole entropy by using the Wald-like approach (Entropy $\propto$ the value of X at the horizon) is consistent with the Cardy formula for warped conformal algebras.

Higher spins. By replacing $sl(2)$ by $sl(3)$ one gets a theory, which may be interpreted as a spin-3 in AdS_2. For a particular choice of the asymptotic conditions one get a W_3 asymptotic symmetry. What else can one get here?

One-loop partition function. It is known to be trivial on constant-dilaton backgrounds. What is it on linear-dilaton backgrounds?

Beyond JT. There are infinitely many 2D dilaton gravities describing asymptotically AdS spaces. Which asymptotic symmetries and holographic duals do they have? Problem: these theories are no longer BF gauge theories.

THANK YOU!