Higher derivatives invariants in 6D $\mathcal{N} = (1, 0)$ supergravity

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Based on:

Butter, Novak & GTM, JHEP 1705 (2017) 133; arXiv:1701.08163

Novak, Ozkan, Pang & GTM arXiv:1706.09330;

Butter, Novak, Ozkan, Pang & GTM *to appear*

see also:

Butter, Kuzenko, Novak & Theisen JHEP 1612 (2016) 072; arXiv:1606.02921

Outline

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5. 6D $\mathcal{N} = (1,0)$ conformal supergravity actions
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7. Conclusion and Outlook

Why higher-derivatives invariants?

- String theory effective action: modified supergravity (SUGRA) by an infinite series of higher derivative quantum corrections

$$L^{low}_{string} = L_{SG} + \sum [\mathcal{D}^p \mathcal{R}^q_{\dots}] + forms + susy\ \ completion$$

SUSY higher-derivatives terms are poorly understood but, e. g.:

- important for phenomenological applications of string theory, see compactifications with fluxes.
- important for black-hole physics within string theory.
  indeed for computing higher-order corrections to black-hole entropy needed for precision tests of AdS/CFT in SUSY (treatable) cases.
- Counterterms for UV divergencies in SUGRA
  see open debate on finiteness in, e.g., 4D $\mathcal{N} = 8$ sugra
  [Stelle, Howe, Kallosh, Bern, ...]

even the simple SUSY $\mathcal{R}^2$ case is not fully understood in general (see for example 6D) but

$\mathcal{R}^2$? in 4D

For instance $\mathcal{R}^2$ gravity attracted attention for over 50 years:

- renormalization of QFT in curved spacetime requires counterterms containing $\mathcal{R}^2$ [Utiyama & DeWitt ('62)]
- In 4D, $\mathcal{R}^2$ terms govern the structure of QFT conformal anomalies relevant in studying renormalization group flows, see 4D *a*-theorem [Komargodski and Schwimmer ('11)]
- Renormalizable (not unitary) $\alpha(\mathcal{C}_{abcd})^2 + \beta(\mathcal{R}_{ab})^2 + \gamma\mathcal{R}^2$, [Stelle ('77)]
- $\mathcal{R} + \mathcal{R}^2$ Starobinsky model of inflation [Starobinsky ('80)]
  Interestingly, $\mathcal{R} + \mathcal{R}^2$ SUGRA models are promising inflationary candidates for CMB data.

Gauss-Bonnet: $\mathcal{R}_{abcd}\mathcal{R}^{abcd} - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}^2$

An interesting curvature squared combination is Gauss-Bonnet

- In 4D it is a topological term (Euler characteristic) arising as the Type A conformal anomaly.
- Governs α'-corrections in compactified string theory [Zweibach (85)]......
- In D>4 it is involved in the definition of ghost free critical gravities
- In general its structure for any space-time dimensions and amount of susy is not known. In particular, the dependence upon the extra sugra matter fields, see the dilaton σ and NSNS b_2 2-form

In 4D [Butter-deWit-Kuzenko-Lodato (13)] and 5D [Ozkan-Pang (13)] the Gauss-Bonnet was constructed off-shell.

In 6D a full classification of the $\mathcal{R}^2$ invariants is missing and in particular the GB invariant has never been fully constructed. We will fill this gap

$\mathcal{D}^p\mathcal{R}^q$ with $p + q \geq 2$?

How about interesting invariants beyond $\mathcal{R}^2$?

$\Downarrow$

six derivatives $\propto \mathcal{R}^3 + \mathcal{D}^2\mathcal{R}^2$

6D conformal supergravity invariants were not known

- In 6D they play a special role parametrizing conformal anomalies
- of importance in studying renormalization group flows and dilaton effective action of 6D QFT;
- Structure of anomalies can help understanding mysterious (1,0) and (2,0) 6D CFT as for instance in the context of AdS/CFT, M5-branes, ...

how higher-derivatives SUGRA?

Once convinced about the importance of higher derivative supergravity the question is: how to efficiently construct them?

- Best approach would be to possess a formalism that guarantees manifest supersymmetry in a model independent way

  $$\Downarrow$$

  An off-shell approach to SUGRA, when available, can be used for general supergravity-matter couplings with model independent susy.

- Two possibilities:
  − component fields superconformal tensor calculus
  See "Supergravity" book by [Freedman & Van Proeyen (12)]
  − superspace approaches See classic books [Gates, Grisaru, Roček, Siegel (83)], [Wess-Bagger (92)], [Buchbinder, Kuzenko (98)], [Galperin, Ivanov, Ogievetsky, Sokatchev (2001)]

how off-shell SUGRA?

The two approaches can be linked and powerfully used together through conformal superspace

- manifestly gauge entire superconformal algebra in superspace [Kugo-Uehara (85)] and combine advantages of both approaches
- Constructed first by Butter 4D $\mathcal{N} = 1$ in 2009 and $\mathcal{N} = 2$ in 2011
- developed and extended to 3D $\mathcal{N} - extended$ and 5D $\mathcal{N} = 1$ SUGRA [Butter-Kuzenko-Novak-GTM ('13), ('14)]
- recently 6D $\mathcal{N} = (1, 0)$ [Butter-Kuzenko-Novak-Theisen ('16)] see also [Butter-Novak-GTM ('17)]

how higher derivatives off-shell SUGRA? outline

In superspace one efficiently:

- Describes off-shell supermultiplets, SUGRA, matter
- Provides manifestly supersymmetric off-shell action principles
- powerful cohomological "superform" techniques to construct and classify SUSY invariants including their component reduction.
- reduces to components and derives superconformal tensor calculus

With these techniques, one can in principle have a systematic approach for higher derivative off-shell invariants.

Examples:

- 6D $\mathcal{N} = (1,0)$ (four-derivatives) curvature squared terms;
- 6D $\mathcal{N} = (1,0)$ (six-derivatives) conformal supergravity invariants and informations about $\mathcal{N} = (2,0)$ case.

Review: standard (1,0) Poincaré Supergravity

An interlude: Conformal gravity

- Conformal gravity in six dimensions may be viewed as gauging the entire conformal group SO(6,2), $X_{\underline{a}} = \{P_a, M_{ab}, \mathbb{D}, K_a\}$
- The vielbein $e_a{}^m$ is associated with P_a (diff.=local-translations) gauge connections are associated with the other generators which can be used to construct covariant derivatives

$$\nabla_a = e_a{}^m \partial_m - \frac{1}{2}\omega_a{}^{bc} M_{bc} - b_a \mathbb{D} - f_a{}^b K_b$$

- The covariant derivative algebra is constrained to be expressed entirely in terms of the Weyl tensor C_{abcd}

$$K_f C_{abcd} = 0 \ , \quad C_{abcd} = C_{[ab][cd]} \ , \quad \eta^{ac} C_{abcd} = 0 \ , \quad C_{[abc]d} = 0$$

- $\omega_a{}^{bc}$ and $f_a{}^b$ are composite function of $e_a{}^m$ while b_a is pure gauge

An interlude: Poincaré gravity

So far conformal gravity with the full conformal group gauged.

- Poincaré gravity: conformal gravity coupled to a dilaton field ϕ transforming under dilatation as $\phi' = \mathrm{e}^{2\tau}\phi$.
- choose a gauge in which $b_a = 0$ and $\phi = 1$:
  standard gravity invariant only under diffeomorphisms and Lorentz.
- For example, the Einstein-Hilbert term:
  action for a conformally coupled scalar compensator

$$I = \int \mathrm{d}^6x\, e\, \phi \nabla^a \nabla_a \phi \,, \quad \textit{if}\;\; b_a = 0,\; \phi = 1 \;\Longrightarrow\quad I_{EH} \propto \int \mathrm{d}^6x\, e\, \mathcal{R}$$

- For example, scalar curvature squared:

$$I = \int \mathrm{d}^6x\, e\, \phi^{-1} (\nabla^a \nabla_a \phi)^2 \,, \quad \textit{if}\;\; b_a = 0,\; \phi = 1 \;\Longrightarrow\quad I_{\mathcal{R}^2} \propto \int \mathrm{d}^6x\, e\, \mathcal{R}^2$$

Analogously, a natural way to describe the multiplets of off-shell Poincaré supergravity, and general supergravity-matter couplings, is to couple off-shell conformal SUGRA to compensators see, e.g.:

superconformal tensor calculus

The standard Weyl multiplet of (1,0) conformal SUGRA

Multiplet of local off-shell gauging of $OSp(6,2|1)$, the $\mathcal{N} = (1,0)$ superconformal group in 6D. [Bergshoeff-Sezgin-VanProeyen (86)]
off-shell physical multiplet composed by independent gauge fields

- vielbein $e_m{}^a$;
- the gravitino $\psi_m{}_i^\alpha$;
- SU(2) gauge field $\mathcal{V}_m{}^{ij}$;
- a dilatation gauge field b_m;

and a set of covariant "auxiliary/matter" fields (to close algebra off-shell)

- real anti-self-dual tensor T^-_{abc};
- a chiral fermion $\chi^{\alpha i}$;
- a real scalar field D.

How is this described in superspace?

6D conformal supergravity in conformal superspace

[Butter-Kuzenko-Novak-Theisen (16)]

Take a $\mathcal{N} = (1,0)$ curved superspace $\mathcal{M}^{6|8}$ parametrised by coordinates

$$z^M = (x^m, \theta^\mu_{\mathtt{i}}) , \qquad m = 0,1,2,3,4,5 , \quad \mu = 1,2,3,4 , \quad \mathtt{i} = \underline{1}, \underline{2}$$

Choose the structure group X to contain
$\mathrm{SO}(5,1) + \mathrm{SU}(2)$+(Dilatations)+($S-$susy) + ($K-$boosts).
The superspace covariant derivatives are

$$\nabla_A = E_A{}^M \partial_M - \omega_A{}^{\underline{b}} X_{\underline{b}} = E_A{}^M \partial_M - \frac{1}{2}\Omega_A{}^{ab} M_{ab} - \Phi_A{}^{ij} J_{ij} - B_A \mathbb{D} - \mathfrak{F}_{AB} K^B$$

- $E_A{}^M(z)$ supervielbein associated with $P_A = (P_a, Q^i_\alpha)$, $\partial_M = \partial/\partial z^M$,
- $\Omega_A{}^{cd}(z)$ Lorentz connection,
- $\Phi_A(z)$ SU(2)-connection, - B_A dilatation connection
- $\mathfrak{F}_{AB}$ special superconformal connection, $K^A = (K^a, S^\alpha_i)$

- conformal SUGRA local gauge transformations:

$$\mathcal{K} := \xi^A \nabla_A + \frac{1}{2}\Lambda^{bc} M_{bc} + \Lambda^{ij} J_{ij} + \tau \mathbb{D} + \Lambda_A K^A , \qquad \delta_{\mathcal{K}} \nabla_A = [\mathcal{K}, \nabla_A]$$

6D conformal supergravity in conformal superspace

One constrains the algebra $[\nabla_A, \nabla_B\}$ to be completely determined in terms of the super-Weyl tensor:

$$W^{\alpha\beta} = (\tilde{\gamma}^{abc})^{\alpha\beta} W_{abc}$$

- $W^{\alpha\beta}$ is a dimension-1 primary superfield

$$K^A W^{\beta\gamma} = 0 \ , \quad \mathbb{D} W^{\alpha\beta} = W^{\alpha\beta}$$

  Jacobi/Bianchi Identities impose differential constraints on $W^{\alpha\beta}$

- The standard Weyl multiplet of $6D$ $\mathcal{N} = (1,0)$ conformal supergravity is encoded in the superspace geometry.
  Component fields, identified as $\theta = 0$ projections of the superspace one-forms and descendants of $W^{\alpha\beta}$ [Butter-Novak-GTM (17)], e. g.:

$$T^-_{abc} := -2W_{abc}| \ , \qquad \chi^{\alpha i} := -\frac{3\mathrm{i}}{4}\nabla^i_\beta W^{\alpha\beta}| \ , \qquad D := -\frac{3\mathrm{i}}{16}\nabla^k_\alpha \nabla_{\beta k} W^{\alpha\beta}|$$

The tensor multiplet and dilaton-Weyl multiplet

So far we have considered only the standard Weyl multiplet which possesses the covariant component fields: T^-_{abc}, $\chi^{\alpha i}$ and D

A variant representation of the off-shell conformal supergravity multiplet:

- The dilaton-Weyl multiplet is obtained by coupling the standard Weyl multiplet to a (on-shell) tensor multiplet with scalar field Φ
- Φ may be described by introducing a gauge (NSNS) two-form B_2 in superspace. Its field strength is the super 3-form $H_3 = \mathrm{d}B_2$

$$\begin{aligned} H_{\alpha}^{i}{}_{\beta}^{j}{}_{\gamma}^{k} &= 0 \ , \quad H_{a}{}_{\alpha}^{i}{}_{\beta}^{j} = 2\mathrm{i}\varepsilon^{ij}(\gamma_a)_{\alpha\beta}\Phi \ , \quad H_{ab}{}_{\alpha}^{i} = (\gamma_{ab})_\alpha{}^\beta \nabla_\beta^i \Phi \ , \\ H_{abc} &= -\frac{\mathrm{i}}{8}(\tilde\gamma_{abc})^{\gamma\delta}\nabla_\gamma^k \nabla_{\delta k}\Phi - 4W_{abc}\Phi \ , \end{aligned}$$

  where Φ is primary, $\mathbb{D}\Phi = 2\Phi$, satisfying $\nabla_\alpha^{(i}\nabla_\beta^{j)}\Phi = 0$

- One can express $W_{abc} = -\frac{1}{4}H_{abc} - \frac{\mathrm{i}}{32}(\tilde\gamma_{abc})^{\gamma\delta}\nabla_\gamma^k\nabla_{\delta k}\Phi$ and then ($\sigma := \Phi|$):

$$T^-_{abc} = \frac{1}{2\sigma}H^-_{abc} \ , \quad D = \frac{15}{4\sigma}(\hat\nabla^a\hat\nabla_a\sigma + \frac{1}{3}T^{-abc}H_{abc}) + \text{fermion terms}$$

- T^-_{abc}, $\chi^{\alpha i}$ and D are exchanged with σ, ψ_α^i and b_{mn} ($H_{abc} \simeq 3\nabla_{[a}b_{bc]}$)

Action principles in six dimensions

So far multiplets, geometry and kinematics; how action principles?

- We use the superform approach to engineer invariant actions from closed super six-forms
- This approach has been rediscovered a number of times: [Hasler (1996)]; "Ectoplasm" [Gates-Grisaru-Knutt–Wehlau-Siegel (1997)]; Rheonomic approach [Castellani-D'Auria-Fré (book-1991)]
- The approach has been developed and used in e.g. the study of the properties of UV counterterms in maximally supersymmetric Yang-Mills theories [Bossard-Howe-Stelle ('09, '10, '13)] and $N = 4$ supergravity theories [Bossard-Howe-Lindström-Stelle-Wulff ('11)] ...
- Since 2009 it has been employed and developed also by Butter, Kuzenko, Novak, GTM to construct off-shell higher derivative invariants

Superform approach to constructing actions I

- In 6D, take a closed super 6-form $J = \frac{1}{6!}\mathrm{d}z^{M_6} \wedge \cdots \wedge \mathrm{d}z^{M_1} J_{M_1 \cdots M_6}$,

$$\mathrm{d}J = \frac{1}{6!}\mathrm{d}z^{M_6} \wedge \cdots \wedge \mathrm{d}z^{M_0} \partial_{M_0} J_{M_1 \cdots M_6} = 0$$

- Action principle:

$$S = \int \mathrm{d}^6 x \, {}^*J|_{\theta=0} \ , \qquad {}^*J = \frac{1}{6!}\varepsilon^{mnpqrs} J_{mnpqrs}$$

Under a superdiffeomorphism with $\xi = \xi^A E_A = \xi^M \partial_M$,

$$\delta_\xi J = \mathcal{L}_\xi J \equiv i_\xi \mathrm{d}J + \mathrm{d} i_\xi J = \mathrm{d} i_\xi J \ .$$

- We also require the action to be invariant under the the structure group X and any additional gauge transformations
- This means that J should transform by (at most) an exact form under these transformations

$$\delta_X J = \mathrm{d}\Xi \ , \qquad \text{for some } 5\text{−form } \ \Xi$$

Superform approach to constructing actions II

- Expressing the action in terms of the tangent frame and by using definition of the gravitini $\psi_m{}_i^\alpha(x) := 2\, E_m{}_i^\alpha(z)|$

$$
\begin{aligned}
S &= \int \mathrm{d}^6 x\ \frac{1}{6!}\varepsilon^{m_1\cdots m_6} E_{m_6}{}^{A_6}\cdots E_{m_1}{}^{A_1} J_{A_1\cdots A_6}|_{\theta=0}\ , \\
&\propto \int \mathrm{d}^6 x\, \mathrm{e}\, \varepsilon^{a_1\cdots a_6}\Big[J_{a_1\cdots a_6} + 3\psi_{a_1}{}_i^\alpha J^i_{\alpha\, a_2\cdots a_6} + \frac{15}{4}\psi_{a_2}{}_j^\beta \psi_{a_1}{}_i^\alpha J^{i\,j}_{\alpha\,\beta\, a_3\cdots a_6} \\
&\quad + \frac{5}{2}\psi_{a_3}{}_k^\gamma \psi_{a_2}{}_j^\beta \psi_{a_1}{}_i^\alpha J^{i\,j\,k}_{\alpha\,\beta\,\gamma\, a_4 a_5 a_6} + \mathcal{O}(\psi^4)\Big]|_{\theta=0}
\end{aligned}
$$

Natural way to compute invariants reduced to component fields

the action S is invariant under the full local supergravity gauge transformations $\mathcal{K}$ and any additional gauge transformations

Classifying closed super 6-forms J
one classifies supersymmetric invariants

(1,0) Poincaré EH $\mathcal{R}$ action: $B_4 \wedge F_2$ action principle

$\mathcal{N} = (1,0)$ Einstein-Hilbert $\mathcal{R}$ term? [Bergshoeff-Sezgin-VanProeyen ('86)]

- Use the Dilaton-Weyl multiplet
- consider a "Linear-multiplet" conformal compensator based on a scalar isotriplet G^{ij}, $\nabla_\alpha^{(i}G^{jk)} = 0$ formulated as a closed super 5-form

$$H_5 = \mathrm{d}B_4, \qquad H_{abc}{}_{\alpha}^{i}{}_{\beta}^{j} = -2\mathrm{i}(\gamma_{abc})_{\alpha\beta}G^{ij}$$

- Construct an invariant action based on the closed 6-form J

$$J = B_4 \wedge F_2 - \Sigma \ , \qquad \mathrm{d}\Sigma = -F_2 \wedge H_5$$

F_2, $\mathrm{d}F_2 = 0$ a vector multiplet 2-form based on a superfield $W^{\alpha i}$ and Σ covariant 6-form (constructed from G^{ij}, $W^{\alpha i}$ and descendants)

$\Longrightarrow$ $B_4 \wedge F_2$ action principle:

$$S_{B_4\wedge F_2} = \frac{1}{2}\int \mathrm{d}^6x\, e\left(\frac{1}{4!}\varepsilon^{abcdef} f_{ab} b_{cdef} + X^{ij}G_{ij} + \text{fermions}\right)$$

X^{ij} is a scalar component of the vector multiplet $X^{ij} := \frac{\mathrm{i}}{4}\nabla_\gamma^{(i}W^{\gamma j)}|$

(1,0) Poincaré EH $\mathcal{R}$ action

- Consider a composite vector multiplet of the linear multiplet

$$\begin{aligned}\mathbb{W}^{\alpha i} \quad = \quad & \frac{1}{G}\nabla^{\alpha\beta}\chi^i_\beta + \frac{4}{G}(W^{\alpha\beta}\chi^i_\beta + 10\mathrm{i}X^\alpha_j G^{ji}) - \frac{1}{2G^3}G_{jk}(\nabla^{\alpha\beta}G^{ij})\chi^k_\beta \\ & + \frac{1}{2G^3}G^{ij}E^{\alpha\beta}\chi_{\beta j} + \frac{\mathrm{i}}{16G^5}\varepsilon^{\alpha\beta\gamma\delta}\chi_{\beta j}\chi_{\gamma k}\chi_{\delta l}G^{ij}G^{kl}\end{aligned}$$

  where

$$\nabla^{(i}_\alpha G^{jk)} = 0\ , \quad \chi^i_\alpha = \frac{2}{3}\nabla_{\alpha j}G^{ij}\ , \quad E^{\alpha\beta} = \frac{\mathrm{i}}{8}\varepsilon^{\alpha\beta\gamma\delta}\nabla^k_{[\gamma}\chi_{\delta]k}\ ; \quad X^{\alpha i} := -\frac{\mathrm{i}}{10}\nabla^i_\beta W^{\alpha\beta}$$

- Plug it back in the vector multiplet part of the $B_4 \wedge F_2$ action and, after gauge fixing, you get EH Poincaré SUGRA action

$$S_{EH} = -\frac{1}{2}\int \mathrm{d}^6x\, e\,\mathcal{R} + \cdots$$

(1,0) SUGRA curvature squared invariants?

Novak, Ozkan, Pang & GTM arXiv:1706.09330;
Butter, Novak, Ozkan, Pang & GTM *to appear*

SUSY $6D$ $\mathcal{R}^2$?

supersymmetric extensions of general curvature squared Lagrangian?

$$\mathcal{L}_{R^2} \propto a\,\mathcal{C}^{abcd}\mathcal{C}_{abcd} + b\,\mathcal{R}^{abcd}\mathcal{R}_{abcd} + c\,\mathcal{R}^2 + \cdots$$

- Weyl tensor: $C_{ab}{}^{cd} = \mathcal{R}_{ab}{}^{cd} - \delta_{[a}{}^{[c}\mathcal{R}_{b]}{}^{d]} + \frac{1}{10}\delta_{[a}{}^{[c}\delta_{b]}{}^{d]}\mathcal{R}$
  with $\mathcal{R}_{ab}{}^{cd}$ component Riemann tensor
- Ricci tensor: $\mathcal{R}_a{}^b := \mathcal{R}_{ad}{}^{bd}$
- Ricci scalar: $\mathcal{R} := \mathcal{R}_a{}^a$

All based on a new action principle
[Butter-Novak-Kuzenko-Theisen (16)]
[Novak-Ozkan-Pang-GTM (17)]

A new $B_2 \wedge H_4$ action principle

It turns out that we can construct all invariants by using an action corresponding to the supersymmetrization of $B_2 \wedge H_4$

- 2-form B_2 of tensor multiplet with $H_3 = \mathrm{d}B_2$
- H_4 a closed 4-form $\mathrm{d}H_4 = 0$ based on $B_a{}^{ij} = B_a{}^{(ij)}$

$$H^{i\,j\,k\,l}_{\alpha\,\beta\,\gamma\,\delta} = H_a{}^{j\,k\,l}_{\beta\,\gamma\,\delta} = 0 \ , \quad H_{ab}{}^{k\,l}_{\gamma\,\delta} = \mathrm{i}(\gamma_{abc})_{\gamma\delta} B^{c\,kl}$$

$$H_{abc}{}^{l}_{\delta} = -\tfrac{1}{12}\varepsilon_{abcdef}(\gamma^{de})_\delta{}^\rho \nabla_{\rho p} B^{f\,lp} \ , \quad H_{abcd} = \tfrac{\mathrm{i}}{48}\varepsilon_{abcdef}(\tilde\gamma^e)^{\alpha\beta}\nabla_{\alpha k}\nabla_{\beta l} B^{f\,kl}$$

where $B^{\alpha\beta\,ij} = (\tilde\gamma^a)^{\alpha\beta} B_a{}^{ij}$ is a dimension 3 primary superfield

$$\nabla^{(i}_\alpha B^{\beta\gamma jk)} = -\frac{2}{3}\delta^{[\beta}_\alpha \nabla^{(i}_\delta B^{\gamma]jk)} \ , \quad [\nabla^{(i}_\alpha, \nabla_{\beta k}] B^{\alpha\beta j)k} = -8\mathrm{i}\nabla_{\alpha\beta} B^{\alpha\beta ij}$$

- Construct an invariant action based on the closed 6-form J

$$J = B_2 \wedge H_4 - \Sigma' \ , \qquad \mathrm{d}\Sigma' = H_3 \wedge H_4$$

Σ' is covariant constructed only from field strengths of H_3 and H_4
$\Longrightarrow$ locally superconformal invariant action principle:

$$S_{B_2\wedge H_4} = \int \mathrm{d}^6x\, e\left\{\frac{1}{4}(b_{ab} - \eta_{ab}\sigma)\,C^{ab} + \text{fermions}\right\} \ , \quad C_{ab} := \frac{\mathrm{i}}{12}(\tilde\gamma_a)^{\alpha\beta}\nabla_{\alpha k}\nabla_{\beta l} B_b{}^{kl}|$$

Riemann^2

It was first constructed by [Bergshoeff-Rakowski (87)].
We can reproduce it by using the $B_2 \wedge H_4$ action principle with

$$B^{\alpha\beta ij} = -\frac{\mathrm{i}}{2}\Lambda^{\alpha(i}{}_{\gamma}{}^{\delta}\Lambda^{\beta j)}{}_{\delta}{}^{\gamma}$$

and the primary

$$\begin{aligned}\Lambda^{\alpha i}{}_{\beta}{}^{\gamma} \quad = \quad & X^{i\,\alpha\gamma}_{\beta} - \frac{1}{3}\delta^{\alpha}_{\beta}X^{\gamma i} + \frac{1}{12}\delta^{\gamma}_{\beta}X^{\alpha i} + \frac{\mathrm{i}}{4}\Phi^{-1}\psi^{i}_{\beta}W^{\alpha\gamma} + \frac{\mathrm{i}}{12}\Phi^{-1}\delta^{\alpha}_{\beta}W^{\gamma\delta}\psi^{i}_{\delta} \\ & - \frac{\mathrm{i}}{12}\Phi^{-1}\delta^{\gamma}_{\beta}W^{\alpha\delta}\psi^{i}_{\delta} + \frac{\mathrm{i}}{12}\varepsilon^{\alpha\gamma\delta\rho}\Phi^{-1}\nabla_{\delta(\rho}\psi^{i}_{\beta)} - \frac{\mathrm{i}}{8}\varepsilon^{\alpha\gamma\delta\rho}\Phi^{-2}(\nabla_{\delta(\rho}\Phi)\psi^{i}_{\beta)} \\ & + \frac{\mathrm{i}}{32}\varepsilon^{\alpha\gamma\delta\rho}\Phi^{-2}H_{\rho\beta}\psi^{i}_{\delta} - \frac{1}{16}\varepsilon^{\alpha\gamma\delta\rho}\Phi^{-3}\psi^{i}_{\delta}\psi^{k}_{(\rho}\psi_{\beta)k}\end{aligned}$$

where

$$X^{\alpha i} := -\tfrac{\mathrm{i}}{10}\nabla^{i}_{\beta}W^{\alpha\beta}\,, \quad X^{k\,\alpha\beta}_{\gamma} = -\tfrac{\mathrm{i}}{4}\nabla^{k}_{\gamma}W^{\alpha\beta} - \delta^{(\alpha}_{\gamma}X^{\beta)k}$$

$$\psi^{i}_{\alpha} = \nabla^{i}_{\alpha}\Phi\,, \quad \nabla^{i}_{\alpha}\psi^{j}_{\beta} = -\tfrac{\mathrm{i}}{2}\varepsilon^{ij}(\gamma^{abc})_{\alpha\beta}H^{+}_{abc} - \mathrm{i}\varepsilon^{ij}(\gamma^{a})_{\alpha\beta}\nabla_{a}\Phi$$

Then, in the gauge $\sigma = 1$, $b_m = 0$

$$\mathcal{L}_{\mathrm{Riem}^2} = \mathcal{R}^{abcd}\mathcal{R}_{abcd} + \mathcal{O}(b_{ab}) + \text{fermions}$$

scalar^2

A $\mathcal{R}^2$ invariant was constructed in components by [Ozkan (13)] by using results of [Bergshoeff-Sezgin-VanProeyen (86)]

We can reproduce it from superspace by using the $B_2 \wedge H_4$ action and

$$B^{\alpha\beta ij} = -\frac{\mathrm{i}}{2}\mathbb{W}^{\alpha(i}\mathbb{W}^{\beta j)}$$

with $\mathbb{W}^{\alpha i}$ the same composite vector multiplet used for the EH term.

A new curvature squared invariant

A new curvature squared invariant by using the $B_2 \wedge H_4$ action and the superfield [Butter-Kuzenko-Novak-Theisen (16)] ($Y_\alpha{}^{\beta ij} = -5/2 \nabla_\alpha^{(i} X^{\beta j)}$)

$$B^{\alpha\beta\, ij} = -4 W^{\gamma[\alpha} Y_\gamma{}^{\beta] ij} - 32\mathrm{i}\, X_\gamma{}^{\alpha\delta(i} X_\delta{}^{\beta\gamma j)} + 10\mathrm{i}\, X^{\alpha(i} X^{\beta j)}$$

this leads to a new independent off-shell $\mathcal{R}^2$ invariant [Novak-Ozkan-Pang-GTM (17)]

$$\begin{aligned} S_{\rm new} \;=\; & \frac{1}{32}\int \mathrm{d}^6 x\, e \Big\{ \sigma C_{ab}{}^{cd} C_{cd}{}^{ab} + 3\sigma \mathcal{R}_{ab}{}^{ij} \mathcal{R}^{ab}{}_{ij} + \frac{4}{15}\sigma D^2 - 8\sigma T^{-dab} \hat{\nabla}_d \hat{\nabla}^c T^-_{abc} \\ & + 4\sigma(\hat{\nabla}_c T^{-abc}) \hat{\nabla}^d T^-_{abd} + 4\sigma T^{-abc} T^-_{ab}{}^d T^{-ef}{}_c T^-_{efd} - \frac{8}{45} H_{abc} T^{-abc} D \\ & - 2 H_{abc} C^{ab}{}_{de} T^{-cde} + 4 H_{abc} T^-_d{}^{ab} \hat{\nabla}_e T^{-cde} - \frac{4}{3} H_{abc} T^{-dea} T^{-bcf} T^-_{def} \\ & - \frac{1}{4}\varepsilon^{abcdef} b_{ab} \big(C_{cd}{}^{gh} C_{efgh} - \mathcal{R}_{cd}{}^{ij} \mathcal{R}_{ef\,ij} \big) \Big\} + \text{fermions} \end{aligned}$$

In the gauge $\sigma = 1$, $b_a = 0$

$$S_{\rm new} \;=\; \frac{1}{32}\int \mathrm{d}^6 x\, e \Big\{ \mathcal{R}_{ab}{}^{cd} \mathcal{R}_{cd}{}^{ab} - \mathcal{R}_b{}^d \mathcal{R}_d{}^b + \frac{1}{4}\mathcal{R}^2 + \cdots \Big\}$$

Application: Gauss-Bonnet $\mathcal{N} = (1,0)$ invariant

Constructed the new curvature squared invariant, we can describe an off-shell extension of the Gauss-Bonnet combination in six dimensions:

$$S_{\rm GB} = -3S_{\rm Riem^2} + 128 S_{\rm new}$$

In the gauge $\sigma = 1$, $b_a = 0$

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm GB} \quad = \quad & \mathcal{R}_{abcd}\mathcal{R}^{abcd} - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}^2 \\ & + \tfrac{1}{2}\mathcal{R}_{abcd}H^{abe}H^{cd}{}_e - \mathcal{R}^{ab}H^2_{ab} + \tfrac{1}{6}\mathcal{R}H^2 + \tfrac{1}{144}(H^2)^2 - \tfrac{1}{8}(H^2_{ab})^2 + \tfrac{5}{24}H^4 \\ & - \tfrac{1}{4}\epsilon^{abcdef}b_{ab}\mathcal{R}_{cd}{}^{gh}(\omega_+)\mathcal{R}_{ef\,gh}(\omega_+) + \epsilon^{abcdef}b_{ab}\mathcal{R}_{cd}{}^{ij}\mathcal{R}_{ef\,ij} + \text{fermions} \end{aligned}$$

where

$$\omega_{+m}{}^{cd} := \omega_{+m}{}^{cd} + \tfrac{1}{2}e_m{}^a H_a{}^{cd}$$

$$H^2 := H_{abc}H^{abc} \; , \quad H^2_{ab} := H_a{}^{cd}H_{bcd} \; , \quad H^4 := H_{abe}H_{cd}{}^e H^{acf}H^{bd}{}_f$$

Advantages to know the off-shell (1,0) Gauss-Bonnet invariant:

- possible to add the invariant to general sugra-matter couplings
- complete off-shell descriptions of NSNS b_2-form
- supersymmetry transformations completely under control.

Application: EH + Gauss-Bonnet supergravity

We can now consider the combination $\mathcal{L} = \mathcal{L}_{\rm EH} + \frac{1}{16}\alpha' \mathcal{L}_{\rm GB}$.

off-shell extension of α'-corrected string theory effective action:
On-shell (integrate auxiliary fields as pure EH, no ghosts!) and in a particular gauge:

$$
\begin{aligned}
e^{-1}\mathcal{L} \quad = \quad & e^{-2\varphi}[\mathcal{R} + 4\partial_m\varphi\partial^m\varphi - \tfrac{1}{12}H_{abc}H^{abc}] \\
& + \frac{1}{16}\alpha'\Big[\mathcal{R}_{abcd}\mathcal{R}^{abcd} - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}^2 + \tfrac{1}{2}\mathcal{R}_{abcd}H^{abe}H^{cd}{}_e - \mathcal{R}^{ab}H^2_{ab} + \tfrac{1}{6}\mathcal{R}H^2 \\
& \qquad + \tfrac{1}{144}(H^2)^2 - \tfrac{1}{8}(H^2_{ab})^2 + \tfrac{5}{24}H^4 - \tfrac{1}{4}\epsilon^{abcdef}b_{ab}\mathcal{R}_{cd}{}^{gh}(\omega_+)\mathcal{R}_{ef\,gh}(\omega_+)\Big]
\end{aligned}
$$

- It matches with on-shell string theory derivation of [Liu-Minasian (13)] α'-corrected Type IIA reduced on K3, dual to Heterotic on T4.
- Action possesses an $\mathrm{AdS}_3 \times \mathrm{S}^3$ solution analogue of the famous $\mathrm{AdS}_5 \times \mathrm{S}^5$ solution in IIB string theory.
- First time the α'-corrected KK spectrum of fluctuations around $\mathrm{AdS}_3 \times \mathrm{S}^3$ organized in short and long multiplets of $\mathrm{SU}(1,1|2) \times \mathrm{SL}(2,R) \times \mathrm{SU}(2)$.
  Hints on the dynamics of strings in $\mathrm{AdS}_3\times$ $\mathrm{S}^3\times$ K3 background.
  [Novak-Ozkan-Pang-GTM (17)]

$6D$ (1,0) conformal supergravity actions?

Butter, Novak, & GTM JHEP 1705 (2017) 133; arXiv:1701.08163;

Conformal gravity invariants

We introduced conformal gravity

- The conformal gravity actions may be written as

$$I = \int \mathrm{d}^6 x \, e \, L \,, \quad K_a L = 0 \,, \quad \mathbb{D} L = 6L$$

with

$$\begin{aligned} L^{(1)}_{C^3} &= C_{abcd} C^{aefd} C_e{}^{bc}{}_f \\ L^{(2)}_{C^3} &= C_{abcd} C^{cdef} C_{ef}{}^{ab} \\ L_{C\Box C} &= C^{abcd} \nabla^2 C_{abcd} + \frac{1}{2} (\nabla_e C_{abcd}) \nabla^e C^{abcd} \\ &\quad + \frac{8}{9} (\nabla^d C_{abcd}) \nabla_e C^{abce} \,, \end{aligned}$$

and $\nabla^2 := \nabla^a \nabla_a$

- We seek to find supersymmetric extensions of the previous actions

A-action principle 6-form

- A primary super 6-form can be constructed by choosing the first non-vanishing component [Arias-Linch-Ridgway('14)]

$$J_{abc}{}_{\alpha}^{\ i}{}_{\beta}^{\ j}{}_{\gamma}^{\ k} = 3(\gamma_{abc})_{(\alpha\beta}A_{\gamma)}{}^{ijk}$$

where

$$\nabla^{(i}_{(\alpha}A_{\beta)}{}^{jkl)} = 0\ , \quad K^B A_\alpha{}^{ijk} = 0$$

- Closure, $\mathrm{d}J = 0$, fixes the remaining components of the superform e.g. the top component of the superform is

$$J_{abcdef} = -\frac{\mathrm{i}}{2^4 4!}\varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{abcdef}\nabla_{\alpha i}\nabla_{\beta j}\nabla_{\gamma k}A_\delta{}^{ijk}$$

- The idea is that making $A_\alpha{}^{ijk}$ composite allows one to describe locally superconformal invariants

6D $\mathcal{N} = (1,0)$: C^3

It was proven in [Butter-Kuzenko-Novak-Theisen (16)] that only one cubic composite primary can be constructed for C^3 based on the super-Weyl tensor and its descendant

$$\begin{aligned} A_\alpha{}^{ijk} \;=\; & 5\mathrm{i}\varepsilon_{\alpha\beta\gamma\delta} X^{\beta(i} X^{\gamma j} X^{\delta k)} - 8\mathrm{i}\varepsilon_{\alpha\beta\gamma\delta} X^{\beta(i} X^{j}_{\alpha'}{}^{\gamma\beta'} X^{k)\delta\alpha'}_{\beta'} \\ & + \frac{64\mathrm{i}}{3} \varepsilon_{\alpha\beta\gamma\delta} X^{(i\,\beta\beta'}_{\alpha'} X^{j}_{\beta'}{}^{\gamma\gamma'} X^{k)\delta\alpha'}_{\gamma'} + 4\varepsilon_{\alpha\beta\gamma\delta} Y_\rho{}^{\beta(ij} X^{k)\rho\gamma}_{\eta} W^{\eta\delta} \\ & - 3\varepsilon_{\alpha\beta\gamma\delta} Y_\rho{}^{\beta(ij} X^{\gamma k)} W^{\rho\delta} \end{aligned}$$

where we used descendants of $W^{\alpha\beta}$

$$\begin{aligned} X^{k\,\alpha\beta}_{\gamma} &= -\frac{\mathrm{i}}{4} \nabla^k_\gamma W^{\alpha\beta} - \delta^{(\alpha}_{\gamma} X^{\beta)k} \,, \quad X^{\alpha i} := -\frac{\mathrm{i}}{10} \nabla^i_\beta W^{\alpha\beta} \,, \quad Y := \frac{1}{4} \nabla^k_\gamma X^\gamma_k \,, \\ Y_\alpha{}^{\beta\, ij} &:= -\frac{5}{2} \Big(\nabla^{(i}_\alpha X^{\beta j)} - \frac{1}{4} \delta^\beta_\alpha \nabla^{(i}_\gamma X^{\gamma j)} \Big) = -\frac{5}{2} \nabla^{(i}_\alpha X^{\beta j)} \,, \\ Y_{\alpha\beta}{}^{\gamma\delta} &:= \nabla^k_{(\alpha} X_{\beta)k}{}^{\gamma\delta} - \frac{1}{6} \delta^{(\gamma}_\beta \nabla^k_\rho X_{\alpha k}{}^{\delta)\rho} - \frac{1}{6} \delta^{(\gamma}_\alpha \nabla^k_\rho X_{\beta k}{}^{\delta)\rho} \end{aligned}$$

Plug it in the superform action principle, compute a LARGE NUMBER (good decision of component frame; handle with a computer program; group theory simplifications; ...) of descendant terms and you get: [Butter-Novak-GTM (17)]

6D $\mathcal{N} = (1,0)$: C^3 (bosonic +...)

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \frac{8}{3} C_{abcd} C^{abef} C^{cd}{}_{ef} - \frac{16}{3} C_{abcd} C^{aecf} C^{b}{}_{e}{}^{d}{}_{f} - 2\, C_{abcd} \mathcal{R}^{ab\,ij} \mathcal{R}^{cd}{}_{ij} + 4\, \mathcal{R}_{ab}{}^{ij} \mathcal{R}^{ac}{}_{i}{}^{k} \mathcal{R}^{b}{}_{c\,jk} \\
& - \frac{32}{225} D^3 - \frac{4}{15} D C_{abcd} C^{abcd} + \frac{8i}{5} D \mathcal{R}_{ab}{}^{ij} \mathcal{R}^{ab}{}_{ij} + \frac{128}{15} T^-_{abc} T^{-ade} D C^{b}{}_{d}{}^{c}{}_{e} + \frac{64}{15} T^-_{abc} D \hat{\nabla}^a \hat{\nabla}_d T^{-bcd} \\
& - \frac{4}{5} D \hat{\nabla}^a T^-_{abc} \hat{\nabla}_d T^{-bcd} + \frac{4}{15} D \hat{\nabla}_a T^-_{bcd} \hat{\nabla}^a T^{-bcd} - \frac{4}{3} D \hat{\nabla}_a T^-_{bcd} \hat{\nabla}^b T^{-acd} - \frac{16}{5} T^-_{abc} T^{-abd} T^{-cef} T^-_{def} D \\
& - \frac{32}{3} T^-_{abc} C^{abde} \hat{\nabla}^f C^{c}{}_{def} + \frac{16}{3} C_{abcd} C^{abef} \hat{\nabla}^c T^{-d}{}_{ef} - 16\, T^-_{abc} \hat{\nabla}_d T^{-abe} \hat{\nabla}_e \hat{\nabla}_f T^{-cdf} \\
& - 16\, T^-_{abc} \hat{\nabla}_d T^{-ade} \hat{\nabla}_e \hat{\nabla}_f T^{-bcf} - 48\, T^-_{abc} \hat{\nabla}_d T^{-ade} \hat{\nabla}^b \hat{\nabla}^f T^{-c}{}_{ef} + 16\, \hat{\nabla}^e T^-_{eab} \hat{\nabla}^f T^-_{fcd} \hat{\nabla}^a T^{-bcd} \\
& - 40\, T^-_{abe} T^{-cde} \hat{\nabla}_f T^{-fab} \hat{\nabla}^g T^-_{gcd} + 16\, T^-_{abc} C^{abde} \hat{\nabla}^c \hat{\nabla}^f T^-_{def} - 16\, T^-_{abc} C^{abde} \hat{\nabla}_d \hat{\nabla}^f T^{-c}{}_{ef} \\
& - 4\, C_{abcd} \hat{\nabla}_e T^{-abe} \hat{\nabla}_f T^{-cdf} + 8\, C_{abcd} \hat{\nabla}_e T^{-abf} \hat{\nabla}_f T^{-cde} - \frac{64}{3} T^{-fb}{}_{d} \hat{\nabla}^e C_{eabc} \hat{\nabla}_f T^{-acd} \\
& + 32\, T^{-ab}{}_{d} \hat{\nabla}^e C_{eabc} \hat{\nabla}_f T^{-fcd} - 32\, T^-_{fgc} T^{-fgd} \hat{\nabla}^c T^-_{dab} \hat{\nabla}_e T^{-eab} - 8\, \hat{\nabla}_e T^-_{bad} \hat{\nabla}^e T^{-cad} T^{-fgb} T^-_{fgc} \\
& - 8\, T^-_{abc} T^{-abd} C^{cefg} \hat{\nabla}_e T^-_{dfg} - \frac{8}{3} T^-_{abc} \mathcal{R}^{ab\,ij} \hat{\nabla}_d \mathcal{R}^{cd}{}_{ij} + \frac{28}{3} T^-_{abc} \mathcal{R}^{ad\,ij} \hat{\nabla}_d \mathcal{R}^{bc}{}_{ij} \\
& - \frac{32}{9} \mathcal{R}_{ab}{}^{ij} \mathcal{R}_{cd\,ij} \hat{\nabla}^a T^{-bcd} + 4\, T^-_{efb} T^{-efc} T^-_{gha} T^{-gh}{}_{c} \hat{\nabla}_d T^{-dab} - 8\, T^-_{abc} T^{-abd} T^-_{efg} T^{-efh} C^{cg}{}_{dh} \\
& + 12\, T^-_{abc} T^{-ade} \mathcal{R}^{bc\,ij} \mathcal{R}_{de\,ij} \\
& + \text{fermion terms}
\end{aligned}
$$

How about $C\Box C$?

Superspace ingredients from [Butter-Kuzenko-Novak-Theisen (16)], *B*-action 6-form and gravitational composite:

- based on a primary dimension 3, $\mathrm{B}^{\alpha\beta\, ij} = (\tilde{\gamma}^a)^{\alpha\beta}\mathrm{B}_a{}^{ij} = \mathrm{B}^{\alpha\beta(ij)}$, $\nabla_\alpha^{(i}\mathrm{B}^{\beta\gamma jk)} = -\frac{2}{3}\delta_\alpha^{[\beta}\nabla_\delta^{(i}\mathrm{B}^{\gamma]jk)}$
- yet another closed 6-form J, $\mathrm{d}J = 0$, now from $\mathrm{B}_a{}^{ij}$,
- Superforms action principle $\Longrightarrow$ another components action principle [Butter-Novak-GTM (17)]
- a gravitational composite (same as new curvature squared)

$$\mathrm{B}^{\alpha\beta\, ij} = W^{\gamma[\alpha}Y_\gamma{}^{\beta]\, ij} + 8\mathrm{i}\, X_\gamma{}^{\delta[\alpha(i}X_\delta{}^{\beta]\gamma j)} - \frac{5\mathrm{i}}{2}X^{[\alpha(i}X^{\beta]j)}$$

Plug it in the superform action principle, compute descendants and get:

6D $\mathcal{N} = (1,0)$: $C\Box C$ (bosonic +...)

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \tfrac{1}{3}\, C_{abcd} \hat{\nabla}^2 C^{abcd} - \tfrac{1}{3}\, C_{ab}{}^{cd} C_{cd}{}^{ef} C_{ef}{}^{ab} - \tfrac{4}{3}\, C_{abcd} C^{aecf} C^{b}{}_{e}{}^{d}{}_{f} \\
& - \mathcal{R}^{bc\,ij} \hat{\nabla}^2 \mathcal{R}_{bc\,ij} - 2\, \mathcal{R}_{ab\,i}{}^{j} \mathcal{R}_{ac\,j}{}^{k} \mathcal{R}_{bc\,k}{}^{i} + 2\, C^{abcd} \mathcal{R}_{ab}{}^{ij} \mathcal{R}_{cd\,ij} \\
& + \hat{\mathfrak{f}}_{a}{}^{b} \big(\tfrac{32}{3}\, C^{acde} C_{bcde} - 8\, \mathcal{R}_{bc}{}^{ij} \mathcal{R}^{ac}{}_{ij} \big) - 4\, \hat{\mathfrak{f}}_{a}{}^{a} \big(C_{bcde} C^{bcde} - \mathcal{R}_{bc}{}^{ij} \mathcal{R}^{bc}{}_{ij} \big) \\
& + \tfrac{4}{45}\, D \hat{\nabla}^2 D + \tfrac{8}{225}\, D^3 + \tfrac{2}{15}\, D\, C_{abcd} C^{abcd} - \tfrac{14}{15}\, D\, \mathcal{R}_{ab}{}^{ij} \mathcal{R}^{ab}{}_{ij} + \tfrac{20}{3}\, T^{-\,abe} C_{ab}{}^{cd} \hat{\nabla}^{f} C_{fecd} \\
& + 4\, T^{-\,abe} \hat{\nabla}^{f} C_{ab}{}^{cd} C_{fecd} + 2\, T^{-}_{abc} \hat{\nabla}_{d} \mathcal{R}^{ab\,ij} \mathcal{R}^{cd}{}_{ij} + 4\, T^{-}_{abc} \hat{\nabla}_{d} \mathcal{R}^{ad\,ij} \mathcal{R}^{bc}{}_{ij} \\
& - 4\, T^{-\,abc} \Big(\hat{\nabla}_{a} \hat{\nabla}^{d} \hat{\nabla}^2 T^{-}_{bcd} + \hat{\nabla}^2 \hat{\nabla}_{a} \hat{\nabla}^{d} T^{-}_{bcd} + \tfrac{1}{3}\, \hat{\nabla}_{a} \hat{\nabla}^2 \hat{\nabla}^{d} T^{-}_{bcd} - \tfrac{4}{3}\, \hat{\nabla}_{e} \hat{\nabla}_{a} \hat{\nabla}^{d} \hat{\nabla}^{e} T^{-}_{bcd} \Big) \\
& - \tfrac{16}{3}\, C_{abcd} T^{-\,abe} \hat{\nabla}_{e} \hat{\nabla}_{f} T^{-\,cdf} - \tfrac{8}{3}\, C_{abcd} T^{-\,abe} \hat{\nabla}_{f} \hat{\nabla}_{e} T^{-\,cdf} + \tfrac{16}{3}\, C_{ab}{}^{cd} T^{-\,aef} \hat{\nabla}^{b} \hat{\nabla}_{c} T^{-}_{def} \\
& - 4\, C_{ab}{}^{cd} \hat{\nabla}^{a} T^{-\,bef} \hat{\nabla}_{c} T^{-}_{def} - 6\, C_{ab}{}^{cd} \hat{\nabla}_{e} T^{-\,abf} \hat{\nabla}_{f} T^{-\,cde} - \tfrac{16}{15}\, D\, T^{-}_{abc} \hat{\nabla}^{a} \hat{\nabla}_{d} T^{-\,bcd} \\
& + \tfrac{8}{15}\, D\, \hat{\nabla}^{a} T^{-}_{abc} \hat{\nabla}_{d} T^{-\,bcd} - 2\, T^{-}_{abc} T^{-\,ade} \mathcal{R}^{bc}{}_{ij} \mathcal{R}_{de}{}^{ij} - \tfrac{4}{3}\, C_{abef} C^{cdef} T^{-}_{abg} T^{-\,cdg} \\
& - \tfrac{1}{2}\, \hat{\nabla}^{a_1} T^{-}_{a_1 ab} \hat{\nabla}^{a_2} T^{-}_{a_2 cd} \hat{\nabla}^{a_3} T^{-}_{a_3 ef}\, \varepsilon^{abcdef} - 6\, T^{-}_{ab}{}^{g} \hat{\nabla}^{a_1} T^{-}_{a_1 gc} \hat{\nabla}_{d} \hat{\nabla}^{a_2} T^{-}_{a_2 ef}\, \varepsilon^{abcdef} \\
& + 8\, C_{abcd} T^{-\,ecd} T^{-}_{efg} \hat{\nabla}^{a} T^{-\,bfg} + \tfrac{10}{3}\, T^{-}_{abc} T^{-\,aed} T^{-\,bf}{}_{d} \hat{\nabla}^2 T^{-\,c}{}_{ef} \\
& - 2\, T^{-}_{abc} T^{-\,abd} \hat{\nabla}^{c} T^{-}_{def} \hat{\nabla}_{g} T^{-\,efg} + 4\, T^{-}_{abc} T^{-\,a}{}_{de} \hat{\nabla}^{f} T^{-\,bdg} \hat{\nabla}_{f} T^{-\,ce}{}_{g} \\
& + 2\, C_{abcd} T^{-\,abe} T^{-\,cfg} T^{-\,d}{}_{fh} T^{-}_{eg}{}^{h} + \tfrac{8}{15}\, D\, T^{-}_{abc} T^{-\,abd} T^{-\,cef} T^{-}_{def} \\
& + \text{fermion terms}
\end{aligned}
$$

6D $\mathcal{N} = (2, 0)$ in $\mathcal{N} = (1, 0)$:

How about the conformal supergravity action in the $\mathcal{N} = (2, 0)$ case?

- By comparing our results with the purely gravitational part of known anomalies for the $\mathcal{N} = (2, 0)$ tensor multiplet,
  see heat-kernel calculations [Bastianelli-Frolov-Tseytlin (10)],
  or holographic results [Kulaxizi-Parnachev (09)],
  we can infer that the $\mathcal{N} = (1, 0)$ truncation of the unique $\mathcal{N} = (2, 0)$ conformal supergravity action is given by the combination

$$S_{(2,0)} = \frac{1}{2} S_{C^3} + S_{C \Box C}$$

- We can actually do more, and give a first proof based on supersymmetry that the $\mathcal{N} = (2, 0)$ conformal supergravity action is unique and fix most of its bosonic terms.

6D $\mathcal{N} = (2,0)$ conformal SUGRA

We can construct with the following steps:

- Assume that a (2,0) action exists;
- Reduce the known (2,0) Weyl multiplet to (1,0) [Bergshoeff-Sezgin-VanProeyen (99)];
- Write the most general purely bosonic terms covariant under, diffeomorphisms, Lorentz and R-symmetry group USp(4);
- Reduce USp(4) to SU(2), the (1,0) R-symmetry group;
- Compare with a combination of the S_{C^3} and $S_{C\Box C}$ (1,0) action;
- Note that a $DC^{abcd}C_{abcd}$ term, appearing in both (1,0) invariants, cannot be lifted to (2,0);
- To cancel these two in the (2,0) action reduced to (1,0) one has to choose

$$S_{(2,0)} = \frac{1}{2}S_{C^3} + S_{C\Box C}$$

6D $\mathcal{N} = (2, 0)$

All the terms then naturally lift form (1,0) to (2,0)

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \frac{1}{3} C_{abcd} \hat{\nabla}^2 C^{abcd} + C_{ab}{}^{cd} C^{abef} C_{cdef} - 4\, C_{abcd} C^{aecf} C^{b}{}_{e}{}^{d}{}_{f} \\
& - R(V)_{bc}{}^{ij} \hat{\nabla}^2 R(V)_{bc\, ij} - 2\, R(V)_{ab\, i}{}^{j} R(V)_{ac\, j}{}^{k} R(V)_{bc\, k}{}^{i} + C^{abcd} R(V)_{ab}{}^{ij} R(V)_{cd\, ij} \\
& + \hat{\mathfrak{f}}_a{}^b \Big(\frac{32}{3} C^{acde} C_{bcde} - 8\, R(V)_{bc}{}^{ij} R(V)^{ac}{}_{ij} \Big) - 4\, \hat{\mathfrak{f}}_a{}^a (C_{bcde} C^{bcde} - R(V)_{bc}{}^{ij} R(V)^{bc}{}_{ij}) \\
& + \frac{1}{225} D^{ij}{}_{kl} \hat{\nabla}^2 D^{kl}{}_{ij} - \frac{2}{3375} D^{ij}{}_{kl} D^{kl}{}_{pq} D^{pq}{}_{ij} - \frac{2}{15} D^{ij}{}_{kl} R(V)_{ab}{}^{k}{}_{i} R(V)^{abj}{}_{l} + 4 T_{abc}{}^{ij} \hat{\nabla}_d R(V)^{ab}{}_{jk} R(V)^{cdk}{}_{i} \\
& + 8 T_{abc}{}^{ij} \hat{\nabla}_d R(V)^{ad}{}_{jk} R(V)^{bck}{}_{i} - T_{abc}{}^{ij} \Delta^4 T^{abc}{}_{ij} + \frac{8}{3} C_{abcd} T^{abe}{}_{ij} \hat{\nabla}_e \hat{\nabla}_f T^{cdf\, ij} + \frac{4}{3} C_{abcd} T^{abe}{}_{ij} \hat{\nabla}_f \hat{\nabla}_e T^{cdf\, ij} \\
& - \frac{8}{3} C^{abcd} T_{aef\, ij} \hat{\nabla}_b \hat{\nabla}_c T_d{}^{ef\, ij} + 2\, C_{ab}{}^{cd} \hat{\nabla}^a T^{bef}{}_{ij} \hat{\nabla}_c T_{def}{}^{ij} + 3\, C_{ab}{}^{cd} \hat{\nabla}_e T^{abf}{}_{ij} \hat{\nabla}_f T^{cde\, ij} \\
& - \frac{4}{3} C_{abef} C^{cdef} T_{abg}{}^{ij} T^{cdg}{}_{ij} + 4\alpha\, T_{abc}{}^{ij} T^{ade\, kl} R(V)^{bc}{}_{ik} R(V)_{de\, jl} + 2(1-\alpha) T_{abc}{}^{ij} T^{ade}{}_{ij} R(V)^{bc}{}_{kl} R(V)_{de}{}^{kl} \\
& + \frac{2}{15} D^{ij}{}_{kl} \Big(T_{abc}{}^{kl} \hat{\nabla}^a \hat{\nabla}_d T^{bcd}{}_{ij} - \frac{1}{2} \hat{\nabla}^a T_{abc}{}^{kl} \hat{\nabla}_d T^{bcd}{}_{ij} \Big) - \frac{1}{60} D^{ij}{}_{kl} T_{abc}{}^{kl} T^{abd}{}_{ij} T^{cef}{}_{pq} T_{def}{}^{pq} + \mathcal{O}(T^4)
\end{aligned}
$$

Red terms consistent with known results in literature.
The rest is new although bosonic action is not fixed being α a free parameter and $\mathcal{O}(T^4)$ still undetermined.

Conclusion and Outlook

- The examples given indicate that there exist powerful techniques to construct higher derivative sugra invariants off-shell.
- can potentially improve the classification and construction of new higher derivative invariants in various dimensions
- The new 6D curvature-squared invariant complete an element missing since the 80s, see the Gauss-Bonnet
- Of importance in studying low energy String Theory and α'-corrected AdS/CFT, ...
- The new conformal supergravities in 6D are of importance in studying anomalies in 6D QFT, possibly CFT and AdS/CFT, ...

next?

- Extensions of $\mathcal{N} = (1,0)$ curvature squared? general matter coupled and...
  How about $\mathcal{N} = (1,1)$ (arising from Type IIA/Heterotic)?
  and $\mathcal{N} = (2,0)$ (arising from Type IIB)?
- Complete the construction of the $\mathcal{N} = (2,0)$ conformal supergravity invariant in (2,0) conformal superspace. This would give a final proof of its existence