Lagrangian description of massive higher spin supermultiplets in 3D AdS

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It is well known that for higher spins the notion of 3D spin is defined only for massive fields. In massless case there are no degrees of freedom.

Supersymmetric extension in 3D Minkowski

- On-shell $N = 1$ component description for the usual massive HS
  I. Buchbinder, T.S, Yu. Zinoviev 2015
- Off-shell $N = 1, 2$ superfield description for the topologically massive HS
  S. Kuzenko, M. Tsulaia; S. Kuzenko, D. Ogburn 2016

Supersymmetric extension in 3D AdS

- The conditions defying the $N = 1, 2$ massive HS superfield representations
  S. Kuzenko, J. Novak, G. Tartaglino-Mazzucchelli 2015
- $N = 1$ unfolded component equations I. Buchbinder, T.S, Yu. Zinoviev 2016

Problem: fined supersymmetric Lagrangian description in 3D AdS

Goal

Setup

- In three dimensions N-extended AdS supersymmetry has several incarnations. It is so-called (p, q) supersymmetries where $p \geq q$ are non-negative integers and $N = p + q$ A. Achucarro, P. Townsend 1986
- The simplest case is $(1, 0)$ supersymmetry. It is naturally associated with 3D AdS supergroup

$$OSp(1, 2) \otimes Sp(2)$$

It means that any massive supermultiplets must contain one bosonic and one fermionic degrees of freedom. For arbitrary interger s we have two higher-spin supermultiplets $(s, s + \frac{1}{2})$ and $(s, s - \frac{1}{2})$.

Goal: Using explicit component approach to fined Lagrangian realization of massive higher spin supermultiplets for minimal (1,0) supersymmetry where the algebra of the supercharges has the form

$$\{Q_\alpha, Q_\beta\} \sim P_{\alpha\beta} + \frac{\lambda}{2} M_{\alpha\beta}$$

Plan

1. Massive higher spin boson
2. Massive higher spin fermion
3. Massive higher spin supermultiplets
4. Realization of 3D AdS superalgebra
5. Summary

Massive higher spin boson

Frame-like description of massless (gauge) bosonic fields

Spin	$k=0$	$k=1$	$k\geq 2$
Field variabl.	$(\varphi,\pi^{\alpha\alpha})$	$(A,B^{\alpha\alpha})$	$(f^{\alpha(2k-2)},\Omega^{\alpha(2k-2)})$
Gauge transf.	$-$	$\delta A = D\xi$	$\delta f^{\alpha(2k)} = d\xi^{\alpha(2k)} + e^\alpha{}_\beta\eta^{\alpha(2k-1)\beta}$ $\delta\Omega^{\alpha(2k)} = d\eta^{\alpha(2k)}$

Lagrangians

$$
\begin{aligned}
k=0 \quad \mathcal{L} &= -E\pi_{\alpha\beta}\pi^{\alpha\beta} + \pi_{\alpha\beta}E^{\alpha\beta}d\varphi \\
k=1 \quad \mathcal{L} &= EB_{\alpha\beta}B^{\alpha\beta} - B_{\alpha\beta}e^{\alpha\beta}dA \\
k\geq 2 \quad \mathcal{L} &= (-1)^k[(k-1)\Omega_{\alpha(2k-3)\beta}e^\beta{}_\gamma\Omega^{\alpha(2k-3)\gamma} + \Omega_{\alpha(2k-2)}df^{\alpha(2k-2)}]
\end{aligned}
$$

here

two-form $E^{\alpha\beta} = \frac{1}{4}e^\alpha{}_\gamma e^{\gamma\beta}$, three-form $E = \frac{1}{6}E_{\alpha\beta}e^{\alpha\beta}$

Scheme notation (f_k,Ω_k)

$$
\mathcal{L} = \Omega_k\Omega_k + \Omega_k df_k, \qquad \delta f_k = d\xi_k + \eta_k, \quad \delta\Omega_k = d\eta_k
$$

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one-forms $e^{\alpha\beta},\ d^2=0 \quad \xrightarrow{AdS} \quad e^{\alpha\beta},\ D^2\zeta^a = -\lambda^2 E^\alpha{}_\beta\zeta^\beta$

Frame-like gauge-invariant description of massive bosonic fields
Massive spin s is described as system of massless one $s \geq k \geq 0$ coupled by the Stueckelberg symmetries.
Fields variables

$$\sum_{k=0}^{s}(f_k, \Omega_k) = \sum_{k=2}^{s}(f^{\alpha(2k-2)}, \Omega^{\alpha(2k-2)}) + (A, B^{\alpha\alpha}) + (\varphi, \pi^{\alpha\alpha})$$

Lagrangian

$$\mathcal{L} = \sum_k \Omega_k \Omega_k + \Omega_k D f_k + m(\Omega_k f_{k-1} + f_k \Omega_{k-1}) + m^2 f_k f_k$$

Gauge transformations

$$\begin{aligned} \delta f_k &= D\xi_k + \eta_k + m(\xi_{k-1} + \xi_{k+1}) \\ \delta \Omega_k &= D\eta_k + m(\eta_{k-1} + \eta_{k+1}) + m^2 \xi_k \end{aligned}$$

Gauge transformations leaving the Lagrangian to be invariant

For higher spin components $1 \leq k \leq s-1$

$$\begin{aligned}
\delta f^{\alpha(2k)} &= D\xi^{\alpha(2k)} + e^{\alpha}{}_{\beta}\eta^{\alpha(2k-1)\beta} + a_k e_{\beta(2)}\xi^{\alpha(2k)\beta(2)} \\
&\quad + \frac{(k+1)a_{k-1}}{k(k-1)(2k-1)} e^{\alpha(2)}\xi^{\alpha(2k-2)} \\
\delta\Omega^{\alpha(2k)} &= D\eta^{\alpha(2k)} + \frac{(k+2)a_k}{k} e_{\beta(2)}\eta^{\alpha(2k)\beta(2)} \\
&\quad + \frac{a_{k-1}}{k(2k-1)} e^{\alpha(2)}\eta^{\alpha(2k-2)} + \frac{s^2M^2}{4k^2(k+1)^2} e^{\alpha}{}_{\beta}\xi^{\alpha(2k-1)\beta}
\end{aligned}$$

For low spin components

$$\delta A = D\xi + \frac{a_0}{4} e_{\alpha(2)}\xi^{\alpha(2)} \qquad \delta\varphi = -2Ms\xi$$

$$\delta B^{\alpha(2)} = 2a_0\eta^{\alpha(2)} \qquad \delta\pi^{\alpha(2)} = \frac{Msa_0}{2}\xi^{\alpha(2)}$$

Here $M^2 = m^2 + (s-1)^2\lambda^2, \qquad {a_0}^2 = \frac{(s+1)(s-1)}{3}[M^2 - \lambda^2]$

$${a_k}^2 = \frac{k(s+k+1)(s-k-1)}{2(k+1)(k+2)(2k+3)}[M^2 - (k+1)^2\lambda^2]$$

Massive higher spin boson

Formalism of gauge invariant objects (curvatures)

In order to achieve gauge invariance for all curvatures one should introduce the so called extra fields $B^{\alpha(2k)}, \pi^{\alpha(2k)}$, $2 \leq k \leq s-1$

$$\delta B^{\alpha(2k)} = \eta^{\alpha(2k)} \qquad \delta \pi^{\alpha(2k)} = \xi^{\alpha(2k)}$$

If we denote schematically (B_k, π_k) then the structures of full set of curvatures can be written

$$\begin{aligned} \mathcal{R}_k &= D\Omega_k + m(\Omega_{k-1} + \Omega_{k+1}) + m^2 f_k \\ \mathcal{T}_k &= Df_k + \Omega_k + m(f_{k-1} + f_{k+1}) \\ \mathcal{B}_k &= DB_k + \Omega_k + m(B_{k-1} + B_{k+1}) + m^2 \pi_k \\ \Pi_k &= D\pi_k + f_k + B_k + m(\pi_{k-1} + \pi_{k+1}) \end{aligned}$$

Lagrangian in terms of curvatures

$$\mathcal{L} = \frac{1}{m} \sum_k (\mathcal{R}_k \Pi_k + \mathcal{T}_k \mathcal{B}_k) + \text{low spin curvatures}$$

$$\mathcal{L} = -\frac{1}{2}\sum_{k=1}^{s-1}(-1)^{k+1}[\mathcal{R}_{\alpha(2k)}\Pi^{\alpha(2k)} + \mathcal{T}_{\alpha(2k)}\mathcal{B}^{\alpha(2k)}] + \frac{a_0}{2sM}e_{\alpha(2)}\mathcal{B}^{\alpha(2)}\Phi$$

$$\begin{aligned}
\mathcal{R}^{\alpha(2k)} &= D\Omega^{\alpha(2k)} + \frac{(k+2)a_k}{k}e_{\beta(2)}\Omega^{\alpha(2k)\beta(2)} \\
&\quad + \frac{a_{k-1}}{k(2k-1)}e^{\alpha(2)}\Omega^{\alpha(2k-2)} + \frac{s^2M^2}{4k^2(k+1)^2}e^{\alpha}{}_{\beta}f^{\alpha(2k-1)\beta} \\
\mathcal{T}^{\alpha(2k)} &= Df^{\alpha(2k)} + e^{\alpha}{}_{\beta}\Omega^{\alpha(2k-1)\beta} + a_k e_{\beta(2)}f^{\alpha(2k)\beta(2)} \\
&\quad + \frac{(k+1)a_{k-1}}{k(k-1)(2k-1)}e^{\alpha(2)}f^{\alpha(2k-2)} \\
\mathcal{B}^{\alpha(2k)} &= DB^{\alpha(2k)} - \Omega^{\alpha(2k)} + \frac{s^2M^2}{4k^2(k+1)^2}e^{\alpha}{}_{\beta}\pi^{\alpha(2k-1)\beta} + \frac{a_{k-1}}{k(2k-1)}e^{\alpha(2)}B^{\alpha(2k-2)} \\
&\quad + \frac{(k+2)}{k}a_k e_{\beta(2)}B^{\alpha(2k)\beta(2)} \\
\Pi^{\alpha(2k)} &= D\pi^{\alpha(2k)} - f^{\alpha(2k)} + e^{\alpha}{}_{\beta}B^{\alpha(2k-1)\beta} + \frac{(k+1)a_{k-1}}{k(k-1)(2k-1)}e^{\alpha(2)}\pi^{\alpha(2k-2)} \\
&\quad + a_k e_{\beta(2)}\pi^{\alpha(2k)\beta(2)}
\end{aligned}$$

Massive higher spin boson

$$
\begin{aligned}
\mathcal{R}^{\alpha(2)} &= D\Omega^{\alpha(2)} + 3a_1 e_{\beta(2)}\Omega^{\alpha(2)\beta(2)} + \frac{s^2M^2}{16}e^{\alpha}{}_{\gamma}f^{\alpha\gamma} \\
&\quad -{a_0}^2 E^{\alpha}{}_{\beta}B^{\alpha\beta} + \frac{sMa_0}{2}E^{\alpha(2)}\varphi \\
\Phi &= D\varphi + 2MsA - \frac{sMa_0}{2}e_{\alpha(2)}\pi^{\alpha(2)}
\end{aligned}
$$

Massive higher spin fermion

Frame-like description of massless (gauge) fermionic fields

Spin	$1/2$	$k+1/2,\ k \geq 1$
Field variabl.	ϕ^α	$\Phi^{\alpha(2k-1)}$
Gauge transf.	$-$	$\delta\Phi^{\alpha(2k-1)} = d\xi^{\alpha(2k-1)}$

Lagrangians

$$\begin{aligned} 1/2 \quad \mathcal{L} &= \frac{i}{2}\phi_\alpha E^\alpha{}_\beta d\phi^\beta \\ k+1/2 \quad \mathcal{L} &= (-1)^k \frac{i}{2}\Phi_{\alpha(2k-1)} d\Phi^{\alpha(2k-1)} \end{aligned}$$

here

two-form $\quad E^{\alpha\beta} = \frac{1}{4} e^\alpha{}_\gamma e^{\gamma\beta}$

Scheme notation Φ_k

$$\mathcal{L} = \Phi_k d\Phi_k, \qquad \delta\Phi_k = d\xi_k$$

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one-forms $\quad e^{\alpha\beta},\ d^2 = 0 \quad \xrightarrow{AdS} \quad e^{\alpha\beta},\ D^2\zeta^a = -\lambda^2 E^\alpha{}_\beta \zeta^\beta$

Massive higher spin fermion

Frame-like gauge-invariant description of massive fermionic fields
Massive spin $s + 1/2$ is described as system of massless one $s \geq k \geq 0$
Fields variables

$$\sum_{k=0}^{s} \Phi_k = \sum_{k=1}^{s} \Phi^{\alpha(2k-1)} + \phi^{\alpha}$$

Lagrangian

$$\mathcal{L} = \sum_k \Phi_k D \Phi_k + m(\Phi_k \Phi_{k-1} + \Phi_k \Phi_k)$$

Gauge transformations

$$\delta\Phi_k = D\xi_k + m(\xi_k + \xi_{k-1} + \xi_{k+1})$$

Massive higher spin fermion

Gauge transformations leaving the Lagrangian to be invariant
For higher spin components $0 \leq k \leq s-1$

$$\begin{aligned}\delta\Phi^{\alpha(2k+1)} &= D\xi^{\alpha(2k+1)} + \frac{(2s+1)M_1}{(2k+1)(2k+3)} e^{\alpha}{}_{\beta}\xi^{\alpha(2k)\beta} \\ &+ \frac{c_k}{k(2k+1)} e^{\alpha(2)}\xi^{\alpha(2k-1)} + c_{k+1} e_{\beta(2)}\xi^{\alpha(2k+1)\beta(2)}\end{aligned}$$

For low spin components

$$\delta\phi^{\alpha} = c_0\xi^{\alpha}$$

Here

$$M_1{}^2 = m_1{}^2 + (s-\frac{1}{2})^2\lambda^2, \qquad c_0{}^2 = 2s(s+1)[M_1{}^2 - \frac{\lambda^2}{4}]$$

$$c_k{}^2 = \frac{(s+k+1)(s-k)}{2(k+1)(2k+1)}[M_1{}^2 - (2k+1)^2\frac{\lambda^2}{4}]$$

Massive higher spin fermion

Formalism of gauge invariant objects (curvatures)
In order to achieve gauge invariance for all curvatures one should introduce the so called extra fields $\phi^{\alpha(2k+1)}$, $1 \leq k \leq s-1$

$$\delta\phi^{\alpha(2k+1)} = \xi^{\alpha(2k+1)}$$

If we denote schematically ϕ_k then the structures of full set of curvatures can be written

$$\begin{aligned} \mathcal{F}_k &= D\Phi_k + m(\Phi_k + \Phi_{k-1} + \Phi_{k+1}) \\ \mathcal{C}_k &= D\phi_k + \Phi_k + m(\phi_k + \phi_{k-1} + \phi_{k+1}) \end{aligned}$$

Lagrangian in terms of curvatures

$$\mathcal{L} = \frac{1}{m}\sum_k \mathcal{F}_k \mathcal{C}_k$$

Massive higher spin fermion

$$\mathcal{L} \quad = \quad -\frac{i}{2}\sum_{k=0}^{s-2}(-1)^{k+1}\mathcal{F}_{\alpha(2k+1)}\mathcal{C}^{\alpha(2k+1)}$$

$$\begin{aligned}
\mathcal{F}^{\alpha(2k+1)} &= D\Phi^{\alpha(2k+1)} + \frac{(2s+1)M_1}{(2k+1)(2k+3)}e^{\alpha}{}_{\beta}\Phi^{\alpha(2k)\beta} \\
&\quad + \frac{c_k}{k(2k+1)}e^{\alpha(2)}\Phi^{\alpha(2k-1)} + c_{k+1}e_{\beta(2)}\Phi^{\alpha(2k+1)\beta(2)} \\
\mathcal{F}^{\alpha} &= D\Phi^{\alpha} + \frac{(2s+1)M_1}{3}e^{\alpha}{}_{\beta}\Phi^{\beta} + c_1 e_{\beta(2)}\Phi^{\alpha\beta(2)} - c_0{}^2 E^{\alpha}{}_{\beta}\phi^{\beta} \\
\mathcal{C}^{\alpha(2k+1)} &= D\phi^{\alpha(2k+1)} - \Phi^{\alpha(2k+1)} + \frac{d_k}{(2k+1)}e^{\alpha}{}_{\beta}\phi^{\alpha(2k)\beta} \\
&\quad + \frac{c_k}{k(2k+1)}e^{\alpha(2)}\phi^{\alpha(2k-1)} + c_{k+1}e_{\beta(2)}\phi^{\alpha(2k+1)\beta(2)}
\end{aligned}$$

Massive higher spin supermultiplets

For minimal (1,0) AdS_3 supersymmetry we have $(s+\frac{1}{2}, s)$ and $(s, s-\frac{1}{2})$

General scheme of supersymmetric construction

- We start with full set of initial curvatures for spin-s and spin-$(s+1/2)$. Let Ω^A, Φ^A is a set of fields and R^A, F^A is a set of curvatures.
- We deform initial curvatures by background gravitino Ψ^α

$$\delta\Psi^\alpha = D\zeta^\alpha + \frac{\lambda}{2} e^\alpha{}_\beta \zeta^\beta \quad \xrightarrow{background} \quad D\zeta^\alpha = -\frac{\lambda}{2} e^\alpha{}_\beta \zeta^\beta$$

Deformed curvatures	Global supertransf.
$\hat{R}^A = D\Omega^A + (e\Omega)^A + (\Psi\Phi)^A,$	$\delta_\zeta \Omega^A = \zeta^\alpha \frac{\delta(\Psi\Phi)^A}{\delta\Psi^\alpha}$
$\hat{F}^A = D\Phi^A + (e\Phi)^A + (\Psi\Omega)^A,$	$\delta_\zeta \Phi^A = \zeta^\alpha \frac{\delta(\Psi\Omega)^A}{\delta\Psi^\alpha}$

The form of supertransformations comes from the requirement

$$\delta\hat{R}^A \sim (F\zeta)^A, \qquad \delta\hat{F}^A \sim (R\zeta)^A$$

- The supersymmetric Lagrangian for given supermultiplets is the sum of Lagrangians for boson and fermion where initial curvatures are replaced by deformed ones $R, F \to \hat{R}, \hat{F}$.

$$\hat{\mathcal{L}} = \sum \hat{R}^A \hat{R}^A + \hat{F}^A \hat{F}^A$$

- Possible arbitrariness is fixed by the condition that the Lagrangian must be invariant under the supertransformations:

$$\delta\hat{\mathcal{L}} = \sum [\hat{R}^A \delta\hat{R}^A + \hat{F}^A \delta\hat{F}^A] = \sum R^A (F\zeta)^A = 0$$

Such a construction can be interpreted as a supersymmetric theory in terms of background fields of supergravity.

Massive higher spin supermultiplets

Supertransformation for bosonic spin-s fields

$$\begin{aligned}
\delta\Omega^{\alpha(2k)} &= \frac{isM}{k(k+1)}\beta_k\Phi^{\alpha(2k-1)}\zeta^\alpha + \frac{isM}{k(k+1)}\alpha_k\Phi^{\alpha(2k)\beta}\zeta_\beta \\
\delta f^{\alpha(2k)} &= i\beta_k\Phi^{\alpha(2k-1)}\zeta^\alpha + i\alpha_k\Phi^{\alpha(2k)\beta}\zeta_\beta \\
\delta\Omega^{\alpha(2)} &= \frac{isM}{2}\beta_1\Phi^\alpha\zeta^\alpha + \frac{isM}{2}\alpha_1\Phi^{\alpha(2)\beta}\zeta_\beta - \frac{i{c_0}^2}{8}\beta_1 e^{\alpha(2)}\phi^\beta\zeta_\beta \\
\delta A &= i\alpha_0\Phi^\alpha\zeta_\alpha + \frac{i{c_0}^2}{4a_0}\beta_1 e_{\alpha(2)}\phi^\alpha\zeta^\beta \\
\delta\varphi &= -\frac{i{c_0}^2}{2a_0}\phi^\gamma\zeta_\gamma \\
\delta B^{\alpha(2k)} &= i\frac{isM}{k(k+1)}\beta_k\phi^{\alpha(2k-1)}\zeta^\alpha + i\frac{isM}{k(k+1)}\alpha_k\phi^{\alpha(2k)\beta}\zeta_\beta \\
\delta\pi^{\alpha(2k)} &= i\beta_k\phi^{\alpha(2k-1)}\zeta^\alpha + i\alpha_k\phi^{\alpha(2k)\beta}\zeta_\beta
\end{aligned}$$

$${\alpha_k}^2 = k(s+k+1)[M+(k+1)\lambda]\hat{\alpha}^2 \qquad {\beta_k}^2 = \frac{(k+1)(s-k)}{k(2k+1)}[M-k\lambda]\hat{\beta}^2$$

$$M_1 = M + \frac{\lambda}{2}, \qquad \hat{\beta} = \frac{\hat{\alpha}}{\sqrt{2}}$$

Massive higher spin supermultiplets

Supertransformation for fermion spin-$(s+1/2)$

$$
\begin{aligned}
\delta\Phi^{\alpha(2k+1)} &= \frac{\alpha_k}{(2k+1)}\Omega^{\alpha(2k)}\zeta^\alpha + 2(k+1)\beta_{k+1}\Omega^{\alpha(2k+1)\beta}\zeta_\beta \\
&+ \frac{sM}{2k(k+1)(2k+1)}\alpha_k f^{\alpha(2k)}\zeta^\alpha + \frac{sM}{(k+2)}\beta_{k+1} f^{\alpha(2k+1)\beta}\zeta_\beta \\
\delta\Phi^\alpha &= 2\beta_1\Omega^{\alpha\beta}\zeta_\beta + \frac{{c_0}^2}{2sM}\beta_1 e_{\beta(2)}B^{\beta(2)}\zeta^\alpha + \frac{sM}{2}\beta_1 f^{\alpha\beta}\zeta_\beta \\
&+ \frac{{c_0}^2}{2a_0}\beta_1 A\zeta^\alpha - \frac{{c_0}^2}{4a_0}\beta_1\varphi e^\alpha{}_\beta\zeta^\beta \\
\delta\phi^\alpha &= 2\beta_1 B^{\alpha\beta}\zeta_\beta + \frac{sM}{2}\beta_1\pi^{\alpha\beta}\zeta_\beta - \frac{{c_0}^2}{4sMa_0}\beta_1\varphi\zeta^\alpha \\
\delta\phi^{\alpha(2k+1)} &= 2(k+1)\beta_{k+1}B^{\alpha(2k+1)\beta}\zeta_\beta + \frac{\alpha_k}{(2k+1)}B^{\alpha(2k)}\zeta^\alpha \\
&+ \frac{sM}{(k+2)}\beta_{k+1}\pi^{\alpha(2k+1)\beta}\zeta_\beta + \frac{sM}{2k(k+1)(2k+1)}\alpha_k\pi^{\alpha(2k)}\zeta^\alpha
\end{aligned}
$$

Results

- The curvature deformation procedure by background gravitino field allows us to supersymmetrize system of massive bosonic (M) and fermionic (M_1) HS.

$$(s, s+\frac{1}{2}): \quad M_1 = M + \frac{\lambda}{2}$$

$$(s, s-\frac{1}{2}): \quad M_1 = M - \frac{\lambda}{2}$$

- Invariance of Lagrangian under supertransformations is fulfilled only up to some on-shell equations.

Let parameters $\eta^{\alpha\beta}, \xi^{\alpha\beta}$ and ζ^{α} correspond to Lorentz transformations, translations and supertransformations respectively.

$$\hat{R}^A = (d+\omega)\Omega^A + (e\Omega)^A + (\Psi\Phi)^A, \qquad \hat{F}^A = (d+\omega)\Phi^A + (e\Phi)^A + (\Psi\Omega)^A$$

$$\delta_\eta \Omega^A = \eta^{\alpha\beta}\frac{\delta(\omega\Omega)^A}{\delta\omega^{\alpha\beta}}, \qquad \delta_\eta \Phi^A = \eta^{\alpha\beta}\frac{\delta(\omega\Phi)^A}{\delta\omega^{\alpha\beta}}$$

$$\delta_\xi \Omega^A = \xi^{\alpha\beta}\frac{\delta(e\Omega)^A}{\delta e^{\alpha\beta}}, \qquad \delta_\xi \Phi^A = \xi^{\alpha\beta}\frac{\delta(e\Phi)^A}{\delta e^{\alpha\beta}}$$

$$\delta_\zeta \Omega^A = \zeta^{\alpha}\frac{\delta(\Psi\Phi)^A}{\delta\Psi^{\alpha}}, \qquad \delta_\zeta \Phi^A = \zeta^{\alpha}\frac{\delta(\Psi\Omega)^A}{\delta\Psi^{\alpha}}$$

For bosonic fields

$$\begin{aligned}
\delta_\eta \hat{\Omega}^{\alpha(2k)} &= \eta^\alpha{}_\beta \hat{\Omega}^{\alpha(2k-1)\beta} \\
\delta_\xi \hat{\Omega}^{\alpha(2k)} &= \frac{(k+2)a_k}{k} \xi_{\beta(2)} \hat{\Omega}^{\alpha(2k)\beta(2)} + \frac{a_{k-1}}{k(2k-1)} \xi^{\alpha(2)} \hat{\Omega}^{\alpha(2k-2)} \\
&\quad + \frac{sM}{2k(k+1)} \xi^\alpha{}_\beta \hat{\Omega}^{\alpha(2k-1)\beta} \\
\delta_\zeta \hat{\Omega}^{\alpha(2k)} &= \frac{isM}{k(k+1)} \beta_k \Phi^{\alpha(2k-1)} \zeta^\alpha + \frac{isM}{k(k+1)} \alpha_k \Phi^{\alpha(2k)\beta} \zeta_\beta \\
\delta_\zeta \hat{f}^{\alpha(2k)} &= 0
\end{aligned}$$

here we introduced new field variables

$$\hat{\Omega}^{\alpha(2k)} = \Omega^{\alpha(2k)} + \frac{sM}{2k(k+1)} f^{\alpha(2k)}, \qquad \hat{f}^{\alpha(2k)} = \Omega^{\alpha(2k)} - \frac{sM}{2k(k+1)} f^{\alpha(2k)}$$

One can see that the $\hat{f}^{\alpha(2k)}$ fields are inert under the supertransformations. It just means that we have (1,0) supersymmetry.

Realization of AdS_3 superalgebra

Commutator of two supertransformations

$$
\begin{aligned}
[\delta_{\zeta_1}, \delta_{\zeta_2}]\hat{\Omega}^{\alpha(2k)} \quad = \quad & isM\hat{\alpha}^2[\frac{2(k+2)a_k}{k}\hat{\Omega}^{\alpha(2k)\gamma\beta}\zeta_{1\beta}\zeta_{2\gamma} + \frac{a_{k-1}}{k(2k-1)}\hat{\Omega}^{\alpha(2k-1)}\zeta_1{}^{\alpha}\zeta_2{}^{\alpha} \\
& + \frac{sM}{k(k+1)}\hat{\Omega}^{\alpha(2k-1)\gamma}\zeta_1{}^{\alpha}\zeta_{2\gamma} + \lambda\hat{\Omega}^{\alpha(2k-1)\gamma}\zeta_1{}^{\alpha}\zeta_{2\gamma}] - (1 \leftrightarrow 2)
\end{aligned}
$$

It is equivalent to

$$
\{Q_\alpha, Q_\beta\} \sim P_{\alpha\beta} + \frac{\lambda}{2}M_{\alpha\beta} \tag{1}
$$

We see, the algebra of the supertransformations is closed. It is worth emphasizing that we did not apply the equations of motion to obtain the relation both in bosonic and in fermionic sectors. This situation is analogous to one for massless higher-spin fields in the three-dimensional frame-like formalism.

Summary

- Using explicit component approach we have constructed Lagrangian realization of massive higher spin supermultiplets for minimal (1,0) supersymmetry.
- The supersymmetrization is achieved by deformation of the curvatures by background gravitino field and hence the supersymmetric Lagrangians are formulated with help of background fields of three-dimensional supergravity.
- Supertransformations are closed without any need in some auxiliary fields. The price we have to pay is that the Lagrangians are invariant under the supertransformations up to the terms proportional to some on-shell (the spin-1 and spin-0 auxiliary fields) equations only.