AdS/CFT in 3.99 Dimensions and Higher-Spin Gravity at One Loop

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Based on papers with Tung Tran and Murat Gunaydin

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Main Messages

Free vector model (free scalar) and Critical Vector Model (Wilson-Fisher) are conjectured to be dual to Higher-Spin Theories (Klebanov, Polyakov; Sezgin, Sundell)

Wilson-Fisher fixed point can be accessed via large-N or $4-\epsilon$ expansion

Therefore, the duality should make sense in non-integer dimensions as well (Klebanov, Polyakov), allowing to probe both large-N and $4-\epsilon$ expansions from the higher-spin theory side

It is difficult to compute anything in higher-spin theories that are not yet constructed, especially in fractional d

We will explore the duality for any d based on one-loop vacuum bubbles (a-anomaly, sphere free energy) and reproduce the results in free CFT and Wilson-Fisher CFT as quantum effects in higher-spin theories

General Results

The most basic higher-spin AdS/CFT duality conjecture (Klebanov, Polyakov; Sezgin, Sundell) says that

- free vector model (fancy name for free scalars) should be dual to a higher-spin theory whose spectrum contains totally-symmetric massless fields
- critical vector model (Wilson-Fisher) should be dual to the same theory for $\Delta = 2$ boundary conditions on $\Phi(x)$

$$J_{a_1 \ldots a_s} = \phi \partial_{a_1} \ldots \partial_{a_s} \phi \qquad \leftrightarrow \qquad \delta \Phi_{\mu_1 \ldots \mu_s}(x) = \nabla_{\mu_1} \xi_{\mu_2 \ldots \mu_s}$$

There is a similar proposal relating free/critical (Gross-Neveu) fermion and the Type-B theory with mixed-symmetry fields

General Results

We need to prove that

$$Z_{CFT}[A] = Z_{AdS}[A]$$

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where Z_{CFT} corresponds to free/critical vector model

$$Z_{CFT} = \exp W[A] = \left\langle \sum_i O_i(x) A_i(x) \right\rangle$$

and O_i are (almost) conserved higher-spin currents

$$O_i : \qquad J_{a_1 \dots a_s}(x) = \phi \partial_{a_1} \dots \partial_{a_s} \phi + \dots$$

We need to prove that

$$Z_{CFT}[A] = Z_{AdS}[A]$$

is the

$$Z_{AdS} = \int \prod_i D\Phi_i|_{\Phi|_{\partial AdS}=A_i(x)} \exp S[\Phi]$$

the most delicate part is that $S[\Phi]$ is not known in full detail

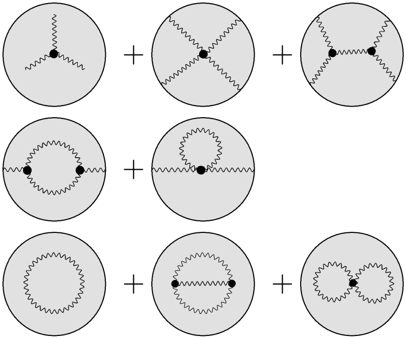
$$S[\Phi] = \sum_s \int \Phi_s(-\nabla^2 + m_s^2)\Phi_s + \mathcal{O}(\Phi^3) + \mathcal{O}(\Phi^4) + \ldots$$

Nevertheless, a considerable part of the duality proposal is a consequence of the higher-spin symmetries

General Results

There are several parts that should be accounted for by AdS/CFT

$$W_{AdS}[A] = S[AdS] +$$



Classical Part

Trees

Legged Loops

Vacuum Bubbles

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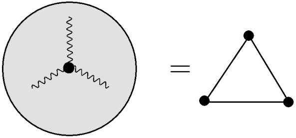
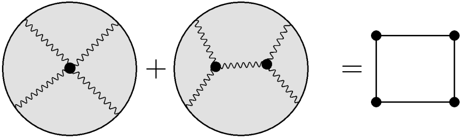
Let's have a look at them one by one

As a matter of principle one can reconstruct the vertices by requiring them to reproduce the right correlators (inverse scattering problem)



Bekaert, Erdmenger, Ponomarev, Sleight; Kessel, Lucena−Gomez, E.S., Taronna; E.S.; Sleight, Taronna; Metsaev



Bekaert, Erdmenger, Ponomarev, Sleight; Taronna, Sleight

The quartic and higher vertices are non-local or too non-local... Bekaert, Erdmenger, Ponomarev, Sleight; Taronna, Sleight. There seems to be no obstructions to this programme at higher orders.

General Results

Gauge Invariance implies Current Conservation, i.e. given

$$0 = \delta S = (\delta_0 + g\delta_1 + \ldots)(S_2 + gS_3 + ...)$$

where S_2 is Fronsdal action and $\delta_0 \Phi = \nabla \xi$.

By dropping boundary terms one can easily see that the correlation functions extracted from

$$\langle O_1(x_1) \ldots O_n(x_n) \rangle_{\text{conn.}} = \frac{\delta^n}{\delta A_1(x) \ldots \delta A_n(x_n)} W_{AdS}[A] \Big|_{A=0}$$

are those of conserved (higher-spin) currents

$$\langle J_{s_1}(\vec{x}_1) \ldots J_{s_n}(\vec{x}_n) \rangle \qquad \qquad \partial \cdot J = 0$$

which is not surprising for global symmetries $J_1 = J_a$ and for the stress-tensor $J_2 = T_{ab}$

General Results

HS Current Conservation implies Free CFT, i.e. given a CFT with stress-tensor J_2 and at least one higher-spin current J_s, one can prove Maldacena, Zhiboedov; Boulanger, Ponomarev, E.S., Taronna; Alba, Diab, Stanev that

- there are infinitely many higher-spin currents and spin is unbounded;
- correlation function (higher-spin algebra) corresponds to free CFT (which CFT, depends on the spectrum)

This essentially proves the duality no matter how the bulk theory is realized. Loops still need to be shown to vanish (be proportional to the tree result)

It is still might be interesting to see directly in the bulk how gravity gets quantized

General Results

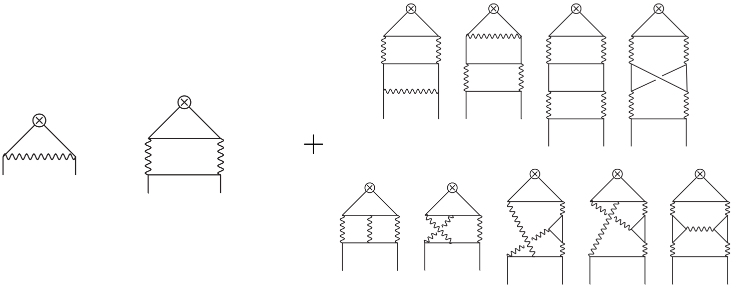
Duality between critical vector model, Gross-Neveu models and higher-spin theories is more interesting due to importance of the former for physics: 'critical indices of Ising model from quantum gravity'

Anomalous dimensions of higher-spin currents should correspond to 'masses' of higher-spin fields (Girardello, Poratti, Zaffaroni)

Large-N expansion of these models (bulk coupling $\sim N^{-1}$) is much harder then $4-\epsilon$, $2+\epsilon$ and the theories are incomparable with the free versions. NLO results (Manashov, E.S.; Manashov, E.S., Strohmaier)

Large-N expansion makes sense for non-integer d and is a useful cross-check of the ϵ expansions

For example, (Manashov, E.S.; Manashov, E.S., Strohmaier)



=Two-loop in Higher-Spin Gravity?

How come that one and the same theory is dual to the most trivial CFT and to the (one of) the most nontrivial CFT's?

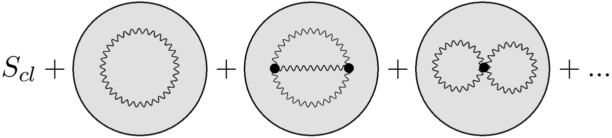
Another general result (Hartman, Rastelli; Giombi, Yin; Bekaert, Joung, Mourad) is that the difference between $\Delta = d-2$ and $\Delta = 2$ boundary conditions is given exactly by those diagrams that are present in the critical model and absent in the free one.

It is assumed that the free boson/higher-spin duality works fine including loops, so we need to look at the difference only

The proof is formal and does not take into account that the diagrams on the CFT side are divergent, which is exactly what leads to the nontrivial physics of the vector models

Vacuum Bubbles

Vacuum bubbles contain a lot of non-trivial information too:



The leading piece is the classical action and is not available, still it makes sense to look at the subleading terms

The one-loop determinant can be computed! (Giombi, Klebanov, Safdi, Tseytlin, Beccaria, Joung, Lal, Bekaert, Boulanger, Gunaydin, E.S, Tung, ...)

Vacuum Bubbles

There are many different quantities that can be extracted from the vacuum diagrams. Let us restrict to the Eucilidean AdS vs. CFT on Sphere case:

$$F_{CFT} = \begin{cases} a \log R & d - \text{even} \\ \text{number} & d - \text{odd} \end{cases}$$

For example, for the free scalar field

$$a_\phi^4 = \frac{1}{90}\,, \qquad a_\phi^6 = -\frac{1}{756}\,, \qquad a_\phi^8 = \frac{23}{113400}$$

and sphere free energy

$$F_\phi^3 = \frac{1}{16}(2\log 2 - \frac{3\zeta(3)}{\pi^2})$$
$$F_\phi^5 = \frac{-1}{2^8}(2\log 2 + \frac{2\zeta(3)}{\pi^2} - \frac{15\zeta(5)}{\pi^4})$$

Vacuum Bubbles

Sphere Free Energy (F-energy) turns out to cover many interesting and deep results.

It has been a long quest to find some measure of degrees of freedom in general QFT: monotonic along RG and stationary at fixed points.

$d = 2$ Zamolodchikov c-theorem

$d = 4$ a-theorem (Cardy; Komargodski, Schwimmer)

$d = 3$ what to do in odd dimensions? Sphere Free Energy (Myers et al; Jafferis et al; Klebanov et al; Casini, Huerta):

$$F = -\log Z_{S^d}$$

Vacuum Bubbles

All the cases above are particular cases of the Generalized Sphere Free Energy (Klebanov, Pufu, Safdi; Giombi, Klebanov)

$$\tilde{F} = (-)^{(d-1)/2} \log Z_{S^d} \to \sin(\tfrac{\pi d}{2}) \log Z_{S^d}$$

in particular even dimensions are covered as well: $\tilde{F} = (-1)^{d/2} \pi a/2$ and $c = -3a$ in $2d$.

For example, for a free scalar field (Diaz, Dorn; Giombi, Klebanov)

$$\tilde{F}^{\phi} = \frac{1}{\Gamma(d+1)} \int_0^1 u \sin(\pi u) \Gamma\left(\frac{d}{2} + u\right) \Gamma\left(\frac{d}{2} - u\right) du \,.$$

The formula makes sense for all d, including non-integer

Vacuum Bubbles

Generalized Sphere Free Energy can also be computed for the critical vector model in $4-\epsilon$ expansion or in large-N (Giombi, Klebanov)

$$\begin{aligned}\delta F &= F_{UV} - F_{IR} \\ &= -\frac{1}{\sin\left(\frac{\pi d}{2}\right)\Gamma(d+1)}\int_0^{2-d/2} u\sin(\pi u)\Gamma\left(\frac{d}{2}-u\right)\Gamma\left(\frac{d}{2}+u\right)du\end{aligned}$$

where the result corresponds to the change due to a double-trace deformation $(\phi^2)^2$

the computation makes sense in all dimensions d

Vacuum Bubbles

On the AdS side the leading correction is one-loop determinant, which can be computed via zeta-function as

$$F = -\zeta(0)\log\Lambda l - \frac{1}{2}\zeta'(0)$$

Subtleties involve

- coefficient of $\log$ must vanish, otherwise finite part is ill-defined;
- we need to sum over $s = 0, 1, 2, ...$ and the sums are not convergent

$$\frac{1}{360} + \sum_s \left(\frac{1}{180} - \frac{s^2}{24} + \frac{5s^4}{24}\right) = 0$$

- $\sum_s \zeta'(0)$ is a technical challenge

Vacuum Bubbles

Thanks to Camporesi and Higuchi, we can derive the zeta-function for arbitrary-spin field in Euclidian AdS space:

$$\zeta = \frac{\mathrm{vol}(\mathbb{H}^{d+1})}{\mathrm{vol}(S^d)} v_d g(s) \int_0^\infty d\lambda \, \frac{\mu(\lambda)}{\left[\frac{1}{4}(d-2\Delta)^2 + \lambda^2\right]^z}$$

where $g(s)$ counts the number of field components and $\mu(\lambda)$ is a spectral density. In flat space $\mu \sim p^{d-1}$, but in AdS_{2k+1}:

$$d \text{ even}: \quad \mu^B(\lambda) = w_d \left(\left(\frac{d-2}{2} + s \right)^2 + \lambda^2 \right) \prod_{j=0}^{\frac{d-4}{2}} (j^2 + \lambda^2)$$

$$d \text{ odd}: \quad \mu^B(\lambda) = w_d \lambda \tanh(\pi\lambda) \left(\left(\frac{d-2}{2} + s \right)^2 + \lambda^2 \right) \prod_{j=1/2}^{\frac{d-4}{2}} (j^2 + \lambda^2)$$

Vacuum Bubbles

There are two cases: minimal Type-A $s = 0, 2, 4, ...$
non-minimal Type-A $s = 0, 1, 2, 3, ...$

Summary of the results so far (Giombi, Klebanov, Safdi):

- $\zeta(0) = 0$ both for min and non-min Type-A;
- $\zeta'(0) = 0$ for the non-minimal Type-A;
- $-\frac{1}{2}\zeta'(0)$ gives a-anomaly or F_ϕ of the free scalar field

This was done for a number of integer dimensions

The general proof is lacking

General Proof

Spectral density for any dimension is known (Camporesi, Higuchi)

$$\mu(\lambda) = \left(\left(\frac{d-2}{2} + s \right)^2 + \lambda^2 \right) \left| \frac{\Gamma\left(\frac{d-2}{2} + i\lambda \right)}{\Gamma(i\lambda)} \right|^2$$

The general proof involves three main ingredients. Laplace transform

$$\frac{1}{(\lambda^2 + \nu^2)^z} = \frac{\sqrt{\pi}}{\Gamma(z)} \int_0^\infty d\beta\, e^{-\beta\nu} \left(\frac{\beta}{2\lambda} \right)^{z-\frac{1}{2}} J_{z-\frac{1}{2}}(\lambda\beta)\,.$$

see also (Bae, Joung, Lal). Modified regularization

$$\lim_{z\to 0} \frac{\beta^{z-\frac{1}{2}} J_{z-\frac{1}{2}}(\beta\lambda)}{(2\lambda)^{z-\frac{1}{2}}} = \frac{2\cos(\beta\lambda)}{\sqrt{\pi}\beta} + \mathcal{O}(z)\,.$$

which leads to some deficit for $\zeta'(0)$, which can be shown to vanish for Type-A (Bae, Joung, Lal)

As a result one arrives at the expected $0 = 0$, but still nontrivial equalities

$$\zeta_{non-min}(0) = \zeta'_{non-min}(0) = \zeta_{min}(0)$$

The interesting case is of the minimal Type-A where we arrive at the intermediate form

$$\tilde{\zeta}'^{A}_{\mathsf{min}}(0) = -\int_0^\infty d\beta \frac{e^{-\beta(2-d)}(1+e^{2\beta})^2}{\beta(e^{2\beta}-1)^d}$$

This integral representation still needs to be regularized

General Proof

Exactly this intermediate form can be shown to arise in the computation of the generalized sphere energy in any d, (Giombi, Klebanov):

$$F = \frac{1}{2} \log \det \left| -\nabla^2 + \frac{(d-2)}{4(d-1)} R \right|$$

We also extended this result to non-integer dimensions and to $\Delta = 2$ boundary conditions, i.e. for the critical vector model dual

It has been already observed that (Giombi, Klebanov, Tseytlin) that certain intermediate AdS results match those on the CFT side and then the same regularization should be applied

The simplest example is identification of R in the $\log R$ of the regularized volume of AdS_{2n+1}-spaces and $\log R$ in the a anomaly extracted from the sphere free energy

Summary

(Generalized) Sphere Free energy F captures important information about CFT's (e.g., a-anomaly)

We reproduced F as a one-loop effect in the dual higher-spin theory for all dimensions, including non-integer ones

$$\tilde{F}^{\phi}_{\text{min.}} = \frac{1}{\Gamma(d+1)} \int_0^1 du \sin(\pi u) \Gamma\left(\frac{d}{2} - u\right) \Gamma\left(\frac{d}{2} + u\right) .$$

This gives a proof for a pattern observed in a number of integer dimensions and extends it to non-integer d

Summary

Changing boundary conditions we get F for large-N Wilson-Fisher CFT

$$\delta\tilde{F} = \tilde{F}_{IR} - \tilde{F}_{UV} = \frac{1}{\Gamma(d+1)} \int_0^{d/2-2} u \sin(\pi u) \Gamma\left(\frac{d}{2} - u\right) \Gamma\left(\frac{d}{2} + u\right) du\,.$$

In particular, in AdS_4/CFT^3 for Wilson-Fisher

$$\delta\tilde{F} = -\frac{\zeta(3)}{8\pi^2}$$

In particular, around $AdS_{4.99}/CFT^{3.99}$ for Wilson-Fisher

$$\delta\tilde{F} = -\frac{\pi}{567}\epsilon^3 - \frac{13\pi}{6912}\epsilon^4 + \ldots$$

The ϵ-expansion of F of the Wilson-Fisher CFT is reproduced as one-loop effect in higher-spin gravity

Summary

It would be interesting to reconsider the Type-B (dual of free/critical fermion) puzzle: F is nice for d odd, but does not match free fermion

Other results that support fractional AdS/CFT:

- extremality of ϕ^3 in AdS_4 is properly compensated by the zer of the coupling $g \sim (d-3)$, (Bekaert, Erdmenger, Ponomarev, Sleight)
- Mellin boostrap (Gopakumar, Kaviraj, Sen, Sinha) operates in terms on unitary conformal blocks that can be directly interpreted as a combination of Witten diagrams in AdS. Allowed to get ϵ^3 results for anomalous dimensions of higher-spin currents.

Thank you!