Walls of mass-deformed Kähler nonlinear sigma models on $SO(2N)/U(N)$, $N > 3$

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Contents

- moduli matrices of walls on the Grassmann manifold [Y.Isozumi& M.Nitta & K.Ohashi & N.Sakai (2004)]
- moduli matrices of walls on $SO(2N)/U(N)$ ($N \leq 3$) [M.Arai & SS (2011)] moduli matrices of magnetic monopoles on $SO(2N)/U(N)$ [M.Eto& T.Fujimori & S.B.Gudnason & Y.Jiang & K.Konishi & M.Nitta & K.Ohashi (2011)]
- moduli matrices of walls on $SO(2N)/U(N)$ ($N > 3$) [B-H. Lee & C. Park & SS (2017)] in the convetion and the formalism used in [M.Eto& T.Fujimori & S.B.Gudnason & Y.Jiang & K.Konishi & M.Nitta & K.Ohashi (2011)]
- elementary walls labelled by simple roots [N.Sakai & D.Tong (2005)],[B-H. Lee & C. Park & SS (2017)].

What we know $\cdots$

- The strong gauge coupling limit of $U(N_C)$ gauge theory $(N_F > N_C)$ becomes the complex Grassmann manifold G_{N_F,N_C} [Y.Isozumi& M.Nitta & K.Ohashi & N.Sakai (2004)].
- It is shown that the vector multiplet part of BPS eq. does not produce additional moduli parameters. It is proven in the case of compact Kähler base space and domain walls in $U(1)$ and non-Abelian gauge theories, etc. [Mundet I Riera(2000)], [Cieliebak, Rita Gaio, Salamon(2000)], [Sakai,Yang(2005)], [Sakai,Tong(2005),K.S.M.Lee(2003)].

Lagrangian

•4D Lagrangian (massless) [K.Higashijima & M.Nitta (1999)]

$$\mathcal{L}_{\rm 4D} = -|D_m \phi_a{}^i|^2 + |F_a{}^i|^2 + \frac{1}{2}(D_a{}^b \phi_b{}^i \bar{\phi}_i{}^a - D_a{}^a)$$
$$+\Big[(F_0)^{ab}\phi_b{}^i J_{ij}\phi^{Tj}{}_a + (\phi_0)^{ab} F_b{}^i J_{ij}\phi^{Tj}{}_a + (\phi_0)^{ab}\phi_b{}^i J_{ij} F^{Tj}{}_a + c.c.\Big]$$
$$(m = 0, 1, 2, 3, \quad i, j \cdots = 1, \cdots, N_F(= 2N), \quad ab, \cdots = 1, \cdots, N_C(= N))$$

•dimensional reduction [J.Scherk & J.H.Schwarz (1979)]
The fields in the x^3-direction are moving along orbits of the Killing vectors $i\phi M$ and $-\bar{M}\bar{\phi}$.

$$\frac{\partial \phi_a{}^i}{\partial x^3} = i\phi_a{}^j M_j{}^i, \quad \frac{\partial \bar{\phi}_i{}^a}{\partial x^3} = -i\bar{M}_i{}^j \bar{\phi}_j{}^a, \quad A_3 = \Sigma$$

•M is a linear combination of Cartan generators. $\rightarrow$ SUSY remains intact.

Lagrangian (cont'd)

•mass-deformed 3D Lagrangian

$$\mathcal{L}_{\mathrm{3D}} =$$
$$= -(\overline{D_\mu \phi})_i{}^a (D^\mu \phi)_a{}^i - |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b \phi_b{}^i|^2 + |F_a{}^i|^2 + \frac{1}{2}(D_a{}^b \phi_b{}^i \bar{\phi}_i{}^a - D_a{}^a)$$
$$+\Big[(F_0)^{ab}\phi_b{}^i J_{ij}\phi^{Tj}{}_a + (\phi_0)^{ab} F_b{}^i J_{ij}\phi^{Tj}{}_a + (\phi_0)^{ab}\phi_b{}^i J_{ij} F^{Tj}{}_a + c.c.\Big]$$
$$(\mu = 0, 1, 2, \quad i, j = 1, \cdots, 2N, \quad a, b = 1, \cdots, N)$$

D-term constraint	$\phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0$
F-term constraints	$\phi_a{}^i J_{ij} \phi^{Tj}{}_b = 0 \quad + \quad$ complex conj.

•J: invariant tensor of $O(2N)$

$$J_{2N} = \sigma^1 \otimes I_N$$

•potential

$$V = |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b \phi_b{}^i|^2 + 4|(\phi_0)^{ab}\phi_b{}^i|^2$$

Lagrangian (cont'd)

•mass matrix
(convention and formalism used in [M.Eto& T.Fujimori & S.B.Gudnason & Y.Jiang & K.Konishi & M.Nitta & K.Ohashi (2011)])

In this basis, Cartan generators are

$$H_n = \left(\begin{array}{c|c} h_n & \\ \hline & -h_n \end{array}\right), \quad (n = 1, \cdots, N)$$

with $N \times N$ matrix h_n which has an only component 1 in (n, n)-element.

$$\underline{m} := (m_1, m_2 \cdots, m_N)$$
$$m_1 > m_2 > \cdots > m_N \quad \text{w/o loss of generality}$$

$$\underline{H} := (H_1, H_2, \cdots, H_N)$$

$$M = \underline{m} \cdot \underline{H}$$

Lagrangian (cont'd)

•vacuum

$$\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i = 0$$
$$(\phi_0)^{ab} = 0$$

Σ can be diagonalized by $U(N)$ transformation

$$\Sigma = \mathrm{diag}(\Sigma_1, \Sigma_2, \cdots, \Sigma_N)$$

therefore the vacua are labelled by

$$(\Sigma_1, \Sigma_2, \cdots, \Sigma_N) = (\pm m_1, \pm m_2, \cdots, \pm m_N)$$

$\rightarrow$ # of vauca$= 2^{N-1}$

of vacua → Euler's characteristics

[E.Witten (1982)][K.Hori& C.Vafa (2000)][C.U.Sanchez& A.L.Cali & J.L.Moreschi (1997)][S.B.Gudnason & Y.Jiang & K.Konishi (2010)]

orientational moduli space $\mathcal{M}$	$\chi(\mathcal{M})$
$\frac{SO(2N)}{U(N)}$	2^{N-1}
$\frac{Sp(N)}{U(N)}$	2^N
$\mathbf{C}P^{N-1} = \frac{SU(N)}{SU(N-1)\times U(1)}$	N
$Gr_{N,k} = \frac{SU(N)}{SU(N-k)\times SU(k)\times U(1)}$	${}_N C_k$
$Q^{2N-2} = \frac{SO(2N)}{SO(2N-2)\times SO(2)}$	$2N$

BPS equation

The BPS equation for wall solutions is derived from the Bogomol'nyi completion of the Hamiltonian. It is assumed that fields are static and all the fields depend only on the $x_1 \equiv x$ coordinate. It is also assumed that there is Poincare invariance on the two-dimensional worldvolume of walls to set $A_0 = A_2 = 0$. The energy is saturated when

$$(D\phi)_a{}^i \mp (\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0.$$

We choose the upper sign for the BPS equation without loss of generality.

moduli matrices

•BPS equation

$$(D\phi)_a{}^i - (\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0$$
$$(D\phi)_a{}^i = \partial\phi_a{}^i - iA_a{}^b \phi_b{}^i$$

By introducing complex matrix functions $S_a{}^b(x)$ and $f_a{}^i(x)$ defined by

$$\Sigma_a{}^b - iA_a{}^b \equiv (S^{-1}\partial S)_a{}^b,\ \phi_a{}^i \equiv (S^{-1})_a{}^b f_b{}^i,$$

the BPS eq. is solved as

$$\phi_a{}^i = (S^{-1})_a{}^b H_{0b}^j (e^{Mx})_j{}^i.$$

H_0 : moduli matrix
All the quantities are invariant under the transformation

$$S_a'{}^b = V_a{}^c S_c{}^b, \quad H_{0a}'{}^i = V_a{}^c H_{0c}^i, V \in GL(N, \mathbf{C}).$$

The V defines an equivalent class of (S, H_0). $\rightarrow$ worldvolume symmetry
[Y.Isozumi & M.Nitta & K.Ohashi & N.Sakai(2004)]

moduli matrices (cont'd)

•constraints

$$\begin{array}{c} \phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0 \\ \phi_a{}^i J_{ij} \phi^{Tj}{}_b = 0, \text{complex conj.} \end{array} \quad \rightarrow \quad \begin{array}{c} H_{0a}^{\ i} (e^{2Mx})_i{}^j H_{0j}^{\dagger\ b} = (S\bar{S})_a{}^b \equiv \Omega_a{}^b \\ H_{0a}^{\ i} J_{ij} H_b^{Tj} = 0, \text{complex conj.} \end{array}$$

•moduli space

$$\begin{array}{l} H_{0a}'^{\ i} = V_a{}^c H_{0c}^{\ i}, V \in GL(N, \mathbf{C}) \\ H_{0a}^{\ i} J_{ij} H_b^{Tj} = 0 \end{array}$$

$\rightarrow$ Moduli space is $SO(2N)/U(N)$.

Gauge invariant quantities

•trace of Σ

$$\begin{aligned} \mathrm{tr}\Sigma &= \mathrm{tr}\left(\Sigma_1 + \Sigma_2 + \cdots + \Sigma_N\right) \\ &= \frac{1}{2}\frac{\partial \det \Omega}{\det \Omega} \end{aligned}$$

•tension

$$\begin{aligned} T &= \int \partial \mathrm{tr}(\phi M \bar{\phi}) \\ &= \frac{1}{2}\partial \ln \det \Omega \end{aligned}$$

elementary walls in Gr_{N_F,N_C}

In [Isozumi& Nitta & Ohashi & Sakai (2004)], walls are algebraically constructed from elementary walls. On the Grassmann manifold, an elementary wall connects two nearest vacua of the same color index changing the flavor by one unit. An elementary wall interpolating two vacua $\langle A \rangle$ and $\langle B \rangle$ in the flavor i and $i+1$ in the same color is $H_{0\langle A \leftarrow B \rangle} = H_{0\langle A \rangle} e^{E_i(r)}$ where $E_i(r) \equiv e^r E_i (r \in \mathbf{C})$. The E_i of an elementary wall carrying tension $T_{\langle A \leftarrow B \rangle}$ is defined by

$$[cM, E_i] = c(m_i - m_{i+1})E_i = T_{\langle i \leftarrow i+1 \rangle},$$

where c is a constant, M is the mass matrix and E_i is an $N_f \times N_f$ square matrix generating an elementary wall. The E_i has an nonzero component only in the $(i, i+1)$-th element.

$\Longrightarrow$ This definition is not compatible with $SO(2N)/U(N)$.

elementary walls in $SO(2N)/U(N)$

We can generalize the formula. [M.Arai,SS(2011)][M.Eto & S.B.Gudnason & Y.Jiang & K.Konishi & M.Nitta & K.Ohashi (2011)][B-H.Lee & C.Park & SS (2017)]

$$[cM, E_i] = c(\underline{m} \cdot \underline{\alpha}_i)E_i = T_{\langle i \leftarrow i+1 \rangle}$$

$\underline{\alpha}_i$: simple roots
$\underline{m} := (m_1, \cdots, m_N)$

where α_i are simple roots of E_i, which are positive step operators of $SO(2N)$. We restrict ourselves to the case where $m_i > m_{i+1}$ then the vector $\underline{m}$ is a vector in the interior of the positive Weyl chamber,

$$\underline{m} \cdot \underline{\alpha}_i > 0.$$

positive step operators & simple roots

$$E_i = \begin{array}{c} \\ i \\ \\ i+N+1 \end{array} \begin{array}{c} \begin{array}{cc} \quad i+1 \quad & \quad i+N \quad \end{array} \\ \left(\begin{array}{c|c} 1 & \\ \hline & -1 \end{array}\right) \end{array} \qquad (i = 1, \cdots, N-1)$$

$$E_N = \begin{array}{c} \\ i \\ i+1 \\ \\ \end{array} \begin{array}{c} \begin{array}{cc} \quad i+N \quad & \quad i+N+1 \quad \end{array} \\ \left(\begin{array}{c|c} & \quad 1 \\ & -1 \quad \\ \hline & \end{array}\right) \end{array}$$

$$\begin{aligned}
\alpha_1 &= (1, -1, 0, 0, \cdots, 0, 0, 0) \\
\alpha_2 &= (0, 1, -1, 0, \cdots, 0, 0, 0) \\
\alpha_3 &= (0, 0, 1, -1, \cdots, 0, 0, 0) \\
&\cdots \\
\alpha_{N-1} &= (0, 0, 0, 0, \cdots, 0, 1, -1) \\
\alpha_N &= (0, 0, 0, 0, \cdots, 0, 1, 1)
\end{aligned}$$

walls

- elementary walls

$$H_{0\langle a\leftarrow b\rangle} = H_{0\langle a\rangle}\, e^{E_{(a\leftarrow b)}(r)}$$

- compressed wall with a level n

$$H_{0\langle a\leftarrow b\rangle} = H_{0\langle a\rangle}\, e^{[E_{a_1},[E_{a_2},[E_{a_3},\cdots,[E_{a_n},E_{a_{n+1}}]](r)}$$

  corresponding root $g_{n+1} = g_1 + g_2 + \cdots + g_n$

- multiwalls

$$H_{0\langle a\leftarrow b\rangle} = H_{0\langle a\rangle}\, e^{E_{a_1}(r_1)} e^{E_{a_2}(r_2)} \ldots e^{E_{a_n}(r_n)}$$

- Walls are penetrable if

$$[E_{a_i}, E_{a_j}] = 0.$$

  corresponding roots $g_i \cdot g_j = 0$

ordering of elementary walls in $SO(2N)/U(N)$

Quotation [N.Sakai& D.Tong (2005)]:
The positions of the elementary domain walls are not allowed to be arbitary : unlike solitons of higher co-dimensions, domain walls must obey at least some ordering on the line. In the abelian theory, this ordering is absolute and domain walls are not able to pass through each other [6,10]. In the non-abelian theory, there is (literally) room for manoeuvre and certain domain walls are allowed to pass through each other [17]. Such walls are said to be "penetrable".

[6] Gauntlett & Tong & Townsend PRD64 025010 (2001) arXiv:hep-th/0012178
[10] Tong PRD66 025013 (2002) arXiv:hep-th/0202012
[17] Isozumi & Nitta & Ohashi & Sakai PRD70 125014 (2004) arXiv:hep-th/0405194

ordering of elementary walls in $SO(2N)/U(N)$ (cont'd)

$\bullet CP^N$ is Abelian.
$\bullet SO(4)/U(2) \simeq CP^1$, $SO(6)/U(3) \simeq CP^3$

$SO(2N)/U(N)$, $N \leq 3 \rightarrow$ Abelian gauge theory
$SO(2N)/U(N)$, $N > 3 \rightarrow$ non-Abelian gauge theory

In non-Abelian gauge theory, there are penetrable walls.

walls on $SO(2N)/U(N)$

In $SO(2N)$, there are

${}_{2N}C_2$ generators

N Cartan generators

$2N^2 - 2N$ root generators

We are only interested in positive roots. So there are $N^2 - N$ root generators, which generate walls.

Walls on $SO(6)/U(3)$

•moduli matrices of vacua
# of vacua : 2^2

$$H_{0\langle 1\rangle} = \left(\begin{array}{ccc|ccc} 1 & & & 0 & & \\ & 1 & & & 0 & \\ & & 1 & & & 0 \end{array}\right) \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (m_1, m_2, m_3),$$

$$H_{0\langle 2\rangle} = \left(\begin{array}{ccc|ccc} 1 & & & 0 & & \\ & 0 & & & 1 & \\ & & 0 & & & 1 \end{array}\right), \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (m_1, -m_2, -m_3)$$

$$H_{0\langle 3\rangle} = \left(\begin{array}{ccc|ccc} 0 & & & 1 & & \\ & 1 & & & 0 & \\ & & 0 & & & 1 \end{array}\right), \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (-m_1, m_2, -m_3)$$

$$H_{0\langle 4\rangle} = \left(\begin{array}{ccc|ccc} 0 & & & 1 & & \\ & 0 & & & 1 & \\ & & 1 & & & 0 \end{array}\right), \quad (\Sigma_1, \Sigma_2, \Sigma_3) = (-m_1, -m_2, m_3)$$

The vacua are labelled in descending order of the summation of Σ_i.

Walls on $SO(6)/U(3)$ (cont'd)

•elementary wall operators

$$E_1 = \left(\begin{array}{ccc|ccc} 0 & 1 & & & & \\ & 0 & & & & \\ & & 0 & & & \\ \hline & & & 0 & & \\ & & & -1 & 0 & \\ & & & & & 0 \end{array}\right), \quad E_2 = \left(\begin{array}{ccc|ccc} 0 & & & & & \\ & 0 & 1 & & & \\ & & 0 & & & \\ \hline & & & 0 & & \\ & & & & 0 & \\ & & & & -1 & 0 \end{array}\right),$$

$$E_3 = \left(\begin{array}{ccc|ccc} & & & 0 & & \\ & & & & 0 & 1 \\ & & & & -1 & 0 \\ \hline & & & & & \\ & & & & & \\ & & & & & \end{array}\right).$$

Walls on $SO(6)/U(3)$ (cont'd)

•single walls, elementary walls

$$H_{0\langle 1\leftarrow 2\rangle} = H_{0\langle 1\rangle} e^{E_3(r)} = \left(\begin{array}{ccc|ccc} 1 & & & 0 & & \\ & 1 & & & 0 & e^r \\ & & 1 & & -e^r & 0 \end{array}\right)$$

$$H_{0\langle 2\leftarrow 3\rangle} = H_{0\langle 2\rangle} e^{E_1(r)} = \left(\begin{array}{ccc|ccc} 1 & e^r & & 0 & & \\ & 0 & & -e^r & 1 & \\ & & 0 & & & 1 \end{array}\right)$$

$$H_{0\langle 3\leftarrow 4\rangle} = H_{0\langle 3\rangle} e^{E_2(r)} = \left(\begin{array}{ccc|ccc} 0 & & & 1 & & \\ & 1 & e^r & & 0 & \\ & & 0 & & -e^r & 1 \end{array}\right)$$

$e^{E_i(r)}$: elementary wall operator

Walls on $SO(6)/U(3)$ (cont'd)

•single walls, compressed walls of level one

$$\tilde{E}_4 = [E_3, E_1], \ \tilde{E}_5 = [E_1, E_2]$$

$\sim$ is used to distinguish the operators from elementary wall operators.

$$H_{0\langle 1 \leftarrow 3 \rangle} = H_{0\langle 1 \rangle} e^{\tilde{E}_4(r_1)}$$
$$H_{0\langle 2 \leftarrow 4 \rangle} = H_{0\langle 2 \rangle} e^{\tilde{E}_5(r_1)}$$

•single walls, compressed walls of level two

$$\tilde{E}_6 = [E_3, \tilde{E}_5] = [\tilde{E}_4, E_2]$$

$$H_{0\langle 1 \leftarrow 4 \rangle} = H_{0\langle 1 \rangle} e^{\tilde{E}_6(r_1)}$$

$E_1,\ E_2,\ E_3,\ \tilde{E}_4,\ \tilde{E}_5,\ \tilde{E}_6 \rightarrow 6 (= 3^2 - 3)$ root generators

Walls on $SO(6)/U(3)$ (cont'd)

•double walls
: constructed by multiplying a single wall moduli matrix and a single wall operator

$$H_{0\langle 1\leftarrow 2\leftarrow 3\rangle} = H_{0\langle 1\leftarrow 2\rangle} e^{E_1(r_2)} = \left(\begin{array}{ccc|ccc} 1 & e^{r_2} & & 0 & & \\ & 1 & & & 0 & e^{r_1} \\ & & 1 & e^{r_1+r_2} & -e^{r_1} & 0 \end{array}\right)$$

$$H_{0\langle 2\leftarrow 3\leftarrow 4\rangle} = H_{0\langle 2\leftarrow 3\rangle} e^{E_2(r_2)} = \left(\begin{array}{ccc|ccc} 1 & e^{r_1} & e^{r_1+r_2} & 0 & & \\ & 0 & & -e^{r_1} & 1 & \\ & & 0 & & -e^{r_2} & 1 \end{array}\right)$$

$$H_{0\langle 1\leftarrow 2\leftarrow 4\rangle} = H_{0\langle 1\leftarrow 2\rangle} e^{\tilde{E}_5(r_2)} = \left(\begin{array}{ccc|ccc} 1 & & e^{r_2} & 0 & & \\ & 1 & & -e^{r_1+r_2} & 0 & e^{r_1} \\ & & 1 & & -e^{r_1} & 0 \end{array}\right)$$

$$H_{0\langle 1\leftarrow 3\leftarrow 4\rangle} = H_{0\langle 1\leftarrow 3\rangle} e^{E_2(r_2)} = \left(\begin{array}{ccc|ccc} 1 & & & 0 & e^{r_1+r_2} & -e^{r_1} \\ & 1 & e^{r_2} & & 0 & \\ & & 1 & e^{r_1} & & 0 \end{array}\right)$$

Walls on $SO(6)/U(3)$ (cont'd)

•triple wall

$$H_{0\langle 1\leftarrow 2\leftarrow 3\leftarrow 4\rangle} = H_{0\langle 1\leftarrow 2\leftarrow 3\rangle} e^{E_2(r_3)} = \left(\begin{array}{ccc|ccc} 1 & e^{r_2} & e^{r_2+r_3} & 0 & & \\ & 1 & e^{r_3} & & -e^{r_1+r_3} & e^{r_1} \\ & & 1 & e^{r_1+r_2} & -e^{r_1} & 0 \end{array}\right)$$

Walls on $SO(6)/U(3)$ (cont'd)

• $SO(6)$ Cartan generators

$$H_1 = \mathrm{diag}(1, 0, 0, -1, 0, 0)$$
$$H_2 = \mathrm{diag}(0, 1, 0, 0, -1, 0)$$
$$H_3 = \mathrm{diag}(0, 0, 1, 0, 0, -1)$$

• simple roots of $SO(6)$

$$\underline{\alpha}_1 := (1, -1, 0)$$
$$\underline{\alpha}_2 := (0, 1, -1)$$
$$\underline{\alpha}_3 := (0, 1, 1)$$

Walls on $SO(6)/U(3)$ (cont'd)

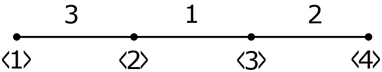
•$g_{\langle i \leftarrow j \rangle}$: root of wall $\langle i \rangle \leftarrow \langle j \rangle$
•elementary walls

$$g_{\langle 1 \leftarrow 2 \rangle} = \underline{\alpha}_3$$
$$g_{\langle 2 \leftarrow 3 \rangle} = \underline{\alpha}_1$$
$$g_{\langle 3 \leftarrow 4 \rangle} = \underline{\alpha}_2$$

•compressed walls

$$g_{\langle 1 \leftarrow 3 \rangle} = \underline{\alpha}_3 + \underline{\alpha}_1$$
$$g_{\langle 2 \leftarrow 4 \rangle} = \underline{\alpha}_1 + \underline{\alpha}_2$$
$$g_{\langle 1 \leftarrow 4 \rangle} = \underline{\alpha}_1 + \underline{\alpha}_2 + \underline{\alpha}_3$$

$\rightarrow$ We can guess multiwalls.



Walls on $SO(8)/U(4)$

•vacua
# of vacua : 2^3

$$\begin{aligned}
H_{0\langle 1\rangle} &: (\Sigma_1,\Sigma_2,\Sigma_3,\Sigma_4)=(m_1,m_2,m_3,m_4)\\
H_{0\langle 2\rangle} &: (\Sigma_1,\Sigma_2,\Sigma_3,\Sigma_4)=(m_1,m_2,-m_3,-m_4)\\
H_{0\langle 3\rangle} &: (\Sigma_1,\Sigma_2,\Sigma_3,\Sigma_4)=(m_1,-m_2,m_3,-m_4)\\
H_{0\langle 4\rangle} &: (\Sigma_1,\Sigma_2,\Sigma_3,\Sigma_4)=(m_1,-m_2,-m_3,m_4)\\
H_{0\langle 5\rangle} &: (\Sigma_1,\Sigma_2,\Sigma_3,\Sigma_4)=(-m_1,m_2,m_3,-m_4)\\
H_{0\langle 6\rangle} &: (\Sigma_1,\Sigma_2,\Sigma_3,\Sigma_4)=(-m_1,m_2,-m_3,m_4)\\
H_{0\langle 7\rangle} &: (\Sigma_1,\Sigma_2,\Sigma_3,\Sigma_4)=(-m_1,-m_2,m_3,m_4)\\
H_{0\langle 8\rangle} &: (\Sigma_1,\Sigma_2,\Sigma_3,\Sigma_4)=(-m_1,-m_2,-m_3,-m_4)
\end{aligned}$$

Walls on $SO(8)/U(4)$ (cont'd)

• simple roots of $SO(8)$

$$\underline{\alpha}_1 := (1, -1, 0, 0)$$
$$\underline{\alpha}_2 := (0, 1, -1, 0)$$
$$\underline{\alpha}_3 := (0, 0, 1, -1)$$
$$\underline{\alpha}_4 := (0, 0, 1, 1)$$

• elementary walls

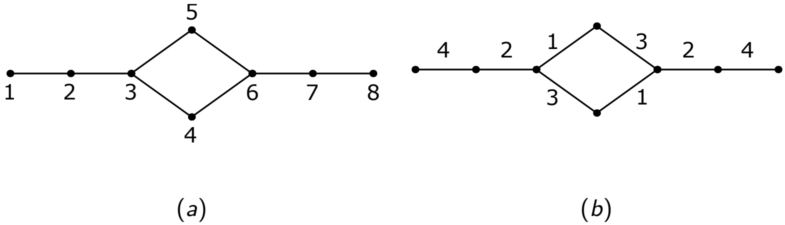
$$g_{\langle 3\leftarrow 4\rangle} = g_{\langle 5\leftarrow 6\rangle} = \underline{\alpha}_1$$
$$g_{\langle 2\leftarrow 3\rangle} = g_{\langle 6\leftarrow 7\rangle} = \underline{\alpha}_2$$
$$g_{\langle 3\leftarrow 5\rangle} = g_{\langle 4\leftarrow 6\rangle} = \underline{\alpha}_3$$
$$g_{\langle 1\leftarrow 2\rangle} = g_{\langle 7\leftarrow 8\rangle} = \underline{\alpha}_4$$

• penetrable walls

$$\underline{\alpha}_i \cdot \underline{\alpha}_j = 0$$

Walls on $SO(8)/U(4)$ (cont'd)

•vacua and elementary walls of $SO(8)/U(4)$

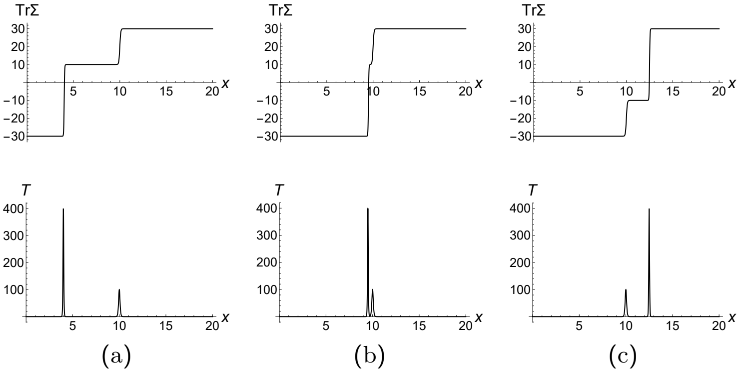


(a)Vacua of $SO(8)/U(4)$. The numbers indicate the label of vacua.
(b)Elementary walls of $SO(8)/U(4)$. The numbers indicate the subscript i of simple roots $\underline{\alpha}_i$, $(i = 1, \cdots, 4)$.

Walls on $SO(8)/U(4)$ (cont'd)

•penetrable walls

double wall $\langle 3 \leftarrow 5 \leftarrow 6 \rangle$ in $SO(8)/U(4)$, which consists of two penetrable walls. $m_1 = 70$, $m_2 = 50$, $m_3 = 30$, $m_4 = 20$. (a)$r_1 = 80$, $r_2 = 100$ (b)$r_1 = 200$, $r_2 = 100$ (c)$r_1 = 250$, $r_2 = 100$.



Walls on $SO(8)/U(4)$ (cont'd)

•compressed wall

Double wall $\langle 2 \leftarrow 3 \leftarrow 5 \rangle$ can be compressed. The moduli matrix is

$$
\begin{aligned}
H_{0\langle 2\leftarrow 3\leftarrow 5\rangle} &= H_{0\langle 2\rangle} e^{E_2(r_1)} e^{E_1(r_2)} \\
&= \left(\begin{array}{cccc|cccc}
1 & e^{r_2} & & & 0 & & & \\
 & 1 & e^{r_1} & & & 0 & & \\
 & & 0 & & e^{r_1+r_2} & -e^{r_1} & 1 & \\
 & & & 0 & & & & 1
\end{array}\right)
\end{aligned}
$$

Under the worldvolume symmetry transformation, $H_{0\langle 2\leftarrow 3\leftarrow 5\rangle}$ can be transformed to

$$
\begin{aligned}
H_{0\langle 2\leftarrow 3\leftarrow 5\rangle} &\rightarrow \begin{pmatrix}
1 & -e^{r_2} & & \\
 & 1 & & \\
 & & 1 & \\
 & & & 1
\end{pmatrix}
\left(\begin{array}{cccc|cccc}
1 & e^{r_2} & & & 0 & & & \\
 & 1 & e^{r_1} & & & 0 & & \\
 & & 0 & & e^{r_1+r_2} & -e^{r_1} & 1 & \\
 & & & 0 & & & & 1
\end{array}\right) \\
&= \left(\begin{array}{cccc|cccc}
1 & & -e^{r_1+r_2} & & 0 & & & \\
 & 1 & e^{r_1} & & & 0 & & \\
 & & 0 & & e^{r_1+r_2} & -e^{r_1} & 1 & \\
 & & & 0 & & & & 1
\end{array}\right)
\end{aligned}
$$

In the limit where $r_1 + r_2 = r$ (finite) and $r_1 \rightarrow -\infty$, double wall $H_{0\langle 2\leftarrow 3\leftarrow 5\rangle}$ becomes compressed wall of level one $H_{0\langle 2\leftarrow 5\rangle} = H_{0\langle 2\rangle} e^{[E_2, E_1](r)}$.

Walls on $SO(10)/U(5)$

•vacua
# of vacua : 2^4

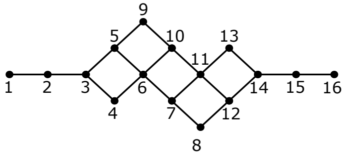
•simple roots of $SO(10)$

$$\underline{\alpha}_1 := (1, -1, 0, 0, 0)$$
$$\underline{\alpha}_2 := (0, 1, -1, 0, 0)$$
$$\underline{\alpha}_3 := (0, 0, 1, -1, 0)$$
$$\underline{\alpha}_4 := (0, 0, 0, 1, -1)$$
$$\underline{\alpha}_5 := (0, 0, 0, 1, 1)$$

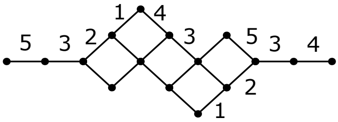
•elementary walls

$$g_{\langle 4 \leftarrow 5 \rangle} = g_{\langle 7 \leftarrow 8 \rangle} = g_{\langle 9 \leftarrow 10 \rangle} = g_{\langle 12 \leftarrow 13 \rangle} = \underline{\alpha}_1$$
$$g_{\langle 3 \leftarrow 4 \rangle} = g_{\langle 6 \leftarrow 7 \rangle} = g_{\langle 10 \leftarrow 11 \rangle} = g_{\langle 13 \leftarrow 14 \rangle} = \underline{\alpha}_2$$
$$g_{\langle 2 \leftarrow 3 \rangle} = g_{\langle 7 \leftarrow 9 \rangle} = g_{\langle 8 \leftarrow 10 \rangle} = g_{\langle 14 \leftarrow 15 \rangle} = \underline{\alpha}_3$$
$$g_{\langle 3 \leftarrow 6 \rangle} = g_{\langle 4 \leftarrow 7 \rangle} = g_{\langle 5 \leftarrow 8 \rangle} = g_{\langle 15 \leftarrow 16 \rangle} = \underline{\alpha}_4$$
$$g_{\langle 1 \leftarrow 2 \rangle} = g_{\langle 9 \leftarrow 12 \rangle} = g_{\langle 10 \leftarrow 13 \rangle} = g_{\langle 11 \leftarrow 14 \rangle} = \underline{\alpha}_5$$

Walls on $SO(10)/U(5)$ (cont'd)



vacua

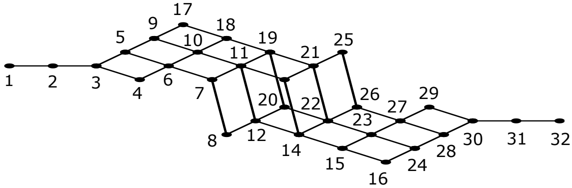


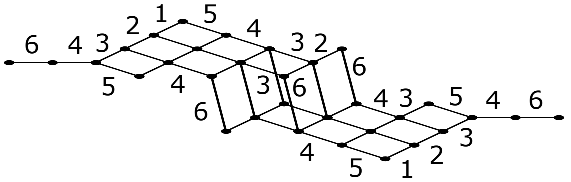
elementary walls

As before a pair of facing sides of each parallelogram are the same and a pair of adjacent sides of each parallelogram are orthogonal.

Walls on $SO(12)/U(6)$

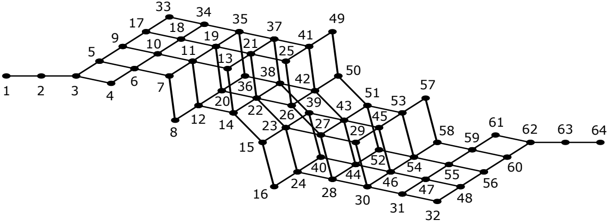
•vacua and elementary walls

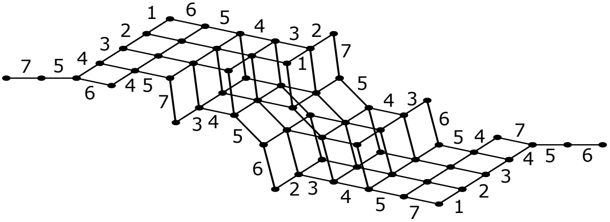




Walls on $SO(14)/U(7)$

•vacua and elementary walls





Generalization

• $N = 2$

$$\underline{2} \leftarrow \langle 2 \rangle$$

• $N = 3$

$$\underline{3} \leftarrow \langle 2 \rangle \leftarrow \underline{1} \leftarrow \langle 3 \rangle \leftarrow \underline{2}$$

• $N = 4$

$$\underline{4} \leftarrow \underline{2} \leftarrow \langle 3 \rangle \leftarrow \{\underline{1}, \underline{3}\} \leftarrow \cdots \leftarrow \{\underline{1}, \underline{3}\} \leftarrow \langle 6 \rangle \leftarrow \underline{2} \leftarrow \underline{4}$$

• $N = 5$

$$\underline{5} \leftarrow \cdots \leftarrow \{\underline{2}, \underline{4}\} \leftarrow \langle 6 \rangle \leftarrow \{\underline{1}, \underline{3}\} \leftarrow \cdots$$
$$\cdots \leftarrow \{\underline{1}, \underline{3}\} \leftarrow \langle 11 \rangle \leftarrow \{\underline{2}, \underline{5}\} \leftarrow \cdots \leftarrow \underline{4}$$

• $N = 6$

$$\underline{6} \leftarrow \cdots \leftarrow \{\underline{2}, \underline{4}\} \leftarrow \langle 11 \rangle \leftarrow \{\underline{1}, \underline{3}, \underline{6}\} \leftarrow \cdots$$
$$\cdots \leftarrow \{\underline{1}, \underline{3}, \underline{6}\} \leftarrow \langle 22 \rangle \leftarrow \{\underline{2}, \underline{4}\} \leftarrow \cdots \leftarrow \underline{6}$$

• $N = 7$

$$\underline{7} \leftarrow \cdots \leftarrow \{\underline{2}, \underline{4}, \underline{7}\} \leftarrow \langle 22 \rangle \leftarrow \{\underline{1}, \underline{3}, \underline{5}\} \leftarrow \cdots$$
$$\cdots \leftarrow \{\underline{1}, \underline{3}, \underline{5}\} \leftarrow \langle 43 \rangle \leftarrow \{\underline{2}, \underline{4}, \underline{6}\} \leftarrow \cdots \leftarrow \underline{6}$$

Generalization (cont'd)

$\bullet N = 4m - 2 \ (m \geq 2)$

$$\underline{N}(= 4m - 2) \leftarrow \cdots$$

$$\cdots \leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4(m-1)}}_{(2m-2)} \right\} \leftarrow \langle A \rangle \leftarrow$$

$$\leftarrow \left\{ \underbrace{\underbrace{\underline{1}, \underline{3}, \cdots, \underline{2(2m-2)-1}}_{(2m-2)}, \underline{4m-2}}_{(2m-1)} \right\} \leftarrow \cdots$$

$$\cdots \leftarrow \left\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-5}, \underline{4m-2}}_{(2m-1)} \right\} \leftarrow \langle B \rangle \leftarrow$$

$$\leftarrow \left\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4(m-1)}}_{(2m-2)} \right\} \leftarrow \cdots$$

$$\cdots \leftarrow \underline{N}(= 4m - 2)$$

Generalization (cont'd)

$\bullet N = 4m - 1 \ (m \geq 2)$

$$
\begin{aligned}
&\underline{N}(= 4m-1) \leftarrow \cdots \\
&\cdots \leftarrow \Bigg\{ \underbrace{\underbrace{\underline{2}, \underline{4}, \cdots, \underline{4(m-1)}}_{2(m-1)}, \underline{4m-1}}_{(2m-1)} \Bigg\} \leftarrow \langle A \rangle \leftarrow \\
&\leftarrow \Bigg\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{2(2m-2)-1}, \underline{2(2m-1)-1}}_{2m-1} \Bigg\} \cdots \\
&\cdots \leftarrow \Bigg\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-5}, \underline{4m-3}}_{2m-1} \Bigg\} \leftarrow \langle B \rangle \leftarrow \\
&\leftarrow \Bigg\{ \underbrace{\underline{2}, \underline{4}, \cdots, 4m-2}_{(2m-1)} \Bigg\} \leftarrow \cdots \\
&\leftarrow \underline{N-1}(= 4m-2)
\end{aligned}
$$

Generalization (cont'd)

$\bullet N = 4m \ (m \geq 2)$

$$
\begin{aligned}
&\underline{N}(=4m) \leftarrow \cdots \\
&\cdots \leftarrow \Big\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-2}}_{(2m-1)} \Big\} \leftarrow \langle A \rangle \leftarrow \\
&\leftarrow \Big\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{2(2m)-1}}_{(2m)} \Big\} \leftarrow \cdots \\
&\cdots \leftarrow \Big\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-1}}_{(2m)} \Big\} \leftarrow \langle B \rangle \leftarrow \\
&\leftarrow \Big\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m-2}}_{(2m-1)} \Big\} \leftarrow \cdots \\
&\cdots \leftarrow \underline{N}(=4m)
\end{aligned}
$$

Generalization (cont'd)

$\bullet N = 4m + 1 \ (m \geq 2)$

$$\begin{aligned}
&\underline{N}(= 4m+1) \leftarrow \cdots \\
&\cdots \leftarrow \Big\{ \underbrace{\underline{2}, \underline{4}, \cdots, \underline{4m}}_{(2m)} \Big\} \leftarrow \langle A \rangle \leftarrow \\
&\leftarrow \Big\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{2(2m)-1}}_{(2m)} \Big\} \leftarrow \cdots \\
&\cdots \leftarrow \Big\{ \underbrace{\underline{1}, \underline{3}, \cdots, \underline{4m-1}}_{(2m)} \Big\} \leftarrow \langle B \rangle \leftarrow \\
&\leftarrow \Big\{ \underbrace{\underbrace{\underline{2}, \underline{4}, \cdots, \underline{2(2m-1)}}_{(2m-1)}, \underline{4m+1}}_{2m} \Big\} \leftarrow \cdots \\
&\cdots \leftarrow \underline{N-1}(= 4m)
\end{aligned}$$

Summary

- SO(2N) constraint is imposed to the moduli matrices of walls.

- Walls are constructed in nonlinear sigma models on $SO(2N)/U(N)$, $N \geq 2$ (by using Mathematica).

- Penetrable walls are observed in nonlinear sigma models on $SO(2N)/U(N)$ with $N > 3$.

Outlook

- $Sp(N)$ constraint (work in progress)
- composite solitons
- multiple constraints
- recurrence relations

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- $Sp(N)$ constraint (work in progress)
- composite solitons
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Thank you !

Спасибо !