Constrained BRST-BFV & BRST-BV Lagrangian formulations for half-integer HS fields on $R^{1,d-1}$

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Based on research with J.Buchbinder; with J.Buchbinder, H.Takata
J.Buchbinder, A.R., in progress; J.Buchbinder, A.R.,H.Takata in progress

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Motivations: (un)constrained HS formulations on $\mathbb{R}^{1,d-1}$, (A)dS$_d$, SFT

Starting (M. Fierz, W. Pauli; V. Ginzburg; E.Fradkin; L. Singh, C. Hagen; C. Fronsdal, M.Vasiliev) the problems of HS field theory have attracted & attracting the significant attention as one from directions of the research in LHC to find new matter & interactions, (TS for $k=1$ row in Young tableau $Y(s_1, ..., s_k)$), (MS $k>1$) $\mathbf{s} = (s_1, s_2, ...)$ (massive and massless:$m=0$) HS fields :

$$\Phi_{(\mu)_{s_1},(\nu)_{s_2},\ldots,(\rho)_{s_k}} \longleftrightarrow \begin{array}{|c|c|ccc|c|c|c|c|c|}\hline \mu_1 & \mu_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \mu_{s_1} \\ \hline \nu_1 & \nu_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \nu_{s_2} & & \\ \hline \cdots & \cdots & \cdots & \cdot & \cdot & \cdot & \cdots & & & \\ \hline \end{array} = Y(s_1, ..., s_k) .$$

in view of connection to SuperString Field Theory (SFT): (E. Witten (1986); C. Thorn(1989)) through special tensionless limit for intercept $(\alpha' \to \infty)$: (A. Sagnotti, M. Tsulaia, (2004)).

$$\boxed{\Longrightarrow \texttt{SFT} \stackrel{\alpha'\to\infty}{\longrightarrow} \{\infty\} \ \texttt{set of HS fields in s/string spectrum}}$$

As shown, Tsulaia & Sagnotti (2004) the free dynamic for massless TS, MS HS fields follows from the tensionless limit for free open bosonic strings leads to triplet-like LF and after imposing of appropriate off-shell constraints on the field and gauge parameters to the Fronsdal formulation for integer HS fields (on $\mathbb{R}^{1,d-1}$) in terms of $\Phi^{(\mu)_s}$, $\delta\Phi^{(\mu)_s} = -\partial^{\{\mu_s}\xi^{(\mu\})_{s-1}}$ **for** $\Phi'' = 0$ **and** $\xi' = 0$ - *holonomic off-shell constraints.*
Tensionless limit: (for $[a_{\mu_k}, a^+_{\nu_l}] = -\delta_{kl}\eta_{\mu_k\nu_l}$, $\mu_k = 0, 1, ..., d-1$, $k, l \in \mathbb{N}_0$, $\mathrm{diag}\eta_{\mu\nu} = (+, -, ..., -)$)

$$\lim_{\alpha'\to\infty} Q_V \;= Q = \eta_0 l_0 + \sum_{k>0}^{\infty}\Big[\eta^+_k l_k + \eta_k l^+_k + \imath\eta^+_k\eta_k\mathcal{P}_0\Big] = \eta_0 l_0 - \imath\mathcal{P}_0 M + \Delta Q,$$

$$\text{for } \Big(l_k,\, l^+_k,\, l_0\Big) = \Big(-ia^{\mu_k}\partial_{\mu_k},\, -ia^{+\mu_k}\partial_{\mu_k},\, \partial^2\Big);\ \boxed{\{\eta^+_k, \mathcal{P}_l\} = \delta_{kl}, \{\eta_0,\, \mathcal{P}_0\} = \imath};$$

$$\underset{algebra}{\overset{Virasoro}{}}:\ \lim_{\alpha'\to\infty}[L_k, L^+_l] = [l_k, l^+ l_l] = l_0\delta_{lk}$$

$Q^2 = 0\ \forall\ d$ as compared to $Q^2_V = 0$, when $d = 26$ (1983).

Motivations: SFT $\rightarrow$ reducible bosonic HS fields

The String Field equation, for vacuum $(\mathcal{P}_0, \mathcal{P}_k, \eta_k)|0\rangle = 0$, $k > 0$ in $D = 26$
$\rightarrow Q^2 = 0$ being subject to tower of reducible gauge symmetries,

$$Q_V|\Phi\rangle = 0,\ \delta|\Phi\rangle = Q_V|\Lambda_0\rangle,\ \delta|\Lambda_0\rangle = Q_V|\Lambda_1\rangle, \ldots \tag{1}$$

turn in tensionless limit after expanding as Q the string field $|\Phi\rangle$, $|\Lambda_0\rangle$, ...

$$|\Phi\rangle = |\varphi_1\rangle + \eta_0|\varphi_2\rangle, \quad |\Lambda_k\rangle \ = |\Lambda^0_{1k}\rangle + \eta_0|\Lambda^1_k\rangle, \ldots \tag{2}$$

in terms of η_0-independent equations

$$l_0|\varphi_1\rangle - \Delta Q|\varphi_2\rangle = 0, \quad \Delta Q|\varphi_1\rangle - M|\varphi_2\rangle = 0, \tag{3}$$

$$\delta|\varphi_1\rangle \ = \Delta Q|\Lambda^0_{10}\rangle - M|\Lambda^1_0\rangle, \quad \delta|\varphi_2\rangle \ = l_0|\Lambda^0_{10}\rangle - \Delta Q|\Lambda^1_0\rangle, \tag{4}$$

(!) In case of TS $\varphi_{\mu_1\ldots\mu_s}$ due to homogeneity in ghost number distribution

$$\begin{aligned} |\varphi_1\rangle &= \varphi_{(\mu)_s} a^{\mu_1+}...a^{\mu_s+}|0\rangle + \eta_1^+\mathcal{P}_1^+ D_{(\mu)_{s-2}} a^{\mu_1+}...a^{\mu_{s-2}+}|0\rangle, \\ |\varphi_2\rangle &= \mathcal{P}_1^+ C_{(\mu)_{s-1}} a^{\mu_1+}...a^{\mu_{s-1}+}|0\rangle, \quad |\Lambda_0\rangle \ = \mathcal{P}_1^+ \Lambda_{(\mu)_{s-1}} a^{\mu_1+}...a^{\mu_{s-1}+}|0\rangle. \end{aligned}$$

1.) Triplet formulation in terms of $\varphi_{(\mu)_s}, C_{(\mu)_{s-1}}, D_{(\mu)_{s-2}}$ reducible repr. $iso(1, d-1)$ (for $s = 1, 2$ -Bengtsson, 1984)
EoM (3) can be written schematically (with and) without oscillators as,

$$\partial^2\varphi_{(\mu)_s} - (\partial C)_{(\mu)_s} = 0, \quad (\partial\cdot\varphi)_{(\mu)_{s-1}} - (\partial D)_{(\mu)_{s-1}} = C_{(\mu)_{s-1}}, \tag{5}$$

$$\partial^2 D_{(\mu)_{s-2}} - (\partial\cdot C)_{(\mu)_{s-2}} = 0, \quad \delta\Big(\varphi, C, D\Big) \ = \Big(\partial, \partial^2, \partial\cdot\Big)\Lambda,$$

$$l_0|\varphi\rangle_s - l_1^+|C\rangle_{s-1} = 0, \quad l_1|\varphi\rangle_s - l_1^+|D\rangle_{s-2} = |C\rangle_{s-1}, \tag{6}$$
$$l_0|D\rangle_{s-2} - l_1|C\rangle_{s-1} = 0$$
$$\delta\Big(|\varphi\rangle_s, |C\rangle_{s-1}, |D\rangle_{s-2}\Big) \;=\Big(l_1^+, l_0, l_1\Big)|\Lambda\rangle_{s-1} \tag{7}$$

System (5) or (6) is Lagrangian and is derived from the action

$$S(\Phi) \;= \int d\eta_{0s}\langle\Phi|Q|\Phi\rangle_s, \delta|\Phi\rangle_s = Q|\Lambda_0\rangle_s, \quad \text{for gh}(|\Phi\rangle, |\Lambda_0\rangle) = (0,-1) \tag{8}$$

(Francia, Sagnotti 2003) - massless reducible representations, $ISO(1,d-1)$ of HS fields with $(s, s-2, ..., 1/0)$
Imposing BRST-extended ($[Q, \mathcal{L}_{11}] = 0$) traceless constraints $\mathcal{L}_{11}$ on $|\Phi\rangle, |\Lambda_0\rangle$ $[Q, \mathcal{L}_{11}] = 0$:

$$\mathcal{L}_{11}\Big(|\Phi\rangle, |\Lambda_0\rangle\Big) = \Big(l_{11} + \eta_1 \mathrm{P}_1\Big)\Big(|\Phi\rangle, |\Lambda_0\rangle\Big) = (0,0), \text{ for } l_{11} = 1/2a^\mu a_\mu. \tag{9}$$

2.) $\Longrightarrow$ to Fronsdal (1978) formulation literally with traceless $\Lambda_{(\mu)_{s-1}}$, and only surviving doubly traceless HS field $\varphi_{(\mu)_s}$.

3.) if we try to enlarge triplet F. up to minimal without higher derivatives GI LF incorporating Eq. (9) in so-called Unconstrained or quartet formulation, then addition of 1 compensator $\delta\alpha_{(\mu)_{s-3}} = (Tr\Lambda)_{(\mu)_{s-3}}$ with GI extension of 2 traceless constraints on fields

$$(Tr\varphi)_{(\mu)_{s-2}} - D_{(\mu)_{s-2}} + (\partial\alpha)_{(\mu)_{s-2}} = 0, \quad (TrD)_{(\mu)_{s-4}} + (\partial\cdot\alpha)_{(\mu)_{s-4}} = 0,$$

$$\Leftrightarrow \; l_{11}|\varphi\rangle_s - |D\rangle_{s-2} + l_1^+|\alpha\rangle_{s-3} = 0, \;\; l_{11}|D\rangle_{s-2} + l_1|\alpha\rangle_{s-3} = 0 \qquad (10)$$

makes joint system (5) and (10) by Lagrangian (with non-gauge HS tensors $\lambda_{1(\mu)_{s-2}}$, $\lambda_{2(\mu)_{s-4}}$ vanishing on shell determining by the EoM for the rest fields:

$$S(\Phi,\alpha,\lambda_1,\lambda_2) \;=\; S(\Phi) + \Big\{{}_{s-2}\langle\lambda_1|\Big(l_{11}|\varphi\rangle_s - |D\rangle_{s-2} + l_1^+|\alpha\rangle_{s-3}\Big) \qquad (11)$$

$$+{}_{s-4}\langle\lambda_2|\Big(l_{11}|D\rangle_{s-2} + l_1|\alpha\rangle_{s-3}\Big) + h.c.\Big\}),$$

$$\delta\Big(|\varphi\rangle_s, |C\rangle_{s-1}, |D\rangle_{s-2}, |\alpha\rangle_{s-3}, |\lambda_i\rangle_{s-2i}\Big) = \Big(l_1^+,\, l_0,\, l_1,\, l_{11}, 0\Big)|\Lambda\rangle_{s-1} \qquad (12)$$

(J.Buchbinder, A. Galajinskii, V.Krykhtin, 2007).

Motivations: SFT $\rightarrow$ bosonic HS fields

Summary 1: the triplet-like formalism for reducible HS theories exist for mixed-symmetric tensor fields (firstly, derived by Tsulaia & Sagnotti)
Summary 2: addition off-shell BRST extended traceless and Young symmetry constraints (on all the field vector and sequence of Gauge parameters) commuting with BRST operator leads to constrained GI LF of the form (8) for spin $(s_1, s_2 \ldots, s_k)$ fields (Barnich, Grigoriev, Semikhatov, Tipunin 2004; Alkalaev Grigoriev,Tipunin, 2008)
Summary 3: One may consider the problem from the beginning to find BRST-like action (8), so that the whole set of irreps Poincare group conditions follows as Lagrangian EoM, i.e.;

$$\partial^2 \varphi_{\mu\nu\ldots} = 0, \quad \partial^\mu \varphi_{\mu\nu\ldots} = 0, \quad \eta^{\mu\nu} \varphi_{\mu\nu\ldots} = 0 \qquad (13)$$

should be considered equally.
Sum. 3 is the essence of Unconstrained BRST-BFV approach for Lagrangian formulation construction for HS fields on $\mathbb{R}^{1,d-1}$ & AdS(d)
Summary 4: the same procedure of tensionless limit for the superstring theory leads to analogous triplet-like (Tsulaia & Sagnotti (2004)) , then to quartet-like formulations initially for reducible half-integer $ISO(1, d-1)$ reps and then irreps.

• We will call the any Lagrangian formulation (LF) for given HS field (or fields) by Unconstrained LF for given HS field (or fields), if there are no (holonomic or not-holonomic) constraints on the field(s) and reducible gauge parameters which can not be produced from the Lagrangian action. In opposite case we will call the Lagrangian formulation (LF) for given HS field (or fields) by Constrained LF for given HS field (or fields).

E.g. almost whole LFs for HS fields on $R^{1,d-1}$, AdS$_d$ obtained in frame-like formalism (E.Skvortsov, M.Vasiliev, Yu.Zinoviev, M.Grigoriev, D.Ponomarev) appear by Constrained LF due to algebraic constraints on field and g.parameters. The BRST and BRST-BV approaches developed in the recent papers by R.Metsaev in metric-like formalism appear by the Constrained LF as well.
The unconstrained LF as well were developed for (half-)integer HS fields on $R^{1.d-1}$ in Campoleoni A, Francia D, Mourad J and Sagnotti A, 2009, 2010 NPB

We develop within BRST-BFV approach for HS fields the Unconstrained LF, which after partial gauge-fixing may lost "Unconstrained" property.
A. R., PEPAN (2017) arXiv:1604.00620 (MAS $Y[s_1, s_2]$ on $R^{1,d-1}$ developed earlier by X.Bekaert, N.Boulanger, S, Cnockaert 2004;
A. R., NPB (2013) arXiv:1211.1273;
I. Buchbinder, A. R., NPB (2012) arXiv:1110.5044;
P.Moshin, A. R., JHEP 0710 (2007) 040 arXiv:0707.0386 ;
A. R., NPB 869 (2013) 523 arXiv:1211.1273;
and on AdS(d) space: I.Buchbinder, V.Krykhtin, P.Lavrov NPB 2006 hep-th/0608005;
I.L. Buchbinder, V.A. Krykhtin, A. R., NPB B787 2007 hep-th/0703049;
C. Burdik, A. R.On representations of Higher Spin symmetry algebras for mixed-symmetry HS fields on AdS-spaces. Lagrangian formulation arXiv:1111.5516

What are BRST-BFV and BRST-BV approaches to derive of (un)constrained Lagrangians for HS fields?

Within stringy-inspired BRST-BFV approach (S. Ouvry, J. Stern, A. Bengtsson, A. Pashnev, M. Tsulaia, J. Buchbinder, V. Krykhtin, A.R.)
In opposite to direct problem of Gen. Ham. Quants of the Constrained dyn.systems the aim of the inverse problem consists in the CONSTRUCTION OF LF FOR HS FIELD WITH GIVEN (m, s)

Irreps conditions ISO(1,d-1), SO(2,d-1)	$\xrightarrow{\text{SFT}}$	(Super)algebra$\{o_I(x)\} : \mathcal{H}$ $[o_I, o_J\} = f^K_{IJ}(o)o_k + \Delta_{ab}(g_0)$

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$$\xrightarrow[\text{Burdik, Pashnev}]{\texttt{conversion}} \boxed{\begin{array}{c}O_I = o_I + o'_I : \mathcal{H} \bigotimes \mathcal{H}'\\ [O_I, O_J\} = F^K_{IJ}(o', O)O_K\end{array}}$$

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$$\xrightarrow[\text{Henneaux}]{\texttt{BFV}} \boxed{\begin{array}{c}\texttt{BRST operator for } \{O_I\}\text{:}Q'(x)\\ Q' = C^I O_I + \frac{1}{2}C^I C^J F^K_{IJ}\mathcal{P}_K + \text{more}\end{array}}$$

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Irreps conditions ISO(1,d-1), SO(2,d-1) $\xrightarrow{\text{SFT}}$ (Super)algebra$\{o_I(x)\} : \mathcal{H}$, $[o_I, o_J\} = f^K_{IJ}(o)o_k + \Delta_{ab}(g_0)$

$\xrightarrow[\text{Burdik, Pashnev}]{\text{conversion}}$ $O_I = o_I + o'_I : \mathcal{H} \bigotimes \mathcal{H}'$, $[O_I, O_J\} = F^K_{IJ}(o', O)O_K$

$\xrightarrow[\text{Henneaux}]{\text{BFV}}$ BRST operator for $\{O_I\}$:$Q'(x)$, $Q' = C^I O_I + \frac{1}{2} C^I C^J F^K_{IJ} \mathcal{P}_K + \text{more}$

$\xrightarrow{\text{LF}}$

$$Q' = Q + (g_0 + h + \text{more})C_g + \ldots : Q^2 = 0$$

mass-shell : $Q|\Psi\rangle = 0, \mathrm{gh}(|\Psi\rangle) = 0 \Leftarrow$ action : $S = \int d\eta_0 \langle\Psi|KQ|\Psi\rangle$

spin: $(g_0 + \text{more})(|\Psi\rangle, |\Lambda\rangle, \ldots) = -h(|\Psi\rangle, |\Lambda\rangle, \ldots)$

gauge transfs: $\delta|\Psi\rangle = Q|\Lambda\rangle, \delta|\Lambda\rangle = Q|\Lambda^1\rangle, \ldots$

At 2-3rd steps the Stuckelberg and gauge fields are appeared automatically to obtain GI LF for basic field

What are BRST-BFV and BRST-BV approaches to derive of (un)constrained Lagrangians for HS fields?

For BRST-BV method the requirement $gh(\Psi_{(\mu)_s....}, \Psi_{(\mu)_s....}) = 0$ for HS components in $|\Psi\rangle, |\Lambda\rangle, ..$ (used in BRST-BFV) are weaken:

$gh(\Psi_{(\mu)_s....}, \Psi_{(\mu)_s....}) > 0 -$ `ghost fields` $gh(\Psi_{(\mu)_s....}, \Psi_{(\mu)_s....}) < 0$ `antifields`

`jointly in` $|\Psi_{\text{gen}}\rangle \Rightarrow$ $S_{BV} = \int d\eta_0 \langle\Psi_{\text{gen}}|KQ|\Psi_{\text{gen}}\rangle = S(\Psi) + \text{"}more\text{"}$

minimal Batalin-Vilkovisky action encoding (free) classical action $S(\Psi)$ and gauge algebra in "more".

For constrained case there are no second-class constraints in the HS symmetry algebra, no conversion, $K = \hat{1}$, the spectrum of the components fields is smaller, there are off-shell (BRST-extended) constraints but the constrained dynamic will be equivalent to the unconstrained one!

The constrained BRST-BFV and BRST-BV approach for LF and minimal BV action was known only for integer spin cases(!) Barnich, Grigoriev, Semikhatov, Tipunin 2004; Alkalaev Grigoriev,Tipunin, 2008, 2011); R.Metsaev 20012-2017

What problems to solve: unknown methods to derive LF and BV actions for fermionic HS fields?

- What can we state for (1) unconstrained BRST-BV method for integer HS fields?;
- What can we state for (2) constrained BRST-BFV and BRST-BV approaches for half-integer HS fields?;
- Does it possible to derive 1) the Fang-Fronsdal (1978) Lagrangian formulation for massless spin-tensor $\Psi_{(\mu)_n}$;
  2) the triplet formulation for massless spin-tensor $\Psi_{(\mu)_n}$ (D. Francia, A. Sagnotti 2003,2005);
  3) the unconstrained quartet formulation as it was done for integer TS HS fields on $R^{1,d-1}$ (J.Buchbinder, A. Galajinskii, V.Krykhtin, 2007)?

1) the Fang-Fronsdal (1978) Lagrangian formulation for massless spin-tensor $\Psi_{(\mu)_n}$ $(s=(n+1/2))$: $\gamma^{\mu_1}\Psi_{(\mu)_n}=0$, $\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\xi_{(\mu)_{n-1}}=0$; $\delta\Psi^{(\mu)_n}=-\partial^{\{\mu_n}\xi^{(\mu)_{n-1}\}}$

$$\mathcal{S}_{c|(n)}(\Psi)=(-1)^n\int d^dx\overline{\Psi}^{(\nu)_n}\Big\{-\imath\gamma^{\mu}\partial_{\mu}\Psi_{(\nu)_n}+\frac{n(n-1)}{4}\eta_{\nu_{n-1}\nu_n}(\imath\gamma^{\mu}\partial_{\mu})\eta^{\mu\rho}\Psi_{(\nu)_{n-2}\mu\rho}$$

$$-n\gamma_{\nu_n}(\imath\gamma^{\mu}\partial_{\mu})\gamma^{\mu_n}\Psi_{(\nu)_{n-1}\mu_n}-n(\imath\partial_{\nu_n})\gamma^{\mu_n}\Psi_{(\nu)_{n-1}\mu_n}+n(\imath\partial^{\mu_n})\gamma_{\nu_n}\Psi_{(\nu)_{n-1}\mu_n}$$

$$+\frac{n(n-1)}{2}\Big(\gamma_{\nu_{n-1}}(\imath\partial_{\nu_n})\eta^{\mu_{n-1}\mu_n}\Psi_{(\nu)_{n-2}\mu_{n-1}\mu_n}-\eta_{\nu_{n-1}\nu_n}\gamma^{\mu_{n-1}}(\imath\partial^{\mu_n})\Psi_{(\nu)_{n-2}\mu_{n-1}\mu_n}\Big)\Big\}$$

What problems to solve: unknown methods to derive LF and BV actions for fermionic HS fields?

2) triplet formulation

$$\mathcal{S}_{c|(n)}(\Psi,\chi_1,\chi) \; = \; {}_n\langle\tilde{\Psi}|t_0|\Psi\rangle_n - {}_{n-2}\langle\tilde{\chi}|t_0|\chi\rangle_{n-2} + {}_{n-1}\langle\tilde{\chi}_1|\tilde{\gamma}t_0\tilde{\gamma}|\chi_1\rangle_{n-1} - \Big({}_{n-1}\langle\tilde{\chi}_1|\tilde{\gamma}\{l_1|\Psi\rangle_n - l_1^+|\chi\rangle_n\} + h.c.\Big),$$

$$\delta\Big(|\Psi\rangle_n, |\chi_1\rangle_{n-2}, |\chi_1\rangle_{n-1}\Big) \; = \; \Big(l_1^+, l_1, \overset{=-\imath\gamma^\mu\partial_\mu}{\tilde{\gamma}t_0}\Big)|\xi\rangle_{n-1}$$

3) Unconstrained quartet formulation with compensator $|\varsigma\rangle_{n-2}$: $\delta|\varsigma\rangle_{n-2} = t_1|\xi\rangle_{n-1}$

$$\mathcal{S}_{(n)} \; = \; \mathcal{S}_{c|(n)}(\Psi,\chi_1,\chi) + \mathcal{S}_{\mathrm{add}|(n)}(\lambda)$$

$$\mathcal{S}_{\mathrm{add}|(n)}(\lambda) = {}_{n-1}\langle\tilde{\lambda}_1|\Big(t_1|\Psi\rangle_n - \tilde{\gamma}|\chi_1\rangle_{n-1} + l_1^+\tilde{\gamma}|\varsigma\rangle_{n-2}\Big) + {}_{n-2}\langle\tilde{\lambda}_2|\Big(|\chi\rangle_{n-2} + \frac{1}{2}t_1\tilde{\gamma}|\chi_1\rangle_{n-1} + \frac{1}{2}t_0\tilde{\gamma}|\varsigma\rangle_{n-2}\Big) + {}_{n-3}\langle\tilde{\lambda}_3|\Big(t_1|\chi\rangle_{n-2} + l_1\tilde{\gamma}|\varsigma\rangle_{n-2}\Big) + h.c.,$$

2 ways to get Constrained BRST-BFV LF for (half-)integer HS fields on $R^{1,d-1}$: Reduction & self-consistency

Statement

The Constrained BRST-BFV Lagrangian Formulations for (half-)integer HS fields on $R^{1,d-1}$ subject to $Y(s_1,...,s_k)$ constructed, first, by means of reduction from the Unconstrained BRST-BFV Lagrangian Formulation and, second, in self-consistent way by extracting from the (half-)integer HS symmetry algebra $\mathcal{A}^{(f)}(Y(k),\mathbb{R}^{1,d-1})$ the second-class (algebraic) constraint subsystem, from which the only half should be consistently imposed on the Hilbert space vectors, are equivalent.

Thus, for the fermionic ($m=0$) HS field the irreps $ISO(1,d-1)$:

$$\imath\gamma^{\mu}\partial_{\mu}\Psi_{(\mu^1)_{n_1},(\mu^2)_{n_2},...,(\mu^k)_{n_k}}=0\,,\quad \gamma^{\mu^i_{l_i}}\Psi_{(\mu^1)_{n_1},(\mu^2)_{n_2},...,(\mu^k)_{n_k}}=0\,,$$
$$\Psi_{(\mu^1)_{n_1},...,\{(\mu^i)_{n_i}\underbrace{,...,\mu^j_1...}\mu^j_{l_j}\}...\mu^j_{n_j},...(\mu^k)_{n_k}}=0,\quad i<j,\ 1\le l_j\le n_j,$$

correspond to the constraints on basic vector (spinor)

$$|\Psi\rangle=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{n_1}\cdots\sum_{n_k=0}^{n_{k-1}}\frac{\imath^{\sum_i n_i}}{n_1!\times...\times n_k!}\Psi_{(\mu^1)_{n_1},(\mu^2)_{n_2},...,(\mu^k)_{n_k}}\prod_{i=1}^{k}\prod_{l_i=1}^{n_i}a_i^{+\mu^i_{l_i}}|0\rangle,$$

$$\tilde{t}_0|\Psi\rangle=\tilde{t}_i|\Psi\rangle=t_{rs}|\Psi\rangle=0,\ \left(\tilde{t}_0\,,\tilde{t}_i\,,t_{rs}\right)=\left(-i\gamma^{\mu}\partial_{\mu}\,,\gamma_{\mu}a^{\mu}_i\,,a^{+}_{\mu r}a^{\mu}_s\right),\quad r<s,$$

Properly Constrained gauge-invariant Lagrangians for half-integer HS fields subject to $Y(s_1, ...s_k)$

$$\texttt{spin:} \quad g_0^i|\Psi\rangle = (n_i + \tfrac{d}{2})|\Psi\rangle\,, \text{ with } g_0^i = -\tfrac{1}{2}\{a_i^{\mu^i}, a^+_{i\mu^i}\}$$

The HS symmetry superalgebra (A.R. NPB 2013)

$$\mathcal{A}^f(Y(k), \mathbb{R}^{1,d-1}) = \{o_I\} = \{o_A; o_{\overline{a}}; o_{\underline{a}}; g_0^i\} \equiv \{t_0, l_0, l_i, l_i^+; t_i, t_{rs}, l_{ij} = \frac{1}{2}a_{\mu i}a^\mu_j; t_i^+, t^+_{rs}, l^+_{ij}; g_0^i\}$$

contains isometry subalgebra of the (differential) 1-class constraints o_A:

$\{o_A, o_B\} \sim o_C$, subsystem of the (algebraic) 2-nd class constraints $o_{\overline{a}}, o_{\underline{a}}$:

$\{o_{\overline{a}}, o_{\underline{a}}\} \underset{\Delta_{\overline{a}\underline{a}}\neq 0}{=} \Delta_{\overline{a}\underline{a}}(g_0^i) + (o_{\overline{a}}, o_{\underline{a}})$ which splits into subsystems of the 1-class constraints $o_{\overline{a}}$, $o_{\underline{a}}$ and g_0^i.

Here, the Grassmann-odd $\tilde{\gamma}^\mu$-matrices are introduced in $t_0, t_i, t_i^+ = \tilde{\gamma}^\mu(-i\partial_\mu, a_{i\mu}, a^+_{i\mu})$ and H.C. $^+$.is determined with help of $\langle\ |\ \rangle$ odd scalar product in $\mathcal{H}$

$$\{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2\eta^{\mu\nu}, \qquad \{\tilde{\gamma}^\mu, \tilde{\gamma}\} = 0, \qquad \tilde{\gamma}^2 = -1, \quad \text{so that } \gamma^\mu = \tilde{\gamma}^\mu\tilde{\gamma},$$

$$\langle\tilde{\Phi}|\Psi\rangle = \delta_{kl}\prod_{i=1}^{k}\delta_{n_i,p_i}\frac{(-1)^{\sum_j s_j}}{s_1!...s_k!}\int d^dx\Phi^+_{(\mu^1)_{n_1},...,(\mu^k)_{n_k}}\tilde{\gamma}_0\Psi^{(\mu^1)_{n_1},...,(\mu^k)_{n_k}}$$

Constrained GI Lagrangians for half-integer HS fields subject to $Y(s_1, ...s_k)$

$$\mathcal{A}^f(Y(k), \mathbb{R}^{1,d-1}) = \Big(T^k \oplus T^{k*} \oplus [T^k, T^{k*}]\Big) \Supset osp(k|2k), \ \ \{T^{k(*)}; [\ ,\ ]\} = \{l_k^{(+)}; l_0 = -t_0^2\}. \quad (14)$$

The construction of the Hermitian constrained BRST operator $Q_c(o_A)$, spin operator $\sigma^i_c(g)$ and BRST-extended (algebraic) independent constraints $\{\mathcal{O}_a\} \subset \{\mathcal{O}_{\overline{a}} = o_{\overline{a}} + f_{\overline{a}}(\eta_i^{(+)}, P_i^{(+)})\}$ does not require the conversion procedure and realized in the Hilbert space $\mathcal{H}_c = \mathcal{H} \otimes \mathcal{H}^{o_A}_{gh}$ augmented by only Hamiltonian ghost oscillators to o_A is straightforward (as for Lie algebra and operator of number particles in $\mathcal{H}_c$)

$$[q_0, p_0] = \{\eta_0, \mathcal{P}_0\} = \imath, \ \ \{\eta_i, \mathcal{P}^+_j\} = \delta_{ij}, \ \ (p_0, \mathcal{P}_0)^+ = (p_0, -\mathcal{P}_0), \quad (15)$$

$$Q_c(o_A) = q_0 t_0 + \eta_0 l_0 + \eta_i^+ l^i + l^{i+}\eta_i + \imath\Big(\sum_i \eta_i^+ \eta^i - q_0^2\Big)\mathcal{P}_0, \quad (16)$$

$$\sigma^i_c(g) = g^i_0 + \eta_i^+ P_i - \eta_i P_i^+ : \ \ (\sigma^i_c(g))^+ = \sigma^i_c(g), \quad (17)$$

with except for $\mathcal{O}_a = o_a + f_a(\eta_i^{(+)}, P_i^{(+)})$ for $o_a = (t_i, t_{rs})$, because of $l_{ij} = \frac{1}{4}\{t_i, t_j\}$ which should found from the generating equations additional to $Q_c^2 = [Q_c, \sigma^i_c(g)] = 0$

$$[Q_c(o_A), \mathcal{T}_i] = 0, \ \ [Q_c(o_A), \mathcal{T}_{rs}] = 0, \ \ [\sigma^i_c(g), \mathcal{T}_i] = \mathcal{T}_i, \ \ [\sigma^i_c(g), \mathcal{T}_{rs}] = \mathcal{T}_{rs} \quad (18)$$

providing compatibility of the L. dynamics for HS field with fixed spin subject to initial

Constrained GI Lagrangians for half-integer HS fields subject to $Y(s_1, ...s_k)$

The solution of (18):

$$\mathcal{T}_i = t_i - \imath\eta_i p_0 - 2q_0\mathcal{P}_i, \quad \mathcal{L}_{lm} = l_{lm} + \frac{1}{2}\eta_{\{m}\mathcal{P}_{l\}}, \; l \leq m \tag{19}$$

$$\mathcal{T}_{rs} = t_{rs} - \eta_r^+\mathcal{P}_s - \mathcal{P}_r^+\eta_s, \; r < s \tag{20}$$

From the spectral problem

$$Q_c|\chi_c\rangle = 0, \qquad \sigma_c^i|\chi_c\rangle = \left(n^i + \frac{d-2}{2}\right)|\chi_c\rangle, \qquad (\varepsilon, gh_H)(|\chi_c\rangle) = (1,0),$$

$$\delta|\chi_c\rangle = Q_c|\chi_c^1\rangle, \qquad \sigma_c^i|\chi_c^1\rangle = \left(n^i + \frac{d-2}{2}\right)|\chi_c^1\rangle, \qquad (\varepsilon, gh_H)(|\chi_c^1\rangle) = (0,-1),$$

$$\ldots\ldots \qquad \ldots\ldots\ldots\ldots \qquad \ldots\ldots$$

$$\delta|\chi_c^{s_c-1}\rangle = Q_c|\chi_c^{s_c}\rangle, \quad \sigma_c^i|\chi_c^{s_c}\rangle = \left(n^i + \frac{d-2}{2}\right)|\chi_c^{s_c}\rangle, \quad (\varepsilon, gh_H)(|\chi_c^{s_c}\rangle) = (s_c+1, -s_c),$$

$$\Big(\mathcal{T}_i, \mathcal{T}_{rs}\Big)|\chi_c^l\rangle = 0, \qquad l = 0, 1, ..., s_c.$$

$$|\chi_c\rangle \;=\; \sum_n q_0^{n_{b0}}\eta_0^{n_{f0}} \prod_{i,j}(\eta_i^+)^{n_{fi}}(\mathcal{P}_j^+)^{n_{pj}}|\Psi(a_i^+)^{n_{b0}n_{f0};(n)_{fi}(n)_{pj}}\rangle.$$

the solution is written as the second-order equations of motion and sequence of the reducible gauge transformations with off-shell constraints:

$$Q_c|\chi^0_c\rangle_{(n)_k} = 0,\ \delta\Big(|\chi^0_c\rangle_{(n)_k}, ..., |\chi^{s_c}_c\rangle_{(n)_k}\Big) = Q_c\Big(|\chi^1_c\rangle_{(n)_k}, \dots, |\chi^{s_c+1}_c\rangle_{(n)_k}\Big),\ \delta|\chi^{s_c+1}_c\rangle_{(n)_k} =$$
$$\Big(\mathcal{T}_i, \mathcal{T}_{rs}\Big)|\chi^l_c\rangle_{(n)_k} = 0, \quad l = 0, 1, ..., s_c, \text{ for } s_c = k.$$

The corresponding BRST-like constrained gauge-invariant action looks

$$\mathcal{S}^{(2)}_{c|(n)_k} = \int d\eta_{0\,(n)_k} \langle\tilde{\chi}^0_c|Q_c|\chi^0_c\rangle_{(n)_k}, \tag{21}$$

Doing standard removing of second-order operator by partial gauge-fixing based on the decomposition in q_0, η_0

$$Q_c = q_0 t_0 + \eta_0 l_0 + -\imath(q_0^2 - \eta_i^+\eta_i)\mathcal{P}_0 + \Delta Q_c, \quad \Delta Q_c = \eta_i^+ l_i + \eta_i l_i^+,$$
$$|\chi^l_c\rangle = \sum_{e=0}^{k} q_0^e(|\chi^{l(e)}_{0|c}\rangle + \eta_0|\chi^{l(e)}_{1|c}\rangle),\ gh_H(|\chi^{l(e)}_{m|c}\rangle) = -(l + e + m + 1),\ m = 0, 1$$

we find all components in powers of q_0, η_0 vector are removed except for 2 fields for each level

$$|\chi^s_c\rangle_{(n)_k} = |\chi^{s(0)}_{0|c}\rangle_{(n)_k} + q_0|\chi^{s(1)}_{0|c}\rangle_{(n)_k} - i\eta_0\tilde{t}_0|\chi^{s(1)}_{0|c}\rangle_{(n)_k},$$

Statement : The first-order constrained gauge-invariant Lagrangian formulation for half-integer HS field, $\Psi_{(\mu^1)_{n_1},\dots,(\mu^k)_{n_k}}(x)$ with generalized spin $(s)_k = (n+\frac{1}{2})_k$, is determined by the action,

$$\mathcal{S}_{c|(n)_k} = \left({}_{(n)_k}\langle\tilde{\chi}^0_{0|c}| \ {}_{(n)_k}\langle\tilde{\chi}^1_{0|c}|\right) \begin{pmatrix} t_0 & \Delta Q_c \\ \Delta Q_c & t_0\eta_i^+\eta_i \end{pmatrix} \begin{pmatrix} |\chi^0_{0|c}\rangle_{(n)_k} \\ |\chi^1_{0|c}\rangle_{(n)_k} \end{pmatrix}$$

invariant with respect to the sequence of the reducible gauge transformations (for $s_c - 1 = (k-1)$-being by the the stage of reducibility):

$$\delta \begin{pmatrix} |\chi^{l(0)}_{0|c}\rangle_{(n)_k} \\ |\chi^{l(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix} = \begin{pmatrix} \Delta Q_c & t_0\eta_i^+\eta_i \\ t_0 & \Delta Q_c \end{pmatrix} \begin{pmatrix} |\chi^{l+1(0)}_{0|c}\rangle_{(n)_k} \\ |\chi^{l+1(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix}, \ \delta \begin{pmatrix} |\chi^{k(0)}_{0|c}\rangle_{(n)_k} \\ |\chi^{k(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix} = 0$$

(for $l = -1, 0, ..., k-1$ and $|\chi^{-1(m)}_{0|c}\rangle = 0$, $m = 0, 1$) with off-shell algebraically independent BRST-extended constraints imposed on the whole set of field and gauge parameters:

$$\mathcal{T}_i\left(|\chi^{l(0)}_{0|c}\rangle_{(n)_k} + q_0|\chi^{l(1)}_{0|c}\rangle_{(n)_k}\right) = 0, \ \mathcal{T}_{rs}|\chi^{l(m)}_{0|c}\rangle_{(n)_k} = 0 \ l = 0, 1, ..., k; \ m = 0, 1. \quad (22)$$

Note, the survived term $-i\eta_0\tilde{t}_0|\chi^{s(1)}_0\rangle_{(n)_k}$ does not give any contribution in (22) due to EoM. It is the first basic result of the work.

The crucial point that we established via analysis of the respective Q, Q_c-complexes in $\mathcal{H}_{tot}$, $\mathcal{H}_c$: Unconstrained LF and Constrained LF are equivalent!

From Constrained BRST-BFV Lagrangian formulation to Constrained BRST-BV for half-integer HS fields

1) Introduce the **correspondence** among gauge parameter vectors, $|\chi_c^{l(e)}\rangle_{(n)_k}$, $l = 1, ..., k$, and respective ghost vectors, denoted as , $|C_c^{l(e)}\rangle_{(n)_k}$ with components spin-tensors from the minimal sector of field-antifield space, on which acts the Lagrangian ghost number, gh_L, distribution by the rule:

$$\Psi^{1(e_{b0})0_{f0};(n^1)_{fi}(n^1)_{pj}}_{(\nu^1)_{p_1^1}\cdots(\nu^k)_{p_k^1}} = C^{1(e_{b0})0_{f0};(n^1)_{fi}(n^1)_{pj}}_{(\nu^1)_{p_1^1}\cdots(\nu^k)_{p_k^1}}\mu_l \Longrightarrow |\chi_c^{1(e)}\rangle_{(n)_k} = |C_c^{1(e)}\rangle_{(n)_k}\mu_l,$$

$$\Psi^{2(e_{b0})0_{f0};(n^2)_{fi}(n^2)_{pj}}_{(\nu^1)_{p_1^2}\cdots(\nu^k)_{p_k^2}} = C^{2(e_{b0})0_{f0};(n^2)_{fi}(n^2)_{pj}}_{(\nu^1)_{p_1^2}\cdots(\nu^k)_{p_k^2}}\mu_2\mu_1 \Rightarrow |\chi_c^{2(e)}\rangle_{(n)_k} = |C_c^{2(e)}\rangle_{(n)_k}\mu_2\mu_1, \ ,$$

. . . .

$$\Psi^{k(e_{b0})0_{f0};(n^k)_{fi}(n^k)_{pj}}_{(\nu^1)_{p_1^k}\cdots(\nu^k)_{p_k^k}} = C^{k(e_{b0})0_{f0};(n^k)_{fi}(n^k)_{pj}}_{(\nu^1)_{p_1^k}\cdots(\nu^k)_{p_k^k}}\mu_k...\mu_1 \Rightarrow |\chi_c^{k(e)}\rangle_{(n)_k} = |C_c^{k(e)}\rangle_{(n)_k}\mu_k...\mu_1,$$

with use of the Grassmann-odd scalars $\mu_1, ..., \mu_{k-1}$: $\{\mu_m, \mu_n\} = 0, \ m, n = 1, ..., k$, and $(\varepsilon, gh_{\mathrm{H}}, gh_{\mathrm{L}})\mu_k = (1, 0, -1)$: so that $\varepsilon\big(|C_c^{k(e)}\rangle_{(n)_k}\big) = \varepsilon\big(|\chi_c^{0(e)}\rangle_{(n)_k}\big)$.

2) Introduce so-called **total ghost number**, gh_{tot} as the sum of Lagrangian and Hamiltonian ghost numbers

$$gh_{\mathrm{tot}} = gh_H + gh_L, \qquad \big(gh_{\mathrm{tot}}, gh_H, gh_L\big)C^{l(e_{b0})0_{f0};(n^l)_{fi}(n^l)_{pj}}_{(\nu^1)_{p_1^l}\cdots(\nu^k)_{p_k^l}} = (l - e, -e, l),$$

$$\big(gh_{\mathrm{tot}}, gh_H, gh_L\big)|C_c^{l(e)}\rangle_{(n)_k} = (-e, -l - e, l).$$

Constrained BRST-BV for half-integer HS fields

we may organize *generalized field vectors* for $e=0,1$

$$|\chi^{0(e)}_{c}\rangle_{(n)_k} \to |\chi^{0(e)}_{\mathrm{gen}|c}\rangle_{(n)_k} = |\chi^{0(e)}_{c}\rangle_{(n)_k} + \sum_{m=1}^{k} |C^{m(e)}_{c}\rangle_{(n)_k},\ \big(\varepsilon, gh_{\mathrm{tot}}\big)|\chi^{0(e)}_{\mathrm{gen}|c}\rangle = (1,-e),$$

3) the spin-tensor antifields, $\Phi^*_{A_{\min}}$ with respective gradings:

$$\Phi^*_{A_{\min}} = \Big(\Psi^{*(\nu^1)_{p_1}\ldots(\nu^k)_{p_k}}_{0(e_{b0})0_{f0};(n)_{fi}(n)_{pj}},\ C^{*(\nu^1)_{p^l_1}\ldots(\nu^k)_{p^l_k}}_{l(e_{b0})0_{f0};(n^l)_{fi}(n^l)_{pj}}\Big),\ l=1,...,k,$$
$$\big(\varepsilon, gh_{\mathrm{tot}}, gh_H, gh_L\big)\Phi^*_{A_{\min}} = \big(1+\varepsilon(\Phi^{A_{\min}}), -1-gh_L(\Phi^{A_{\min}}), 0, -1-gh_L(\Phi^{A_{\min}})\big),$$

as the external sources to the left, s, and right generators, $\overleftarrow{s}$, of *Lagrangian BRST-transformations* of the constrained classical, $\Psi^{0(e_{b0})0_{f0};(n)_{fi}(n)_{pj}}_{(\nu^1)_{p_1}\ldots(\nu^k)_{p_k}}$, and ghost, $C^{l(e_{b0})0_{f0};(n^l)_{fi}(n^l)_{pj}}_{(\nu^1)_{p^l_1}\ldots(\nu^k)_{p^l_k}}$, $l=1,...,k$, fields parameterizing as $\Phi^{A_{\min}}$ the configuration space, $\mathcal{M}_{\min}$ in terms of Fock space vectors:

Constrained BRST-BV for half-integer HS fields

$$s\begin{pmatrix} |C^{l(0)}_c\rangle_{(n)_k} \\ |C^{l(1)}_c\rangle_{(n)_k}\end{pmatrix} = \begin{pmatrix} \Delta Q_c & t_0\eta^+_i\eta_i \\ t_0 & \Delta Q_c\end{pmatrix}\begin{pmatrix} |C^{l+1(0)}_{0|c}\rangle_{(n)_k} \\ |C^{l+1(1)}_{0|c}\rangle_{(n)_k}\end{pmatrix}\theta_{kl},\ \ \theta_{kl} = 1, k > l$$

$$\Big({}_{(n)_k}\langle\tilde{C}^{l(0)}_c|, {}_{(n)_k}\langle\tilde{C}^{l(1)}_c|\Big)\overleftarrow{s} = \Big({}_{(n)_k}\langle\tilde{C}^{l+1(0)}_c|, {}_{(n)_k}\langle\tilde{C}^{l+1(1)}_c|\Big)\begin{pmatrix} \Delta Q_c & t_0 \\ t_0\eta^+_i\eta_i & \Delta Q_c\end{pmatrix}\theta_{kl},$$

$$\delta_B|C^{l(e)}_c\rangle_{(n)_k} \stackrel{def}{=} \mu s|C^{l(e)}_c\rangle_{(n)_k},\ \ \delta_B\big({}_{(n)_k}\langle\tilde{C}^{l(e)}_c|\big) \stackrel{def}{=} {}_{(n)_k}\langle\tilde{C}^{l(e)}_c|\overleftarrow{s}\mu,$$

$$\big(\varepsilon, gh_{\rm tot}, gh_H, gh_L\big)\big[\mu,\ s\big] = \big[(1,-1,0,-1),(1,1,0,1)\big],\ \text{for}\ |C^{0(e)}_c\rangle \equiv |\chi^{0(e)}_c\rangle, l = 0, ..., k$$

The homogeneity in ε, gh_{tot} - gradings requires the following rule of organizing for any field vector $|C^{l(e)}_c\rangle_{(n)_k}$ its antifield vector $|C^{*l(e)}_c\rangle_{(n)_k}$ by changes in each monomial

$$\Big((\tilde{\gamma})^m; \eta^+_i, \mathcal{P}^+_j; C^{l(e_{b0})0_{f0};(n^l)_{fi}(n^l)_{pj}}_{(\nu^1)_{p^l_1}\dots(\nu^k)_{p^l_k}}\Big) \mapsto \Big((\tilde{\gamma})^{1+m}; \mathcal{P}^+_i,\ \eta^+_j; C^{*(\nu^1)_{p^l_1}\dots(\nu^k)_{p^l_k}}_{l(e_{b0})0_{f0};(n^l)_{fi}(n^l)_{pj}}\Big),$$

$$\text{e.g. for}\ \Big(\tilde{\gamma}|\Psi(a^+)\rangle_{(n)_k}, \mathcal{P}^+_i\tilde{\gamma}|\chi^{0(1)i}(a^+)\rangle_{(n)_k}\Big) \mapsto \Big(|\Psi^*(a^+)\rangle_{(n)_k}, \eta^+_i|\chi^{*0(1)i}(a^+)\rangle_{(n)_k}\Big)$$

$$\big(\varepsilon,\ gh_{\rm tot},\ gh_H,\ gh_L\big)\big[|\Psi^*(a^+)\rangle; |\chi^{*0(1)i}(a^+)\rangle\big] = (1,-1,0,-1).$$

Constrained BRST-BV for half-integer HS fields

Therefore, the vectors, $|C_c^{*l(e)}\rangle_{(n)_k}$, on $\mathcal{H}_c$ for each $e=0,1$ (and for $|C_c^{*0(e)}\rangle_{(n)_k} \equiv |\chi_c^{*0(e)}\rangle_{(n)_k}$) being homogeneous in $(\varepsilon, gh_{\mathrm{tot}})$-gradings can be combined into the vectors:

$$|\chi^{*0(e)}_{\mathrm{gen}|c}\rangle_{(n)_k} = |\chi_c^{*0(e)}\rangle_{(n)_k} + \sum_{m=1}^{k} |C_c^{*m(e)}\rangle_{(n)_k}, \ \ (\varepsilon, gh_{\mathrm{tot}})|\chi^{*0(e)}_{\mathrm{gen}|c}\rangle = (1, e-1),\ e=0,1,$$

which we will call as the *generalized antifield vectors*. As the result we come to the following minimal BV action

$$\begin{aligned} S_{c|(n)_k} \ &= \ \mathcal{S}_{c|(n)_k} + \sum_{l=0}^{k-1}\Big\{ \Big({}_{(n)_k}\langle \tilde{C}_c^{l+1(0)}|, {}_{(n)_k}\langle \tilde{C}_c^{l+1(1)}|\Big) \begin{pmatrix} \Delta Q_c & t_0 \\ t_0, \eta_i^+\eta_i & \Delta Q_c \end{pmatrix} \begin{pmatrix} |C^{*l(0)}_{0|c}\rangle_{(n)_k} \\ |C^{*l(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix} \\ &+ \ \Big({}_{(n)_k}\langle \tilde{C}_c^{*l(0)}|, {}_{(n)_k}\langle \tilde{C}_c^{*l(1)}|\Big) \begin{pmatrix} \Delta Q_c & t_0\eta_i^+\eta_i \\ t_0 & \Delta Q_c \end{pmatrix} \begin{pmatrix} |C^{l+1(0)}_{0|c}\rangle_{(n)_k} \\ |C^{l+1(1)}_{0|c}\rangle_{(n)_k} \end{pmatrix} \Big\}, \end{aligned}$$

we see, that it is invariant with respect to the **Lagrangian BRST-transformations** (23):

$$\delta_B S_{c|(n)_k} = 0 \ \ \text{when} \ \ \delta_B |\chi^{*0(e)}_{\mathrm{gen}|c}\rangle_{(n)_k} = 0.$$

$$\delta_B |C_c^{l(e)}\rangle_{(n)_k} \overset{def}{=} \mu s |C_c^{l(e)}\rangle_{(n)_k},\ \ \delta_B \Big({}_{(n)_k}\langle \tilde{C}_c^{l(e)}|\Big) \overset{def}{=} {}_{(n)_k}\langle \tilde{C}_c^{l(e)}| \overleftarrow{s}\mu,$$

subject to the off-shell BRST-extended constraints as for BRST-BFV case for the field vectors, whereas for antifield one should impose *antifield BRST-extended constraints* $\mathcal{T}^*, \mathcal{T}^*$

Example of constrained Fang-Fronsdal, triplet & unconstrained quartet Lagrangians for totally-symmetric half-integer HS fields

Now. it is straightforwardly to get from the Constrained BRST-BFV LF for massless (massive) ts $\Psi_{(\mu)_n}$ $(s = (n + 1/2))$ all the formulations for:

$$|\Psi\rangle = \sum_{n=0}^{\infty} \frac{\imath^n}{n!} \Psi_{(\mu)_n} a^{+\mu_1} \ldots a^{+\mu_n} |0\rangle.$$

$$Q_c = q_0 t_0 + \eta_0 l_0 + \eta_1^+ l_1 + l_1^+ \eta_1 + \imath(\eta_1^+ \eta_1 - q_0^2)\mathcal{P}_0,$$

$$\{\mathcal{T}_1, \mathcal{L}_{11}\} = \{t_1 - \imath\eta_1 p_0 - 2q_0\mathcal{P}_1,\ l_{11} + \eta_1\mathcal{P}_1\},$$

$$\widehat{\sigma}_c(g) = g_0 + \eta_1^+\mathcal{P}_1 - \eta_1\mathcal{P}_1^+$$

$$\mathcal{T}_i\Big(|\chi^0_{0|c}\rangle_n + q_0|\chi^1_{0|c}\rangle_n\Big) = 0, \quad \mathcal{T}_i|\chi^{1(0)}_{0|c}\rangle_n = 0.,$$

$$|\chi^0_{0|c}\rangle_n = |\Psi\rangle_n + \eta_1^+\mathcal{P}_1^+|\chi\rangle_{n-2} = |\Psi\rangle_n + \frac{\imath^{n-2}}{(n-2)!}\eta_1^+\mathcal{P}_1^+\chi^{(\mu)_{n-2}}\prod_{k=1}^{n-2} a^+_{\mu_k}|0\rangle,$$

$$|\chi^1_{0|c}\rangle_n = \mathcal{P}_1^+\tilde{\gamma}|\chi_1\rangle_{n-1} = \frac{\imath^{n-1}}{(n-1)!}\mathcal{P}_1^+\tilde{\gamma}\chi_1^{(\mu)_{n-1}}\prod_{k=1}^{n-1} a^+_{\mu_k}|0\rangle,$$

$$|\chi^{1(0)}_{0|c}\rangle_n = \mathcal{P}_1^+|\xi\rangle_{n-1} = \frac{\imath^{n-1}}{(n-1)!}\mathcal{P}_1^+\xi^{(\mu)_{n-1}}\prod_{k=1}^{n-1} a^+_{\mu_k}|0\rangle,$$

Example of constrained Fang-Fronsdal, triplet & unconstrained quartet Lagrangians for totally-symmetric half-integer HS fields

leads to

$$(t_1)^3|\Psi\rangle_n = 0, \quad \tilde{\gamma}|\chi_1\rangle_{n-1} = t_1|\Psi\rangle_n, \quad |\chi\rangle_{n-2} = -\frac{1}{2}(t_1)^2|\Psi\rangle_n$$

and to the Fang-Fronsdal LF and etc....

$$\mathcal{S}_{c|(n)} = {}_n\langle\tilde{\Psi}|\Big(t_0 - \frac{1}{4}(t_1^+)^2 t_0 t_1^2 - t_1^+ t_0 t_1 + l_1^+ t_1 + t_1^+ l_1 + \frac{1}{2} t_1^+ l_1^+ t_1^2 + \frac{1}{2}(t_1^+)^2 l_1 t_1\Big)|\Psi\rangle_n,$$

$$\delta|\Psi\rangle_n = l_1^+|\xi\rangle_{n-1}.$$

$$|\chi^0_{\text{gen}|c}\rangle_n = |\chi^0_c\rangle_n + |C^{1(0)}_c\rangle_n, \ \left(\varepsilon, gh_{\text{tot}}\right)|\chi^0_{\text{gen}|c}\rangle = (1,0).$$

with untouched $|\chi^1_{\text{gen}|c}\rangle_n = |\chi^1_{0|c}\rangle_n$

The corresponding antifield spin-tensors $\Psi^{*(\nu)_n}, \chi_1^{*(\mu)_{n-1}}, \chi^{*(\mu)_{n-2}}, C^{*(\mu)_{n-1}}$ with

$$(\varepsilon, gh_L)\Psi^* = (\varepsilon, gh_L)\chi^*_1 = (\varepsilon, gh_L)\chi^* = (0,-1) \text{ and } (\varepsilon, gh_L)C^* = (1,-2)$$

are combined into generalized antifield vectors

$$|\chi^{*0}_{\text{gen}|c}\rangle_n = |\chi^{*0}_c\rangle_n + |C^{*1(0)}_c\rangle_n = \tilde{\gamma}\Big(|\Psi^*(a^+)\rangle_n + \mathcal{P}^+_1\eta^+_1|\chi^*(a^+)\rangle_{n-2}\Big) + \tilde{\gamma}\eta^+_1|C^*\rangle_{n-1}$$

$$|\chi^{*1}_{\text{gen}|c}\rangle_n = |\chi^{*1}_c\rangle_n = \eta^+_1|\chi^*_1(a^+)\rangle_{n-1}, \quad \left(\varepsilon, gh_{\text{tot}}\right)|\chi^{*e}_{\text{gen}|c}\rangle = (1, e-1), e = 0,1$$

$$\text{with } \left(gh_L, gh_H\right)|A^*\rangle = (-1,0), \text{ for } A \in \{\Psi, \chi, \chi_1\}, \quad \left(gh_L, gh_H\right)|C^*\rangle = (-2,0)$$

for the ghost- and $\tilde{\gamma}$- independent antifield vectors $|\Psi^*(a^+)\rangle_n$, $|\chi^*(a^+)\rangle_{n-2}$, $|\chi^*_1(a^+)\rangle_{n-1}$, $|C^*\rangle_{n-1}$ The minimal Batalin-Vilkovisky action takes the form

$$S_{c|(n)} \quad = \quad \mathcal{S}_{c|(n)} + \Big\{\Big({}_n\langle\tilde{\chi}^{*0}_c|\left(\eta_1 l^+_1 + \eta^+_1 l_1\right) + {}_n\langle\tilde{\chi}^{*1}_c|t_0\Big)|C^{1(0)}_c\rangle_n + h.c\Big\}.$$

The functional $S_{c|(n)}$ is invariant with respect to the Lagrangian BRST-transformations

$$\delta_B S_{c|(n)} = 0 \ \text{ for } \ \delta_B\Big(|\chi^0_{0|c}\rangle_n, |\chi^1_{0|c}, \rangle_n, |C^{1(0)}_c\rangle_n\Big) = .\mu\Big(\eta_1 l^+_1 + \eta^+_1 l_1,\, t_0, 0\Big)|C^{1(0)}_c\rangle_n,$$

Minimal BV actions for Fang-Fronsdal, triplet & unconstrained quartet LF

the field, antifield vectors are subject to the off-shell BRST extended constraint

$$\mathcal{T}_1 \sum_{m=0}^{1} q_0^m |\chi_c^e\rangle_n = 0, \quad \mathcal{T}_1^* \Big(\sum_{e=0}^{1} q_0^{1-e} |\chi_c^{*e}\rangle_n \Big) \big/ \{ q_0^2 \mathcal{P}_1 |\chi_c^{*0}\rangle_n = 0 \} = 0,$$

$$\mathcal{T}_1 |C_c^{1(0)}\rangle_n = 0, \quad \mathcal{T}_1^* |C_c^{*1(0)}\rangle_n \big/ \{ q_0^2 \mathcal{P}_1 |\chi_c^{*1(0)}\rangle_n = 0 \} = 0.$$

which resolution has the form

$$(t_1)^3 \Big(|\Psi\rangle, |\Psi^*\rangle \Big) = 0, \qquad t_1 \Big(|\Psi\rangle_n, \, |\Psi^*\rangle_n \Big) = \tilde{\gamma} \Big(|\chi_1\rangle_{n-1}, \, -|\chi_1^*\rangle_{n-1} \Big),$$

$$t_1 \Big(|C_c\rangle_{n-1}, \, |C_c^*\rangle_{n-1} \Big) = 0, \qquad \Big(|\chi\rangle_{n-2}, \, |\chi^*\rangle_{n-2} \Big) = \frac{1}{2} (t_1)^2 \Big(-|\Psi\rangle_n, \, |\Psi^*\rangle_n \Big).$$

In the ghost-independent form in terms of the triplets of field $|\Psi\rangle_n$, $|\chi_1\rangle_{n-1}$, $|\chi\rangle_{n-2}$ and antifield $|\Psi^*\rangle_n$, $|\chi_1^*\rangle_{n-1}$, $|\chi^*\rangle_{n-2}$ vectors and singlets $|C_c^{1(0)}\rangle_{n-1}$, $|C_c^{*1(0)}\rangle_{n-1}$ read

$$S_{c|(n)} \big(\Psi^{(*)}, \chi_1^{(*)}, \chi^{(*)} \big) \; = \; \mathcal{S}_{c|(n)} \big(\Psi, \chi_1, \chi \big) + \Big\{ \Big({}_n\langle \tilde{\Psi}^* | \tilde{\gamma} l_1^+ + {}_{n-2}\langle \tilde{\chi}^* | \tilde{\gamma} l_1 + {}_{n-1}\langle \tilde{\chi}_1^* | t_0 \Big) |C\rangle_{n-1} + h.c. \Big\},$$

$$\delta_B \Big(|\Psi\rangle_n, |\chi_1\rangle_{n-2}, |\chi_1\rangle_{n-1}, |C\rangle_{n-1} \Big) \; = \; \mu \Big(l_1^+, l_1, \tilde{\gamma} t_0, \, 0 \Big) |C\rangle_{n-1},$$

and independent field $|\Psi\rangle_n$, ghost $|C_c^{1(0)}\rangle_{n-1}$ and antifield $|\Psi^*\rangle_n$, $|C_c^{*1(0)}\rangle_{n-1}$ vectors

Minimal BV actions for Fang-Fronsdal, triplet & unconstrained quartet LF

$$S_{c|(n)} = \mathcal{S}_{c|(n)}\big(|\Psi\rangle\big) + \Big\{{}_n\langle\tilde{\Psi}^*|\tilde{\gamma}\Big(l_1^+ + \frac{1}{2}(t_1^+)^2 l_1 - t_1^+ t_0\Big)|C\rangle_{n-1} + h.c.\Big\},$$

$$\delta_B\Big(|\Psi\rangle_n,\, |C\rangle_{n-1}\Big) = \mu\Big(l_1^+,\, 0\Big)|C\rangle_{n-1},$$

The minimal Fang-Fronsdal BV action (with triple-gammatraceless for $\Psi^{(*)}$ and gammatraceless for $C^{(*)}$)

$$S_{c|(n)}(\Psi, C, \Psi^*) = \mathcal{S}_{c|(n)}(\Psi) + (-1)^n \int d^d x \Big[\overline{\Psi}^{(\nu)_n}\Big\{n\Big(\gamma_{\nu_n}\gamma^\mu\partial_\mu - \partial_{\nu_n}\Big)C_{(\nu)_{n-1}} + \frac{1}{2}n(n-1)\eta_{\nu_{n-1}\nu_n}\partial^{\mu_n}C_{(\nu)_{n-2}\mu_n}\Big\} + h.c.\Big]$$

$$= \mathcal{S}_{c|(n)}(\Psi) + (-1)^n \int d^d x \Big\{n\overline{\Psi}'\not{p}(\imath C) - n\overline{\Psi}\cdot p(\imath C) + \frac{1}{2}n(n-1)\overline{\Psi}'' p\cdot(\imath C) + h.c\Big\}$$

$$\delta_B\Big(\Psi^{(\mu)_n},\, C^{(\mu)_{n-1}}\Big) = \mu\Big(-\sum_{i=1}^{n}\partial^{\mu_i}C^{\mu_1\cdots\mu_{i-1}\mu_{i+1}\cdots\mu_n},\, 0\Big).$$

and for the unconstrained quartet formulation

$$S_n = S_{c|(n)}\big(\Psi^{(*)}, \chi_1^{(*)}, \chi^{(*)}\big) + \mathcal{S}_{\mathrm{add}|(n)}(\lambda) + \Big({}_{n-2}\langle\tilde{\varsigma}^*(a)|t_1|C\rangle_{n-1} + h.c.\Big),$$

$$\delta_B\Big(|\Psi\rangle_n, |\chi_1\rangle_{n-2}, |\chi_1\rangle_{n-1}, |\varsigma\rangle_{n-2},\, |C\rangle_{n-1}\Big) = \mu\Big(l_1^+, l_1, \tilde{\gamma}t_0,\, \tilde{\gamma}t_1,\, 0\Big)|C\rangle_{n-1}$$

Conclusions

- The constrained BRST-BFV method to construct gauge-invariant Lagrangian Formulations for mixed-symmetric half-integer HS fields subject to $Y(s_1, ..., s_k)$ in $\mathbb{R}^{1,d-1}$ space is suggested ;
- The equivalence among dynamics in constrained and unconstrained BRST-BFV Lagrangian Formulations for the same MS HS field as well as with respective solutions for initial Poincare group irreps conditions are established;
- The constrained BRST-BV method to find minimal field-antifield BV actions with off-shell algebraic constraints for MS half-integer HS fields subject to $Y(s_1, ..., s_k)$ in $\mathbb{R}^{1,d-1}$ space is developed ;
- The Fang-Fronsdal, triplet formulations with off-shell (gamma-traceless) constraints for totally-symmetric of spin $(n + 1/2)$ HS field and unconstrained quartet GI Lagrangians are derived;
- The field-antifield minimal BV actions for above Lagrangian formulations for totally-symmetric of spin $(n + 1/2)$ HS field are derived on a base of constrained BRST-BV method.

Outlook

- development of the constrained BRST-BFV and BRST-BV Lagrangian formulations for HS fields on AdS_d starting from totally-symmetric cases to get Fang-Fronsdal, triplet, quartet formulations ;
- Construction on a base of constrained BRST-BFV(BV) Lagrangians for totally-symmetric HS fields with integer and half-integer spins the SUSY Lagrangian formulation (developed on component levels for some supermultiplets by S.Kusenko, J.Buchbinder, Yu.Zinoviev, T.Snegirev,....) for supermultiplet of given superspin(s) with triplets of integer and half-integer HS fields;
- Development of the interacting Lagrangians both for SUSY Lagrangian formulation (like Wess-Zumino model) and without SUSY but with constrained integer and half-integer HS fields.

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Thank you very much