LIGHT-CONE HIGHER-SPIN THEORIES IN FLAT SPACE

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MOTIVATION

HIGHER SPIN INTERACTIONS IN FLAT SPACE: NO-GO RESULTS

[Weinberg'64; Aragone, Deser'79; Berends, Burgers, van Dam'85; …]

WAY OUT?

Light-cone deformation procedure results into <u>additional local cubic vertices</u> compared to manifestly covariant approaches.

[Bengtsson, Bengtsson, Brink'83; Bengtsson, Bengtsson, Linden'87]

[Bengtsson'14]

In particular, a two-derivative interaction with gravity (minimal coupling) <u>does exist</u>, contrary to covariant approaches (by the Aragone-Deser argument).

MOTIVATION

FURTHER ANALYSIS

Deformation procedure was partially solved at the order g^2

This fixes all coupling constants in cubic vertices in terms of a single one

[Metsaev'91]

Satisfy Weinberg's equivalence principle (coupling is universal)

Agree with a "flat limit" of cubic vertices found from AdS/CFT

[Bekaert, Erdmenger, Ponomarev, Sleight'15; Skvortsov'16]

[Taronna, Sleight'16]

(nothing of this can be seen in covariant approaches)

MOTIVATION

This couple of points suggest that
a consistent higher spin theory may exist in flat space

GOAL

Revisit higher-spin interactions in flat space focusing on methods that do not require manifest Lorentz covariance (Lorentz tensors).

PRIMARY TOOL

Light-cone deformation procedure

MANIFEST LORENTZ INVARIANCE

FREE THEORIES UIR's of Poincare group

INTERACTIONS Generators are deformed non-linearly. Consistency requirement: still generate the Poincare algebra

LORENTZ TENSORS

All Poincare symmetry is manifest

Introduces extra d. o. f.
Massless fields = gauge invariance

<u>Fewer local interactions</u>

DIRECT ANALYSIS

Manual control of Poincare symmetry

Only physical d. o. f.

<u>More local interactions</u>

CHIRAL HIGHER-SPIN THEORY FROM LIGHT-CONE

BASICS OF LIGHT-CONE

Light-cone gauge

$$\phi^{+\cdots} = 0$$

Light-cone time

$$x^+ = \frac{1}{\sqrt{2}}(x^3 + x^0)$$

∂^- is time derivative, ∂^+ is not and can be inverted

$\phi^\rho{}_\rho{}^{\cdots} = 0$ and $\partial_\rho \phi^{\rho\cdots} = 0$ are algebraic consequences

This allows to eliminate all unphysical degrees of freedom

ALTERNATIVELY Fundamentally define a theory in the light-cone gauge

BASICS OF LIGHT-CONE: FREE THEORY

The action

(λ is helicity)

$$S_2 \equiv \int d^4x L_2, \qquad L_2 = -\frac{1}{2}\sum_\lambda \partial_a \Phi^{-\lambda}\partial^a \Phi^\lambda$$

Higher-spin fields look like scalars

Difference: only in spin part of angular momentum

$$S^{+a}\cdot \Phi^\lambda = 0, \qquad S^{ab}\partial_a \cdot \Phi^\lambda = 0 \qquad S^{x\bar{x}}\cdot \Phi^\lambda = -\lambda \Phi^\lambda$$

Noether charges generate associated transformation via the commutator

$$P_2^i = \int d^3x^\perp T^{i,+}, \qquad J_2^{ij} = \int d^3x^\perp L^{ij,+}$$

$$[\Phi^\lambda, P_2^i] = P_2^i \Phi^\lambda, \qquad [\Phi^\lambda, J_2^{ij}] = J_2^{ij}\Phi^\lambda$$

BASICS OF LIGHT-CONE: INTERACTIONS

Deform <u>dynamical</u> generators

$$D: \qquad H \equiv P^-, \qquad J \equiv J^{x-}, \qquad \bar{J} \equiv J^{\bar{x}-}$$

Remaining are not deformed, called <u>kinematical</u> $\qquad K$

BASICS OF LIGHT-CONE: INTERACTIONS

Deformation

$$H = H_2 + \sum_n H_n$$

$$H_n = \frac{1}{n!}\sum_{\lambda_i}\int d^{3n}q^{\perp}\delta^3(\sum_{i=1}^{n} q_i^{\perp})h_n^{\lambda_1\ldots\lambda_n}\prod_{i=1}^{n}\Phi^{\lambda_i}(q_i^{\perp})$$

$$(q^{\perp}\equiv\{q,\bar{q},q^+\},\quad \beta\equiv q^+)$$

Kinematical constraints (solved only once)

+ Fix transverse momentum dependence

$$\bar{\mathbb{P}}_{ij}\equiv\bar{q}_i\beta_j-\bar{q}_j\beta_i,\qquad \mathbb{P}_{ij}\equiv q_i\beta_j-q_j\beta_i$$

+ Impose some homogeneity conditions on h

BASICS OF LIGHT-CONE: INTERACTIONS

<u>Dynamical constraints</u> (main difficulty)

$$[H, J] = 0 \qquad \Rightarrow \qquad [H_2, J_n] + [H_3, J_{n-1}] + \cdots + [H_{n-1}, J_3] + [H_n, J_2] = 0$$

Reminiscent of the Noether procedure

CUBIC INTERACTIONS

$$[H_2, J_3] + [H_3, J_2] = 0$$

SOLUTION

$$h_3^{\lambda_1\lambda_2\lambda_3} = C^{\lambda_1\lambda_2\lambda_3}\frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} + \bar{C}^{-\lambda_1-\lambda_2-\lambda_3}\frac{\mathbb{P}_{12}^{-\lambda_1-\lambda_2-\lambda_3}}{\beta_1^{-\lambda_1}\beta_2^{-\lambda_2}\beta_3^{-\lambda_3}}$$

where C are arbitrary coupling constants

[Bengtsson, Bengtsson, Brink'83; Bengtsson, Bengtsson, Linden'87; Metsaev'91]

DERIVATIVES

$$N(\partial) = |\lambda_1 + \lambda_2 + \lambda_3|$$

Individual helicities can be negative. These vertices <u>violate bounds</u> on the number of derivatives in <u>covariant approaches</u>. In particular

$$\{s_1, s_2, s_3\} = \{s, s, 2\}, \qquad \{\lambda_1, \lambda_2, \lambda_3\} = \{s, -s, 2\} \qquad \Rightarrow \qquad N(\partial) = 2$$

<u>Light-cone allows to couple minimally higher spins to gravity!</u>

QUARTIC ORDER ANALYSIS

$$[H_2, J_2] + [H_3, J_3] + [H_4, J_2] = 0$$

One can adjust: $C^{\lambda_1\lambda_2\lambda_3}, \quad \bar{C}^{\lambda_1\lambda_2\lambda_3}, \quad h_4^{\lambda_1\ldots\lambda_4}, \quad j_4^{\lambda_1\ldots\lambda_4}.$

A KEY OBSERVATION

only [H3,J3] has a non-vanishing contribution to the q-independent part of the equation. So, this part of [H3,J3] should <u>vanish separately.</u>

[Metsaev'91]

CHIRAL HIGHER-SPIN THEORY

$$[H_3, J_3]|_{q=0} = 0$$

Receives contributions only from antiholomorphic vertices

$$C^{\lambda_1\lambda_2\lambda_3}\frac{\bar{\mathbb{P}}_{12}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}}$$

SOLUTION

$$C^{\lambda_1\lambda_2\lambda_3} = \frac{\ell^{\lambda_1+\lambda_2+\lambda_3-1}}{(\lambda_1+\lambda_2+\lambda_3-1)!}$$

[Metsaev'91]

Moreover, if we have only antiholomorphic vertices, the remaining terms in the consistency condition are zero, hence the consistency condition is satisfied (to all orders).
This leads us to a chiral higher spin theory.

CHIRAL HIGHER-SPIN THEORY

COMPLETE ACTION

$$S = -\int d^4x \partial_\mu \Phi^{-\lambda} \partial^\mu \Phi^\lambda$$
$$+ g \int d^4x \frac{\ell^{\lambda_1+\lambda_2+\lambda_3-1}}{(\lambda_1+\lambda_2+\lambda_3-1)!} \frac{\bar{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3}}{(\partial_1^+)^{\lambda_1}(\partial_2^+)^{\lambda_2}(\partial_3^+)^{\lambda_3}} \Phi^{\lambda_1}\Phi^{\lambda_2}\Phi^{\lambda_3}$$

1) Consistent to all orders in coupling constant
2) Contains lower derivative couplings, absent in covariant approaches. In particular, minimal coupling to gravity
3) Obeys generalised Weinberg's equivalence principle: coupling to gravity is universal.
4) Has vanishing four-point amplitude. Expected to hold for n-points.
5) Avoids no-go's (in somewhat degenerate manner).

[Ponomarev, Skvortsov'16]

UNIVERSAL PROPERTIES OF HOLOMORPHIC THEORIES

LIGHT-CONE AND SYMMETRIES

NOETHER PROCEDURE

g^2 consistency conditions define a Lie algebra

(via deformation of an Abelian algebra of a free theory)

LIGHT-CONE DEFORMATION PROCEDURE?

<u>Different story</u>: gauge symmetry is completely fixed!

<u>Option</u>: promote vertices to the covariant form and find the algebra

[Sleight, Taronna'16]

Is there any algebraic structure relevant to the light-cone theory <u>irrespectively of any extensions</u>?

(ANTI)HOLOMORPHIC CUBIC THEORIES

CLASS OF THEORIES

$$S = \sum_{\lambda_1,\lambda_2} \int dq_1 dq_2 \delta(q_1+q_2)\delta_{a_1 a_2}(q_2^- \beta_2 + q_2 \bar{q}_2)\Phi^{\lambda_1|a_1}(q_1)\Phi^{\lambda_2|a_2}(q_2)$$

$$+ \sum_{\lambda_1,\lambda_2,\lambda_3} \int dq_1 dq_2 dq_3 \delta(q_1+q_2+q_3) h_{\lambda_1|a_1\ \lambda_2|a_2\ \lambda_3|a_3}(\bar{\mathbb{P}}, \beta_i)$$

$$\times \Phi^{\lambda_1|a_1}(q_1)\Phi^{\lambda_2|a_2}(q_2)\Phi^{\lambda_3|a_3}(q_3)$$

+ subsectors of parity-invariant theories

+ consistent on their own

LIE ALGEBRA

Define the inner product

$$(\Phi, \Psi) \equiv \sum_{\lambda_1, \lambda_2} \int dq_1 dq_2 \delta(q_1 + q_2) \delta^{\lambda_1 + \lambda_2, 0} \delta_{a_1 a_2} \Phi^{\lambda_1 | a_1}(q_1) \Psi^{\lambda_2 | a_2}(q_2)$$

Raising/lowering indices = swaps ingoing and outgoing particles

OBSERVATION

$$f^{\lambda_1 | a_1}{}_{\lambda_2 | a_2 \; \lambda_3 | a_3}(q_1; q_2, q_3) \equiv h^{\lambda_1 | a_1}{}_{\lambda_2 | a_2 \; \lambda_3 | a_3}(\bar{\mathbb{P}}, \beta_i) \frac{\beta_2 \beta_3}{\bar{\mathbb{P}} \beta_1}$$

<u>obey the Jacobi identity</u>

That is EOM's are

$$(\partial^- \partial^+ + \partial \bar{\partial}) \Phi^a - g f^a{}_{bc} \partial^+ \left(\frac{\bar{\partial}}{\partial^+} \Phi^b \Phi^c \right) = 0$$

EXAMPLES: SELF-DUAL YANG MILLS

$$f^{\lambda_1|a_1}{}_{\lambda_2|a_2\ \lambda_3|a_3}(q_1;q_2,q_3) = \delta(-q_1+q_2+q_3)\delta^{\lambda_1,1}\delta_{\lambda_2;1}\delta_{\lambda_3;1}f^{a_1}{}_{a_2a_3}$$

Gives: loop extension of the internal Lie algebra $\mathfrak{g}$

$$\begin{aligned} f^{a_1}{}_{a_2a_3} \qquad &\rightarrow \qquad \mathfrak{g} \\ f^{a_1}{}_{a_2a_3}(q_1;q_2,q_3) \qquad &\rightarrow \qquad \mathfrak{g}\otimes C^\infty(\mathbb{R}^{3,1}) \end{aligned}$$

$$[T_a,T_b] = f^c{}_{ab}T_c \qquad \rightarrow \qquad [T_a(x),T_b(x)] = f^c{}_{ab}T_c(x)$$

EXAMPLES: SELF-DUAL GRAVITY

$$f^{\lambda_1|a_1}{}_{\lambda_2|a_2\ \lambda_3|a_3}(q_1;q_2,q_3)=\delta(-q_1+q_2+q_3)\delta^{\lambda_1,2}\delta_{\lambda_2;2}\delta_{\lambda_3;2}\frac{\bar{\mathbb{P}}}{\beta_2\beta_3}\beta_1$$

beta dependence can be removed by rescaling polarisation vectors

Gives: affine extension of area-preserving diffeomorphisms $\mathrm{SDiff}(\mathbb{R}^2)\otimes C^\infty(\mathbb{R}^2)$

$$[F,G]=\partial_x F\partial_- G-\partial_x G\partial_- F$$

[Monteiro, O'Connell'11]

EXAMPLES: CHIRAL HIGHER-SPIN THEORY

$$f^{\lambda_1|a_1}{}_{\lambda_2|a_2\ \lambda_3|a_3}(q_1;q_2,q_3) = \frac{\delta(-q_1+q_2+q_3)}{(-\hat{\lambda}_1+\hat{\lambda}_2+\hat{\lambda}_3)!}\frac{\bar{\mathbb{P}}^{-\hat{\lambda}_1+\hat{\lambda}_2+\hat{\lambda}_3}}{\beta_2^{\hat{\lambda}_2}\beta_3^{\hat{\lambda}_3}}\beta_1^{\hat{\lambda}_1}$$

$$\hat{\lambda}_i \equiv \lambda_i - 1$$

Remove beta dependence, define

$$F(x^\mu;z) \equiv \sum_\lambda F(x^\mu)z^\lambda$$

Gives

$$[F,G] = \sinh\frac{1}{z}\left(\partial_x^F\partial_-^G - \partial_x^G\partial_-^F\right)FG$$

Algebra: $\mathfrak{g}_{HS}\otimes C^\infty(\mathbb{R}^2)$

CHIRAL HIGHER-SPIN ALGEBRA

$$[F, G] = \sinh \frac{1}{z} \left(\partial_x^F \partial_-^G - \partial_x^G \partial_-^F \right) FG$$

Properties

$$[T_{\lambda_1}, T_{\lambda_2}] = T_{\lambda_1+\lambda_2-2} + T_{\lambda_1+\lambda_2-4} + \dots + T_{\lambda_1+\lambda_2-1038} + \dots$$

$$[T_\lambda, T_2] = T_\lambda + T_{\lambda-2} + \dots$$

VS

Higher spin algebra <u>in AdS</u>

$$[T_{s_1}, T_{s_2}] = T_{s_1+s_2-2} + T_{s_1+s_2-4} + \dots + T_{|s_1-s_2|+2}$$

$$[T_s, T_2] = T_s$$

CONTEXT: COLOUR-KINEMATICS DUALITY

Yang-Mills amplitudes can be written in a cubic form

$$\mathcal{A} = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

$$c_s = f^{a_4}{}_{a_3 b} f^{b}{}_{a_2 a_1}, \quad c_s = f^{a_4}{}_{a_2 b} f^{b}{}_{a_1 a_3}, \quad c_s = f^{a_4}{}_{a_1 b} f^{b}{}_{a_3 a_2}$$

$$c_s + c_t + c_u = 0, \qquad n_s + n_t + n_u = 0$$

Signals existence of some kinematic Lie algebra

[Bern, Carrasco, Johansson'08]

For self-dual Yang-Mills was identified as an algebra of area-preserving diffeos

[Monteiro, O'Connell'11]

CONTEXT: SELF-DUAL YANG-MILLS

$$(\partial^- \partial^+ + \partial\bar{\partial})\Phi^a - g f^a{}_{bc} \partial^+ \left(\frac{\bar{\partial}}{\partial^+} \Phi^b \Phi^c \right) = 0$$

Can be rewritten as

$$\partial^- A^{a|x} - \partial^x A^{a|-} + g f^a{}_{bc} A^{b|-} A^{c|x} = 0, \qquad \partial^+ A^{a|-} + \partial^{\bar{x}} A^{a|x} = 0,$$

$$A^{a|x} \equiv \Phi^a$$

Go even further: undoing the light-cone gauge

$$\begin{aligned} F^{-x|a} &= 0 \\ F^{+\bar{x}|a} &= 0 \\ F^{+-|a} + F^{\bar{x}x|a} &= 0 \end{aligned}$$

Reveals connection to self-duality conditions

$$F^a = i * F^a$$

SELF-DUALITY AND CONSEQUENCES

INFINITE HIDDEN SYMMETRIES & INTEGRABILITY

SDYM: $$su(N) \otimes C[\lambda, \lambda^{-1}, x - \lambda^{-1}x^{+}, x^{-} + \lambda^{-1}\bar{x}]$$

Directly: *[Chau, Ge, Wu, Sinha, Dolan, Crane ...]*

Relating to Riemann-Hilbert problem: *[Ueno, Nakamura, ...]*

Via twistors: *[Penrose, Atiyah, Hitchin, Singer, Ward, ...]*

SDGR: $$\mathrm{SDiff}(\mathbb{R}^2) \otimes C[\lambda, \lambda^{-1}]$$

[Plebanski, Boyer, Takasaki,...]

SELF-DUALITY AND CONSEQUENCES

RELATION TO 2D SIGMA MODELS

SDGR

$$\partial_+ K_{\bar{x}} - \partial_{\bar{x}} K_+ = 0$$
$$\partial_+ K_{\bar{x}} - \partial_{\bar{x}} K_+ + g[K_+, K_{\bar{x}}] = 0$$

where K is a connection of $\mathrm{SDiff}(\mathbb{R}^2)$ in (x^-, x) plane

[Park'90; Husain'94]

INTEGRABILITY & HIDDEN SYMMETRIES

Directly extends to all other (anti)holomorphic theories

SDGR	$\mathrm{SDiff}(\mathbb{R}^2) \otimes C[\lambda, \lambda^{-1}]$
CHS	$\mathfrak{g}_{HS} \otimes C[\lambda, \lambda^{-1}]$

CONJECTURE

Cubic (anti)holomorphic theories

$$[H, J] = 0 \qquad \Leftrightarrow \qquad \text{Jacobi identity}$$

Essentially proven: it is understood how to solve both sides systematically

[Ponomarev, Skvortsov'16]

More direct arguments? Extension to theories with higher vertices?

General (anti)holomorphic theories

$$[H, J] = 0 \qquad \Leftrightarrow \qquad L_\infty \text{ relations}$$

Knowing RHS one can solve for H. What about underlying 2D structure?

SUMMARY

GENERAL

Lorentz tensors may be constraining

- massive fields
- AdS
- other dimensions

SUMMARY

ON THE CHIRAL THEORY

- There is a chiral higher spin theory, consistent to all orders. Avoids no-go's!
- It features extra local lower-derivative interactions, e.g. minimal coupling to gravity
- Generalised equivalence principle holds
- We found a simple way to derive the formula of Metsaev.
- "Agreement" with AdS/CFT
- Vanishing 4-point function

SUMMARY

UNIVERSAL PROPERTIES OF (ANTI)HOLOMORPHIC THEORIES

- All cubic (anti)holomorphic theories are SDYM theories
- Extend colour-kinematics duality to higher spins (in this sector)
- Integrability & infinite symmetries carry over to all (anti)holomorphic theories
- Conjecture: Lorentz invariance = L infinity relations
- Possibly will allow to find all theories of this class