Price's theorem in Gauge/Gravity Duality

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Items to discuss:

1. Price's Theorem for distorted Black Holes
2. Relaxation to the spherical symmetry for SMBHs
3. Small BHs and Price's Theorem in the $\mathrm{G}_{\mathrm{auge}}/\mathrm{G}_{\mathrm{ravity}}\mathrm{D}_{\mathrm{uality}}$
4. Summary and conclusions

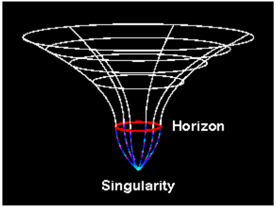
Price's Theorem for distorted Black Holes

No Hair Theorem

J. Droste Doct.Thes. Leiden 1916

Schwarzschild Black Hole (1916): Mass M

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$



Kerr-Newmann Black Holes (1963-1965):

$$M, \qquad Q, \qquad J$$

No Hair Theorem (Wheeler 1967-1969)

Price's Theorem

A part of the game in science consists of solid claims, which are unproved, and theorems, which never been proved to the whole extent.

One of the arguments behind the black hole "no-hair" theorem is the Price's theorem (Price 1971-72)

According to Price a BH becomes "bald" very fast, loosing all but three hair:
the mass, the charge and the momentum.

Hence the only spherical topology of the BH horizon is admitted for non-rotating BHs!

Q: What about distorted non-rotating BHs then? Do they really exist in Nature?

A few words on distorted BHs

The Israel's uniqueness theorem (Israel 1967) selects the Schwarzschild solution as the only static vacuum BH solution of the Einstein eqs. in an asymtotically flat (AF) spacetime.

For a real astrophysical problem the Schwarzschild solution is highly idealized (even if we do not take into account a possible BH rotation).

Any presence of matter, e.g. in accretion disk, distorts the metric!

If a static distribution of matter is localized outside the BH horizon, the spacetime at the vicinity of the horizon remains vacuum spacetime.

The solution of a BH type, describing such a configuration, is called a distorted BH. (Weyl 1917; Geroch&Hartle 1982).

A distortion of the static/stationary BH horizon is the modelling, to some extent, the interaction of a BH with the external matter.

Price's Theorem

Q: Do distorted non-rotating BHs are just mathematical solutions or they correspond to real astrophysical objects?

This is not a purely academic question; people now focus on SMBHs as astrophysical objects on which many issues of BH's theory may be experimentally verified. (Though they're rotating, we can think about them as of non-rotating in the first approximation.)

The interest to experiments with Sg A* is twofold:

- once the no-hair theorem will be verified, it definitely works for astrophysical BHs;
- once the no-hair theorem does not work for astrophysical BHs, we should think about an extension(?) of the Einstein's General Relativity.

Consequently, the Price's theorem, proved for only a small distortion over the spherical background (Thorne 1970-72), is correct or not.

Relaxation to the spherical symmetry for SMBHs

Relaxation to the spherical symmetry

The relaxation from the skew to the spherical geometry of a (non-rotating in the first approximation) BH horizon is computed accordingly to

$$\omega \simeq 0.37 M^{-1} \qquad (\hbar = c = G = 1)$$

where M is the BH mass.

This result comes from an analytic formula of computing the frequency of gravitational perturbations (QNMs) with an angular momentum (multipole number) $l \geq 2$ for the Schwarzschild BH of mass M:

$$\Re\,(\omega M) \approx \left[\frac{(l-1)(l+2)}{27} - \left(\frac{1}{2\sqrt{27}}\right)^2\right]^{1/2}, \quad \Im\,(\omega M) \approx \frac{1}{2\sqrt{27}}$$

The approximation has been justified by numerics (from Chandrasekhar 1975 to improving numerics of Cho 2011), or by analytical (the WKB and other) methods (Iyer 1986; Siopsis 2008).

Relaxation to the spherical symmetry

Two remarks on the Master Formula

$$\Re(\omega M) \approx \left[\frac{(l-1)(l+2)}{27} - \left(\frac{1}{2\sqrt{27}}\right)^2\right]^{1/2}, \quad \Im(\omega M) \approx \frac{1}{2\sqrt{27}}$$

- It does not take into account (small) deviations in the imaginary part of the l-wave frequencies. These deviations are classified by a positive number n (the overtone number) and are sharply indicated (from Chandrasekhar 1975) in numerics;
- Account of the BH (small) charge and momentum does not essentially modify the result. In most cases (including AdS) the Master Formula works great.

In usual Unit System taking $l = 2$ we have the early announced

$$\omega \sim 0.37 \frac{c^3}{MG} \approx 7.5 \cdot 10^4 \sec^{-1} \frac{M_\odot}{M}$$

with the Sun mass $M_\odot \sim 2 \cdot 10^{30}$ kg.

Relaxation to the spherical symmetry

According to the Price's theorem all distortions from the spherically-symmetric case have to be radiated with gravitational waves out of the BH.

The characteristic time associated with the obtained frequency (we have chosen this frequency as the most undamped one, whose imaginary part, hence the damping strength, is minimal) is

$$\tau \sim \omega^{-1} \sim 1.3(3) \cdot 10^{-4} \frac{M}{M_\odot} \,\mathrm{sec}$$

For the standard (non-supermassive) black holes of tens of the solar mass

$$\tau \sim 1.3(3) \cdot 10^{-3} \,\mathrm{sec}$$

The relaxation to the "perfect" state time for the SMBH with the mass $M \sim 10^9 \, M_\odot$ is

$$\tau \sim 1.3(3) \cdot 10^{5} \,\mathrm{sec} \sim 37 \,\mathrm{hrs}$$

Small BHs and Price's Theorem in the Gauge/Gravity Duality

What is the QGP?

The Quark-Gluon Plasma is a state of the nuclear matter:

- At the QCD strong coupling constant.
- At non-zero temperature and finite charges densities.
- Having the transverse to the HIC direction momentum flow.
- And with damping the transverse momentum.

Physics of QGP

The QGP is a fluid with non-trivial shear viscosity!

Hence the theory of QGP is hydrodynamics of thermal QCD at the strong coupling constant and the finite chemical potentials.

What is the QGP?

QGP – QFT-type Theory

The hydrodynamic limit of thermal QFT (Shuryak'79; Bjorken'83; Hosoya,Sakagami,Takao'84; Hosoya & Kajantie'85). Phenomenology.

Transport coefficients are determined by summing the Feynman diagrams (BFKL'75-78; Jeon'94, McLerran & Venugopalan'94;. . . ; Christiansen et.al 2014, Jimenes-Alba & Yee 2015)

QGP – Experiment

Early times (1980-2000) (Summarized by CERN (SPS) WA98 Group: Aggarwal et al, PRL 85 (2000) 3595, Indirect QGP observation).

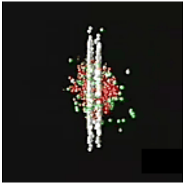
Observation (2000-2004) (RHIC (PHOBOS, STAR, PHENIX Groups): Summarized in Gyulassy&McLerran, NPA 750, 30 (2005)).

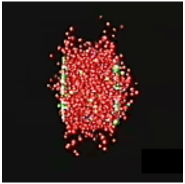
Data improving (RHIC (PHOBOS, STAR, PHENIX, BRAHMS), CERN LHC (ALICE, SMS, ATLAS)), JINR NICA

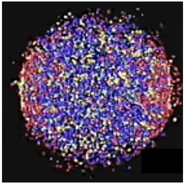
What is the QGP?

Borrowed from RHIC Web-page


1. Ions about to collide*


2. Ion collision


3. Quarks, gluons freed


4. Plasma created

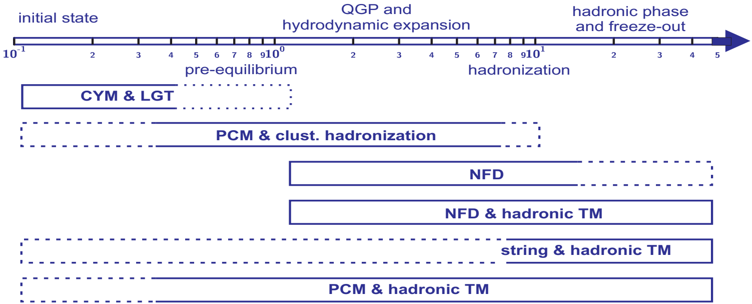
The QGP parameters for fun:

- Time scales: $10^{-24} \div 10^{-22}$ sec
- Temperature: $5.5 \cdot 10^{12}$ K
- Energy density: $1.8 \cdot 10^{15}$ g$\cdot$cm^{-3}
- Bulk pressure: $0.52 \cdot 10^{20}$ bar
- Shear viscosity/entropy density: $\geq 6.08 \cdot 10^{-13}$ K$\cdot$s

The QGP Theory: A remark

Due to the QGP complexity perhaps a right way is to develop a unified approach from different, sutable at the different stages of the QGP evolution, approaches

Fig. is borrowed from H. Song PhD, Ohio SU 2009

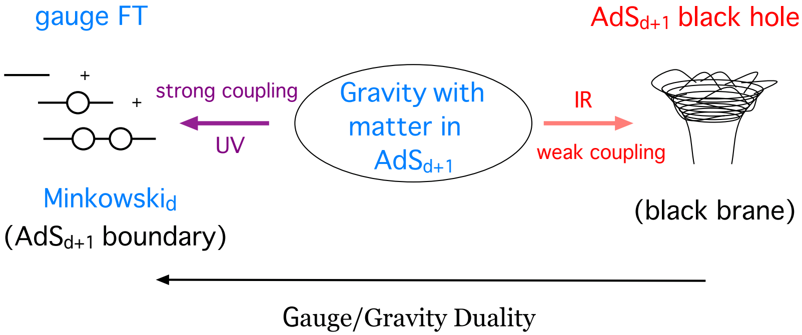


PCM≡ PartonClasterModels; NFD≡NuclearFluidDynamics; TM≡TransportModels

We are still far from the ultimate theory of the QGP.

From Strings to the QGP

Gauge/Gravity duality is a plain version of the AdS/CFT correspondence



The black hole is the strong coupling limit of the gauge theory!

From Strings to the QGP

Within Gauge/Gravity duality properties of a gauge theory at the strong coupling constant (QGP) are encoded in properties of a Black Hole in anti-de Sitter space.

Q: How Black Hole models the QGP fluid viscosity?

When a ball falls into a liquid the surface waves are generated. Due to viscosity they quickly decay, and the liquid gets to the equilibrium back.

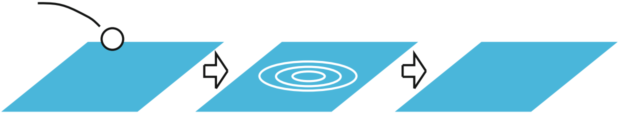


Fig. is borrowed from M. Natsuume, AdS/CFT Duality User Guide, Springer 2015

From Strings to the QGP

When we drop something into a Black Hole the shape of the horizon is changed. A part of perturbations is absorbed by the Black Hole; another part is radiated out as gravitational waves. Very quickly the BH returns to the equilibrium state. Effectively, it is due to viscosity of the BH (stretched) horizon.

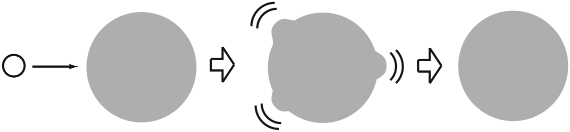


Fig. is borrowed from M. Natsuume, AdS/CFT Duality User Guide, Springer 2015

A Black Hole behaves like a viscous liquid!

From Strings to the QGP

For a wide range of Black Holes (Black Branes) it turns out that

$$\eta \sim \lim_{\omega \to 0} \sigma_{\mathrm{BH}} = A$$

where η is the effective "shear viscosity" of the BH horizon; ω is the frequency of the scattered by the BH waves; A is the BH area.

According to Bekenstein the entropy density-area relation for a BH is

$$s_{\mathrm{BH}} = \frac{k_B}{4G\hbar} A$$

Hence,

$$\frac{\eta}{s} \sim \frac{4G\hbar}{k_B}$$

For most of Black Branes the proportionality coefficient is $(16\pi G)^{-1}$ (Kovtun, Son, Starinets 2005)

$$\eta/s = \frac{1}{4\pi} = 0.08 \qquad (c = \hbar = k_B = 1)$$

Price's Theorem in the GGD

Let's check consequencies of the Price's theorem for the GGD.

First, let us estimate the BH horizon value r_+. Applying the gauge/gravity duality to the QGP, notice the characteristic temperature of the QGP, $T_{QGP} \sim 220\,\mathrm{MeV} \sim 2.553 \cdot 10^{12}\,\mathrm{K}$.

The radial location of the small neutral AdS_4 Schwarzschild BH horizon is very close to the radial coordinate of the flat-space Schwarzschild BH horizon. Hence, the small BH mass can be computed from

$$T = \frac{\hbar c^3}{8\pi M G k_B} \sim 2.553 \cdot 10^{12}\,\mathrm{K} \quad \leftrightsquigarrow \quad M = 2.4\ M_\odot \cdot 10^{-20}$$

The radius of the horizon for the BH of this mass is

$$r_+ \approx 7.0869 \cdot 10^{-17}\,\mathrm{m} = 7.0869 \cdot 10^{-2}\,\mathrm{fm}$$

Price's Theorem in the GGD

If we naively use the obtained

$$\tau \sim \omega^{-1} \sim 1.3(3) \cdot 10^{-4} \frac{M}{M_\odot} \sec$$

we get the following relaxation to the "perfect" spherical geometry time:

$$\tau \sim 3.2 \cdot 10^{-24} \sec$$

Even the naively computed relaxation time is in the remarkable agreement with the characteristic lifetime of the QGP:

$$\tau_{QGP} \sim \frac{1\mathrm{fm}}{c} \approx 3.33564 \cdot 10^{-24} \sec$$

Hence, effects of the non-uniformity on the BH horizon are important in the gauge/gravity duality.

Price's Theorem in the GGD

Now let's improve the result on account of numerically computed values of the QNMs for the AdS-Schwarzschild BH.

A brief analysis of Tables 11 and 15 in Cardoso et al gr-qc/0305037 leads to the frequency $\hat{\omega} \sim 4$ of the less damped gravitational mode in the inverse AdS length κ units ($\hat{\omega} \equiv \omega/\kappa$).

This frequency is computed for a small BH with the horizon location $\kappa r_+ = 0.2$.

From $r_+ \approx 7.0869 \cdot 10^{-17}\,\mathrm{m}$ we recover $\kappa \sim 2.82 \cdot 10^{15}\,\mathrm{m}^{-1}$.

Therefore,

$$\omega \sim 4\kappa \approx 1.128 \cdot 10^{16}\mathrm{m}^{-1} \sim 3.384 \cdot 10^{24}\,\mathrm{sec}^{-1}$$

and

$$\tau \sim 0.3 \cdot 10^{-24}\,\mathrm{sec}$$

Price's Theorem in the GGD

Choosing the frequency values of the most undamped even ($\hat{\omega} \approx 3.56571$) and odd ($\hat{\omega} \approx 4.91594$) gravitational modes (Cardoso et al gr-qc/0305037) we get the following relaxation time to the spherical geometry:

$$\tau \approx 0.27 \div 0.37 \cdot 10^{-24}\,\mathrm{sec}$$

This improved estimation is of the one order less than the characteristic QGP lifetime

$$\tau_{QGP} \sim \frac{1\mathrm{fm}}{c} \approx 3.33564 \cdot 10^{-24}\,\mathrm{sec}$$

However, at the early stage of the QGP formation the characteristic length is of 0.1 fm (Casalderrey-Solana et al CUP 2014).

The early-time QGP is in the highly non-equilibrium state and the difference of the BH horizon geometry from the pure spherical one is worth to be taken into account within the gauge/gravity duality.

Summary and conclusions

Summary and conclusions

The Price's Theorem is one of the stories of Folklore in physics: the complete proof of the theorem is still missing. Nevertheless,

applying the Price's theorem in the Gauge/Gravity Duality context we got a remarkable agreement with the experimentally estimated life-time of the QGP!

This observation is new, and is an additional rationale to apply Gauge/Gravity Duality to the QGP theory.

Another important observation is:

to describe the most interesting early-time highly non-equilibrium state of the QGP within the G/GD one has to deal with time-dependent BH solutions with a skew shape of the horizon on the gravity side of the correspondence.

(Moskalets&AJN 2017 for the Vaidya-type BH with a skew horizon)

Summary and conclusions

However, the conclusions before are applicable for non-astrophysical BHs.

The performed (currently in the data processing stage) and planned experiments on restoring the shape of the SMBH Sg A* horizon shadow,

the Event Horizon Telescope

may give a new evidence on the (in)correctness of the Price's theorem too.

In its turn it gives us new features of the astrophysical/mathematical BHs correspondence.