Supersymmetric description of the dynamical systems with applications to the cosmological problems, quanputers and zeta-functions

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- Why supersymmetry is So universal?
- It describes universal connection of a system to the environment.
Supermathematics unifies discrete and continual aspects of mathematics.

"... all things physical are information-theoretic in origin ..."
John A. Wheeler

We say that we find **New Physics** (NP) when either we find a phenomenon which is forbidden by SM in principal - this is the qualitative level of NP - or we find a significant deviation between precision calculations in SM of an observable quantity and a corresponding experimental value.
In 1900, the British physicist Lord Kelvin is said to have pronounced: "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement." Within three decades, quantum mechanics and Einstein's theory of relativity had revolutionized the field. Today, no physicist would dare assert that our physical knowledge of the universe is near completion. To the contrary, each new discovery seems to unlock a Pandora's box of even bigger, even deeper physics questions.

In the Universe, matter has manly two geometric structures, homogeneous, [Weinberg, 1972] and hierarchical, [Okun, 1991] .
The homogeneous structures are naturally described by real numbers with an infinite number of digits in the fractional part and usual archimedean metrics. The hierarchical structures are described with p-adic numbers with an infinite number of digits in the integer part and non-archimedean metrics, [Koblitz, 1977].

A discrete, finite, regularized, version of the homogenous structures are homogeneous lattices with constant steps and distance rising as arithmetic progression. The discrete version of the hierarchical structures is hierarchical lattice-tree with scale rising in geometric progression.

There is an opinion that present day theoretical physics needs (almost) all mathematics, and the progress of modern mathematics is stimulated by fundamental problems of theoretical physics.

Let us consider a general dynamical system described by the following system of the ordinary differential equations [Arnold, 1978]

$$\dot{x}_n = v_n(x),\ 1 \leq n \leq N, \tag{1}$$

$\dot{x}_n$ stands for the total derivative with respect to the parameter t. When the number of the degrees of freedom is even, and

$$v_n(x) = \varepsilon_{nm}\frac{\partial H_0}{\partial x_m},\ 1 \leq n, m \leq 2M, \tag{2}$$

the system (1) is Hamiltonian one and can be put in the form

$$\dot{x}_n = \{x_n, H_0\}_0, \tag{3}$$

where the Poisson bracket is defined as

$$\{A, B\}_0 = \varepsilon_{nm}\frac{\partial A}{\partial x_n}\frac{\partial B}{\partial x_m} = A\frac{\overleftarrow{\partial}}{\partial x_n}\varepsilon_{nm}\frac{\overrightarrow{\partial}}{\partial x_m}B, \tag{4}$$

and summation rule under repeated indices has been used.

Let us consider the following Lagrangian

$$L = (\dot{x}_n - v_n(x))\psi_n \tag{5}$$

and the corresponding equations of motion

$$\dot{x}_n = v_n(x), \dot{\psi}_n = -\frac{\partial v_m}{\partial x_n}\psi_m. \tag{6}$$

The system (6) extends the general system (1) by linear equation for the variables ψ. The extended system can be put in the Hamiltonian form [Makhaldiani, Voskresenskaya, 1997]

$$\dot{x}_n = \{x_n, H_1\}_1, \dot{\psi}_n = \{\psi_n, H_1\}_1, \tag{7}$$

where first level (order) Hamiltonian is

$$H_1 = v_n(x)\psi_n \tag{8}$$

and (first level) bracket is defined as

$$\{A, B\}_1 = A(\frac{\overleftarrow{\partial}}{\partial x_n}\frac{\overrightarrow{\partial}}{\partial \psi_n} - \frac{\overleftarrow{\partial}}{\partial \psi_n}\frac{\overrightarrow{\partial}}{\partial x_n})B. \tag{9}$$

Note that when the Grassmann grading [Berezin, 1987] of the conjugated variables x_n and ψ_n are different, the bracket (9) is known as Buttin bracket[Buttin, 1996].
In the Faddeev-Jackiw formalism [Faddeev, Jackiw, 1988] for the Hamiltonian treatment of systems defined by first-order Lagrangians, i.e. by a Lagrangian of the form

$$L = f_n(x)\dot{x}_n - H(x), \tag{10}$$

motion equations

$$f_{mn}\dot{x}_n = \frac{\partial H}{\partial x_m}, \tag{11}$$

for the regular structure function f_{mn}, can be put in the explicit hamiltonian (Poisson; Dirac) form

$$\dot{x}_n = f^{-1}_{nm}\frac{\partial H}{\partial x_m} = \{x_n, x_m\}\frac{\partial H}{\partial x_m} = \{x_n, H\}, \tag{12}$$

where the fundamental Poisson (Dirac) bracket is

$$\{x_n, x_m\} = f^{-1}_{nm},\ f_{mn} = \partial_m f_n - \partial_n f_m. \tag{13}$$

The system (6) is an important example of the first order regular hamiltonian systems. Indeed, in the new variables,

$$y_n^1 = x_n, y_n^2 = \psi_n, \tag{14}$$

lagrangian (5) takes the following first order form

$$\begin{aligned} &L = (\dot{x}_n - v_n(x))\psi_n \Rightarrow \frac{1}{2}(\dot{x}_n\psi_n - \dot{\psi}_n x_n) - v_n(x)\psi_n \\ &= \frac{1}{2}y_n^a \varepsilon^{ab}\dot{y}_n^b - H(y) \\ &= f_n^a(y)\dot{y}_n^a - H(y), f_n^a = \frac{1}{2}y_n^b\varepsilon^{ba}, H = v_n(y^1)y_n^2, \\ &f_{nm}^{ab} = \frac{\partial f_m^b}{\partial y_n^a} - \frac{\partial f_n^a}{\partial y_m^b} = \varepsilon^{ab}\delta_{nm}; \end{aligned} \tag{15}$$

corresponding motion equations and the fundamental Poisson bracket are

$$\dot{y}_n^a = \varepsilon_{ab}\delta_{nm}\frac{\partial H}{\partial y_m^b} = \{y_n^a, H\}, \{y_n^a, y_m^b\} = \varepsilon_{ab}\delta_{nm}. \tag{16}$$

To the canonical quantization of this system corresponds

$$[\hat{y}^a_n, \hat{y}^b_m] = i\hbar\varepsilon_{ab}\delta_{nm},\ \hat{y}^1_n = y^1_n,\ \hat{y}^2_n = -i\hbar\frac{\partial}{\partial y^1_n} \tag{17}$$

In this quantum theory, classical part, motion equations for y^1_n, remain classical.

Nabu – Babylonian God
of Wisdom and Writing.

The Hamiltonian mechanics (HM) is in the fundamentals of mathematical description of the physical theories [Faddeev, Takhtajan, 1990]. But HM is in a sense blind; e.g., it does not make a difference between two opposites: the ergodic Hamiltonian systems (with just one integral of motion) [Sinai, 1993] and (super)integrable Hamiltonian systems (with maximal number of the integrals of motion).
Nabu mechanics (NM) [Nambu, 1973, Whittaker, 1927] is a proper generalization of the HM, which makes the difference between dynamical systems with different numbers of integrals of motion explicit (see, e.g.[Makhaldiani, 2007]).

In the canonical formulation, the equations of motion of a physical system are defined via a Poisson bracket and a Hamiltonian, [Arnold, 1978]. In Nambu's formulation, the Poisson bracket is replaced by the Nambu bracket with $n+1, n \geq 1$, slots. For $n = 1$, we have the canonical formalism with one Hamiltonian. For $n \geq 2$, we have Nambu-Poisson formalism, with n Hamiltonians, [Nambu, 1973], [Whittaker, 1927].

The system of N vortexes can be described by the following system of differential equations, [Aref, 1983, Meleshko,Konstantinov, 1993]

$$\dot{z}_n = i \sum_{m \neq n}^{N} \frac{\gamma_m}{z_n^* - z_m^*}, \quad 1 \leq n \leq N, \tag{18}$$

where $z_n = x_n + iy_n$ are complex coordinate of the centre of n-th vortex, for $N = 3$, and the quantities

$$\begin{aligned} u_1 &= ln|z_2 - z_3|^2, \\ u_2 &= ln|z_3 - z_1|^2, \\ u_3 &= ln|z_1 - z_2|^2 \end{aligned} \tag{19}$$

reduce to the following system

$$\begin{aligned} \dot{u}_1 &= \gamma_1(e^{u_2} - e^{u_3}), \\ \dot{u}_2 &= \gamma_2(e^{u_3} - e^{u_1}), \\ \dot{u}_3 &= \gamma_3(e^{u_1} - e^{u_2}), \end{aligned} \tag{20}$$

The system (20) has two integrals of motion

$$H_1 = \sum_{i=1}^{3} \frac{e^{u_i}}{\gamma_i}, H_2 = \sum_{i=1}^{3} \frac{u_i}{\gamma_i}$$

and can be presented in the Nambu–Poisson form, [Makhaldiani, 1997,2]

$$\dot{u}_i = \omega_{ijk} \frac{\partial H_1}{\partial u_j} \frac{\partial H_2}{\partial u_k} = \{x_i, H_1, H_2\} = \omega_{ijk} \frac{e^{u_j}}{\gamma_j} \frac{1}{\gamma_k},$$

where

$$\omega_{ijk} = \epsilon_{ijk} \rho, \rho = \gamma_1 \gamma_2 \gamma_3$$

and the Nambu–Poisson bracket of the functions A, B, C on the three-dimensional phase space is

$$\{A, B, C\} = \omega_{ijk} \frac{\partial A}{\partial u_i} \frac{\partial B}{\partial u_j} \frac{\partial C}{\partial u_k}. \tag{21}$$

This system is superintegrable: for $N = 3$ degrees of freedom, we have maximal number of the integrals of motion $N - 1 = 2$.

As an example of the infinite dimensional Nambu-Poisson dynamics, let me conside the following extension of Schr$\ddot{o}$dinger quantum mechanics [Makhaldiani, 2000]

$$iV_t = \Delta V - \frac{V^2}{2}, \tag{22}$$

$$i\psi_t = -\Delta\psi + V\psi. \tag{23}$$

An interesting solution to the equation for the potential (22) is

$$V = \frac{4(4-d)}{r^2}, \tag{24}$$

where d is the dimension of the spase. In the case of $d = 1$, we have the potential of conformal quantum mechanics.

The variational formulation of the extended quantum theory, is given by the following Lagrangian

$$L = (iV_t - \Delta V + \frac{1}{2}V^2)\psi. \tag{25}$$

The momentum variables are

$$P_v = \frac{\partial L}{\partial V_t} = i\psi, P_\psi = 0. \tag{26}$$

As Hamiltonians of the Nambu-theoretic formulation, we take the following integrals of motion

$$\begin{aligned} H_1 &= \int d^d x(\Delta V - \frac{1}{2}V^2)\psi, \\ H_2 &= \int d^d x(P_v - i\psi), \\ H_3 &= \int d^d x P_\psi. \end{aligned} \tag{27}$$

We invent unifying vector notation, $\phi = (\phi_1, \phi_2, \phi_3, \phi_4) = (\psi, P_\psi, V, P_v)$. Then it may be verified that the equations of the extended quantum theory can be put in the following Nambu-theoretic form

$$\phi_t(x) = \{\phi(x), H_1, H_2, H_3\}, \tag{28}$$

where the bracket is defined as

$$\{A_1, A_2, A_3, A_4\} = i\varepsilon_{ijkl} \int \frac{\delta A_1}{\delta\phi_i(y)} \frac{\delta A_2}{\delta\phi_j(y)} \frac{\delta A_3}{\delta\phi_k(y)} \frac{\delta A_4}{\delta\phi_l(y)} dy$$
$$= i \int \frac{\delta(A_1, A_2, A_3, A_4)}{\delta(\phi_1(y), \phi_2(y), \phi_3(y), \phi_4(y))} dy = i det(\frac{\delta A_k}{\delta\phi_l}). \quad (29)$$

The basic building blocks of M theory are membranes and $M5-$branes. Membranes are fundamental objects carrying electric charges with respect to the 3-form C-field, and $M5$-branes are magnetic solitons. The Nambu-Poisson 3-algebras appear as gauge symmetries of superconformal Chern-Simons nonabelian theories in $2 + 1$ dimensions with the maximum allowed number of $N = 8$ linear supersymmetries.
The Bagger and Lambert [Bagger, Lambert, 2007] and, Gustavsson [Gustavsson, 2007] (BLG) model is based on a 3-algebra,

$$[T^a, T^b, T^c] = f_d^{abc} T^d \tag{30}$$

where T^a, are generators and f_{abcd} is a fully anti-symmetric tensor.

Given this algebra, a maximally supersymmetric Chern-Simons lagrangian is:

$$L = L_{CS} + L_{matter},$$
$$L_{CS} = \frac{1}{2}\varepsilon^{\mu\nu\lambda}(f_{abcd}A^{ab}_{\mu}\partial_{\nu}A^{cd}_{\lambda} + \frac{2}{3}f_{cdag}f^{g}_{efb}A^{ab}_{\mu}A^{cd}_{\nu}A^{ef}_{\lambda}), \qquad (31)$$

$$L_{matter} = \frac{1}{2}B^{Ia}_{\mu}B^{\mu I}_{a} - B^{Ia}_{\mu}D^{\mu}X^{I}_{a}$$
$$+\frac{i}{2}\bar{\psi}^{a}\Gamma^{\mu}D_{\mu}\psi_{a} + \frac{i}{4}\bar{\psi}^{b}\Gamma_{IJ}x^{I}_{c}x^{J}_{d}\psi_{a}f^{abcd}$$
$$-\frac{1}{12}tr([X^I, X^J, X^K][X^I, X^J, X^K]),\ I = 1, 2, ..., 8, \qquad (32)$$

where A^{ab}_{μ} is gauge boson, ψ^a and $X^I = X^I_a T^a$ matter fields. If $a = 1, 2, 3, 4$, then we can obtain an $SO(4)$ gauge symmetry by choosing $f_{abcd} = f\varepsilon_{abcd}, f$ being a constant. It turns out to be the only case that gives a gauge theory with manifest unitarity and $N = 8$ supersymmetry.

The action has the first order form so we can use the formalism of the first section. The motion equations for the gauge fields

$$f^{nm}_{abcd}\dot{A}^{cd}_m(t,x) = \frac{\delta H}{\delta A^{ab}_n(t,x)}, f^{nm}_{abcd} = \varepsilon^{nm} f_{abcd} \tag{33}$$

take canonical form

$$\dot{A}^{ab}_n = f^{abcd}_{nm}\frac{\delta H}{\delta A^{cd}_m} = \{A^{ab}_n, A^{cd}_m\}\frac{\delta H}{\delta A^{cd}_m} = \{A^{ab}_n, H\},$$
$$\{A^{ab}_n(t,x), A^{cd}_m(t,y)\} = \varepsilon_{nm} f^{abcd}\delta^{(2)}(x-y) \tag{34}$$

Nambu-Poisson dynamics of an extended particle with spin in an accelerator

The quasi-classical description of the motion of a relativistic (nonradiating) point particle with spin in accelerators and storage rings includes the equations of orbit motion

$$\begin{aligned}
&\dot{x}_n = f_n(x),\ f_n(x) = \varepsilon_{nm}\partial_m H,\ n, m = 1, 2, ..., 6;\\
&x_n = q_n,\ x_{n+3} = p_n,\ \varepsilon_{n,n+3} = 1,\ n = 1, 2, 3;\\
&H = e\Phi + c\sqrt{\wp^2 + m^2c^2},\ \wp_n = p_n - \frac{e}{c}A_n
\end{aligned} \tag{35}$$

and Thomas-BMT equations
[Tomas, 1927, Bargmann, Michel,Telegdi, 1959] of classical spin motion

$$\begin{aligned}
&\dot{s}_n = \varepsilon_{nmk}\Omega_m s_k = \{H_1, H_2, s_n\},\ H_1 = \Omega \cdot s,\ H_2 = s^2,\\
&\{A, B, C\} = \varepsilon_{nmk}\partial_n A\partial_m B\partial_k C,
\end{aligned} \tag{36}$$

$$\Omega_n = \frac{-e}{m\gamma c}((1+k\gamma)B_n - k\frac{(B\cdot\wp)\wp_n}{m^2c^2(1+\gamma)} + \frac{1+k(1+\gamma)}{mc(1+\gamma)}\varepsilon_{nmk}E_m\wp_k) \qquad (37)$$

where, parameters e and m are the charge and the rest mass of the particle, c is the velocity of light, $k=(g-2)/2$ quantifies the anomalous spin g factor, γ is the Lorentz factor, p_n are components of the kinetic momentum vector, E_n and B_n are the electric and magnetic fields, and A_n and Φ are the vector and scalar potentials;

$$B_n = \varepsilon_{nmk}\partial_m A_k,\ \ E_n = -\partial_n\Phi - \frac{1}{c}\dot{A}_n,$$
$$\gamma = \frac{H - e\Phi}{mc^2} = \sqrt{1+\frac{\wp^2}{m^2c^2}} \qquad (38)$$

The spin motion equations we put in the Nambu-Poisson form. Hamiltonization of this dynamical system according to the general approach of the previous sections we will put in the ground of the optimal control theory of the accelerator.

The general method of Hamiltonization of the dynamical systems we can use also in the spinning particle case. Let us invent unified configuration space $q = (x, p, s),\ x_n = q_n,\ p_n = q_{n+3},\ s_n = q_{n+6},\ n = 1, 2, 3$; extended phase space, (q_n, ψ_n) and hamiltonian

$$H = H(q, \psi) = v_n\psi_n,\ n = 1, 2, ...9; \tag{39}$$

motion equations

$$\begin{aligned} \dot{q}_n &= v_n(q), \\ \dot{\psi}_n &= -\frac{\partial v_m}{\partial q_n}\psi_m \end{aligned} \tag{40}$$

where the velocities v_n depends on external fields as in previous section as control parameters which can be determined according to the optimal control criterium.

EDM are one of the keys to understand the origin of our Universe [Sakharov, 1967]. Andrei Sakharov formulated three conditions for baryogenesis:
1. Early in the evolution of the universe, the baryon number conservation must be violated sufficiently strongly,
2. The C and CP invariances, and T invariance thereof, must be violated, and
3. At the moment when the baryon number is generated, the evolution of the universe must be out of thermal equilibrium.
CP violation in kaon decays is known since 1964, it has been observed in B-decays and charmed meson decays. The Standard Model (SM) accommodates CP violation via the phase in the Cabibbo-Kobayashi-Maskawa matrix.
CP and P violation entail nonvanishing P and T violating electric dipole moments (EDM) of elementary particles $\vec{d} = d\vec{s}$.

Although extremely successful in many aspects, the SM has at least two weaknesses: neutrino oscillations do require extensions of the SM and, most importantly, the SM mechanisms fail miserably in the expected baryogenesis rate.
Simultaneously, the SM predicts an exceedingly small electric dipole moment of nucleons $10^{-33} < d_n < 10^{-31} e \cdot cm$, way below the current upper bound for the neutron EDM, $d_n < 2.9 \times 10^{-26} e \cdot cm$. In the quest for physics beyond the SM one could follow either the high energy trail or look into new methods which offer very high precision and sensitivity.
Supersymmetry is one of the most attractive extensions of the SM and S. Weinberg emphasized [Weinberg, 1993]: "Endemic in supersymmetric (SUSY) theories are CP violations that go beyond the SM. For this reason it may be that the next exciting thing to come along will be the discovery of a neutron electric dipole moment."

The SUSY predictions span typically $10^{-29} < d_n < 10^{-24} e \cdot cm$ and precisely this range is targeted in the new generation of EDM searches [Roberts, Marciano, 2010]. There is consensus among theorists that measuring the EDM of the proton, deuteron and helion is as important as that of the neutron. Furthermore, it has been argued that T-violating nuclear forces could substantially enhance nuclear EDM [Flambaum, Khriplovich, Sushkov, 1986]. At the moment, there are no significant direct upper bounds available on d_p or d_d. Non-vanishing EDMs give rise to the precession of the spin of a particle in an electric field. In the rest frame of a particle

$$\dot{s}_n = \varepsilon_{nmk}(\Omega_m s_k + d_m E_k),\ \Omega_m = -\mu B_m, \tag{41}$$

where in terms of the lab frame fields

$$\begin{aligned} B_n &= \gamma(B_n^l - \varepsilon_{nmk}\beta_m E_k^l), \\ E_n &= \gamma(E_n^l + \varepsilon_{nmk}\beta_m B_k^l) \end{aligned} \tag{42}$$

Now we can apply the Hamiltonization and optimal control theory methods to this dynamical system.

In QFT existence of a given theory means, that we can control its behavior at some scales (short or large distances) by renormalization theory [Collins, 1984].
If the theory exists, than we want to solve it, which means to determine what happens on other (large or short) scales. This is the problem (and content) of Renormdynamics.
The result of the Renormdynamics, the solution of its discrete or continual motion equations, is the effective QFT on a given scale (different from the initial one).
We can invent scale variable λ and consider QFT on $D+1+1$ dimensional space-time-scale. For the scale variable $\lambda \in (0,1]$ it is natural to consider q-discretization, $0<q<1,\ \lambda_n = q^n,\ n = 0,1,2,\ \ldots$ and p - adic, nonarchimedian metric, with $q^{-1} = p$ - prime integer number.
The field variable $\varphi(x,t,\lambda)$ is complex function of the real, x, t, and p - adic, λ, variables. The solution of the UV renormdynamic problem means, to find evolution from finite to small scales with respect to the scale time $\tau = \ln \lambda/\lambda_0 \in (0,-\infty)$. Solution of the IR renormdynamic problem means to find evolution from finite to the large scales, $\tau = \ln \lambda/\lambda_0 \in (0,\infty)$.

Every (good) school boy/girl knows what is

$$\frac{d^n}{dx^n} = \partial^n = (\partial)^n, \tag{43}$$

but what is its following extension

$$\frac{d^\alpha}{dx^\alpha} = \partial^\alpha \ , \quad \alpha \in \Re \ ? \tag{44}$$

Let us consider the integer derivatives of the monomials

$$\frac{d^n}{dx^n}x^m = m(m-1)...(m-(n-1))x^{m-n}, \quad n \leq m, \\ = \frac{\Gamma(m+1)}{\Gamma(m+1-n)}x^{m-n}. \tag{45}$$

L.Euler (1707 - 1783) invented the following definition of the fractal derivatives,

$$\frac{d^\alpha}{dx^\alpha}x^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\alpha)}x^{\beta-\alpha}. \tag{46}$$

J.Liouville (1809-1882) takes exponents as a base functions,

$$\frac{d^\alpha}{dx^\alpha}e^{ax} = a^\alpha e^{ax}. \tag{47}$$

The following Cauchy formula

$$I^n_{0,x}f = \int_0^x dx_n \int_0^{x_{n-1}} dx_{n-2} ... \int_0^{x_2} dx_1 f(x_1) = \frac{1}{\Gamma(n)} \int_0^x dy (x-y)^{n-1} f(y)($$

permits analytic extension from integer n to complex α,

$$I^\alpha_{0,x}f = \frac{1}{\Gamma(\alpha)} \int_0^x dy (x-y)^{\alpha-1} f(y) \quad (49)$$

J.H. Holmgren invented (in 1863) the following integral transformation,

$$D^{-\alpha}_{c,x}f = \frac{1}{\Gamma(\alpha)} \int\limits_c^x |x-t|^{\alpha-1} f(t) dt. \quad (50)$$

It is easy to show that

$$
\begin{aligned}
D_{c,x}^{-\alpha}x^m &= \frac{\Gamma(m+1)}{\Gamma(m+1+\alpha)}(x^{m+\alpha}-c^{m+\alpha}), \\
D_{c,x}^{-\alpha}e^{ax} &= a^{-\alpha}(e^{ax}-e^{ac}),
\end{aligned} \tag{51}
$$

so, $c=0$, when $m+\alpha\geq 0$, in Holmgren's definition of the fractal calculus, corresponds to the Euler's definition, and $c=-\infty$, when $a>0$, corresponds to the Liouville's definition.
Holmgren's definition of the fractal calculus reduce to the Euler's definition for finite c, and to the Liouvill's definition for $c=\infty$,

$$
\begin{aligned}
D_{c,x}^{-\alpha}f = D_{0,x}^{-\alpha}f - D_{0,c}^{-\alpha}f, \\
D_{\infty,x}^{-\alpha}f = D_{-\infty,x}^{-\alpha}f - D_{-\infty,\infty}^{-\alpha}f.
\end{aligned} \tag{52}
$$

We considered the following modification of the $c = 0$ case [Makhaldiani, 2003],

$$D_{0,x}^{-\alpha} f = \frac{|x|^\alpha}{\Gamma(\alpha)} \int\limits_0^1 |1-t|^{\alpha-1} f(xt)dt, = \frac{|x|^\alpha}{\Gamma(\alpha)} B(\alpha,\ \partial x) f(x)$$
$$= |x|^\alpha \frac{\Gamma(\partial x)}{\Gamma(\alpha+\partial x)} f(x),\ f(xt) = t^{x\frac{d}{dx}} f(x). \quad (53)$$

As an example, consider Euler B-function,

$$B(\alpha,\beta) = \int_0^1 dx|1-x|^{\alpha-1}|x|^{\beta-1} = \Gamma(\alpha)\Gamma(\beta) D_{01}^{-\alpha} D_{0x}^{1-\beta} 1 = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \quad (54)$$

We can define also FC as

$$D^\alpha f = (D^{-\alpha})^{-1} f = \frac{\Gamma(\partial x+\alpha)}{\Gamma(\partial x)} (|x|^{-\alpha} f),\ \partial x = \delta + 1,\ \delta = x\partial \quad (55)$$

For the Liouville's case,

$$D_{-\infty,x}^\alpha f = (D_{-\infty,x})^\alpha f = (\partial_x)^\alpha f, \quad (56)$$

$$\partial_x^{-\alpha} f = \frac{1}{\Gamma(\alpha)} \int_0^\infty dt t^{\alpha-1} e^{-t\partial_x} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^\infty dt t^{\alpha-1} f(x-t)$$
$$= \frac{1}{\Gamma(\alpha)} \int_{-\infty}^x dt (x-t)^{\alpha-1} f(t) = D_{-\infty,x}^{-\alpha} f. \quad (57)$$

The integrals can be calculated as

$$D^{-n}f = (D^{-1})^n f, \tag{58}$$

where

$$D^{-1}f = x\frac{\Gamma(\partial x)}{\Gamma(1+\partial x)}f = x\frac{1}{\partial x}f = x(\partial x)^{-1}f = (\partial)^{-1}f = \int_0^x dt f(t). \tag{59}$$

Let us consider Weierstrass C.T.W. (1815 - 1897) fractal function

$$f(t) = \sum_{n\geq 0} a^n e^{i(b^n t+\varphi_n)}, \quad a<1, \quad ab>1. \tag{60}$$

For fractals we have no integer derivatives,

$$f^{(1)}(t) = i\sum (ab)^n e^{i(b^n t+\varphi_n)} = \infty, \tag{61}$$

but the fractal derivative,

$$f^{(\alpha)}(t) = \sum (ab^\alpha)^n e^{i(b^n t+\pi\alpha/2+\varphi_n)}, \tag{62}$$

when $ab^\alpha = a' < 1$, is another fractal (60).
Question: what if $ab = p$ is prime number? Can we define integer derivatives in this case?

p-adic analog of the fractal calculus (311) ,

$$D_x^{-\alpha} f = \frac{1}{\Gamma_p(\alpha)} \int_{Q_p} |x-t|_p^{\alpha-1} f(t)dt, \tag{63}$$

where $f(x)$ is a complex function of the p-adic variable x, with p-adic Γ–function

$$\Gamma_p(\alpha) = \int_{Q_p} dt|t|_p^{\alpha-1}\chi(t) = \frac{1-p^{\alpha-1}}{1-p^{-\alpha}}, \tag{64}$$

was considered by V.S. Vladimirov [Vladimirov,1988].
The following modification of p-adic FC is given in [Makhaldiani, 2003]

$$\begin{aligned} D_x^{-\alpha} f &= \frac{|x|_p^\alpha}{\Gamma_p(\alpha)} \int_{Q_p} |1-t|_p^{\alpha-1} f(xt)dt \\ &= |x|_p^\alpha \frac{\Gamma_p(\partial|x|)}{\Gamma_p(\alpha+\partial|x|)} f(x). \end{aligned} \tag{65}$$

Last expression is applicable for functions of the type $f(x) = f(|x|)$.
For a functions of the form

$$f(x) = \sum a_n |x|_p^n, \tag{66}$$

we have

$$D_x^{-\alpha} f = \sum a_n \frac{\Gamma_p(n+1)}{\Gamma_p(n+1+\alpha)} |x|_p^{n+\alpha}. \tag{67}$$

The basic object of q-calculus [Gasper, Rahman, 1990] is q-derivative

$$D_q f(x) = \frac{f(x) - f(qx)}{(1-q)x} = \frac{1 - q^{x\partial}}{(1-q)x} f(x), \tag{68}$$

where either $0 < q < 1$ or $1 < q < \infty$. In the limit $q \to 1, D_q \to \partial_x$.
Now we define the fractal q-calculus,

$$\begin{aligned} & D_q^\alpha f(x) = (D_q)^\alpha f(x) \\ & = ((1-q)x)^{-\alpha}(f(x) + \sum_{n\geq 1}(-1)^n \frac{\alpha(\alpha-1)...(\alpha-n+1)}{n!} f(q^n x)). \end{aligned} \tag{69}$$

For the case $\alpha = -1$, we obtain the integral

$$D_q^{-1} f(x) = (1-q)x(1-q^{x\partial})^{-1} f(x) = (1-q)x \sum_{n\geq 0} f(q^n x). \tag{70}$$

In the case of $1 < q < \infty$, we can give a good analytic sense to these expressions for prime numbers $q = p = 2, 3, 5, ..., 29, ..., 137, ...$ This is an *algebra-analytic quantization* of the q-calculus and corresponding physical models. Note also, that p-adic calculus is the natural tool for the physical models defined on the fractal(space)s like Bete lattice (or Brua-Tits trees, in mathematical literature).
Note also a symmetric definition of the calculus

$$D_{qs} f(x) = \frac{f(q^{-1}x) - f(qx)}{(q^{-1} - q)x} f(x). \tag{71}$$

Usual finite difference calculus is based on the following (left) derivative operator

$$D_- f(x) = \frac{f(x) - f(x-h)}{h} = (\frac{1 - e^{-h\partial}}{h}) f(x). \quad (72)$$

We define corresponding fractal calculus as

$$D_-^\alpha f(x) = (D_-)^\alpha f(x). \quad (73)$$

In the case of $\alpha = -1$, we have usual finite difference sum as regularization of the Riemann integral

$$D_-^{-1} f(x) = h(f(x) + f(x-h) + f(x-2h) + ...). \quad (74)$$

(I believe that) the fractal calculus (and geometry) are the proper language for the quantume (field) theories, and discrete versions of the fractal calculus are proper regularizations of the fractal calculus and field theories.

A hypergeometric series, in the most general sense, is a power series in which the ratio of successive coefficients indexed by n is a rational function of n,

$$f(x) = \sum_{n \geq 0} a_n x^n, \; a_{n+1} = R(n) a_n, \; R(n) = \frac{P(\alpha, n)}{Q(\beta, n)} \tag{75}$$

so

$$\begin{aligned} &P(\alpha, \delta) f(x) = Q(\beta, \delta)(f(x) - f(0))/x, \\ &f(x) - f(0) = x R(\delta) f(x), f(x) = (1 - x R(\delta))^{-1} f(0), \delta = x \partial_x \end{aligned} \tag{76}$$

Hypergeometric functions have many particular special functions as special cases, including many elementary functions, the Bessel functions, the incomplete gamma function, the error function, the elliptic integrals and the classical orthogonal polynomials, because the hypergeometric functions are solutions to the hypergeometric differential equation, which is a fairly general second-order ordinary differential equation.

In a generalization given by Eduard Heine (1821 - 1881) in the late nineteenth century, the ratio of successive terms, instead of being a rational function of n, are considered to be a rational function of q^n

$$
\begin{aligned}
&f(x) = \sum_{n \geq 0} a_n x^n, \ a_{n+1} = R(q^n) a_n, \ R(n) = \frac{P(\alpha, q^n)}{Q(\beta, q^n)}, \\
&P(\alpha, q^\delta) f(x) = Q(\beta, q^\delta)(f(x) - f(0))/x, \\
&f(x) - f(0) = xR(q^\delta) f(x), f(x) = (1 - xR(q^\delta))^{-1} f(0), \delta = x\partial_x
\end{aligned}
\tag{77}
$$

Another generalization, the elliptic hypergeometric series, are those series where the ratio of terms is an elliptic function (a doubly periodic meromorphic function) of n.
There are a number of new definitions of hypergeometric series, by Aomoto, Gelfand and others; and applications for example to the combinatorics of arranging a number of hyperplanes in complex N-space.

Formal solutions for the the hypergeometric functions (76,77), we put in the fieldtheoretic form,

$$f(x) = G(x)f(0),$$
$$G(x) =< \psi(x)\phi(0) >= \frac{\delta^2 \ln Z}{\delta J(x)\delta I(0)} = (1 - xR)^{-1},$$
$$Z = \int d\psi d\phi e^{-S+I\phi+J\psi} = e^{I(1-xR)^{-1}J},$$
$$S = \int \psi(1 - xR)\phi = \int \psi(Q - xP)\varphi,\ \phi = Q\varphi. \tag{78}$$

When we invent interaction terms, we obtain nontrivial HFT. In terms of the fundamental fields, ψ, φ, we have local field model.

For LFs (see, e.g. [Miller,1977]), we find the following formulas [Makhaldiani, 2011]

$$F_A(a;b_1,...,b_n;c_1,...,c_n;z_1,...,z_n) = \frac{(a)_{\delta_1+...+\delta_n}(b_1)_{\delta_1}...(b_n)_{\delta_n}}{(c_1)_{\delta_1}...(c_n)_{\delta_n}}\ e^{z_1+...+z_n}$$
$$= \frac{(a)_{\delta_1+...+\delta_n}}{(a_1)_{\delta_1}...(a_n)_{\delta_n}}\ F(a_1,b_1;c_1;z_1)...F(a_n,b_n;c_n;z_n)$$
$$= T^{-1}(a)F^n = \Sigma_{m\geq 0}\frac{(a)_{m_1+...+m_n}(b_1)_{m_1}...(b_n)_{m_n}}{(c_1)_{m_1}...(c_n)_{m_n}}\frac{z_1^{m_1}}{m_1!}...\frac{z_n^{m_n}}{m_n!},\ |z_1|+...+|z_n|<1;$$
$$F_B(a_1,...,a_n;b_1,...,b_n;c;z_1,...,z_n) = \frac{(a_1)_{\delta_1}...(a_n)_{\delta_n}(b_1)_{\delta_1}...(b_n)_{\delta_n}}{(c)_{\delta_1+...+\delta_n}}\ e^{z_1+...+z_n}$$
$$= \frac{(c_1)_{\delta_1}...(c_n)_{\delta_n}}{(c)_{\delta_1+...+\delta_n}}\ F(a_1,b_1;c_1;z_1)...F(a_n,b_n;c_n;z_n) = T(c)F^n$$
$$= \Sigma_{m\geq 0}\frac{(a_1)_{m_1}...(a_n)_{m_n}(b_1)_{m_1}...(b_n)_{m_n}}{(c)_{m_1+...+m_n}}\frac{z_1^{m_1}}{m_1!}...\frac{z_n^{m_n}}{m_n!},\ |z_1|<1,...,|z_n|<1; \tag{79}$$

$$
\begin{aligned}
&F_C(a;b;c_1,...,c_n;z_1,...,z_n) = \frac{(a)_{\delta_1+...+\delta_n}(b)_{\delta_1+...+\delta_n}}{(c_1)_{\delta_1}...(c_n)_{\delta_n}}\ e^{z_1+...+z_n} \\
&= \frac{(a)_{\delta_1+...+\delta_n}(b)_{\delta_1+...+\delta_n}}{(a_1)_{\delta_1}...(a_n)_{\delta_n}(b_1)_{\delta_1}...(b_n)_{\delta_n}}\ F(a_1,b_1;c_1;z_1)...F(a_n,b_n;c_n;z_n) \\
&= T^{-1}(a)T^{-1}(b)F^n = T^{-1}(b)F_A \\
&= \Sigma_{m\geq 0}\frac{(a)_{m_1+...+m_n}(b)_{m_1+...+m_n}}{(c_1)_{m_1}...(c_n)_{m_n}}\frac{z_1^{m_1}}{m_1!}...\frac{z_n^{m_n}}{m_n!},\ |z_1|^{1/2}+...+|z_n|^{1/2}<1; \\
&F_D(a;b_1,...,b_n;c;z_1,...,z_n) = \frac{(a)_{\delta_1+...+\delta_n}(b_1)_{\delta_1}...(b_n)_{\delta_n}}{(c)_{\delta_1+...\delta_n}}\ e^{z_1+...+z_n} \\
&= \frac{(a)_{\delta_1+...+\delta_n}(c_1)_{\delta_1}...(c_n)_{\delta_n}}{(a_1)_{\delta_1}...(a_n)_{\delta_n}(c)_{\delta_1+...\delta_n}}\ F(a_1,b_1;c_1;z_1)...F(a_n,b_n;c_n;z_n) \\
&= T^{-1}(a)T(c)F^n = T(c)F_A = T^{-1}(a)F_B \\
&= \Sigma_{m\geq 0}\frac{(a)_{m_1+...+m_n}(b_1)_{m_1}...(b_n)_{m_n}}{(c_1)_{m_1}...(c_n)_{m_n}}\frac{z_1^{m_1}}{m_1!}...\frac{z_n^{m_n}}{m_n!},\ |z_1|<1,...,|z_n|<1. \qquad (80)
\end{aligned}
$$

In the paper ([Lomidze, 1994]) the following formula were proposed

$$det[x_j^{i-1}\int_{x_{j-1/x_j}}^1 u^{i-1}(1-u)^{r_j-1}\prod_{k=0,k\neq j}^n(\frac{x_ju-x_k}{x_j-x_k})^{r_k-1}du]/det[x_j^{i-1}]$$
$$=\frac{\Gamma(r_0)\Gamma(r_1)...\Gamma(r_n)}{\Gamma(r_o+r_1+...+r_n)},\ 0=x_0<x_1<x_2<...<x_n,\ n\geq 1. \quad (81)$$

Let us put the formula in the following factorized form

$$LB_n(x,r)\equiv det[x_j^{i-1}\int_{x_{j-1/x_j}}^1 duu^{i+r_0-2}(1-u)^{r_j-1}\prod_{k=1,k\neq j}^n(\frac{x_ju-x_k}{x_j-x_k})^{r_k-1}]$$
$$=detV_n(x)B_n(r),\ V_n(x)=[x_j^{i-1}],\ B_n(r)=\frac{\Gamma(r_0)\Gamma(r_1)...\Gamma(r_n)}{\Gamma(r_0+r_1+...+r_n)} \quad (82)$$

Now, it is enough to proof this formula for general values of x_i and particular values of r_i, e.g., $r_i=1$, and for general values of r_i and particular values of x_i, e.g. $x_i=p^i,\ 1\leq i\leq n$. In the case of $r_i=1$, right hand side of the formula is equal to the Vandermonde determinant divided by n! The left hand side is the determinant of the matrix with elements
$A_{ij}=x_j^{i-1}(1-(x_{j-1}/x_j)^i)/i$
When we calculate determinant of this matrix, from the row i, we factorize $1/i, 2\leq i\leq n$ which gives the $1/n!$ the rest matrix we calculate transforming the matrix to the form of the Vandermonde matrix.

This is the half way of the proof. Let us take the concrete values of $x_i = p^i,\ 1 \le i \le n$, where p is positive integer and general complex values for $r_i, 0 \le i \le n$, and calculate both sides of the equality. For Vandermonde determinant we find for high values of p the following asymptotic

$$detV = p^N,\ N = \sum_{k=2}^{n} k(k-1) = \frac{n(n^2-1)}{3} \tag{83}$$

The matrix elements are

$$\begin{aligned} B_{ij} &= x_j^{i-1} \int_{x_{j-1}/x_j}^{1} u^{i+r_0-2}(1-u)^{r_j-1} \prod_{k=1,k\neq j}^{n} (\frac{x_j u - x_k}{x_j - x_k})^{r_k-1} du] \\ &= x_j^{i-1}(\prod_{1\le k<j} (\frac{x_j}{x_j - x_k})^{r_k-1} \prod_{j<k\le n} (\frac{x_k}{x_k - x_j})^{r_k-1} \int_{x_{j-1}/x_j}^{1} u^{i+r_0-2}(1-u)^{r_j-1} \\ &\cdot \prod_{1\le k<j} (u - x_k/x_j)^{r_k-1} \prod_{j<k\le n} (1 - x_j/x_k u)^{r_k-1} du \\ &= p^{(i-1)j}(\int_0^1 u^{i+r_0-2+\sum_{k=1}^{j-1}(r_k-1)}(1-u)^{r_j-1} du \\ &= p^{(i-1)j} B(i + \sum_{k=0}^{j-1}(r_k-1), r_j) \end{aligned} \tag{84}$$

For $n = 2$ we have

$$B_{11} = \int_0^1 u^{r_0-1}(1-u)^{r_1-1}du = \frac{\Gamma(r_0)\Gamma(r_1)}{\Gamma(r_0+r_1)},$$
$$B_{22} = p^2\int_0^1 u^{r_0+r_1-1}(1-u)^{r_2-1}du = \frac{\Gamma(r_0+r_1)\Gamma(r_2)}{\Gamma(r_0+r_1+r_2)},$$
$$LB_2/V_2 = B_{11}B_{22}/p^2 = \frac{\Gamma(r_0)\Gamma(r_1)\Gamma(r_2)}{\Gamma(r_0+r_1+r_2)} \qquad (85)$$

For $n = 3$,

$$B_{11} = \int_0^1 u^{r_0-1}(1-u)^{r_1-1} = \frac{\Gamma(r_0)\Gamma(r_1)}{\Gamma(r_0+r_1)} = B(r_0, r_1),$$
$$B_{22} = p^2\int_0^1 u^{r_0+r_1-1}(1-u)^{r_2-1} = p^2\frac{\Gamma(r_0+r_1)\Gamma(r_2)}{\Gamma(r_0+r_1+r_2)},$$
$$B_{33} = p^6\int_0^1 u^{r_0+r_1+r_2-1}(1-u)^{r_3-1} = p^6\frac{\Gamma(r_0+r_1+r_2)\Gamma(r_3)}{\Gamma(r_0+r_1+r_2+r_3)}$$
$$LB_3/V_3 = B_{11}B_{22}B_{33}/p^8 = \frac{\Gamma(r_0)\Gamma(r_1)\Gamma(r_2)\Gamma(r_3)}{\Gamma(r_0+r_1+r_2+r_3)} \qquad (86)$$

Now it is obvious the last step of the proof [Makhaldiani, 2011]

$$LB_n(x,r) = detV_n(x)B(r_0,r_1)...B(r_0+r_1+...+r_{n-1},r_n)$$
$$= detV_n(x)B_n(r)$$
$$V_n(x) = [x_j^{i-1}],\ B_n(r) = \frac{\Gamma(r_0)\Gamma(r_1)..\Gamma(r_n)}{\Gamma(r_0+r_1+...+r_n)} \qquad (87)$$

Note that this proof is based on the factorization assumption (82). The proof without this assumption given by I.R.Lomidze is given in [Lomidze, Makhaldiani, 2012].

Let us consider the following action

$$S = \frac{1}{2} \int\limits_{Q_v} dx \Phi(x) D_x^\alpha \Phi, \ v = 1, 2, 3, 5, ..., 29, ..., 137, ... \tag{88}$$

Q_1 is real number field, Q_p, p - prime, are p-adic number fields. In the momentum representation

$$S = \frac{1}{2} \int\limits_{Q_v} du \tilde{\Phi}(-u)|u|_v^\alpha \tilde{\Phi}(u), \ \Phi(x) = \int\limits_{Q_v} du \chi_v(ux) \tilde{\Phi}(u),$$
$$D^{-\alpha} \chi_v(ux) = |u|_v^{-\alpha} \chi_v(ux). \tag{89}$$

The statistical sum of the corresponding quantum theory is

$$Z_v = \int d\Phi e^{-\frac{1}{2}\int \Phi D^\alpha \Phi} = det^{-1/2} D^\alpha = (\prod_u |u|_v)^{-\alpha/2}. \tag{90}$$

For (symmetrized, 4-tachyon) Veneziano amplitude we have (see, e.g. [Kaku, 2000])

$$B_s(\alpha,\beta) = B(\alpha,\beta) + B(\beta,\gamma) + B(\gamma,\alpha) = \int_{-\infty}^{\infty} dx|1-x|^{\alpha-1}|x|^{\beta-1}, \quad \alpha+\beta+\gamma = 1 \tag{91}$$

For the p-adic Veneziano amplitude we take

$$B_p(\alpha,\beta) = \int_{Q_p} dx|1-x|_p^{\alpha-1}|x|_p^{\beta-1} = \frac{\Gamma_p(\alpha)\Gamma_p(\beta)}{\Gamma_p(\alpha+\beta)} \tag{92}$$

Now we obtain the N-tachyon amplitude using fractal calculus. We consider the dynamics of particle given by multicomponent generalization of the action (104), $\Phi \to x^\mu$.

For the closed trajectory of the particle passing through N points, we have

$$A(x_1, x_2, ..., x_N) = \int dt \int dt_1 ... \int dt_N \delta(t - \Sigma t_n)$$
$$v(x_1, t_1; x_2, t_2) v(x_2, t_2; x_3, t_3) ... v(x_N, t_N; x_1, t_1)$$
$$= \int dx(t) \Pi(\int dt_n \delta(x^\mu(t_n) - x_n^\mu)) exp(-S[x(t)])$$
$$= \int \Pi(dk_n^\mu \chi(k_n x_n)) \tilde{A}(k), \tag{93}$$

where

$$\tilde{A}(k) = \int dx V(k_1) V(k_2) ... V(k_N) exp(-S),$$
$$V(k_n) = \int dt \chi(-k_n x(t)) \tag{94}$$

is vertex function.
Motion equation

$$D^\alpha x^\mu - i \Sigma k_n^\mu \delta(t - t_n) = 0, \tag{95}$$

in the momentum representation

$$|u|^\alpha \tilde{x}^\mu(u) - i \Sigma_n k_n^\mu \chi(-u t_n) = 0 \tag{96}$$

have the solution

$$\tilde{x}^\mu(u) = i \Sigma k_n^\mu \frac{\chi(-u t_n)}{|u|^\alpha}, \quad u \neq 0, \tag{97}$$

the constraint

$$\Sigma_n k_n = 0, \tag{98}$$

and the zero mod $\tilde{x}_n^\mu(0)$, which is arbitrary. Integration in (93) with respect to this zero mod gives the constraint (98). On the solution of the equation (95)

$$x^\mu(t) = iD_t^{-\alpha}\Sigma_n k_n^\mu \delta(t - t_n) = \frac{i}{\Gamma(\alpha)}\Sigma_n k_n^\mu |t - t_n|^{\alpha - 1}, \tag{99}$$

the action (104) takes value

$$S = -\frac{1}{\Gamma(\alpha)}\Sigma_{n<m} k_n k_m |t_n - t_m|^{\alpha-1},$$
$$\tilde{A}(k) = \int \Pi_{n=1}^N dt_n exp(-S) \tag{100}$$

In the limit, $\alpha \to 1$, for p-adic case we obtain

$$x^\mu(t) = -i\frac{p-1}{p\ lnp}\Sigma_n k_n^\mu ln|t - t_n|,$$
$$S[x(t)] = \frac{p-1}{p\ lnp}\Sigma_{n<m} k_n k_m\ ln|t_n - t_m|,$$
$$\tilde{A}(k) = \int \Pi_{n=1}^N dt_n \Pi_{n<m} |t_n - t_m|^{\frac{p-1}{p\ lnp} k_n k_m}. \tag{101}$$

Now in the limit $p = q^{-1} \to 1$ we obtain the proper expressions of the real case

$$x^{\mu}(t) = -i\Sigma_n k_n^{\mu} ln|t - t_n|,$$
$$S[x(t)] = \Sigma_{n<m} k_n k_m \; ln|t_n - t_m|,$$
$$\tilde{A}(k) = \int \Pi_{n=1}^{N} dt_n \Pi_{n<m} |t_n - t_m|^{k_n k_m}. \qquad (102)$$

By fractal calculus and vector generalization of the model (104), fundamental string amplitudes were obtained in [Makhaldiani, 1987].

The ring of (rational) adeles can be defined as the restricted product

$$A_Q = R\prod_p' Q_p \tag{103}$$

of all the real numbers and the p-adic completions Q_p, or in other words as the restricted product of all completions of the rationals. In this case the restricted product means that for an adele $a = (a_1, a_2, a_3, a_5,)$ all but a finite number of the a_p are p-adic integers.
The group of invertible elements of the adele ring is the idele group. As a locally compact abelian group, the adeles have a nontrivial translation invariant measure. Similarly, the group of ideles has a nontrivial translation invariant measure.
Let us consider the following action

$$S = \frac{1}{2}\int_{Q_v} dx\Phi(x)D_x^\alpha\Phi,\ \ v = 1, 2, 3, 5, ... \tag{104}$$

In the momentum representation

$$S = \frac{1}{2}\int\limits_{Q_v} du\tilde{\Phi}(-u)|u|_v^{\alpha}\tilde{\Phi}(u), \quad (105)$$

where

$$\Phi(x) = \int\limits_{Q_v} du\chi_v(ux)\tilde{\Phi}(u),$$
$$D^{-\alpha}\chi_v(ux) = |u|_v^{-\alpha}\chi_v(ux). \quad (106)$$

The statistical sum of the corresponding quantum theory is

$$Z_v = \int d\Phi e^{-\frac{1}{2}\int \Phi D^{\alpha}\Phi} = det^{-1/2}D^{\alpha} = (\prod_u |u|_v)^{-\alpha/2}. \quad (107)$$

Adels $a \in A$ are constructed by real $a_1 \in Q_1$ and p-adic $a_p \in Q_p$ numbers (see e.g. [Gelfand et al, 1966])

$$a = (a_1, a_2, a_3, a_5, ..., a_p, ...), \tag{108}$$

with restriction that $a_p \in Z_p = \{x \in Q_p, |x|_p \leq 1\}$ for all but a finite set F of primes p.

A is a ring with respect to the componentwise addition and multiplication.
A prinsipal adel is a sequence $r = (r, r, ..., r, ...)$, $r \in Q$-rational number.
Norm on adels is defined as

$$|a| = \prod_{p \geq 1} |a_p|_p. \tag{109}$$

Note that the norm on principal adels is trivial.
In the adelic generalization of the model (104),

$$\Phi(x) = \prod_{p \geq 1} \Phi_p(x_p), \quad dx = \prod_{p \geq 1} dx_p, \quad D_x^\alpha = \sum_{p \geq 1} D_{x_p}^\alpha, \tag{110}$$

where by $D_{x_1}^\alpha$ we denote fractal derivative (314), x_1 is real and $| \;|_1$ is real norm.

If

$$\int dx_p |\Phi(x_p)|^2 = 1, \tag{111}$$

then

$$\int dx |\Phi(x)|^2 = 1, \quad S = \sum_{p \geq 1} S_p, \tag{112}$$

so

$$Z = \prod_{p \geq 1} Z_p = \prod_{p \geq 1} (\prod_u |u|_p)^{-\alpha/2} = (\prod_u \prod_{p \geq 1} |u|_p)^{-\alpha/2} = 1,$$
$$\lambda \sim lnZ = 0, \tag{113}$$

if $u \in Q$.

The cosmological constant problem is one of the most serious paradoxes in modern particle physics and cosmology [Weinberg, 1989]. Some astronomical observations indicate that the cosmological constant is many orders of magnitude smaller than estimated in modern theoretical elementary particles physics.
In his attempt, [Einstein, 1917] to apply the general relativity to the whole universe, A. Einstein invented a new term involving a free parameter λ, the cosmological constant (CC),

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \lambda g_{\mu\nu} - 8\pi G T_{\mu\nu}. \tag{114}$$

With this modification he finds a static solution for the universe filled with dust of zero pressure and mass density

$$\rho = \frac{\lambda}{8\pi G}. \tag{115}$$

The geometry of the universe was that of a sphere S_3 with proper circumference $2\pi r$, where

$$r = \lambda^{-1/2}, \tag{116}$$

so the mass of the universe was

$$M = \rho 2\pi^2 r^3 = 4\pi^2 r/\kappa = \frac{\pi}{2G} r \tag{117}$$

It is interesting to describe hadrons with a similar geometric picture corresponding to the low energy QCD.

Any contributions to the energy density of the vacuum acts just like CC. By Lorentz invariance, in the vacuum,

$$< T_{\mu\nu} >= - < \rho > g_{\mu\nu}, \tag{118}$$

so

$$\lambda_{eff} = \lambda + 8\pi G < \rho >, \tag{119}$$

or the total vacuum energy density

$$\rho_V =< \rho > + \frac{\lambda}{8\pi G} = \frac{\lambda_{eff}}{8\pi G}. \tag{120}$$

The experimental upper bound on λ_{eff} or ρ_V is provided by measurements of cosmological redshifts as a function of distance. From the present expansion rate of the universes

$$\frac{dlnR}{dt} \equiv H_0 = 100h\frac{km}{secMpc},\ h = 0.7 \pm 0.07 \tag{121}$$

we have

$$H_0^{-1} = (1 \div 2) \times 10^{10} ye,\ |\lambda_{eff}| \leq H_0^2, \\ |\rho_V| \leq 10^{-29} g/cm^2 \simeq 10^{-47} GeV^4. \tag{122}$$

The quantum oscillator with hamiltonian

$$H = \frac{1}{2}P^2 + \frac{1}{2}\omega^2 x^2, \tag{123}$$

has the energy spectrum

$$E_n = \hbar\omega(n + 1/2), \ n = 0, 1, 2, ... \tag{124}$$

with the lowest, vacuum, value $E_0 = \hbar\omega/2$. Normal modes of a quantum field of mass m are oscillators with frequencies $\omega(k) = \sqrt{k^2 + m^2}$. Summing the zero-point energies of all normal modes of the field up to a wave number cut-off $\Lambda >> m$ yields a vacuum energy density

$$< \rho >= \int_0^\Lambda \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \simeq \frac{\Lambda^4}{16\pi^2}. \tag{125}$$

If we take $\Lambda = (8\pi G)^{-1/2}$, then

$$< \rho >\simeq 2^{-10}\pi^{-4}G^{-2} = 2 \times 10^{71} GeV^4. \tag{126}$$

We saw that

$$|<\rho>+\frac{\lambda}{8\pi G}| \leq 10^{-47}GeV^4 \simeq (10^{-3}eV)^4, \tag{127}$$

so the two terms must cancel to better than 100 decimal places! If we take Λ_{QCD}, $<\rho>\simeq 10^{-6}GeV^4 \simeq (300MeV)^4$, the two terms must cancel better to than 40 decimal places. Since the cosmological upper bound on $<\rho_{eff}>$ is vastly less than any value expected from particle theory, theorists assumed that (for some unknown reason) this quantity is zero.

A minimal realization of the algebra of supersymmetry

$$\begin{aligned} &\{Q, Q^+\} = H, \\ &\{Q, Q\} = \{Q^+, Q^+\} = 0, \end{aligned} \tag{128}$$

is given by a point particle in one dimension, [Witten, 1998]

$$\begin{aligned} &Q = a(-iP + W), \\ &Q^+ = a^+(iP + W), \end{aligned} \tag{129}$$

where $P = -i\partial/\partial x$, the superpotential $W(x)$ is any function of x, and spinor operators a and a^+ obey the anticommuting relations

$$\begin{aligned} &\{a, a^+\} = 1, \\ &a^2 = (a^+)^2 = 0. \end{aligned} \tag{130}$$

There is a following representation of operators a, a^+ and σ by the Pauli spin matrices

$$
\begin{aligned}
a &= \frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \\
a^+ &= \frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \\
\sigma &= \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\end{aligned} \tag{131}
$$

From formulae (128) and (129) then we have

$$H = P^2 + W^2 + \sigma W_x. \tag{132}$$

The simplest nontrivial case of the superpotential $W = \omega x$ corresponds to the supersimmetric oscillator with Hamiltonian

$$H = H_B + H_F, \quad H_B = P^2 + \omega^2 x^2, \quad H_F = \omega\sigma, \tag{133}$$

wave function

$$\psi = \psi_B \psi_F, \tag{134}$$

and spectrum

$$\begin{aligned} H_B \psi_{Bn} = \omega(2n+1)\psi_{Bn}, \\ H_F \psi_+ = \omega \psi_+, \quad H_F \psi_- = -\omega \psi_-. \end{aligned} \tag{135}$$

The ground state energies of the bosonic and fermionic parts are

$$E_{B0} = \omega, \quad E_{F0} = -\omega, \tag{136}$$

so the vacuum energy of the supersymmetric oscillator is

$$< 0|H|0 >= E_0 = E_{B0} + E_{F0} = 0, \quad |0 >= \psi_{B0}\psi_{F0}. \tag{137}$$

Note that the spectrum of this supersymmetric oscillator coincides with the natural numbers, n=0,1,2,... If you like, we can say that this is number-theoretic motivation of the sypersymmetric models.

Let us see on this toy - solution of the CC problem from the quantum statistical viewpoint. The statistical sum of the supersymmetric oscillator is

$$Z(\beta) = Z_B Z_F, \tag{138}$$

where

$$Z_B = \sum_n e^{-\beta E_{Bn}} = e^{-\beta\omega} + e^{-\beta\omega(2+1)} + ...$$
$$Z_F = \sum_n e^{-\beta E_{Fn}} = e^{\beta\omega} + e^{-\beta\omega}. \tag{139}$$

In the low temperature limit,

$$Z(\beta) = 1 + O(e^{-\beta 2\omega}) \to 1, \quad \beta = T^{-1}, \tag{140}$$

so CC

$$\lambda \sim lnZ \to 0. \tag{141}$$

Now we would like to consider a model of multiparticle production based on the d-dimensional sphere, and (try to) motivate the values of the NBD parameter k. The volum of the d-dimensional sphere with radius r, in units of hadron size r_h is

$$v(d,r) = \frac{\pi^{d/2}}{\Gamma(d/2+1)}(\frac{r}{r_h})^d \tag{142}$$

Note that,

$$v(0,r) = 1,\ v(1,r) = 2\frac{r}{r_h},$$
$$v(-1,r) = \frac{1}{\pi}\frac{r_h}{r} \tag{143}$$

If we identify this dimensionless quantity with corresponding coulomb energy formula,

$$\frac{1}{\pi} = \frac{e^2}{4\pi}, \tag{144}$$

we find $e = \pm 2$.

For less then -1 even integer values of d, and $r \neq 0,\ v = 0$. For negative odd integer $d = -2n + 1$

$$v(-2n+1, r) = \frac{\pi^{-n+1/2}}{\Gamma(-n+3/2)} (\frac{r_h}{r})^{2n-1},\ n \geq 1, \tag{145}$$

$$v(-3, r) = -\frac{1}{2\pi^2} (\frac{r_h}{r})^3,\ v(-5, r) = \frac{3}{4\pi^3} (\frac{r_h}{r})^5 \tag{146}$$

Note that,

$$v(2, r)v(3, r)v(-5, r) = \frac{1}{\pi},\ v(1, r)v(2, r)v(-3, r) = -\frac{1}{\pi} \tag{147}$$

We postulate that after collision,it appear intermediate state with almost spherical form and constant energy density. Than the radius of the sphere rise, dimension decrease, volume remains constant. At the last moment of the expansion, when the crossection of the one dimensional sphere - string become of order of hadron size, hadronic string divide in k independent sectors which start to radiate hadrons with geometric (Boze-Einstein) distribution, so all of the string final state radiate according to the NBD distribution.

So, from the volume of the hadronic string,

$$v = \pi(\frac{r}{r_h})^2 \frac{l}{r_h} = \pi k, \tag{148}$$

we find the NBD parameter k,

$$k = \frac{\pi^{d/2-1}}{\Gamma(d/2+1)}(\frac{r}{r_h})^d \tag{149}$$

Knowing, from experimental date, the parameter k, we can restrict the region of the values of the parameters d and r of the primordial sphere (PS),

$$r(d) = (\frac{\Gamma(d/2+1)}{\pi^{d/2-1}}k)^{1/d} r_h,$$
$$r(3) = (\frac{3}{4}k)^{1/3} r_h,\ r(2) = k^{1/2} r_h,\ r(1) = \frac{\pi}{2} k r_h \tag{150}$$

If the value of $r(d)$ will be a few r_h, the matter in the PS will be in the hadronic phase. If the value of r will be of order $10r_h$, we can speak about deconfined, quark-gluon, Glukvar, phase. From the formula (150), we see, that to have for the r, the value of order $10r_h$, in $d = 3$ dimension, we need the value for k of order 1000, which is not realistic.

So in our model, we need to consider the lower than one, fractal, dimensions. It is consistent with the following intuitive picture. Confined matter have point-like geometry, withe dimension zero. Primordial sphere of Glukvar have nonzero fractal dimension, which is less than one,

$$k = 3,\ r(0.7395)/r_h = 10.00,$$
$$k = 4,\ r(0.8384)/r_h = 10.00 \quad (151)$$

From the experimental data we find the parameter k of the NBD as a function of energy, $k = k(s)$. Then, by our spherical model, we construct fractal dimension of the Glukvar as a function of $k(s)$.
If we suppose that radius of the primordial sphere r is of order (or less) of r_h. Than we will have higher dimensional PS, e.g.

d	r/r_h	k
3	1.3104	3.0002
4	1.1756	3.0003
6	1.1053	2.9994
8	1.1517	3.9990

With extra dimensions gravitation interactions may become strong at the LHC energies,

$$V(r) = \frac{m_1 m_2}{m^{2+d}} \frac{1}{r^{1+d}} \tag{152}$$

If the extra dimensions are compactified with(in) size R, at $r >> R$,

$$V(r) \simeq \frac{m_1 m_2}{m^2 (mR)^d} \frac{1}{r} = \frac{m_1 m_2}{M_{Pl}^2} \frac{1}{r}, \tag{153}$$

where (4-dimensional) Planck mass is given by

$$M_{Pl}^2 = m^{2+d} R^d, \tag{154}$$

so the scale of extra dimensions is given as

$$R = \frac{1}{m} (\frac{M_{Pl}}{m})^{\frac{2}{d}} \tag{155}$$

If we take $m = 1TeV, (GeV^{-1} = 0.2fm)$

$$\begin{aligned} & R(d) = 2 \cdot 10^{-17} \cdot (\frac{M_{Pl}}{1TeV})^{\frac{2}{d}} \cdot cm, \\ & R(1) = 2 \cdot 10^{15} cm, \\ & R(2) = 0.2\ cm\ ! \\ & R(3) = 10^{-7} cm\ ! \\ & R(4) = 2 \cdot 10^{-9} cm, \\ & R(6) \sim 10^{-11} cm \end{aligned} \tag{156}$$

Note that lab measurements of $G_N (= 1/M_{Pl}^2, M_{Pl} = 1.2 \cdot 10^{19} GeV)$ have been made only on scales of about 1 cm to 1 m; 1 astronomical unit(AU) (mean distance between Sun and Earth) is $1.5 \cdot 10^{13} cm$; the scale of the periodic structure of the Universe, $L = 128 Mps \simeq 4 \cdot 10^{26} cm$. It is curious which (small) value of the extra dimension corresponds to L?

$$\begin{aligned} d &= 2\frac{\ln \frac{M_{Pl}}{m}}{\ln(mL)} = 0.74,\ m = 1TeV, \\ &= 0.81,\ m = 100GeV, \\ &= 0.07,\ m = 10^{17} GeV. \end{aligned} \tag{157}$$

We will call RDF functions $g_n = f_n(t)$ which are solutions of the RD motion equations

$$\dot{g}_n = \beta_n(g), 1 \leq n \leq N. \tag{158}$$

In the simplest case of one coupling constant the function $g = f(t)$ is constant, $g = g_c$ when $\beta(g_c) = 0$, or is invertible (monotone). Indeed,

$$\dot{g} = f'(t) = f'(f^{-1}(g)) = \beta(g). \tag{159}$$

Each monotone interval ends by UV and IR fixed points and describes corresponding phase of the system.
Note that the simplest case of the classical dynamics, the Hamiltonian system with one degree of freedom, is already two dimensional, so we have no analog of one charge renormdynamics.

The regular Hamiltonian systems of the classical mechanics are defined on the even dimensional phase space, so there is no analog of the three dimensional renormdynamics for the coupling constants of the SM. The fixed points of renormdynamics belong to a set of solutions of the polynomial system of equations $\beta_n(g) = 0, 1 \leq n \leq N$, in the perturbative renormdynamics. Describing the solutions is the task of contemporary algebraic and computational geometry.

The expression of the β-function can be obtained in the following way. The canonical dimensions of the bare fields and constants in the d-dimensional space-time are

$$[m] = 1, \quad [A] = \frac{d-2}{2}, [\Psi] = \frac{d-1}{2}, [g_b] = d - [A] - 2[\Psi] = \frac{4-d}{2} = \varepsilon,$$
$$d = 4 - 2\varepsilon, [a_b] = [g_b^2] = 2\varepsilon, \quad a_b = \mu^{2\varepsilon} Z a,$$
$$0 = da_b/dt = d(\mu^{2\varepsilon} Z a)/dt = \mu^{2\varepsilon}(\varepsilon Z a + \frac{\partial(Za)}{\partial a}\frac{da}{dt}) \Downarrow$$
$$\frac{da}{dt} = \beta(a,\varepsilon) = \frac{-\varepsilon Z a}{\frac{\partial(Za)}{\partial a}} = -\varepsilon a + \beta(a), \ \beta(a) = a^2 \frac{d}{da}(Z_1), \tag{160}$$

where

$$\beta(a,\varepsilon) = \frac{d-4}{2} a + \beta(a) \tag{161}$$

is $d-$dimensional $\beta-$function and Z_1 is the residue of the first pole in ε expansion

$$Z(a,\varepsilon) = 1 + Z_1 \varepsilon^{-1} + ... + Z_n \varepsilon^{-n} + ... \tag{162}$$

Since Z does not depend explicitly on μ, the β-function is the same in all MS-like schemes, i.e. within the class of renormalization schemes which differ by the shift of the parameter μ.

The higher residue of the pole expansion can be defined from (160),

$$0 = \varepsilon Za + \frac{\partial(Za)}{\partial a}\frac{da}{dt} = \varepsilon(a + \frac{aZ_1}{\varepsilon} + \frac{aZ_2}{\varepsilon^2} + ...) + (1 + \frac{(aZ_1)'}{\varepsilon} + \frac{(aZ_2)'}{\varepsilon^2} + ...)(-\varepsilon a + \beta(a))$$
$$= \beta - a(aZ_1)' + aZ_1 + \frac{1}{\varepsilon}(aZ_2 - a(aZ_2)' + \beta(aZ_1)') + ... \Rightarrow$$
$$\beta(a) = a^2\frac{dZ_1}{da};\ a^2\frac{dZ_{n+1}}{da} = \beta(a)\frac{d(aZ_n)}{da},\ n \geq 1, \quad (163)$$

where the last equation includes also the previous one, if we define, $Z_0 = 1$.
Than we find

$$Z_n = B^n Z_0,\ Bf = \int da\frac{\beta}{a^2}\frac{d}{da}(af) = \int dZ_1\frac{d(af)}{da},$$
$$Z = (1 - B/\varepsilon)^{-1}Z_0 = \varepsilon(\varepsilon - B)^{-1}Z_0 \quad (164)$$

These formal considerations are correct in the nonstandard (non-archimedean) analysis [Davis, 1977].
There is another way of finding Z,

$$Z(a) = \frac{a_b}{a\mu^{2\varepsilon}},\ \int^a \frac{da}{-\varepsilon a + \beta(a)} = t. \quad (165)$$

For any multiplicative renormalized quantity A, $A_b = Z(\varepsilon, a)A$,

$$\dot{A} = -\gamma(\varepsilon, a)A, \quad \gamma = \dot{Z}/Z, \quad A = \exp(-\int^a da\frac{\gamma(\varepsilon, a)}{-\varepsilon a + \beta(a)}), \quad (166)$$

for $A = a$, $\gamma = \varepsilon - \beta(a)/a$.

Dual symmetry

Let us take for renormalization constant in critical dimenshion

$$0 \leq Z(a,0) = 1 - a \leq 1, \quad 0 \leq a \leq 1, \quad a_b = a(1-a)$$
$$\Rightarrow \quad a_\pm = \frac{1 \pm \sqrt{1-4a_b}}{2}, \quad 0 \leq a_b \leq \frac{1}{4}. \tag{167}$$

The solutions for the observable values of the coupling constant $a_\pm$, has the property

$$a_+ a_- = a_b, \tag{168}$$

which is similar with the Dirac's quantization

$$\alpha_e \alpha_m = \frac{1}{4} = (4\pi)^2 a_b = e_b^2, \quad e_b = \pm\frac{1}{2}, \quad a_b = \frac{n^2}{64\pi^2} \leq \frac{1}{4} \Rightarrow |n| \leq 12. \tag{169}$$

This condition gives the restriction on the charge, correspondingly, mass, size,..., of the cumulative quasiparticles $\varphi-$Fluktons.
We have a dual symmetry with respect to the change $a_\pm \to a_\mp$, or $a \leftrightarrow Z$. If $a = a_-$ is not small, we can expend with respect to $a_+ = a_b/a = Z = 1 - a$.
At the selfdual point/scale $a_+ = a_- = n/2$; at opposite RD fixed points $a_{1c} = 1/(2na_{2c})$
If we consider a_b as effective coupling constant at the n-th RD iteration, $a_b = a_n$, take $a = a_{n+1}$, we will have

$$a_n = a_{n+1}(1 - a_{n+1}), \quad 0 < a_n < a_{n+1} < \frac{1}{2}, \quad a_n \to \frac{1}{2}. \tag{170}$$

The quantitative values and qualitative content of the given field theory depend on the scale (parameter, e.g. $\mu-$renormalization point, $g = g(\mu),\ A = A(\mu)$). In QCD e.g. the effective action has the following form:

$$S(\mu) = \frac{1}{g^2(\mu)} \int d^D x \mathcal{L}(A(\mu)), \tag{171}$$

variation with respect to the change of scale gives

$$\delta S = -2\frac{\beta(g)}{g\mu}\delta\mu S + \frac{1}{g^2}\int d^D x \frac{\delta \mathcal{L}}{\delta A}\delta A \tag{172}$$

and the following two statements are equivalent:

$$\delta S = 0,\ \beta(g) = 0 \Leftrightarrow \delta S = 0,\ \frac{\delta \mathcal{L}}{\delta A} = 0. \tag{173}$$

So, from renorminvariance of the effective action follows that at the conformal symmetric points, the motion equations for fields are satisfied. Generalization for the several coupling constants and other models is obvious.

In string theory, the connection between conformal invariance of the effective theory on the parametric world sheet and the motion equations of the fields on the embedding space is well known [Ketov, 2000]. A more recent topic in this direction is AdS/CFT Duality [Maldacena, 1988]. In this approach for QCD coupling constant the following expression was obtained [Brodsky, de Tèramond, Deur, 2010]

$$\alpha_{AdS}(Q^2) = \alpha(0)e^{-Q^2/4k^2}. \tag{174}$$

A corresponding β-function is

$$\beta(\alpha_{AdS}) = \frac{d\alpha_{AdS}}{\ln Q^2} = -\frac{Q^2}{4k^2}\alpha_{AdS}(Q^2) = \alpha_{AdS}(Q^2)\ln\frac{\alpha_{AdS}(Q^2)}{\alpha(0)} \tag{175}$$

So, this renormdynamics of QCD interpolates between the IR fixed point $\alpha(0)$, which we take as $\alpha(0) = 2$, and the UV fixed point $\alpha(\infty) = 0$.

For the QCD running coupling considered in [Diakonov, 2003]

$$\alpha(q^2) = \frac{4\pi}{9\ln(\frac{q^2+m_g^2}{\Lambda^2})}, \tag{176}$$

where $m_g = 0.88GeV,\ \Lambda = 0.28GeV$, the $\beta-$function of renormdynamics is

$$\beta(\alpha) = -\frac{\alpha^2}{k}(1 - c\exp(-\frac{k}{\alpha})),$$
$$k = \frac{4\pi}{9} = 1.40,\ c = \frac{m_g^2}{\Lambda^2} = (3.143)^2 = 9.88, \tag{177}$$

for a nontrivial (IR) fixed point we have

$$\alpha_{IR} = k/\ln c = 0.61 \tag{178}$$

For $\alpha(m) = 2$, at valence quark scale m we predict the gluon (or valence quark) mass as

$$m_g = \Lambda e^{\frac{k}{2\alpha(m)}} = 1.42\Lambda = m_N/3,\ \Lambda = 220MeV. \tag{179}$$

From the nonperturbative $\beta-$functions we see that besides perturbative phase with asymptotic freedom there is also nonperturbative phase with infrared fixed point and rising coupling constant at higher energies.

It is nice to have a nonperturbative $\beta-$function like (177), but it is more important to see which kind of nonperturbative corrections we need to have a phenomenological coupling constant dynamics.
It was noted [Voloshin, Ter-Martyrosian, 1984] that in valence quark parametrization $\alpha_s(m) = 2$, at a valence quark scale m.

Following the pion, the rho is the most prominent meson. Vector mesons play an important role when considering the interaction of hadrons with electromagnetic fields. In the vector meson dominance model the hadrons couple to photons exclusively through intermediate vector mesons. The equality of the ρ meson self-coupling g and the coupling to nucleons $g_{\rho N}$ and pions $g_{\rho\pi}$, the universality of the ρ meson coupling, plays an important role in vector meson dominance [Sakurai, 1969] and is a consequence of the existence of a consistent EFT with ρ mesons, pions, and nucleons. Indeed, one can rewrite the Lagrangian of [Weinberg, 1968] in terms of renormalized fields and couplings, thereby introducing the basic Lagrangian

$$\begin{aligned} L_R = {} & \bar{N}(i\gamma\partial - M)N - \frac{1}{2}\pi(\partial^2 + m^2)\pi - \frac{1}{4}(\partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu)^2 + \frac{1}{2}M^2_\rho \rho^2 \\ & + g\bar{N}\gamma^\mu t_a N \rho^a_\mu + g_{\pi\rho}\epsilon^{abc}\pi^a \partial^\mu \pi^b \rho^c_\mu - g(\rho_\mu \times \rho_\nu)\cdot \partial^\mu \rho^\nu \\ & - \frac{g^2}{4}(\rho_\mu \times \rho_\nu)^2 \end{aligned} \quad (180)$$

Requiring that the results are UV finite introduces relations between the couplings of the theory [Djukanovic et al, 2004], $g_{\pi\rho} = g$. The coupling g is directly related to the width of the ρ meson.

In the previous $\pi\rho N$ model of pion-nucleon interaction [Di Giacomo, Paffuti, Rossi, 1992]

$$L_{\pi N} = g(\bar{N}\gamma^{\mu}t_a N + \epsilon^{abc}\pi^b\partial^{\mu}\pi^c)\rho^a_{\mu}, \tag{181}$$

pion interacts with nucleon through the exchange of the vector meson $\rho(m_\rho = 750\ MeV, T = 1)$, the amplitude of $\rho^0 \to \pi^+\pi^-$ decay is

$$M = g\varepsilon^{\mu}(k_{\pi^-} - k_{\pi^+})_{\mu}, \tag{182}$$

the decay width is

$$\Gamma = \frac{1}{2m_\rho}|M|^2(1 - \frac{4m_\pi^2}{m_\rho^2})^{\frac{1}{2}}\frac{1}{8\pi} = \frac{g^2}{48\pi}m_\rho(1 - \frac{4m_\pi^2}{m_\rho^2})^{\frac{3}{2}} \tag{183}$$

and for fine structure coupling constant we have

$$\alpha_{\pi\rho N} = \frac{g^2}{4\pi} = \frac{\Gamma}{m_\rho}\frac{12}{(1 - \frac{4m_\pi^2}{m_\rho^2})^{\frac{3}{2}}} = \frac{12.}{5(1 - \frac{4\times 14^2}{75^2})^{\frac{3}{2}}} = 3.006 = 3.0.. \tag{184}$$

for $\Gamma = \Gamma_{\rho\pi\pi} = 150MeV, m_\pi = 140MeV, m_\rho = 770MeV$. So, in this strong coupling model the expansion parameter is a prime number, $\alpha_g = 3$.

In the 1870's G.J. Stoney [Stoney, 1881], the physicist who coined the term "electron" and measured the value of elementary charge e, introduced as universal units of Nature for L, T, M :

$$l_S = \frac{e}{c^2}\sqrt{G},\ t_S = \frac{e}{c^3}\sqrt{G},\ m_S = \frac{e}{\sqrt{G}} \tag{185}$$

M. Planck introduced [Planck, 1899] as universal units of Nature for L, T, M:

$$m_P = \sqrt{\frac{hc}{G}} = \frac{m_S}{\sqrt{\alpha}},\ l_P = \frac{h}{cm_P} = \frac{l_S}{\sqrt{\alpha}} = 11.7 l_S,\ t_P = \frac{l_P}{c} = \frac{t_S}{\sqrt{\alpha}} \tag{186}$$

Stoney's fundamental constants are more fundamental just because they are less than Planck's constants :) Due to the value of $\alpha^{-1} = 137$, we can consider relativity theory and quantum mechanics as deformations of the classical mechanics when deformation parameter $c = 137$ (in units $e = 1, \hbar = 1$) and $\hbar = 137$ (in units $e = 1, c = 1$), correspondingly. These deformations have an analytic sense of p-adic convergent series. The number 137 has a very interesting geometric sense,

$$137 = 11^2 + 4^2, \tag{187}$$

so, $\sqrt{137}$ is the hypotenuse length of a triangle with other sides of lengths 11 and 4.

The Babylonians used a base 60 number system which is still used for measuring time - 60 seconds in a minute, 60 minutes in an hour - and for measuring angle - 360 degrees in a full turn. The base 60 number system has its origin in the ration of the Sumerian mina (m) and Akkadian shekel (s), $m/s \simeq 60 = 3 \cdot 4 \cdot 5$.
We also can consider base 137 system for fundamental theories.

For the nuclear physics strong coupling phenomena description we may take as a base $p = 13$.
For the hadronic physics, valence scale QCD, and graphen strong coupling phenomena description we may take as a base $p = 2$.
For the weak coupling physics SM m_Z scale and MSSM unification scale phenomena description we may take as a base $p = 29$.

There are different opinions about the number of fundamental constants [Duff, Okun, Veneziano, 2001].
According to Okun, there are three fundamental dimensionful constants in Nature: Planck's constant, $\hbar$; the velocity of light, c; and Newton's constant, G.
According to Veneziano, there are only two: the string length L_s and c.
According to Duff, there are not fundamental constants at all.
Usually $L_s = l_p$, so, the fundamental area is $L_s^2 = 137 l_s^2$.
The value $s_s = l_s^2 -$ Stoney area, is more like on a fundamental area :)

In mathematics we have two kind of structures, discrete and continuous one. If a physical quantity has discrete values, it might have no dimension. If the values are continuous - the quantity might have a dimension, a unit of measure. These structures may depend on scale, e.g. on macroscopic scale condensed state of matter (and time) is well described as continuous medium, so we use dimensional units of length (and time). On the scale of atoms, the matter has a discrete structure, so we may count lattice sites and may not use a unit of length. If at small (e.g. at Plank) scale space (and/or time) is discrete, then we do not need a unit of length (time) for measuring, there is a fundamental length and we can just count.

The reduction of the dimensionless couplings in GUTs is achieved by searching for RD integrals of motion-renormdynamic invariant (RDI) relations among them holding beyond the unification scale. Finiteness results from the fact that there exist RDI relations among dimensional couplings that guarantee the vanishing of all beta-functions in certain GUTs even to all orders. In this case the number of the independent motion integrals N is equal to the number of the coupling constants. Note that in superintegrable dynamical systems the number of the integrals is $\leq N-1$, so the RD of the finite field theories is trivial, coupling constants do not run, they have fixed values, the renormdynamics is more than superintegrable, it is hyperintegrable. Developments in the soft supersymmetry breaking sector of GUTs and FUTs lead to exact RDI relations, i.e. reduction of couplings, in this dimensionful sector of the theory, too. Based on the above theoretical framework phenomenologically consistent FUTs have been constructed. The main goal expected from a unified description of interactions by the particle physics community is to understand the present day large number of free parameters of the SM in terms of a few fundamental ones. In other words, to achieve reduction of couplings at a more fundamental level.

This mechanism indicates that with time inversion symmetry we can have only composed scalar fields. With the discovery of the Higgs particle with mass $125\ GeV$, a nice number $m_W/m_H \simeq 2/3$ appear, which, at least for me, indicates for composed nature of W and H, with a same mass of about $40\ GeV$ two and three valence constituents correspondingly. The fermion constituents ψ_n^a of W and scalar constituents φ_n^a of H compose scalar super multiplet (φ_n^a, ψ_n^a) with a flavor index n and color index a. Another notation is (h, sh)-(He, She:).

With exact SUSY we have cofinement by dimentional counting: superspace dimension is zero on the hadronic scale, hadrons are pointlike, color is confined inside hadrons. For SM QCD this picture indicates that at the hadronic scale we have effective SQCD, which contains scalar quarks.

The 40 GeV constituents may be good candidates in dark matter particles. Coupling constant unification at $\alpha_u^{-1} = 29.0$ and scale $10^{16} GeV$ in MSSM [Makhaldani, 2014] has a relict on the SM scale: $\alpha_2^{-1}(m) = 29.0$ at $m = 41 GeV$.
If we extrapolate the SM value of $\alpha^{-1}(m_Z)$ to electron masse scale, we find $\alpha^{-1}(m_e) = 137.0$
SQCD with valence quarks and their superpartner scalar $h-$quarks all of them with valence mass about 40 GeV may produce the observable W and H particles. With exact SUSY we have cofinement by dimentional counting: superspace dimension is zero on the hadronic scale, hadrons are pointlike, color is confined inside hadrons. For SM QCD this picture indicates that at the hadronic scale we have effective SQCD, which contains scalar quarks (as diquarks in the minimal case, see [S. Catto, 2015]).
Note that in condensed state physics we have an analogous phenomenon, a localization of a particle in the random environment which can be described as superspace dynamics of the pointlike particle on the scale larger than the size of the localized particle. For QCD this picture corresponds to the localization of the valence quarks in the random (gluon) vacuum inside hadrons. Mathematically rigorous description can be done in the SQCD model.

Phenomenological approach to the nonrelativistic potential-model study of Υ and ψ spectra leads to a static Coulombic Power-law potential of the form

$$V(r) = a(r)r^{2-d(r)} \sim \begin{array}{c} 1/r,\ r \sim 0.1fm \\ r,\ r \sim 1.fm \end{array} \quad (188)$$

E.g. in the case of the Υ and small r

$$V(r) = \frac{4}{3}\frac{\alpha_s}{r},\ \alpha_s = \frac{2\pi}{b\ln r\Lambda},\ b = 9. \quad (189)$$

This behavior corresponds not only to the running fine structure constant but also to the running space dimension. Confinement-the point-like hadrons on the scales higher than hadronic, corresponds to the zero dimensional space for hadron constituents.
RD equations of QCD beyond the critical dimention has explicit dependence on the space dimension. When the dimension becomes running we should consider two dimensional renormdinamics

$$\begin{array}{l} \dot{a_1} = \beta_1(a_1, a_2),\ a_1 = a, \\ \dot{a_2} = \beta_2(a_1, a_2),\ a_2 = d \end{array} \quad (190)$$

In the Standard Model of Particle Physics (SM), the values of the coupling constants and masses of particles depends on the scale according to the Renormdynamic motion equations. One charge a, one mass m RD equations are

$$\begin{aligned} \dot{\alpha} &= \beta(\alpha), \\ \dot{m} &= \gamma(\alpha) m \end{aligned} \tag{191}$$

For the electron and nucleon masses, electrodynamic and pion-nucleon fine structure constants we have an empirical relation:

$$m_e/\alpha \simeq m_N/\alpha_{\pi N} \tag{192}$$

We take the relation $m/\alpha = const$ as an integral of renormdynamic motion equations for m and α, find exact form of the β function in the minimal mass parametrization

$$\begin{aligned} &\gamma(\alpha) = \gamma_1 \alpha + \gamma_2 \alpha^2 + ... = \gamma_1 A, \\ &A = f^{-1}(\alpha) = \alpha + \gamma_2/\gamma_1 \alpha^2 + \gamma_3/\gamma_1 \alpha^3 + ..., \\ &\alpha = f(A) = A + f_2 A^2 + f_3 A^3 + ... \end{aligned} \tag{193}$$

From the integral of motion, in the minimal mass parametrization: $\gamma(\alpha) = \gamma_1 \alpha$, we obtain

$$\begin{aligned} &(\ln \alpha)^{\cdot} = (\ln m)^{\cdot} \Rightarrow \beta(\alpha)/\alpha = \gamma(\alpha) \\ &= \gamma_1 \alpha \Rightarrow \beta(\alpha) = \beta_2 \alpha^2,\ \beta_2 = \gamma_1 \end{aligned} \tag{194}$$

so, we have the following algebraic equations for the flavor and color content of the theory

$$\begin{aligned} &\beta_n = 0,\ n \geq 3, \\ &\beta_2 = \gamma_1 \end{aligned} \tag{195}$$

and prediction for the dimension of space-time: $D = 4$. Solution of the motion equations are

$$\begin{aligned} &\alpha(t) = \frac{\alpha_0}{1 - \alpha_0 \beta_2 t}, \\ &m(t) = m_0 |\alpha_0^{-1} - \beta_2 t|^{-\gamma_1/\beta_2} = \frac{m_0}{\alpha_0} \alpha(t) \end{aligned} \tag{196}$$

In the multidimentional renormdynamics, when we have several (N) coupling constants and masses, we assume that there are maximal number $(N-1)$ integrals of motion H_n. If the number of integrals is N, we not have dynamics, we have only statics - finite field theory,
$\alpha_n = const,\ n = 1, ..., N.$
The idea of reduction to the one dimensional renormdynamics is simple:

$$\frac{d\alpha_n}{dt} = \beta_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha_N) \Rightarrow \frac{d\alpha_n}{d\alpha} = B_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha),\ \alpha = \alpha_N,$$
$$B_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha) = \beta_n(\alpha_1, ..., \alpha_{(N-1)}, \alpha)/\beta_N(\alpha_1, ..., \alpha_{(N-1)}, \alpha),$$
$$\alpha_n = \sum_{k\geq 1} f_{nk}\alpha^k, \quad n = 1, 2, ..., N-1 \tag{197}$$

Note that the procedure of reduction of the higher order dynamical system, e.g. second order Euler-Lagrange motion equations, to the first order dynamical systems, in the case to the Hamiltonian motion equations, can be continued using fractal calculus. E.g. first order system can be reduced to the half order one,

$$\begin{aligned} &D^{1/2}q = \psi, \\ &D^{1/2}\psi = p \Leftrightarrow \dot{q} = p. \end{aligned} \tag{198}$$

Another realization is in supermathematics,

$$D_t^{1/2} = \partial_\theta + \theta\partial_t \tag{199}$$

Note that the procedure of reduction of the higher order dynamical system, e.g. second order Euler-Lagrange motion equations, to the first order dynamical systems, in the case to the Hamiltonian motion equations, can be continued using fractal calculus. E.g. first order system can be reduced to the half order one,

$$\begin{aligned} &D^{1/2}q = \psi, \\ &D^{1/2}\psi = p \Leftrightarrow \dot{q} = p. \end{aligned} \tag{200}$$

We define the following dynamical system

$$\begin{aligned} &D^{1/2}q = f(q),\ D^{1/2} = \partial_\theta + \theta\partial_t,\ q(t,\theta) = q_0(t) + \theta q_1(t), \\ &f(q) = f_0(q) + \theta f_1(q) = f_0(q_0) + \theta(f_0'(q_0)q_1 + f_1(q_0)) \end{aligned} \tag{201}$$

which is equivalent to the following dynamical system in component form

$$\begin{aligned} &q_1(t) = f_0(q_0), \\ &\dot{q}_0 = f_0'(q_0)q_1 + f_1(q_0) \end{aligned} \tag{202}$$

For hamiltonian extension of this system, we consider the following lagrangian

$$
\begin{aligned}
S &= \int dt \int d\theta p(t,\theta)(D^{1/2}q - f(q)) \\
&= \int dt p_1(q_1 - f_0(q_0)) + p_0(\dot{q}_0 - f_0'(q_0)q_1 - f_1(q_0)) \\
&= \int dt \int d\theta (\bar{D}^{1/2}pq - pf(q))
\end{aligned} \tag{203}
$$

where we used the following definition of the momentum p and usual integration rules for Grassmann variables θ

$$
p(t,\theta) = p_0(t) + \theta p_1(t); \ \int d\theta = 0, \ \int d\theta\theta = 1 \tag{204}
$$

From the component form of the lagrangian we obtain the following system of motion equations

$$\begin{aligned}
&q_1(t) = f_0(q_0),\\
&\dot{q}_0 = f_0'(q_0)q_1 + f_1(q_0) = f_0'(q_0)f_0(q_0) + f_1(q_0) \equiv \varphi(q_0),\\
&p_1 = f_0'(q_0)p_0,\\
&\dot{p}_0 = -(f_0''(q_0)q_1 + f_1'(q_0))p_0 - f_0'(q_0)p_1\\
&= -(f_0''(q_0)q_1 + f_1'(q_0) + f_0'(q_0)^2)p_0 = -\varphi'(q_0)p_0
\end{aligned} \tag{205}$$

Knowing right equation for momentum in the component form we may fix its form also in superfield form

$$\bar{D}^{1/2}p = pf'(q) \tag{206}$$

where $\bar{D}$ we define from the following consideration

$$\begin{aligned}
&\int_0^T dt \int d\theta p(t,\theta) D^{1/2} q = \int_0^T dt(p_1 q_1 + p_0 \dot{q}_0)\\
&= \int_0^T dt(p_1 q_1 + (p_0 q_0)^{\cdot} - \dot{p}_0 q_0) = \int_0^T dt \int d\theta \bar{D}^{1/2} pq + p_0 q_0|_0^T,\\
&\bar{D}^{1/2} = \partial_\theta - \theta\partial_t
\end{aligned} \tag{207}$$

Note that there is the following shorthand (mass shell) formulation

$$S = \int dt p_0 (\dot{q}_0 - \varphi(q_0)) \tag{208}$$

Now we conceder the following fractal dynamical system

$$D^{-\alpha}q = f(q), \tag{209}$$

where we take the fractal derivative-integral in the Riemann-Liouville form

$$\begin{aligned} {}_0D_t^{-\alpha}q &= \frac{1}{\Gamma(\alpha)}\int_0^t d\tau(t-\tau)^{\alpha-1}q(\tau) = t^\alpha\frac{\Gamma(\delta+1)}{\Gamma(\delta+1+\alpha)}q, \\ \delta &= t\frac{d}{dt} = t\partial_t \end{aligned} \tag{210}$$

Hamiltonian extension of the dynamical system we define

$$\begin{aligned} {}_0D_t^{-\alpha}q &= f(q), \\ {}_tD_T^{-\alpha}p &= pf'(q) \end{aligned} \tag{211}$$

by the following action

$$\begin{aligned} S &= \int_0^T dtp(t)(D^{-\alpha}q - f(q)) = \int_0^T dt(q(t)_tD_T^{-\alpha}p - pf(q)), \\ {}_tD_T^{-\alpha}p &= ({}_0D_T^{-\alpha} -_0 D_t^{-\alpha})p \end{aligned} \tag{212}$$

In the case of the fractal deformation of the usual hamiltonian systems

$$\int_0^T dt(p(t)\dot{q} - H(p,q)) \tag{213}$$

we have

$$S = \int_0^T dt(p(t)D^{-\alpha}q - H(p,q)) \tag{214}$$

For free particle,

$$D^{-\alpha}q = p, \;\; H = \frac{1}{2}p^2 \tag{215}$$

Now we return to our extended system (6) and formulate conditions for the integrals of motion $H(x,\psi)$

$$H = H_0(x) + H_1 + ... + H_N, \tag{216}$$

where

$$H_n = A_{k_1 k_2 ... k_n}(x)\psi_{k_1}\psi_{k_2}...\psi_{k_N}, \ 1 \le n \le N, \tag{217}$$

we are assuming Grassmann valued ψ_n and the tensor $A_{k_1 k_2 ... k_n}$ is skew-symmetric. For integrals (216) we have

$$\dot{H} = \{\sum_{n=0}^{N} H_n, H_1\} = \sum_{n=0}^{N}\{H_n, H_1\} = \sum_{n=0}^{N}\dot{H}_n = 0. \tag{218}$$

Now we see, that each term in the sum (216) must be conserved separately. In particular for Hamiltonian systems (2), zeroth, H_0 and first level H_1, (8), Hamiltonians are integrals of motion. For $n = 0$

$$\dot{H}_0 = H_{0,k}v_k = 0, \tag{219}$$

for $1 \leq n \leq N$ we have

$$\begin{aligned}\dot{H}_n &= \dot{A}_{k_1 k_2 \ldots k_n} \psi_{k_1} \psi_{k_2} \ldots \psi_{k_N} + A_{k_1 k_2 \ldots k_n} \dot{\psi}_{k_1} \psi_{k_2} \ldots \psi_{k_N} + \ldots \\ &+ A_{k_1 k_2 \ldots k_n} \psi_{k_1} \psi_{k_2} \ldots \dot{\psi}_{k_N} \\ &= (A_{k_1 k_2 \ldots k_n, k} v_k - A_{k k_2 \ldots k_n} v_{k_1, k} - \ldots \\ &- A_{k_1 \ldots k_{n-1} k} v_{k_n, k}) \psi_{k_1} \psi_{k_2} \ldots \psi_{k_N} = 0, \end{aligned} \tag{220}$$

and there is one-to-one correspondence between the existence of the integrals (217) and the existence of the nontrivial solutions of the following equations

$$\frac{D}{Dt} A_{k_1 k_2 \ldots k_n} = A_{k_1 k_2 \ldots k_n, k} v_k - A_{k k_2 \ldots k_n} v_{k_1, k} - \ldots - A_{k_1 \ldots k_{n-1} k} v_{k_n, k} = 0. \tag{221}$$

For $n = 1$ the system (221) gives

$$A_{k_1, k} v_k - A_k v_{k_1, k} = 0 \tag{222}$$

and this equation has at list one solution, $A_k = v_k$.

If we have two (or more) independent first order integrals

$$H_1^{(1)} = A_k^1 \Psi_k; \; H_1^{(2)} = A_k^2 \Psi_k, ... \tag{223}$$

we can construct corresponding (reducible) second (or higher)order MBKY tensor(s)

$$\begin{aligned} &H_2 = H_1^{(1)} H_1^{(2)} = A_k^1 A_l^2 \Psi_k \Psi_l = A_{kl} \Psi_k \Psi_l; \\ &H_M = H_1^{(1)} ... H_M^{(M)} = A_{k_1 ... k_M} \Psi_{k_1} ... \Psi_{k_M}, \\ &A_{k_1 ... k_M} = \{A_{k_1}^{(1)} ... A_{k_M}^{(M)}\}, \; 2 \le M \le N \end{aligned} \tag{224}$$

where under the bracket operation, $\{B_{k_1,...,k_N}\} = \{B\}$ we understand complete anti-symmetrization. The system (221) defines a Generalization of the Bochner-Killing-Yano structures of the geodesic motion of the point particle, for the case of the general (1) (and extended (6)) dynamical systems.

Having $A_M, 2 \leq M \leq N$ independent MBKY structures, we can construct corresponding second order Killing tensors and Nambu-Poisson dynamics.
In the superintegrable case, we have maximal number of the motion integrals, N-1.
The structures defined by the system (221) we call the Modified Bochner-Killing-Yano structures or MBKY structures for short, [Makhaldiani, 1999].

Complex Polynomial Equations and Nambu-poisson Dynamics
We consider the following polynomial equation

$$P_N(z) - tz^{N+1} = 0, \ z \in \mathcal{C}, \ t \in (0, \infty) \quad (225)$$

For small times t all zeros but one of this polynomial are near the zeros of the polynomial $P_N(z)$. The extra zero z_{N+1} is far from other zeros, for small t,

$$z_{N+1} = \frac{a_N}{t} + ... \quad (226)$$

In regular case main zeros are linear functions of t, for small t.
For large times all $n+1$ zeros are near the zeros of the equation

$$a_0 - tz^{N+1} = 0, \ z_n = \sqrt[N+1]{a_0/t}\exp(2\pi i\frac{n}{N+1}), \ n = 0, 1, ..., N \quad (227)$$

At a root x_c of multiplicity k we have

$$\frac{P_N^{(k)}(x_c)}{n!}(x - x_c)^k + ... = tx_c^{N+1}, \ x_n(t) = x_c + c_{n,k}t^{1/k}$$
$$c_{n,k} = (\frac{x_c^{N+1}n!}{P_N^{(k)}(x_c)})^{\frac{1}{k}}\exp(2\pi i\frac{n}{k}), \ 0 \le n \le k-1 \quad (228)$$

So we can define the multiplicity of the root k from the time dependence of the roots. It is interesting to know how extra zero approach with time to the other zeros and then all of them organized as sites of symmetric polygon on the circle with decreasing radius. Note that coefficients $a_n,\ 1 \leq n \leq N$ are known functions of zeros but do not depend on t - are invariants - integrals of motion. Having N integrals of motion $H_n,\ 1 \leq n \leq N$ we construct Nambu-Poisson dynamics for the roots [Nambu, 1973], [Makhaldiani, 2007], [Makhaldiani, 2012].

$$\dot{x}_n = \{x_n, H_1, H_2, ..., H_N\},\ 1 \leq n \leq N \tag{229}$$

As an example we consider quadratic deformation of the linear equation

$$\begin{aligned} &a_0 + a_1 z - tz^2 = -t(z - z_1)(z - z_2) = 0, \\ &a_0 = -tz_1 z_2,\ a_1 = t(z_1 + z_2) \end{aligned} \tag{230}$$

As a 'time independent' Hamiltonian we take

$$H = -a_0/a_1 = \frac{z_1 z_2}{z_1 + z_2} \tag{231}$$

the motion equations we find from the time independence of a_0 and a_1

$$\begin{aligned}
\dot{a}_0 &= -z_1 z_2 - t(\dot{z}_1 z_2 + z_1 \dot{z}_2) = 0, \\
\dot{a}_1 &= z_1 + z_2 + t(\dot{z}_1 + \dot{z}_2) = 0, \\
\dot{z}_1 &= \frac{z_1^3 z_2}{a_0(z_1 - z_2)} = \{z_1, H\} = f_{12}\frac{\partial H}{\partial z_2}, \\
\dot{z}_2 &= \frac{z_2^3 z_1}{a_0(z_2 - z_1)} = \{z_2, H\} = f_{21}\frac{\partial H}{\partial z_1}, \\
f_{12} &= \frac{z_1 z_2 (z_1 + z_2)^2}{a_0(z_1 - z_2)} = \frac{a_1^2}{t^3(z_2 - z_1)}
\end{aligned} \tag{232}$$

In the cubic deformation of the quadratic equation

$$a_0 + a_1 z + a_2 z^2 - t z^3 = -t(z - z_1)(z - z_2)(z - z_3) = 0 \qquad (233)$$

we have

$$\begin{aligned}
& a_0 = t z_1 z_2 z_3,\ a_1 = -t(z_1 z_2 + z_2 z_3 + z_3 z_1),\ a_2 = t(z_1 + z_2 + z_3), \\
& \dot{z}_1 = \frac{z_1^4 z_2 z_3}{a_0(z_2 - z_1)(z_1 - z_3)} = \{z_1, H_1, H_2\} = f_{1nm} \frac{\partial H_1}{\partial z_n} \frac{\partial H_2}{\partial z_m}, \\
& f_{123} = \frac{z_1 z_2 z_3 (z_1 z_2 + z_2 z_3 + z_3 z_1)(z_1 + z_2 + z_3)}{a_0(z_2 - z_1)(z_3 - z_2)(z_1 - z_3)} \\
& = \frac{a_1 a_2}{t^3 (z_1 - z_2)(z_1 - z_3)(z_3 - z_2)}, \\
& H_1 = \frac{z_1 z_2 z_3}{z_1 z_2 + z_2 z_3 + z_3 z_1},\ H_2 = \frac{z_1 z_2 + z_2 z_3 + z_3 z_1}{z_1 + z_2 + z_3}
\end{aligned} \qquad (234)$$

Introducing new time variable $\tau = a_1 a_2 t^{-2}/2$ we put the equation in the form

$$\frac{dz_1}{d\tau} = \{z_1, H_1, H_2\} = f_{1nm} \frac{\partial H_1}{\partial z_n} \frac{\partial H_2}{\partial z_m},$$
$$f_{123} = \frac{1}{(z_1 - z_2)(z_1 - z_3)(z_2 - z_3)} \tag{235}$$

For the following generalization of the Weierstrass function $V_n(z)$

$$\int_{V_n(z)}^{\infty} \frac{dV}{\sqrt{P_n(V)}} = z,$$
$$P_n(V) = \frac{4}{(n-2)^2} V^n + C_{n-2} V^{n-2} + ... + C_0, \qquad (236)$$

we have the following series (re)presentation

$$V_n(z) = \wp_n(z, C_{n-2}, ..., C_0) = \frac{1}{z^{2/(n-2)}} - \frac{(n-2)^2}{4(n+2)} C_{n-2} z^{2/(n-2)} + ... \qquad (237)$$

After formulation of the mathematical framework of quantum mechanics (QM), operatorial formulation of QM, Koopman and von Neumann gave operatorial approach to classical Hamiltonian mechanics [Koopman 1991], [von Neumann 1932]. After Wiener introduction of the functional integrals, Dirac and Feynman gave formal functional integral formulation of the quantum theory [Feynman, Hibbs 1965]. Recently Gozzi invented functional integral formulation of the classical theory [Gozzi 2001]. The path-integral formulation of Hamiltonian classical mechanics.
For supersymmetric gauge theories stochastic quantization appears to have one definite advantage: since a gauge fixing term is unnecessary, supersymmetry will not be broken at any step. This holds both for the Abelian and non-Abelian case. It appears at the moment as if stochastic regularization is the only viable candidate for a regularization scheme which manifestly conserves both supersymmetry, chiral symmetry and gauge invariance. However, supersymmetry is related to stochastic quantization also at a much deeper level. As an example, even purely scalar field theories will, when quantized stochastically, display a 'hidden' supersymmetry. This issue, is intimately connected with the existence of a so-called 'Nicolai map' for supersymmetric field theories [Nicolai 1980].

Parisi-Sourlas 'dimensional reduction' of scalar field theories in external random fields [Parisi, Sourlas 1979], is closely related to both supersymmetry and stochastic quantization. This becomes apparent when one establishes the connection to the Nicolai map.
The phenomenon of dynamical 'dimensional reduction' was first noted within the context of critical phenomena associated with spin systems in random external fields. Systems very close to such a situation can in fact be created and studied in the laboratory. From renormalization group theory, the detailed long-distance behaviour of, for example, Ising spin systems can, sufficiently close to a critical point, be understood from the behaviour of a scalar field theory

$$S = \int d^D x (\frac{1}{2}\varphi(-\partial^2 + m^2)\varphi + V(\varphi)),\ V(\varphi) = a\varphi^3 + g\varphi^4 \qquad (238)$$

We start in the simplest possible way by considering the Langevin equation associated with a point particle being subjected to random background noise. This corresponds to the very real physical problem of the Brownian motion of a (classical) particle in a heat bath. Surprisingly, this problem turns out to be equivalent to a supersymmetric quantum mechanical problem. Let us now see why. The Langevin equation for the particle reads

$$\frac{dx}{dt} \equiv \dot{x} = -\frac{\delta S}{\delta x} + \eta(t) \tag{239}$$

where x represents the space coordinate of the particle. Expectation values are, as usual, evaluated as the path integral

$$< x(t_1)... \, x(t_n) >= \int d\eta \; x(t_1)... \, x(t_n) \exp(-\frac{1}{4}\int dt \eta(t)^2) \tag{240}$$

over a Gaussian noise, i.e.

$$< \eta(t_1)\eta(t_2) >= 2\delta(t_1 - t_2) \tag{241}$$

we now attempt to make a change of variables: $\eta \to x$. This involves the Jacobian

$$det(\delta\eta(t)/\delta x(t')) = det((d/dt + V')\delta(t - t')) \tag{242}$$

where we have introduced $V = \delta S/\delta x$.
For partition function Z,

$$\begin{aligned} Z &= \int d\eta \exp(-\frac{1}{4}\int dt\eta(t)^2) \\ &= \int d\eta dx det(d/dt + V')\delta(\dot{x} + V - \eta(t)) \exp(-\frac{1}{4}\int dt\eta(t)^2) \\ &= \int dx det(d/dt + V') \exp(-\frac{1}{4}\int dt(\dot{x} + V)^2) \\ &= \int dx d\psi d\bar{\psi} \exp(-S), \\ S &= \int dt(\frac{1}{4}(\dot{x} + V)^2 - \bar{\psi}(d/dt + V')\psi) \end{aligned} \tag{243}$$

This system is recognized as Witten's example of supersymmetric quantum mechanics.

Solitons are particlelike states, solutions of motion equations and they quantum extensions. Examples are solitons of SinGordon motion equation or barions-skirmions of Skyrme model. In particle theory, the skyrmion was described by Tony Skyrme in 1962 and consists of a quantum superposition of baryons and resonance states.
Skyrmions as topological objects are important in solid state physics. Researchers could read and write skyrmions using scanning tunneling microscopy. The topological charge, representing the existence and non-existence of skyrmions, can represent the bit states "1" and "0".

QCD consists of quarks and gluons. Quarks possess both color (r, g, b) and flavor $(u, d, s,\text{etc.})$, while gluons possess color (r, g, b) and anti-color $(\bar{r}, \bar{g}, \bar{b})$, but not flavor. An open **string** (a string with two endpoints) is ideally suited to account for such quantum numbers at its two ends. For quarks, one end represents color and the other end flavor. For gluons, one end represents color and the other anti-color. In string theory, there are **branes** (higher dimensional extended objects that are generalized membranes) to which the endpoints of an open string are confined. Applying this idea to QCD, we introduce N_c colored branes and N_f flavored branes at which open strings corresponding to quarks and gluons terminate. The energy of a string is given by the sum of the classical energy stored inside the string and the excitation energies of vibration and rotation. Because the classical energy of a string is proportional to its length and because gluons are massless, N_c colored branes should lie on top of one another. On the other hand, quarks possess intrinsic masses, and therefore the endpoints of a quark string, namely, a flavored brane and a colored brane should be separated from each other by a nonvanishing distance U. Then, the intrinsic quark mass m_q can be represented as $m_q = U \times$(string tension), where the string tension is the energy stored inside a unit length of string and is represented: string tension$=1/(2\pi\alpha')$ in terms of α' , historically called the Regge slope.

To describe QCD, we have to prepare Dp-branes and Dq-branes with p, q$\geq$3 for colored branes and flavored branes, respectively, and these branes should be located in the space of more than five dimensions. To evaluate the amplitude for a certain process to occur in the above picture, we have to sum up all the possible two-dimensional world sheets with the weight $\exp(iS)$, where the action S is given by S=(energy)$\times$(time)=(area of the strings world sheet)$/2\pi\alpha'$, following the Feymann path integral formulation.

Cumulative Effect: Production of particles from nuclei in a region, kinematically forbidden for reactions with free nucleons is connected to the existence of **Fluctons** - droplets of dense cold nuclear matter.
Classical fields have canonical, rational for integer D, (mass)dimensions e.g. in electrodynamics

$$L = \int d^D x(\bar{\psi}(\gamma(\partial - eA) - m)\psi - \frac{1}{4}F^2),$$
$$d_\psi = [\psi] = (D-1)/2,\ d_A = (D-2)/2,\ d_e = (4-D)/2 \quad (244)$$

Quantum corrections introduce (anomaly) corrections to the canonical dimensions, so the fields and coupling constants become **fractals.** At fixed points of RD, the fractals are self similar and their compositions present at low energies **unparticles.**

Qualitative picture of the (un)particle(like) objects we will illustrate with the simplest model of scalar field given by the following lagrangian

$$L = L(\Phi, M, \lambda, n) = \frac{1}{2}(\partial_\mu \Phi) - \frac{1}{2}M^2\Phi^2 - V(\Phi),\ \mu = 0, 1, 2, ..., D \quad (245)$$

where self interaction usually we take in the form

$$V(\Phi) = \lambda \Phi^n,\ n = -2, 1, 2, 3, 4, 6 \quad (246)$$

In renormalisible case,

$$n = \frac{2D}{D-2} = 2 + \varepsilon(D),\ \varepsilon(D) = \frac{4}{D-2},$$
$$D = \frac{2n}{n-2} = 2 + \varepsilon(n),\ \varepsilon(n) = \frac{4}{n-2}, \quad (247)$$

sometimes we consider also intermediate values of n and D and other forms of V.

In the free (self non interacting) field (particle) approximation: $\lambda = 0$, but in external gravitational field we have

$$L(g, \Phi, M) = \sqrt{-g}L(\Phi, M, 0), \;\; g = det g_{\mu\nu}(x) \tag{248}$$

Now we will see a nice composite particle mechanism :) Let us take a substitute: $\Phi = \varphi^k$, than we find

$$L(g, \Phi, M) = L((k\varphi^{k-1})^4 g, \varphi, M/k), \;\; g_{\mu\nu}(x) \Rightarrow (k\varphi^{k-1})^{4/D} g_{\mu\nu}(x) \tag{249}$$

Indeed

$$\begin{aligned} L(g, \Phi, M) &= \sqrt{-g}(k^2\varphi^{2(k-1)}\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}M^2\varphi^{2k}) \\ &= \sqrt{-g(k\varphi^{k-1})^4}(\frac{1}{2}(\partial_\mu\varphi)^2) - \frac{1}{2}(\frac{M}{k})^2\varphi^2) \end{aligned} \tag{250}$$

Now, having an experience with constituent - composite particle relation, we turn attention on the self-interaction therm,

$$L = \sqrt{-g(k\varphi^{k-1})^4}(... - \frac{\lambda}{k^2}\varphi^N), \ N = kn - 2(k-1) \tag{251}$$

Most natural value of n for stable systems $(1+1 \to 1+1, \ 2 \to 2)$ is $n = 4$. In this case, $N = 2k + 2$ and only natural value of constituents for which we have a renormalizable interaction is $k = 2 \Rightarrow N = 6$ with corresponding spacetime dimension $D = 3$. The most natural value for fission-fusion interaction $(1 \leftrightarrow 2)$ is $n = 3 \Rightarrow N = k + 2$, for which we have realistic values $k = 2$ and $N = 4, \ D = 4$:) Other interesting values of naturally interpretable monomial (polynomial) interactions generally corresponds to the non-integer, fractional-fractal dimensions of space(time) D, with fractal-flucton-unparticle interpretations of the corresponding states of matter.

The size of particle-like states (solutions of the motion equations) are defined as $l \sim M^{-1}$, because at the boundary region, the linear part of the motion equations dominates and the Yukawa-like asymptotic $\Phi(r) \sim e^{-Mr}$ acts. In a pion-nucleon model for nucleon size we have $l_N \sim m_\pi^{-1} \simeq 1.43$ fm. The amplitude of the state (at maximum) $A \sim \lambda^{-\alpha},\ \alpha = 1/(n-2)$. Indeed, the motion equation do not contains the coupling constant after a scaling substitution $\Phi = \lambda^{-\alpha}\phi$, so a particle-like solution ϕ dos not contains λ and corresponding solution $\Phi = \lambda^{-\alpha}\phi \sim \lambda^{-\alpha}$,

$$\triangle\Phi + M^2\Phi + \lambda n\Phi^{n-1} = \lambda^{-\alpha}(\triangle\phi + M^2\phi + \lambda^{1-(n-2)\alpha}n\phi^{n-1}) = 0. \quad (252)$$

At not so low energies from string theory we may extract the following scalar field theory

$$L = \sqrt{-g}(\frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2}M^2\Phi^2 - \lambda\Phi^3),\ \mu = 0, 1, ..., D-1,\ D = 6+\varepsilon \quad (253)$$

where $\varepsilon \in [0, 20]$. The one loop $\beta-$function is

$$\beta(a) = (D-6)a - \beta_2 a^2,\ a \sim \lambda^2 \quad (254)$$

and it has stable UV fixed point at $a = (D-6)/\beta_2$ and IR fixed point $a = 0$. Beyond this point we have an unparticle $\Phi = \phi^2$ with lagrangian

$$L = \sqrt{-g'}(\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\frac{M}{2})^2\phi^2 - \frac{\lambda}{4}\phi^4),\ \mu = 0, 1, ..., D-1,$$
$$d = 4 - \varepsilon,\ \varepsilon \in [0, 1]. \quad (255)$$

The one loop $\beta-$function is

$$\beta(\lambda) = (d-4)\lambda + b\lambda^2 \tag{256}$$

and it has stable IR fixed point at $\lambda = (4-d)/b$. The UV fixed point is $\lambda = 0$. At this point we have reduction from higher dimentional Φ^3 to lower dimentional ϕ^4.
Another possibilities is an unparticle $\Phi = \varphi^4$ with lagrangian

$$L = \sqrt{-g''}(\frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}(\frac{M}{4})^2\varphi^2 - \frac{\lambda}{16}\varphi^6),\ \mu = 0, 1, ..., d-1. \tag{257}$$

The one loop $\beta-$function is

$$\beta(\lambda) = (d-3)\lambda + c\lambda^2,\ d = 3-\varepsilon,\ \varepsilon \in [0, 2]. \tag{258}$$

The IR fixed point is $\lambda = \varepsilon/c$. UV fixed point is $\lambda = 0$.

Similar consideration gives reduction from higher energy ϕ^4 model to lower energy φ^6 one. Some technical questions remain. One of them concern to the substitution $\Phi = \phi^2$. It restricts Φ as $\Phi \geq 0$. OK, we already have a constraint, that the fields are real valued, we have a restriction

$$\phi^*(x) = \phi(x) = \frac{1}{(2\pi)^D} \int d^D p \exp(ipx)\hat{\phi}(p) \Rightarrow \hat{\phi}^*(p) = \hat{\phi}(-p) \quad (259)$$

To formulate positivity condition is not so easy. We will take another path, we define the interaction as $\Phi^3 = (\Phi^2)^{3/2} \geq 0$. Then the substitution $\Phi^2 = \phi^4$ will works. Bytheway by this definition we made also another improvement: the potential become bounded from below. For the reduction the substitution $\Phi^2 = \phi^4$ also works,

$$L = \sqrt{-g}(\frac{1}{8\Phi^2})(\partial_\mu \Phi^2)^2 - \frac{1}{2}M^2\Phi^2 - \lambda(\Phi^2)^{n/2}, \; n = 3, 4. \quad (260)$$

Note that by substitution

$$(\frac{\Phi}{\Phi_0})^2 = \phi^{2k},\ \phi^2 = \exp(\ln(\Phi^2/\Phi_0^2)/k)$$
$$= 1 + \frac{1}{k}\ln(\frac{\Phi}{\Phi_0})^2 + O(K^{-2}),\ \phi = \pm 1 + O(k^{-1}) \qquad (261)$$

we reduce the field theory to a discrete theory, to a system of bits. Also, changing dimension of space D and nonlinearity n restricted by condition

$$n = \frac{2D}{D-2},\ D = \frac{2n}{n-2},\ \frac{1}{n} + \frac{1}{D} = \frac{1}{2} \qquad (262)$$

we assume that they are functions of scale or coupling constant, due to monotonic property of the coupling constant. We have the following relation

$$\beta_n = -\frac{4}{(D-2)^2}\beta_D,$$
$$\beta_n = \mu\frac{dn}{d\mu} = \frac{dn}{d\lambda}\beta_\lambda,\ \beta_D = \mu\frac{dD}{d\mu} = \frac{dD}{d\lambda}\beta_\lambda. \qquad (263)$$

The idea of computations on quanputers is in finding of the needed (value of the) state (wave function $\psi(t,x)$) from the initial, easy constructible, state ($\psi(0,x)$,) which is superposition of different states, including interesting one, with the same weight. During the computation the weight of the interesting state is growing till the value when we can guess the solution of the problem and then test it, which is much more easier then to find it.
Let us consider the following nonlinear evolution equation

$$iV_t = \Delta V - \frac{1}{2}V^2 + J, \tag{264}$$

extended Lagrangian and Hamiltonian

$$L = \int dx^D (iV_t - \Delta V + \frac{1}{2}V^2 - J)\psi,$$
$$H = \int dx^D (\Delta V - \frac{1}{2}V^2 + J)\psi \tag{265}$$

and corresponding Hamiltonian motion equations

$$iV_t = \Delta V - \frac{1}{2}V^2 + J = \{V, H\},$$
$$i\psi_t = -\Delta\psi + V\psi = \{\psi, H\},$$
$$\{V(t,x), \psi(t,y)\} = \delta^D(x-y) \qquad (266)$$

The solution of the problem is given in the form

$$|T) = U(T)|0),\ \psi(t,x) =< x|t),\ U(T) = Pexp(-i\int_0^T dtH(t)) \qquad (267)$$

Under the programming of the quanputer we understand construction of the potential V, or the corresponding Hamiltonian. For the given potential, we calculate corresponding source J.
The discrete version of the system can be put in the form

$$S_m(n+1) = \Phi_n(S(n)) + J_m(n),$$
$$\Psi_m(n-1) = A_{mk}(S(n))\Psi_k(n),\ A_{mk}(S(n)) = \frac{\partial\Phi_k(S(n))}{\partial S_m(n)} \qquad (268)$$

or, in the regular case, when the matrix A is regular, we obtain explicit form of the corresponding discrete dynamics

$$\begin{aligned} S_m(n+1) &= \Phi_n(S(n)) + J_m(n), \\ \Psi_m(n+1) &= A^{-1}_{mk}(S(n+1))\Psi_k(n), \end{aligned} \tag{269}$$

Now the state vector $S(n)$ and wave vector $\Psi_m(n)$ may correspond not only to the discrete values of the potential $V(n,m) = S_m(n)$, and wave function $\psi(n,m)$

As an example of GRID we take LHC Computing Grid. The LHC Computing Grid (LCG), is an international collaborative project that consists of a grid-based computer network infrastructure incorporating over 170 computing centers in 36 countries. It was designed by CERN to handle the prodigious volume of data produced by Large Hadron Collider (LHC) experiments. The Large Hadron Collider at CERN was designed to prove or disprove the existence of the Higgs boson, an important but elusive piece of knowledge that had been sought by particle physicists for over 40 years. A very powerful particle accelerator was needed, because Higgs bosons might not be seen in lower energy experiments, and because vast numbers of collisions would need to be studied. Such a collider would also produce unprecedented quantities of collision data requiring analysis. Therefore, advanced computing facilities were needed to process the data. A design report was published in 2005. It was announced to be ready for data on 3 October 2008. It incorporates both private fiber optic cable links and existing high-speed portions of the public Internet. At the end of 2010, the Grid consisted of some 200,000 processing cores and 150 petabytes of disk space, distributed across 34 countries.

GRID and Quanputing

The data stream from the detectors provides approximately 300 GByte/s of data, which after filtering for "interesting events", results in a data stream of about 300 MByte/s. The CERN computer center, considered "Tier 0" of the LHC Computing Grid, has a dedicated 10 Gbit/s connection to the counting room. The project was expected to generate 27 TB of raw data per day, plus 10 TB of "event summary data", which represents the output of calculations done by the CPU farm at the CERN data center. This data is sent out from CERN to eleven Tier 1 academic institutions in Europe, Asia, and North America, via dedicated 10 Gbit/s links. This is called the LHC Optical Private Network. More than 150 Tier 2 institutions are connected to the Tier 1 institutions by general-purpose national research and education networks. The data produced by the LHC on all of its distributed computing grid is expected to add up to 10-15 PB of data each year.
Today, without big efforts, we can modify (some) GRID elements in time-invertible form. After development of the quanputer technologies, we can modify (some) GRID elements in quanputer forms.

Social profit of big collaborations

Nowadays there are several big collaborations in science, e.g. LHC. Scientific value of LHC depends on three components, the highest quality of accelerator, highest quality of detectors and distributed data processing. The first two components need good mathematical and physical modeling. Third component and the collaboration as a social structure are not under (anther) the control by scientific methods and corresponding modeling. By definition, scientific collaborations (SC) have a main scientific aim: to obtain answer on the important scientific question(s) and maybe gain extra scientific bonus: new important questions and discoveries. SC is more open information system than e.g. finance or military systems. So, it is possible to describe and optimize SC by scientific methods. Profit from scientific modeling of SC maybe also for other information systems and social structures.

Let us consider the following **discrete dynamics**:

$$S_{n+1} + S_{n-1} = \Phi(S_n), \tag{270}$$

which is obviously a (discrete) time (n) invertible in this implicit form. In the explicit form

$$S_{n+1} = F(S_n, S_{n-1}) = \Phi(S_n) - S_{n-1} \tag{271}$$

it is not obviously time invertible. If we take two step time lattice-make simplest discrete RD step and from one component-scalar S(n) construct two component-spinor $\Psi(n)$, we obtain explicit time invertible dynamics

$$\Psi_{n+1} = \Omega(\Psi_n),\ \Psi_{n+1} = \begin{pmatrix} S_{n+2} \\ S_{n+1} \end{pmatrix},\ \Psi_n = \begin{pmatrix} S_n \\ S_{n-1} \end{pmatrix} \tag{272}$$

This **dynamical mechanism of origin spin** which connects time inversion symmetry and spin was invented when was constructed the theory of quanputers [Makhaldiani, 2011.2].

Computers are physical devices and their behavior is determined by physical laws. The Quantum Computations [Benenti, Casati, Strini, 2004 , Nielsen, Chuang, 2000], Quantum Computing, Quanputing [Makhaldiani, 2007.2], is a new interdisciplinary field of research, which benefits from the contributions of physicists, computer scientists, mathematicians, chemists and engineers. Contemporary digital computer and its logical elements can be considered as a spatial type of discrete dynamical systems [Makhaldiani, 2001]

$$S_n(k+1) = \Phi_n(S(k)), \tag{273}$$

where

$$S_n(k), \quad 1 \leq n \leq N(k), \tag{274}$$

is the state vector of the system at the discrete time step k. Vector S may describe the state and Φ transition rule of some Cellular Automata [Toffoli, Margolus, 1987].The systems of the type (273) appears in applied mathematics as an explicit finite difference scheme approximation of the equations of the physics [Samarskii, Gulin, 1989].

Definition: *We assume that the system (273) is time-reversible if we can define the reverse dynamical system*

$$S_n(k) = \Phi_n^{-1}(S(k+1)). \tag{275}$$

In this case the following matrix

$$M_{nm} = \frac{\partial \Phi_n(S(k))}{\partial S_m(k)}, \tag{276}$$

is regular, i.e. has an inverse. If the matrix is not regular, this is the case, for example, when $N(k+1) \neq N(k)$, we have an irreversible dynamical system (usual digital computers and/or corresponding irreversible gates).

Let us consider an extension of the dynamical system (273) given by the following action function

$$A = \sum_{kn} l_n(k)(S_n(k+1) - \Phi_n(S(k))) \tag{277}$$

and corresponding motion equations

$$\begin{aligned} S_n(k+1) &= \Phi_n(S(k)) = \frac{\partial H}{\partial l_n(k)}, \\ l_n(k-1) &= l_m(k)\frac{\partial \Phi_m(S(k))}{\partial S_n(k)} = l_m(k)M_{mn}(S(k)) = \frac{\partial H}{\partial S_n(k)}, \end{aligned} \tag{278}$$

where

$$H = \sum_{kn} l_n(k)\Phi_n(S(k)), \tag{279}$$

is discrete Hamiltonian. In the regular case, we put the system (278) in an explicit form

$$\begin{aligned} S_n(k+1) &= \Phi_n(S(k)), \\ l_n(k+1) &= l_m(k)M^{-1}_{mn}(S(k+1)). \end{aligned} \tag{280}$$

From this system it is obvious that, when the initial value $l_n(k_0)$ is given, the evolution of the vector $l(k)$ is defined by evolution of the state vector $S(k)$. The equation of motion for $l_n(k)$ - Elenka is linear and has an important property that a linear superpositions of the solutions are also solutions.

Statement: *Any time-reversible dynamical system (e.g. a time-reversible computer) can be extended by corresponding linear dynamical system (quantum - like processor) which is controlled by the dynamical system and has a huge computational power,*

[Makhaldiani, 2001, Makhaldiani, 2002, Makhaldiani, 2007.2, Makhaldiani, 2011.2].

For motion equations (278) in the continual approximation, we have

$$S_n(k+1) = x_n(t_k+\tau) = x_n(t_k) + \dot{x}_n(t_k)\tau + O(\tau^2),$$
$$\dot{x}_n(t_k) = v_n(x(t_k)) + O(\tau), \quad t_k = k\tau,$$
$$v_n(x(t_k)) = (\Phi_n(x(t_k)) - x_n(t_k))/\tau;$$
$$M_{mn}(x(t_k)) = \delta_{mn} + \tau\frac{\partial v_m(x(t_k))}{\partial x_n(t_k)}. \qquad (281)$$

(de)Coherence criterion: *the system is reversible, the linear (quantum, coherent, soul) subsystem exists, when the matrix M is regular,*

$$detM = 1 + \tau\sum_n \frac{\partial v_n}{\partial x_n} + O(\tau^2) \neq 0. \qquad (282)$$

For the Nambu - Poisson dynamical systems (see e.g. [Makhaldiani, 2007])

$$v_n(x) = \varepsilon_{nm_1m_2...m_p}\frac{\partial H_1}{\partial x_{m_1}}\frac{\partial H_2}{\partial x_{m_2}}...\frac{\partial H_p}{\partial x_{m_p}}, \quad p = 1,2,3,...,N-1,$$
$$\sum_n \frac{\partial v_n}{\partial x_n} \equiv divv = 0. \qquad (283)$$

Construction of the reversible discrete dynamical systems

Let me motivate an idea of construction of the reversible dynamical systems by simple example from field theory. There are renormalizable models of scalar field theory of the form (see, e.g. [Makhaldiani, 1980])

$$L = \frac{1}{2}(\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2) - g\varphi^n, \tag{284}$$

with the constraint

$$n = \frac{2d}{d-2} \ , \tag{285}$$

where d is dimension of the space-time and n is degree of nonlinearity. It is interesting that if we define d as a function of n, we find

$$d = \frac{2n}{n-2} \tag{286}$$

the same function !
Thing is that, the constraint can be put in the symmetric implicit form [Makhaldiani, 1980]

$$\frac{1}{n} + \frac{1}{d} = \frac{1}{2} \tag{287}$$

Now it is natural to consider the following symmetric function

$$f(y) + f(x) = c \tag{288}$$

and define its solution

$$y = f^{-1}(c - f(x)). \tag{289}$$

This is the general method, that we will use in the following construction of the reversible dynamical systems. In the simplest case,

$$f(x) = x, \tag{290}$$

we take

$$y = S(k+1), \quad x = S(k-1), \quad c = \tilde{\Phi}(S(k)) \tag{291}$$

and define our reversible dynamical system from the following symmetric, implicit form (see also [Toffoli, Margolus, 1987])

$$S(k+1) + S(k-1) = \tilde{\Phi}(S(k)), \tag{292}$$

explicit form of which is

$$\begin{aligned} S(k+1) &= \Phi(S(k), S(k-1)) \\ &= \tilde{\Phi}(S(k)) - S(k-1). \end{aligned} \tag{293}$$

This dynamical system defines given state vector by previous two state vectors. We have reversible dynamical system on the time lattice with time steps of two units,

$$\begin{aligned} &S(k+2,2) = \Phi(S(k,2)),\\ &S(k+2,2) \equiv (S(k+2), S(k+1)),\\ &S(k,2) \equiv (S(k), S(k-1))). \end{aligned} \tag{294}$$

Starting from a general discrete dynamical system, we obtained reversible dynamical system with internal(spin,bit) degrees of freedom

$$\begin{aligned} S_{ns}(k+2) &\equiv \begin{pmatrix} S_n(k+2) \\ S_n(k+1) \end{pmatrix} = \begin{pmatrix} \Phi_n(\Phi(S(k)) - S(k-1)) - S(k)) \\ \Phi_n(S(k)) - S_n(k-1) \end{pmatrix} \\ &\equiv \Phi_{ns}(S(k)), \quad s = 1,2 \end{aligned} \tag{295}$$

where

$$S(k) \equiv (S_{ns}(k)), \quad S_{n1}(k) \equiv S_n(k), \quad S_{n2}(k) \equiv S_n(k-1) \tag{296}$$

For the extended system we have the following action

$$A = \sum_{kns} l_{ns}(k)(S_{ns}(k+2) - \Phi_{ns}(S(k))) \tag{297}$$

and corresponding motion equations

$$
\begin{aligned}
S_{ns}(k+2) &= \Phi_{ns}(S(k)) = \frac{\partial H}{\partial l_{ns}(k)}, \\
l_{ns}(k-2) &= l_{mt}(k)\frac{\partial \Phi_{mt}(S(k))}{\partial S_{ns}(k)} \\
&= l_{mt}(k)M_{mtns}(S(k)) = \frac{\partial H}{\partial S_{ns}(k)},
\end{aligned} \tag{298}
$$

By construction, we have the following reversible dynamical system

$$
\begin{aligned}
S_{ns}(k+2) &= \Phi_{ns}(S(k)), \\
l_{ns}(k+2) &= l_{mt}(k)M^{-1}_{mtns}(S(k+2)),
\end{aligned} \tag{299}
$$

with classical S_{ns} and quantum l_{ns}(in the external, background S) string bit dynamics.

p-point cluster and higher spin states reversible dynamics, or pit string dynamics

We can also consider p-point generalization of the previous structure,

$$f_p(S(k+p)) + f_{p-1}(S(k+p-1)) + ... + f_1(S(k+1))$$
$$+f_1(S(k-1)) + ... + f_p(S(k-p)) = \tilde{\Phi}(S(k)),$$
$$S(k+p) = \Phi(S(k), S(k+p-1), ..., S(k-p))$$
$$\equiv f_p^{-1}(\tilde{\Phi}(S(k)) - f_{p-1}(S(k+p-1)) - ... - f_p(S(k-p))) \quad (300)$$

and corresponding reversible p-oint cluster dynamical system

$$S(k+p,p) \equiv \Phi(S(k,p)),$$
$$S(k+p,p) \equiv (S(k+p), S(k+p-1), ..., S(k+1)),$$
$$S(k,p) \equiv (S(k), S(k-1), ..., S(k-p+1)), \quad S(k,1) = S(k). \quad (301)$$

So we have general method of construction of the reversible dynamical systems on the time (tame) scale p. The method of linear extension of the reversible dynamical systems (see [Makhaldiani, 2001] and previous section) defines corresponding Quanputers,

$$S_{ns}(k+p) = \Phi_{ns}(S(k)),$$
$$l_{ns}(k+p) = l_{mt}(k) M^{-1}_{mtns}(S(k+p)), \quad (302)$$

p-point cluster and higher spin states reversible dynamics, or pit string dynamics

This case the quantum state function $l_{ns},\ s = 1, 2, ...p$ will describes the state with spin $(p-1)/2$.

Note that, in this formalism for reversible dynamics minimal value of the spin is $1/2$. There is not a place for a scalar dynamics, or the scalar dynamics is not reversible. In the Standard model (SM) of particle physics, [Beringer et al, 2012], all of the fundamental particles, leptons, quarks and gauge bosons have spin. Only scalar particles of the SM are the Higgs bosons. Perhaps the scalar particles are composed systems or quasiparticles like phonon, or Higgs dynamics is not reversible (a mechanism for 'time arrow').

The $NP \stackrel{?}{=} P$ problem will be solved if for some $NP-$ complete problem, e.g. TSP, a polynomial algorithm find; or show that there is not such an algorithm; or show that it is impossible to find definite answer to that question.
TSP means to find minimal length path between N fixed points on a surface, which attends any point ones. We consider a system where N points with quenched positions $x_1, x_2, ..., x_N$ are independently distributed on a finite domain D with a probability density function $p(x)$. In general, the domain D is multidimensional and the points x_n are vectors in the corresponding Euclidean space. Inside the domain D we consider a polymer chain composed of N monomers whose positions are denoted by $y_1, y_2, ..., y_N$. Each monomer y_n is attached to one of the quenched sites x_m and only one monomer can be attached to each site. The state of the polymer is described by a permutation $\sigma \in \Sigma_N$ where Σ_N is the group of permutations of N objecs.

A way to the Solution of the Traveling salesman problem (TSP) with Quanputing

The Hamiltonian for the system is given by

$$H = \sum_{n=1}^{N} V(|y_n - y_{n-1}|) \tag{303}$$

Here V is the interaction between neighboring monomers on the polymer chain. For convenience the chain is taken to be closed, thus we take the periodic boundary condition $x_0 = x_N$. A physical realization of this system is one where the x_n are impurities where the monomers of a polymer loop are pinned. In combinatorial optimization, if one takes $V(x)$ to be the norm, or distance, of the vector x then $H(\sigma)$ is the total distance covered by a path which visits each site x_n exactly once. The problem of finding σ_0 which minimizes $H(\sigma)$ is known as the traveling salesman problem (TSP) [Gutin, Pannen, 2002].

A way to the Solution of the Traveling salesman problem (TSP) with Quanputing

In field theory language to the TSP we correspond the calculation of the following correlator

$$
\begin{aligned}
& G_{2N}(x_1, x_2, ..., x_N) = Z_0^{-1} \int d\varphi(x)\varphi^2(x_1)\varphi^2(x_2)...\varphi^2(x_N)e^{-S(\varphi)} \\
& = \frac{\delta^{2N} F(J)}{\delta J(x_1)^2 ... \delta J(x_N)^2}, \ F(J) = \ln Z(J), \\
& Z(J) = \int d\varphi e^{-\frac{1}{2}\varphi\cdot A\cdot\varphi + J\cdot\varphi} = e^{\frac{1}{2}J\cdot A^{-1}\cdot J}, \ A^{-1}(x, y; m) = e^{-m|x-y|}, \\
& L_{min}(x_1, ..., x_N) = -\frac{d}{dm}\ln G_{2Ns} + O(e^{-am}) \\
& < A^{-1} > \equiv \frac{1}{\Gamma(s)}\int_0^\infty dm m^{s-1}A^{-1}(x, y; m) = \frac{1}{|x-y|^s} \\
& = L_s A^{-1}(x-y; s) \\
& k(d)\Delta_d L_s A^{-1}(x; s) = \delta^d(x) \Rightarrow A(x; s) = k(d)\Delta_d L_s, \\
& s = d-2; \varphi = \varphi(x, m).
\end{aligned}
\tag{304}
$$

If we take relativistic massive scalar field, then $A = \Delta_d + m^2$,

$$A^{-1}(x) \sim |x|^{2-d} e^{-m|x|}, \tag{305}$$

and for $d = 2$, we also have the needed behaviour. Note that G_{2N} is symmetric with respect to its arguments and contains any paths including minimal length one.

Let us consider the integer derivatives of the monomials

$$\frac{d^n}{dx^n}x^m = m(m-1)...(m-(n-1))x^{m-n}, \quad n \le m, \\ = \frac{\Gamma(m+1)}{\Gamma(m+1-n)}x^{m-n}. \tag{306}$$

L.Euler (1707 - 1783) invented the following definition of the fractal derivatives,

$$\frac{d^\alpha}{dx^\alpha}x^\beta = \frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\alpha)}x^{\beta-\alpha}. \tag{307}$$

J.Liouville (1809-1882) takes exponents as a base functions,

$$\frac{d^\alpha}{dx^\alpha}e^{ax} = a^\alpha e^{ax}. \tag{308}$$

The following Cauchy formula

$$I^n_{0,x}f = \int_0^x dx_n \int_0^{x_{n-1}} dx_{n-2} \ldots \int_0^{x_2} dx_1 f(x_1) = \frac{1}{\Gamma(n)} \int_0^x dy (x-y)^{n-1} f(y) \quad (3$$

permits analytic extension from integer n to complex α,

$$I^\alpha_{0,x}f = \frac{1}{\Gamma(\alpha)} \int_0^x dy (x-y)^{\alpha-1} f(y) \quad (310)$$

J.H. Holmgren invented (in 1863) the following integral transformation,

$$D_{c,x}^{-\alpha} f = \frac{1}{\Gamma(\alpha)} \int_c^x |x-t|^{\alpha-1} f(t) dt. \tag{311}$$

It is easy to show that

$$\begin{aligned} D_{c,x}^{-\alpha} x^m &= \frac{\Gamma(m+1)}{\Gamma(m+1+\alpha)} (x^{m+\alpha} - c^{m+\alpha}), \\ D_{c,x}^{-\alpha} e^{ax} &= a^{-\alpha} (e^{ax} - e^{ac}), \end{aligned} \tag{312}$$

so, $c = 0$, when $m + \alpha \geq 0$, in Holmgren's definition of the fractal calculus, corresponds to the Euler's definition, and $c = -\infty$, when $a > 0$, corresponds to the Liouville's definition.
Holmgren's definition of the fractal calculus reduce to the Euler's definition for finite c, and to the Liouvill's definition for $c = \infty$,

$$\begin{aligned} &D_{c,x}^{-\alpha} f = D_{0,x}^{-\alpha} f - D_{0,c}^{-\alpha} f, \\ &D_{\infty,x}^{-\alpha} f = D_{-\infty,x}^{-\alpha} f - D_{-\infty,\infty}^{-\alpha} f, \\ &D_{-\infty,x}^{-\alpha} f = D_{0,x}^{\alpha} f - D_{0,-\infty}^{-\alpha} f. \end{aligned} \tag{313}$$

We considered the following modification of the $c = 0$ case [Makhaldiani, 2003],

$$D_{0,x}^{-\alpha} f = \frac{|x|^\alpha}{\Gamma(\alpha)} \int\limits_0^1 |1-t|^{\alpha-1} f(xt) dt, = \frac{|x|^\alpha}{\Gamma(\alpha)} B(\alpha,\ \partial x) f(x)$$
$$= |x|^\alpha \frac{\Gamma(\partial x)}{\Gamma(\alpha+\partial x)} f(x),\ f(xt) = t^{x\frac{d}{dx}} f(x). \quad (314)$$

As an example, consider Euler B-function,

$$B(\alpha,\beta) = \int_0^1 dx |1-x|^{\alpha-1} |x|^{\beta-1} = \Gamma(\alpha)\Gamma(\beta) D_{01}^{-\alpha} D_{0x}^{1-\beta} 1 = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \quad (315)$$

Negative binomial distribution (NBD) is defined as

$$P(n) = \frac{\Gamma(n+r)}{n!\Gamma(r)} p^n (1-p)^r, \; \sum_{n \geq 0} P(n) = 1, \tag{316}$$

The Bose-Einstein distribution is a special case of NBD with $r = 1$.
NBD provides a very good parametrization for multiplicity distributions in e^+e^- annihilation; in deep inelastic lepton scattering; in proton-proton collisions; in proton-nucleus scattering. Hadronic collisions at high energies (LHC) lead to charged multiplicity distributions whose shapes are well fitted by a single NBD in fixed intervals of central (pseudo)rapidity η.

A Bose-Einstein, or geometrical, distribution is a thermal distribution for single state systems. An useful property of the negative binomial distribution with parameters

$$< n >,\ k$$

is that it is (also) the distribution of a sum of k independent random variables drawn from a Bose-Einstein distribution with mean $< n > /k$,

$$P_n = \frac{1}{< n > +1}(\frac{< n >}{< n > +1})^n$$
$$= (e^{\beta\hbar\omega/2} - e^{-\beta\hbar\omega/2})e^{-\beta\hbar\omega(n+1/2)},\ T = \frac{\hbar\omega}{\ln\frac{<n>+1}{<n>}}$$
$$\sum_{n\geq 0} P_n = 1,\ \sum nP_n =< n >= \frac{1}{e^{\beta\hbar\omega - 1}},\ T \simeq \hbar\omega < n >,\ < n >\gg 1,$$
$$P(x) = \sum_n x^n P_n = (1+ < n > (1-x))^{-1}. \quad (317)$$

Indeed, for

$$n = n_1 + n_2 + ... + n_k, \tag{318}$$

with n_i independent of each other, the probability distribution of n is

$$P_n = \sum_{n_1,...,n_k} \delta(n - \sum n_i) p_{n_1} ... p_{n_k},$$
$$P(x) = \sum_n x^n P_n = p(x)^k \tag{319}$$

This has a consequence that an incoherent superposition of N emitters that have a negative binomial distribution with parameters $k, < n >$ produces a negative binomial distribution with parameters $Nk, N < n >$.
So, for the GF of NBD we have (N=2)

$$F(k, < n >) F(k, < n >) = F(2k, 2 < n >) \tag{320}$$

And more general formula (N=m) is

$$F(k, < n >)^m = F(mk, m < n >) \tag{321}$$

We can put this equation in the closed nonlocal form

$$Q_q F = F^q, \tag{322}$$

where

$$Q_q = q^D, \quad D = \frac{kd}{dk} + \frac{< n > d}{d < n >} = \frac{x_1 d}{dx_1} + \frac{x_2 d}{dx_2} \tag{323}$$

Note that temperature defined in (317) gives an estimation of the Glukvar temperature when it radiates hadrons. If we take $\hbar\omega = 100MeV$, to $T \simeq T_c \simeq 200MeV$ corresponds $< n > \simeq 1.5$ If we take $\hbar\omega = 10MeV$, to $T \simeq T_c \simeq 200MeV$ corresponds $< n > \simeq 20$. A singular behavior of $< n >$ may indicate corresponding phase transition and temperature. At that point we estimate characteristic quantum $\hbar\omega$.
We see that universality of NBD in hadron-production is similar to the universality of black body radiation.

The Riemann zeta function $\zeta(s)$ is defined for complex $s = \sigma + it$ and $\sigma > 1$ by the expansion

$$\begin{aligned}\zeta(s) &= \sum_{n\geq 1} n^{-s},\ Re\ s > 1,\\ &= \delta_x^{-s}\frac{x}{1-x}|_{x\to 1} = \frac{1}{\Gamma(s)}\int_0^\infty t^{s-1}e^{-\delta_x t}\frac{x}{1-x}|_{x\to 1}\\ &= \frac{1}{\Gamma(s)}\int_0^\infty t^{s-1}e^{t\partial_\tau}\frac{1}{e^\tau - 1}|_{\tau\to 0}\\ &= \frac{1}{\Gamma(s)}\int_0^\infty \frac{t^{s-1}dt}{e^t - 1},\ x = e^{-\tau}\end{aligned} \quad (324)$$

All complex zeros, $s = \alpha + i\beta$, of $\zeta(\sigma + it)$ function lie in the critical stripe $0 < \sigma < 1$, symmetrically with respect to the real axe and critical line $\sigma = 1/2$. So it is enough to investigate zeros with $\alpha \leq 1/2$ and $\beta > 0$. These zeros are of three type, with small, intermediate and big ordinates.

The Riemann hypothesis states that the (non-trivial) complex zeros of $\zeta(s)$ lie on the critical line $\sigma = 1/2$.
At the beginning of the XX century Polya and Hilbert made a conjecture that the imaginary part of the Riemann zeros could be the oscillation frequencies of a physical system (ζ - (mem)brane).
After the advent of Quantum Mechanics, the Polya-Hilbert conjecture was formulated as the existence of a self-adjoint operator whose spectrum contains the imaginary part of the Riemann zeros.
The Riemann hypothesis (RH) is a central problem in Pure Mathematics due to its connection with Number theory and other branches of Mathematics and Physics.

The functional equation is

$$\zeta(1-s) = \frac{2\Gamma(s)}{(2\pi)^s}\cos(\frac{\pi s}{2})\zeta(s) \tag{325}$$

From this equation we see the real (trivial) zeros of zeta function:

$$\zeta(-2n) = 0,\ n = 1, 2, ... \tag{326}$$

Also, at s=1, zeta has pole with reside 1.
From Field theory and statistical physics point of view, the functional equation (325) is duality relation, with self dual (or critical) line in the complex plane, at $s = 1/2 + i\beta$,

$$\zeta(\frac{1}{2} - i\beta) = \frac{2\Gamma(s)}{(2\pi)^s}\cos(\frac{\pi s}{2})\zeta(\frac{1}{2} + i\beta), \tag{327}$$

we see that complex zeros lie symmetrically with respect to the real axe.
On the critical line, (nontrivial) zeros of zeta corresponds to the infinite value of the free energy,

$$F = -T\ln\zeta. \tag{328}$$

At the point with $\beta = 14.134725...$ is located the first zero. In the interval $10 < \beta < 100$, zeta has 29 zeros. The first few million zeros have been computed and all lie on the critical line. It has been proved that uncountably many zeros lie on critical line.
The first relation of zeta function with prime numbers is given by the following formula,

$$\zeta(s) = \prod_p (1 - p^{-s})^{-1}, \; Re\ s > 1. \tag{329}$$

Another formula, which can be used on critical line, is

$$\begin{aligned} \zeta(s) &= (1 - 2^{1-s})^{-1} \sum_{n \geq 1} (-1)^{n+1} n^{-s} \\ &= \frac{1}{1 - 2^{1-s}} \frac{1}{\Gamma(s)} \int_0^\infty \frac{t^{s-1} dt}{e^t + 1}, \; Re\ s > 0 \end{aligned} \tag{330}$$

Let us consider the values $q = n, n = 1, 2, 3, ...$ and take sum of the corresponding equations (322), we find

$$\zeta(-D)F = \frac{F}{1-F} \tag{331}$$

In the case of the NBD we know the solutions of this equation.
Now we invent a Hamiltonian H with spectrum corresponding to the set of nontrivial zeros of the zeta function, in correspondence with Riemann hypothesis,

$$\begin{aligned} &-D_n = \frac{n}{2} + iH_n,\ H_n = i(\frac{n}{2} + D_n), \\ &D_n = x_1\partial_1 + x_2\partial_2 + ... + x_n\partial_n,\ H_n^+ = H_n = \sum_{m=1}^{n} H_1(x_m), \\ &H_1 = i(\frac{1}{2} + x\partial_x) = -\frac{1}{2}(x\hat{p} + \hat{p}x),\ \hat{p} = -i\partial_x \end{aligned} \tag{332}$$

The Hamiltonian $H = H_n$ is hermitian, its spectrum is real. The case $n = 1$ corresponds to the Riemann hypothesis.

The case $n = 2$, corresponds to NBD,

$$\zeta(1 + iH_2)F = \frac{F}{1 - F}, \ \zeta(1 + iH_2)|_F = \frac{1}{1 - F},$$
$$F(x_1, x_2; h) = (1 + \frac{x_1}{x_2}(1 - h))^{-x_2} \tag{333}$$

Let us scale $x_2 \to \lambda x_2$ and take $\lambda \to \infty$ in (333), we obtain

$$\zeta(\frac{1}{2} + iH_1(x))e^{-(1-h)x} = \frac{1}{e^{(1-h)x} - 1},$$
$$\frac{1}{\zeta(\frac{1}{2} + iH(x))}\frac{1}{e^{\varepsilon x} - 1} = e^{-\varepsilon x},$$
$$H(x) = i(\frac{1}{2} + x\partial_x) = -\frac{1}{2}(x\hat{p} + \hat{p}x), \ H^+ = H, \varepsilon = 1 - h. \tag{334}$$

Let us take an eigenvector $|n>$ with eigenvalue E_n of H, than

$$< n|\zeta(\frac{1}{2} + iH(x))e^{-(1-h)x} >= \zeta(\frac{1}{2} + iE_n(x)) < n|e^{-(1-h)x} >$$
$$=< n|\frac{1}{e^{(1-h)x} - 1} > \tag{335}$$

For zeros of Zeta function, E_n, the eigenfunctions fulfils the following conditions

$$< n|\frac{1}{e^{(1-h)x} - 1} >= 0, \ < n|e^{-(1-h)x} >\neq 0. \tag{336}$$

For eigenvalues of H, we have

$$H|n>=E_n|n>,\ H=i(\frac{1}{2}+x\partial_x),\ |n>\sim x^{s_n},\ s_n=-\frac{1}{2}-iE_n,$$
$$<n|\frac{1}{e^{(1-h)x}-1}>\sim\zeta(\frac{1}{2}+iE_n),$$
$$<n|e^{-(1-h)x}>\sim\Gamma(\frac{1}{2}+iE_n). \tag{337}$$

We consider a novel method to generate a polynomial expression for each of the Euler sums,

$$E_k = \sum_{n=1}^{N} n^k,\ k \in Z^+(k = 0, 1, 2, ...) \tag{338}$$

One of the way of calculation of the sum

$$E_k(N) = \frac{N^{k+1}}{k+1} + P_k(N),\ P_k = x_k N^k + x_{k-1} N^{k-1} + ... + x_0 \tag{339}$$

we show by explicit calculation of E_2.
For particular values $N = 1,\ 2$ and 3, we have

$$\begin{aligned} x_2 + x_1 + x_0 = 1 - 1/3 = 2/3, \\ 4x_2 + 2x_1 + x_0 = 5 - 8/3 = 7/3, \\ 9x_2 + 3x_1 + x_0 = 14 - 27/3 = 5 \end{aligned} \tag{340}$$

Subtracting from the second equation the first and from third the second, we obtain

$$3x_2 + x_1 = 5/3,$$
$$5x_2 + x_1 = 8/3 \qquad (341)$$

than we have

$$2x_2 = 1 \Rightarrow x_2 = 1/2 \Downarrow$$
$$x_1 = 5/3 - 3x_2 = 1/6 \Rightarrow x_0 = 2/3 - x_1 - x_2 = 2/3 - 1/6 - 1/2 = 0, \Downarrow$$
$$E_3(N) = N^3/3 + N^2/2 + N/6$$
$$= N(N+1)(2N+1)/6 = N(N+1/2)(N+1)/3 \qquad (342)$$

Note that, the right hand side have a sense also for $N \leq 0$ and has zeros at $N = 0, -1/2, -2$.

For general case $E_k(N)$ we have

$$x_k(N) = detV_l(N)/detW_k(N),$$

$$detW_k(N) = det\begin{pmatrix} 1 & N_1 & . & N_1^k \\ 1 & N_2 & . & N_2^k \\ . & . & . & . \\ 1 & N_{k+1} & . & N_{k+1}^k \end{pmatrix},$$

$$detV_l(N) = det\begin{pmatrix} 1 & N_1 & . & N_1^k \\ 1 & N_2 & . & N_2^k \\ \bar{E}_l(N_1) & \bar{E}_l(N_2) & . & \bar{E}_l(N_{k+1}) \\ 1 & N_{k+1} & . & N_{k+1}^k \end{pmatrix},\ X = \begin{pmatrix} x_0 \\ x_1 \\ . \\ x_k \end{pmatrix}$$

$$\bar{E}_k(N_l) = E_k(N_l) - \frac{N_l^{k+1}}{k+1},\ E = \begin{pmatrix} \bar{E}_k(N_1) \\ \bar{E}_k(N_2) \\ . \\ \bar{E}_k(N_{k+1}) \end{pmatrix},$$

$$WX = E,\ X = W^{-1}E \tag{343}$$

As a numbers N_n we can take any different integers, but the simplest choice is: $N_{n+1} = N_n + 1,\ N_1 = 1$, as in considered explicit calculation for E_2. In this case, $E_k(N+1) = E_k(N) + (N+1)^k$.

We propose the following compact form for E_k

$$E_k(N) = \frac{d^k}{dx^k} P(x,N)|_{x\Rightarrow 0} \equiv D^k P = P^{(k)}(0,N),$$
$$P(x,N) = \sum_{n=1}^{N} e^{nx} = \frac{e^{(N+1)x} - e^x}{e^x - 1} \tag{344}$$

We take also the following slightly simpler form of $P(x,N)$, for $k = 1, 2, 3, \ldots$

$$P(x,N) = \sum_{n=0}^{N} e^{nx} = \frac{e^{(N+1)x} - 1}{e^x - 1} \tag{345}$$

As an example, let us calculate $E_1(N)$,

$$E_1(N) = \frac{(N+1)e^{(N+1)x}}{e^x - 1} - \frac{(e^{(N+1)x} - 1)e^x}{(e^x - 1)^2}$$
$$= \frac{(N+1)e^{(N+1)x}(e^x - 1) - (e^{(N+1)x} - 1)e^x}{(e^x - 1)^2} \Downarrow$$
$$\frac{(N+1)(1+(N+1)x...)(x+\frac{x^2}{2}...) - ((N+1)x + \frac{(N+1)^2x^2}{2}...)(1+x...)}{(x+...)^2}$$
$$= (N+1)^2 + (N+1)/2 - (N+1) - (N+1)^2/2 = N(N+1)/2 \quad (346)$$

We can present the derivative operator in the complex integral form

$$f^{(k)}(0) = \frac{k!}{2\pi i} \oint \frac{dz f(z)}{z^{k+1}} \tag{347}$$

In this form the calculation gives

$$\begin{aligned} S(1,N) &= \frac{1}{2\pi i} \oint \frac{dz}{z^2} \frac{(N+1)z + (N+1)^2 z^2/2}{z + z^2/2} \\ &= \frac{1}{2\pi i} \oint \frac{dz}{z^2} \frac{(N+1) + (N+1)^2 z/2}{1 + z/2} \\ &= \frac{1}{2\pi i} \oint \frac{dz}{z} \frac{(N+1)}{z}(1 - z/2) + (N+1)^2/2 \\ &= N(N+1)/2 \end{aligned} \tag{348}$$

By this example we see that the second form of calculation is easier.

The theory of analytic functions of a complex variable occupies a central place in analysis. Riemann considered the unique continuation property to be the most characteristic feature of analytic functions. GPF do possess the unique continuation property, and each class of GPF has almost as much structure as the class of analytic functions. In particular, the operations of complex differentiation and complex integration have meaningful counterparts in the theory of GPF and this theory generalizes not only the Cauchy-Riemann approach to function theory but also that of Weierstrass. Such functions were considered by Picard and by Beltrami, but the first significant result was obtained by Carleman in 1933, and a systematic theory was formulated by Lipman Bers [Bers 1952] and Ilia Vekua (1907-1977), [Vekua 1962]. For more resent results see [Giorgadze 2011].

Analytic function $f = u + iv$ satisfy the partial differential equation $\partial_{\overline{z}} f = 0$, where complex differential operators are defined as

$$\partial_{\overline{z}} = \frac{\partial}{\partial \overline{z}} := \frac{1}{2}(\partial_x + i\partial_y),\ \partial_z = \frac{\partial}{\partial z} := \frac{1}{2}(\partial_x - i\partial_y) \tag{349}$$

Generalized analytic functions $f = u + iv$ satisfy the following generalized Cauchy-Riemann equation [Vekua 1962]

$$\partial_{\overline{z}} f = Af + B\bar{f} + J,\ A = A_0 + iA_1,\ B = B_0 + iB_1,\ J = j_1 + ij_2 \tag{350}$$

or in terms of the real u and imaginary v components canonical form of the elliptic systems of partial differential equations of the first order

$$\begin{aligned} u_x - v_y &= au + bv + j_1,\ a = A_0 + B_0,\ b = -A_1 + B_1, \\ u_y + v_x &= cu + dv + j_2,\ c = A_1 + B_1,\ d = A_0 - B_0, \end{aligned} \tag{351}$$

or in matrix form

$$D\psi = E\psi + J,\ D = \begin{pmatrix} \partial_x & -\partial_y \\ \partial_y & \partial_x \end{pmatrix} = \partial_x - i\sigma_2\partial_y,$$
$$E = \begin{pmatrix} a & b \\ c & d \end{pmatrix},\ \psi = \begin{pmatrix} u \\ v \end{pmatrix},\ J = \begin{pmatrix} j_1 \\ j_2 \end{pmatrix}. \tag{352}$$

In the classical sense by a solution of the system of equations (360) we understand a pair of real continuously differentiable functions $u(x,y),\ v(x,y)$ of the real variables x and y which satisfy this system everywhere in a domain G. Such solutions, however, exist only for a comparatively narrow class of equations.
The formal solution of the canonical equation for GPF (360) is

$$\psi = \psi_0 + RJ,\ R = (D - E)^{-1},\ (D - E)\psi_0 = 0. \tag{353}$$

Let us introduce a 'fundamental' length parameter $l = h^{-1}$, $\mathbf{x}_n = lx_n,\ n = 1,2,\ x_1 = x,\ x_2 = y,\ x_n$ is dimensionless. Then, for the resolvent R, we will have the long-wave and short-wave expansions,

Formal, Long-Wave and Short-Wave Solutions of the Canonical Equations for GPF

$$
\begin{aligned}
R_{LW} &:= (lD - E)^{-1} = -E^{-1} \sum_{n \geq 0} l^n (DE^{-1})^n, \\
R_{ShW} &:= (lD - E)^{-1} = hD^{-1} \sum_{n \geq 0} h^n (ED^{-1})^n, \\
E^{-1} &= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} / \Delta_E, \ \Delta_E = ad - bc, \\
D^{-1} &= \Delta_D^{-1} \begin{pmatrix} \partial_x & \partial_y \\ -\partial_y & \partial_x \end{pmatrix}, \ \Delta_D = \partial_x^2 + \partial_y^2
\end{aligned}
\tag{354}
$$

There is a fairly complete theory of generalized analytic functions; it represents an essential extension of the classical theory preserving at the same time its principal features [Vekua 1962].

From the previous consideration it is natural to make the following four dimensional extention

$$
\begin{aligned}
& D=\partial_x-i\sigma_2\partial_y \Rightarrow D_4=\partial_t-i\sigma_n\nabla_n \\
& =-i(\partial_\tau+\sigma_n\nabla_n)=-iD_{13},\ t=i\tau \\
& D^{-1}=\Delta_4^{-1}(\partial_t+i\sigma_n\nabla_n),\ \Delta_4=\partial_t^2+\Delta_3,\ \Delta_3=\partial_x^2+\partial_y^2+\partial_z^2, \\
& D_{13}^{-1}=\Delta_{13}^{-1}(\partial_\tau-\sigma_n\nabla_n),\ \Delta_{13}=\partial_\tau^2-\Delta_3, \\
& \sigma_n\sigma_m=\delta_{nm}+i\varepsilon_{nmk}\sigma_k
\end{aligned} \tag{355}
$$

In matrix form

$$D_4 = \begin{pmatrix} \partial_t - i\partial_z & -i\partial_x - \partial_y \\ -i\partial_x + \partial_y & \partial_t + i\partial_z \end{pmatrix}$$
$$= 2\begin{pmatrix} \partial_\zeta & -\partial_{\bar\eta} \\ \partial_\eta & \partial_{\bar\zeta} \end{pmatrix},\ \zeta = t + iz,\ \eta = y + ix$$
$$D_{13} = \begin{pmatrix} \partial_\tau + \partial_z & \partial_x - i\partial_y \\ \partial_x + i\partial_y & \partial_\tau - \partial_z \end{pmatrix}$$
$$= 2\begin{pmatrix} \partial_- & \partial_\varsigma \\ \partial_{\bar\varsigma} & \partial_+ \end{pmatrix},\ \pm = \tau \pm z,\ \varsigma = x + iy,$$
$$\Delta_4 = 4(\partial^2_{\zeta\bar\zeta} + \partial^2_{\eta\bar\eta}),\ \Delta_{13} = 4(\partial^2_{-+} - \partial^2_{\varsigma\bar\varsigma}) \qquad (356)$$

In the Minkowski spacetime for analytic functions in matrix form $D_{13}\psi = 0$ or in components

$$\partial_- u + \partial_\varsigma v = 0,\ \partial_+ v + \partial_{\bar\varsigma} u = 0 \Downarrow$$
$$(\partial^2_{-+} - \partial^2_{\varsigma\bar\varsigma})u_n = 0,\ u_1 = u,\ u_2 = v \qquad (357)$$

In euclidian space $D_4\psi = 0$,

$$\begin{array}{l} \partial_\zeta u - \partial_{\bar\eta} v = 0,\ \partial_{\bar\zeta} v + \partial_\eta u = 0 \Downarrow \\ (\partial^2_{\zeta\bar\zeta} + \partial^2_{\eta\bar\eta}) u_n = 0,\ u_1 = u,\ u_2 = v \end{array} \quad (358)$$

So, u_n are harmonic or wave functions.

$$\begin{array}{l} \partial_- u + \partial_\varsigma v = au + bv + j_1, \\ \partial_+ v + \partial_{\bar\varsigma} u = cu + dv + j_2, \end{array} \quad (359)$$

or in matrix form

$$\begin{array}{l} D\psi = E\psi + J,\ D = \left(\begin{array}{cc} \partial_- & \partial_\varsigma \\ \partial_{\bar\varsigma} & \partial_+ \end{array} \right) = \partial_\tau + \sigma_n \nabla_n, \\ E = \left(\begin{array}{cc} a & b \\ c & d \end{array} \right),\ \psi = \left(\begin{array}{c} u \\ v \end{array} \right),\ J = \left(\begin{array}{c} j_1 \\ j_2 \end{array} \right). \end{array} \quad (360)$$

It is curious to imagine that Hamilton knew about neutrinos equation a hundred years before Weyl :) In the extended version, to the $E-$terms corresponds neutrinos mass.

In SM we have three type of lefthanded neutrinos $\nu_n,\ n=e,\mu,\tau$ which interacts weakly with corresponding leptons, lepton number is conserved. Corresponding part of the SM lagrangian is

$$\bar{l}_n\gamma^\mu\nu_n'W_\mu+\bar{\nu}'(\gamma\partial-M)\nu',\ \bar{\nu}'=(\bar{\nu}_e'\bar{\nu}_\mu'\bar{\nu}_\tau') \tag{361}$$

where M is a 3×3 matrix in flavor space. If the matrix is nondiagonal, we diagonalize it by an unitary transformation:

$$\begin{aligned}&\bar{\nu}'(\gamma\partial-M)\nu'=\bar{\nu}_n(\gamma\partial-m_n)\nu_n,\ U^{-1}MU=diad(m_1,m_2,m_3),\\&\nu_n=U^{-1}_{nk}\nu_k',\ \bar{l}_n\gamma^\mu\nu_n'W_\mu=\bar{l}_n\gamma^\mu U_{nk}\nu_kW_\mu\end{aligned} \tag{362}$$

If the Koide formula works for lepton masses, may be it works also for neutrino masses. If the lepton masses are an unique solution of the Koide formula, than neutrino masses are proportional to the lepton masses: $m_n=qM_n,\ n=e,\mu,\tau.$

Koide's mass formula is an empirical relation among the charged lepton masses which holds with a striking precision,

$$Q = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} \simeq \frac{2}{3} \tag{363}$$

were electron mass is $m_e = 0.510998910(13) MeV/c^2$, muon mass is $m_\mu = 105.658367(4) MeV/c^2$, tauon mass is $m_t = 1776.84(17) MeV/c^2$. The digits in parentheses are the uncertainties in the last figures. This gives $Q = 0.666659(10)$. Not only is this result odd in that three apparently random numbers should give a simple fraction, but also that Q is exactly halfway between the two extremes of $1/3$ (should the three masses be equal) and 1 (should one mass dominate). Similar matches have been found for quarks depending on running masses, and for triplets of quarks not of the same flavour. The Koide relation is exact (within experimental error) for the pole masses, which are low-energy quantities defined at different energy scales. It is RD integral of motion.

Under the generalized KF we mean the following formulas [Das, Makhaldiani 2016]

$$Q_n = \frac{m_1 + m_2 + ... + m_n}{(\sqrt{m_1} + \sqrt{m_2} + ... + \sqrt{m_n})^2} = q_n,\ q_3 \simeq \frac{2}{3} \tag{364}$$

In the solvable model of RD for coupling constants and masses, $m_k/\alpha_k = I = const$. So, the KF is equivalent to the corresponding relation for charges

$$Q_n = \frac{e_1^2 + e_2^2 + ... + e_n^2}{(e_1 + e_2 + ... + e_n)^2} = q_n,\ e_k = \sqrt{4\pi\alpha_k} \tag{365}$$

In the $n-$dimensional euklidian space a vector e with coordinates $(e_1, e_2, ..., e_n)$ has the length squire $e_1^2 + ... + e_n^2$. If all components are oriented in one dimension, the length is $(e_1 + ... + e_n)^2$.

If we take a symmetric vector $(1, 1, ..., 1)$ with length $\sqrt{n}$ than $\cos\varphi$ of the angler φ between charge vector $e = (e_1, e_2, ..., e_n)$ and the symmetric vector will be

$$\cos\varphi_n = \frac{e_1 + ... + e_n}{\sqrt{n}\sqrt{e_1^2 + ... + e_n^2}} = \frac{1}{\sqrt{nq_n}},$$
$$\cos\varphi_1 = \cos\varphi_2 = 1, \ \cos\varphi_3 = \frac{1}{\sqrt{2}} \quad (366)$$

If the sequence q_n is bounded, than $\cos\varphi_n \to 0$

RG can be defined as a solution of the following equation

$$x^2 + x - 1 = 0,$$
$$x = \frac{\sqrt{5}-1}{2} \qquad (367)$$

The equation and the solution can be put in the following form

$$x = \frac{1}{1+x} = \frac{1}{1+\frac{1}{1+x}} = ... = \frac{1}{1+\frac{1}{1+...}} = \frac{\sqrt{5}-1}{2} = 0.618034 \qquad (368)$$

We can consider corresponding recurrent relarion

$$x_{n+1} = \frac{1}{1+x_n},\ x_1 = 1,\ x_2 = 1/2,\ x_3 = 2/3,$$
$$x_4 = 3/5 = 0.6,\ x_5 = 5/8 = 0.625...$$
$$\lim_{n\to\infty} x_n = x = \frac{\sqrt{5}-1}{2} \qquad (369)$$

So, the value of the Koide ratio corresponds to $x_3 = 2/3$. We might consider as an exact value of the ratio the GR. The recurrent relation is a discrete RD equation with the fixed point solution the GR.

The first higgs particle has a mass $m_h = 125GeV$, the second one has $m_H = 750GeV$. The two term Koide formula

$$Q_2 = \frac{m_h + m_H}{(\sqrt{m_h} + \sqrt{m_H})^2} = \frac{7}{(1+\sqrt{6})} = 0.588286 \lesssim \frac{2}{3} = 0.666667 \quad (370)$$

We suppose that there is a third higgs particle with mass $m_x = x$, which makes the Koide formula exact and find the mass:

$$\begin{aligned}
&Q_3 = \frac{m_h + m_H + m_x}{(\sqrt{m_h} + \sqrt{m_H} + \sqrt{m_x})^2} = \frac{2}{3}, \\
&3(7 + x) = 2((a + \sqrt{x})^2) = 2a^2 + 4a\sqrt{x} + 2x \Downarrow \\
&x - 4a\sqrt{x} - 2a^2 + 21 \Rightarrow \sqrt{x} = 2a \pm \sqrt{6a^2 - 21} = 13.9978 = 14, \\
&a = 1 + \sqrt{6} = 3.44949,\ 2a = 6.89898,\ \sqrt{6a^2 - 21} = 7.09886 \\
&m_x = 196 m_h = 196 \times 125 = 24.5TeV. \qquad (371)
\end{aligned}$$

If we take the second, negative, root $\sqrt{x}$,

$$\begin{aligned}
&\sqrt{x} = 2a - \sqrt{6a^2 - 21} = -0.199885,\ x = 0.039954, \\
&m_x = 0.039954 m_h = 0.039954 \times 125 = 4.99425 = 5GeV. \qquad (372)
\end{aligned}$$

Let us take Q_2 and find the second mass (charge)

$$\frac{1+x}{(1+\sqrt{x})^2} = \frac{2}{3} \Rightarrow x - 4\sqrt{x} + 1 = 0 \Rightarrow \sqrt{x} = 2 \pm \sqrt{3},$$
$$x_1 = (2+\sqrt{3})^2 = 7 + 4\sqrt{3} = 13.9282 \simeq 14, \;\; x_2 = (2-\sqrt{3})^2 = 7 - 4\sqrt{3}$$
$$m_1 = 125 \times 13.9282 = 1741.03 = 1741 GeV,$$
$$m_2 = 125 \times 0.0717968 = 8.9746 \simeq 9 GeV$$

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