Goldstino superfields in supergravity

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Based on:

- I. Bandos, M. Heller, SMK, L. Martucci & D. Sorokin, "The Goldstino brane, the constrained superfields and matter in $\mathcal{N} = 1$ supergravity," JHEP **1611**, 109 (2016) [arXiv:1608.05908 [hep-th]].
- SMK, I. McArthur & G. Tartaglino-Mazzucchelli, "Goldstino superfields in N=2 supergravity," JHEP **1705**, 061 (2017) [arXiv:1702.02423 [hep-th]].
- E. Buchbinder & SMK, "Three-form multiplet and supersymmetry breaking," arXiv:1705.07700 [hep-th].
- SMK and G. Tartaglino-Mazzucchelli, "New nilpotent $\mathcal{N} = 2$ superfields," arXiv:1707.07390 [hep-th].

General comments

- Cosmological constant is negative (e.g., old minimal SUGRA) or vanishing (e.g., new minimal SUGRA) in unbroken supergravity without scalars.
- According to the latest cosmological data, we live in an expanding universe with a small but positive cosmological constant.
- It is desirable to develop theoretical mechanisms to explain positive cosmological constant.
- It has recently been recognised that such a mechanism is provided by spontaneously broken supergravity.

General comments

Recent interest in $\mathcal{N} = 1$ off-shell supergravity coupled to Goldstino superfields (2015–2017)

- Goldstino superfields contain the Volkov-Akulov Goldstone fermion (Goldstino) and, sometimes, also auxiliary field(s).
- Coupling a Goldstino superfield to off-shell supergravity leads to spontaneously broken local supersymmetry without bringing in new degrees of freedom, except for making the gravitino massive.
  super-Higgs effect — D. Volkov & V. Soroka (1973)
  S. Deser & B. Zumino (1977)
- Absence of scalars is attractive for phenomenological applications.
- Positive contribution to the cosmological constant is generated.

Nilpotent chiral Goldstino superfield

R. Casalbuoni, S. De Curtis, D. Dominici, F. Feruglio & R. Gatto (1989)
Z. Komargodski & N. Seiberg (2009)

X is chiral, $\bar{D}_{\dot{\alpha}} X = 0$, and obeys the nilpotency constraint

$$X^2 = 0 \quad \Longrightarrow \quad X = -\frac{D^\alpha X D_\alpha X}{D^2 X}$$

In addition it is required that $D^2 X$ is nowhere vanishing, $D^2 X \neq 0$.
Dynamics of this supermultiplet is described by the action

$$S_X = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, \bar{X} X - \left\{ f \int \mathrm{d}^4x \mathrm{d}^2\theta \, X + \mathrm{c.c.} \right\}$$

Component fields of X:

$$X| = \varphi = \frac{1}{2F}\psi^2 \ , \qquad D_\alpha X| = \sqrt{2}\psi_\alpha \, , \qquad -\frac{1}{4} D^2 X| = F$$

Goldstino ψ_α and complex auxiliary field F are independent component fields.

Nilpotent chiral Goldstino superfield

Component action

$$S[\psi, F] = \int \mathrm{d}^4x \Big[-\partial_a\Big(\frac{\psi^2}{2F}\Big)\partial^a\Big(\frac{\bar\psi^2}{2\bar F}\Big) - \mathrm{i}\psi^\alpha\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} + F\bar F - f(F+\bar F)\Big]$$

Elimination of the auxiliary fields

$$F = f + \frac{\bar\psi^2}{2\bar F^2}\Box\frac{\psi^2}{2F} = f\left(1 + \frac{1}{4}f^{-4}\bar\psi^2\Box\psi^2 - \frac{1}{16}f^{-8}(\psi^2\bar\psi^2\Box\psi^2\Box\bar\psi^2)\right)$$

Upon elimination of the auxiliaries, the action becomes

$$S[\psi] = -\int \mathrm{d}^4x \left[f^2 + \mathrm{i}\psi^\alpha\partial_{\alpha\dot\alpha}\bar\psi^{\dot\alpha} + \dots \right],$$

up to quartic order in the Goldstino.

At first sight, it appears that off-shell supersymmetry is gone upon elimination of the auxiliary fields. Actually this is not the case.

Short nilpotent chiral Goldstino superfield

E. Ivanov & A. Kapustnikov (1978) M. Roček (1978)

ϕ is chiral, $\bar{D}_{\dot\alpha}\phi = 0$, and obeys the constraints proposed by Roček:

$$\phi^2 = 0 \, ,$$
$$f\phi = -\frac{1}{4}\phi \bar{D}^2\bar{\phi} \, .$$

The auxiliary field F is now a descendant of the Goldstino

$$F = f\Big(1 + f^{-2}\langle\bar u\rangle - f^{-4}\big(\langle u\rangle\langle\bar u\rangle + \frac{1}{4}\bar\psi^2\Box\psi^2\big) + f^{-6}\big(\langle u\rangle^2\langle\bar u\rangle + \mathrm{c.c.}\big)$$
$$+ \frac{1}{4}f^{-6}\big(\langle\bar u\rangle\psi^2\Box\bar\psi^2 + 2\langle u\rangle\bar\psi^2\Box\psi^2 + \bar\psi^2\Box(\psi^2\langle\bar u\rangle)\big)$$
$$- 3f^{-8}\big(\langle u\rangle^2\langle\bar u\rangle^2 + \frac{1}{4}\psi^2\bar\psi^2\Box(\langle u\rangle^2 - \langle u\rangle\langle\bar u\rangle + \langle\bar u\rangle^2) + \frac{1}{16}\psi^2\bar\psi^2\Box\bar\psi^2\Box\psi^2\big)\Big) \, .$$

Notation: $\langle M\rangle = \mathrm{tr}(M) = M_a{}^a$, with $M = (M_a{}^b)$

$$u = (u_a{}^b)\, , \quad u_a{}^b := \mathrm{i}\psi\sigma^b\partial_a\bar\psi\, , \qquad \bar u = (\bar u_a{}^b)\, , \quad \bar u_a{}^b := -\mathrm{i}\partial_a\psi\sigma^b\bar\psi\, .$$

Short nilpotent chiral Goldstino superfield

Goldstino action

$$S_\phi = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,\bar\phi\phi - \left\{ f \int \mathrm{d}^4x\mathrm{d}^2\theta\,\phi + \mathrm{c.c.}\right\}$$
$$= - \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,\bar\phi\phi = -f\int \mathrm{d}^4x\mathrm{d}^2\theta\,\phi$$

Off-shell supersymmetry.

Relation to the (CDCDG & Komargodski-Seiberg) X-model:
Nilpotency condition $X^2 = 0$ is preserved if X is locally rescaled,

$$X \to \mathrm{e}^\tau X\,, \qquad \bar D_{\dot\alpha}\tau = 0\,.$$

Requiring the action

$$S_X = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,\bar X X - \left\{ f \int \mathrm{d}^4x\mathrm{d}^2\theta\,X + \mathrm{c.c.}\right\}$$

to be stationary under such re-scalings of X leads to the equation $-\frac{1}{4}X\bar D^2\bar X = fX$, and hence

$$X = \phi$$

de Sitter supergravity

Old minimal supergravity coupled to a nilpotent chiral scalar X,

$$\bar{\mathcal{D}}_{\dot\alpha} X = 0 \; , \qquad X^2 = 0$$

E. Bergshoeff, D. Freedman, R. Kallosh & A. Van Proeyen (2015)
F. Hasegawa & Y. Yamada (2015)

Complete locally supersymmetric action

$$S = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta \, E \left(-\frac{3}{\kappa^2} \bar S_0 S_0 + \bar X X \right) + \left\{ \int \mathrm{d}^4x \mathrm{d}^2\theta \, \mathcal{E} \left(\frac{\mu}{\kappa^2} S_0^3 - f S_0^2 X \right) + \mathrm{c.c.} \right\}$$

S_0 chiral conformal compensator, $\bar{\mathcal{D}}_{\dot\alpha} S_0 = 0$.
The action is super-Weyl invariant.
Cosmological constant:

$$\Lambda = f^2 - 3 \frac{|\mu|^2}{\kappa^2} \; .$$

- Pure supergravity can be realised as conformal supergravity coupled to a compensating supermultiplet.
- Different off-shell formulations for supergravity correspond to different compensators.
- Super-Weyl transformations P. Howe & R. Tucker (1978)

$$
\begin{aligned}
\delta_\sigma \mathcal{D}_\alpha &= (\bar\sigma - \frac{1}{2}\sigma)\mathcal{D}_\alpha + (\mathcal{D}^\beta\sigma)\, M_{\alpha\beta} \ , \\
\delta_\sigma \bar{\mathcal{D}}_{\dot\alpha} &= (\sigma - \frac{1}{2}\bar\sigma)\bar{\mathcal{D}}_{\dot\alpha} + (\bar{\mathcal{D}}^{\dot\beta}\bar\sigma)\bar{M}_{\dot\alpha\dot\beta} \ , \\
\delta_\sigma \mathcal{D}_{\alpha\dot\alpha} &= \frac{1}{2}(\sigma+\bar\sigma)\mathcal{D}_{\alpha\dot\alpha} + \frac{\rm i}{2}(\bar{\mathcal{D}}_{\dot\alpha}\bar\sigma)\mathcal{D}_\alpha + \frac{\rm i}{2}(\mathcal{D}_\alpha\sigma)\bar{\mathcal{D}}_{\dot\alpha} \\
&\quad + (\mathcal{D}^\beta{}_{\dot\alpha}\sigma)M_{\alpha\beta} + (\mathcal{D}_\alpha{}^{\dot\beta}\bar\sigma)\bar{M}_{\dot\alpha\dot\beta} \ ,
\end{aligned}
$$

where σ is an arbitrary covariantly chiral scalar superfield, $\bar{\mathcal{D}}_{\dot\alpha}\sigma = 0$.

$$
\delta_\sigma S_0 = \sigma S_0 \ , \qquad \delta_\sigma X = \sigma X
$$

Weyl-invariant formulation for Einstein's gravity

Einstein-Hilbert action with a cosmological term

$$S_{\rm EH} = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\, e\, R - \Lambda \int \mathrm{d}^4x\, e$$

Weyl-invariant reformulation S. Deser (1970)

$$S = \frac{1}{2}\int \mathrm{d}^4x\, e\left(\nabla^a\varphi\nabla_a\varphi + \frac{1}{6}R\varphi^2 - \lambda\varphi^4\right) ,$$

where φ is conformal compensator.
Weyl transformation

$$\delta\nabla_a = \sigma\nabla_a + (\nabla^b\sigma)M_{ba} , \qquad \delta\varphi = \sigma\varphi$$

Imposing Weyl gauge condition $\varphi = \frac{\sqrt{6}}{\kappa} = \mathrm{const}$ takes us back to the original action.

Pure de Sitter supergravity

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Using superconformal methods we derive an explicit de Sitter supergravity action invariant under spontaneously broken local $\mathcal{N}=1$ supersymmetry. The supergravity multiplet interacts with a nilpotent Goldstino multiplet. We present a complete locally supersymmetric action including the graviton and the fermionic fields, gravitino and Goldstino, no scalars. In the global limit when the supergravity multiplet decouples, our action reproduces the Volkov-Akulov theory. In the unitary gauge where the Goldstino vanishes we recover pure supergravity with the positive cosmological constant. The classical equations of motion, with all fermions vanishing, have a maximally symmetric solution: de Sitter space.

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I. INTRODUCTION

The cosmological constant is known to be negative or zero in pure supergravity, if there are no scalar fields [1]. Pure supergravity with a positive cosmological constant without scalars was not previously known. In this paper we present the locally $\mathcal{N}=1$ supersymmetric action and transformation rules of such a theory. De Sitter space is a homogeneous solution of the bosonic equations of motion. Supersymmetry is spontaneously broken, so there is no conflict with no-go theorems that prohibit linearly realized supersymmetry [2].[1]

The main motivation for this work is an increasing amount of observational evidence for an accelerating Universe where a positive cosmological constant is a good fit to data. The next step toward a better understanding of dark energy is not expected before the ESA space mission Euclid launches in 2020. It is therefore desirable to find a simple version of de Sitter supergravity as a natural source for the positive cosmological constant.

Volkov-Akulov (VA) Goldstino theory [6] coupled to a supergravity background. The global supersymmetry is realized nonlinearly. This recent development indicates that a scalar independent de Sitter supergravity might exist. Another indication of the existence of such a supergravity was presented in [7], where the proposal to couple the VA Goldstino theory [6] to supergravity was made. However, a complete action and transformation rules that describe this coupling have never been presented. The supersymmetric coupling of the gravitino and Goldstino in $D=10$ at the quadratic level in fermions was studied in [8,9]. The curved superspace formulation of the VA Goldstino theory was studied soon after the discovery of this theory; see for example a review paper [10] or an application of the constrained superfield formalism in superspace in [11]. The relation between the superspace approach and non-linearly realized supersymmetries was investigated in [12].

All earlier theories were not yet developed to the level of a component supergravity action with spontaneously broken local supersymmetry, generalizing the globally

de Sitter supergravity

Bergshoeff *et al.* cited two old papers, probably without noticing that the same value for the cosmological constant was actually derived in these papers.

- *On-shell supergravity*
  S. Deser and B. Zumino, Phys. Rev. Lett. **38**, 1433 (1977)
- *Off-shell supergravity*
  U. Lindström and M. Roček, Phys. Rev. D **19**, 2300 (1979)

At that time, nobody was interested in a positive cosmological constant.

All attempts were targeted at getting a vanishing cosmological constant.
S. W. Hawking, "The cosmological constant is probably zero," Phys. Lett. **134B**, 403 (1984)

Significance of the 2015 work by Bergshoeff *et al.* is that it has renewed interest in spontaneously broken supergravity.

de Sitter supergravity

Broken Supersymmetry and Supergravity

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(Received 5 April 1977)

We consider the supersymmetric Higgs effect, in which a spin-$\frac{1}{2}$ Goldstone fermion is transformed away by a redefinition of the supergravity fields and the spin-$\frac{3}{2}$ gauge field acquires the degrees of freedom appropriate to finite mass. More generally we discuss the consistency and physical applicability of supergravity theories with broken local supersymmetry.

Rigorous supersymmetry implies the existence of supermultiplets made up of fermions and bosons with equal masses. If supersymmetry is to be relevant for the physical world, it must be broken, either softly or spontaneously. Spontaneous breaking of global supersymmetry gives rise to the appearance of one or more Goldstone fermions.[1] When global supersymmetry is promoted to a local invariance by coupling supersymmetric matter to supergravity, the Goldstone fermion disappears as a consequence of a phenomenon analogous to the Higgs effect of ordinary gauge theories. In this Letter we describe this supersymmetric Higgs effect,[2] and consider its possible application to the construction of realistic models.[3] In particular, the supersymmetric Higgs effect gives a possible solution to the problem of the Majorana spin-$\frac{1}{2}$ field λ. Irrespective of the particular field theory in which it arises, it can be characterized, following Volkov and Akulov,[5] by the nonlinear realization of global supersymmetry

$$\delta\lambda = a^{-1}\alpha + ia\bar{\alpha}\gamma^{\mu}\lambda\partial_{\mu}\lambda, \tag{1}$$

where α is the infinitesimal supersymmetry parameter and a is a constant which measures the strength of the spontaneous breaking of supersymmetry. The nonlinear Lagrangian for λ, invariant (up to a divergence) under (1), is given by

$$L_{\lambda} = -(2a^2)^{-1}\det(\delta_{\mu}{}^{\nu} + ia^2\bar{\lambda}\gamma^{\nu}\partial_{\mu}\lambda)$$
$$= -(2a^2)^{-1} - \tfrac{1}{2}i\bar{\lambda}\gamma\cdot\partial\lambda + \dots . \tag{2}$$

The analogy with nonlinear pion dynamics is apparent. However, the chiral group SU(2)$\otimes$SU(2)

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formation with parameter $\alpha(x)$ and to make (2) invariant under it by coupling λ to the supergravity fields $e_\mu{}^a$ and ψ_μ. The complete Lagrangian will be rather complicated. Assuming its existence, one can easily find the first terms in an expansion in the coupling constants a and κ (gravitational constant). The Lagrangian ($e \equiv \det e_\mu{}^a$)

$$L_\lambda = -(2a^2)^{-1}e - \tfrac{1}{2}i\bar{\lambda}\gamma\cdot\partial\lambda - (i/2a)\bar{\lambda}\gamma\cdot\psi + \dots \qquad (3)$$

changes by a divergence under

$$\delta\lambda = a^{-1}\alpha(x) + \dots,$$
$$\delta e_\mu{}^a = -i\kappa\bar{\alpha}\gamma^a\psi_\mu, \qquad (4)$$
$$\delta\psi_\mu = -2\kappa^{-1}\partial_\mu\alpha + \dots.$$

To (3) one must add the usual supergravity Lagrangian[6,7]

$$L_{sg} = -(2\kappa^2)^{-1}eR - \tfrac{1}{2}i\epsilon^{\lambda\mu\nu\rho}\bar{\psi}_\lambda\gamma_5\gamma_\mu D_\nu\psi_\rho, \qquad (5)$$

where

$$D_\mu = \partial_\mu - \tfrac{1}{2}\omega_{\mu,ab}\Sigma^{ab}, \quad \Sigma^{ab} = \tfrac{1}{4}[\gamma^a,\gamma^b], \qquad (6)$$

and R is the contracted Riemann tensor, taken as The simplest and most natural is the corresponding de Sitter space and one knows that the concept of mass is rather delicate there.[9] We next recall the recent observations[10-12] that one can add to the supergravity Lagrangian (5) the sum of a cosmological term and of a spin-$\frac{3}{2}$ mass term

$$ce - \tfrac{1}{2}im\epsilon^{\lambda\mu\nu\rho}\bar{\psi}_\lambda\gamma_5\Sigma_{\mu\nu}\psi_\rho. \qquad (7)$$

Local supersymmetry is valid provided that the two parameters are related by

$$c\kappa^2 = 3m^2. \qquad (8)$$

Indeed, the sum of (5) and (7) is then invariant under a modified supersymmetry transformation, in which the usual transformation law for the spin-$\frac{3}{2}$ field, $\delta\psi_\mu = -2\kappa^{-1}D_\mu\alpha$, is replaced by

$$\delta\psi_\mu = -2\kappa^{-1}\mathfrak{D}_\mu\alpha, \qquad (9)$$

where

$$\mathfrak{D}_\mu \equiv D_\mu + \tfrac{1}{2}m\gamma_\mu \qquad (10)$$

(there is a corresponding change in $\delta\omega_{\mu,ab}$).

The existence of this local supersymmetry[13]

de Sitter supergravity

(cf. Ref. 7). Clearly, there exists a gauge (U gauge) where $\chi = 0 \Rightarrow \mathfrak{A} = 0$ and $\mathfrak{F} = 1/a$.

The action invariant under the transformation (5a) is found by applying the density formula[9]:

$$I = \int d^4x \{\mathfrak{L}_{SG} - \frac{e}{4a} [\mathfrak{F}(\chi, \chi') - \sqrt{2}\, \psi^{A'}{}_{AA'}\chi^A + \mathfrak{A}(\chi, \chi')(\mathcal{S}^* + \tfrac{1}{2}\psi^{A'}{}_{AA'}\psi_{B'}{}^{AB'} + \tfrac{1}{2}\psi_{B'AA'}\psi^{A'AB'}) + \text{c.c.}]\}$$
$$= \int d^4x \left[\mathfrak{L}_{SG} + \frac{\sqrt{2}\, e}{2} (\chi^A D_{AA'}\chi^{A'} + \chi^{A'} D_{AA'}\chi^A) + \cdots\right] , \tag{5b}$$

where $\mathfrak{L}_{SG}$ is the supergravity Lagrangian.[9]

In the U gauge the action (5b) reduces to

$$I_U = \int d^4x \left(\mathfrak{L}_{SG} - \frac{e}{2a^2}\right). \tag{5c}$$

The supergravity Lagrangian $\mathfrak{L}_{SG}$ can be taken to include a separately invariant term,

$$\mathfrak{L}_m = me(\mathcal{S}^* + \tfrac{1}{2}\psi^{A'}{}_{AA'}\psi_{B'}{}^{AB'} + \tfrac{1}{2}\psi_{B'AA'}\psi^{A'AB'} + \text{c.c.}) , \tag{5d}$$

and necessarily contains the auxiliary spin-0 field in the combination $-(e/3)\mathcal{S}\mathcal{S}^*$; integrating out the auxiliary field $\mathcal{S}$ (still in the U gauge) leads to a cosmological term, $e(3m^2 - 1/2a^2)$, which vanishes for $m^2 = 1/6a^2$, in agreement with Ref. 7, and leaves the spin-$\frac{3}{2}$ field with a mass $m = 1/a\sqrt{6}$. In a general gauge, the relevant terms quadratic or lower in $\chi'$are

$$\mathfrak{L}_2 = e\left[-\tfrac{1}{3}\mathcal{S}\mathcal{S}^* + m(\mathcal{S}^* + \mathcal{S}) - \frac{1}{2a^2} - \frac{\sqrt{2}}{2a}(\chi^{A'}\psi^A{}_{AA'} + \chi^A\psi^{A'}{}_{AA'}) - \tfrac{1}{3}(\mathcal{S}^*\chi^2 + \mathcal{S}\chi'^2)\right] .$$

Goldstino superfields

Is there anything unique in the nilpotent Goldstino superfield used by Bergshoeff *et al.* ? Actually not much (except for one technical point).

Its defining constraints

$$\bar{\mathcal{D}}_{\dot\alpha} X = 0 \,, \qquad X^2 = 0$$

are invariant under local rescalings $X \to e^{\tau} X \,, \quad \bar{\mathcal{D}}_{\dot\alpha}\tau = 0$.
Requiring the complete action for supergravity coupled to X,

$$S = \int d^4x d^2\theta d^2\bar\theta \, E \left(-\frac{3}{\kappa^2} \bar{S}_0 S_0 + \bar{X} X \right) + \left\{ \int d^4x d^2\theta \, \mathcal{E} \left(\frac{\mu}{\kappa^2} S_0^3 - f S_0^2 X \right) + \text{c.c.} \right\} \,,$$

to be stationary under such re-scalings gives $X = \phi$, where ϕ is the Goldstino superfield used by Lindström and Roček,

$$\bar{\mathcal{D}}_{\dot\alpha}\phi = 0 \,, \qquad \phi^2 = 0 \,, \qquad f S_0^2 \phi = -\frac{1}{4}\phi(\bar{\mathcal{D}}^2 - 4R)\bar\phi$$

Goldstino superfields

In the presence of supergravity-matter couplings, the nonlinear constraint obeyed by ϕ gets deformed.
Example:

$$S = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\left(-\frac{3}{\kappa^2}\bar{S}_0 S_0\, \mathrm{e}^{-\frac{\kappa^2}{3}K(\Phi,\bar{\Phi})} + \bar{X}X\Upsilon(\Phi,\bar{\Phi})\right) + \left\{\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\left(S_0^3 W(\Phi) - S_0^2 X\mathfrak{F}(\Phi)\right) + \mathrm{c.c.}\right\} ,$$

Requiring the complete action to be stationary under local re-scalings

$$X \to \mathrm{e}^{\tau} X , \quad \bar{\mathcal{D}}_{\dot{\alpha}}\tau = 0$$

leads to deformed nonlinear constraint

$$\mathfrak{F}(\Phi)S_0^2 X = -\frac{1}{4}X(\bar{\mathcal{D}}^2 - 4R)\Big(\bar{X}\Upsilon(\Phi,\bar{\Phi})\Big)$$

Two types of Goldstino superfields

There are two general types of $\mathcal{N} = 1$ Goldstino superfields.
I. Bandos, M. Heller, SMK, L. Martucci & D. Sorokin (2016)

- Irreducible Goldstino superfields
  Every irreducible Goldstino superfield contains only one independent component field – the Goldstino itself, while the other component fields are composites constructed from the Goldstino.
- Reducible Goldstino superfields
  Every reducible Goldstino superfield contains at least two independent fields, one of which is the Goldstino and the other fields are auxiliary (the latter become descendants of the Goldstino on the mass shell).
- Every reducible Goldstino superfield can be represented as an irreducible one plus a "matter" superfield, which contains all the auxiliary component fields. (Example will be provided below.)

Irreducible Goldstino superfields

Scalar Goldstino superfields (all of them are nilpotent)

- Chiral superfield E. Ivanov & A. Kapustnikov, M. Roček (1978)

$$\bar{D}_{\dot{\alpha}}\phi = 0 , \qquad \phi^2 = 0 , \qquad f\phi = -\frac{1}{4}\phi\bar{D}^2\bar{\phi}$$

- Improved complex linear superfield SMK & S. Tyler (2011)

$$-\frac{1}{4}\bar{D}^2\Sigma = f , \qquad \Sigma^2 = 0 , \qquad fD_\alpha\Sigma = -\frac{1}{4}\Sigma\bar{D}^2 D_\alpha\Sigma$$

- Complex linear superfield S. Tyler (2011)

$$\bar{D}^2\Gamma = 0 , \qquad \Gamma^2 = 0 , \qquad f\Gamma = -\frac{1}{4}\Gamma\bar{D}^2\bar{\Gamma}$$

F. Farakos, O. Hulik, P. Koci & R. von Unge (2015)

Irreducible Goldstino superfields

Scalar Goldstino superfields (continued)

- Real superfield

$$\mathcal{V}^2 = 0 , \quad \mathcal{V} D_A D_B \mathcal{V} = 0 , \quad \mathcal{V} D_A D_B D_C \mathcal{V} = 0$$
$$f \mathcal{V} = \frac{1}{16} \mathcal{V} D^\alpha \bar{D}^2 D_\alpha \mathcal{V}$$

I. Bandos, M. Heller, SMK, L. Martucci & D. Sorokin (2016)

Explicit realisation for $\mathcal{V}$ was given long ago:

$$f \mathcal{V} = \bar{\phi} \phi$$

U. Lindström and M. Roček (1979)

Another realisation for $\mathcal{V}$:

$$f \mathcal{V} = \bar{\Sigma} \Sigma$$

SMK & S. Tyler (2011)

Irreducible Goldstino superfields

Spinor Goldstino superfields

For every irreducible spinor Goldstino superfield, its spinor covariant derivatives must be some functions of this superfield and its spacetime derivatives.

- Volkov-Akulov Goldstino E. Ivanov & A. Kapustnikov (1978)

$$D_\alpha \Lambda_\beta = -f\varepsilon_{\alpha\beta} - \mathrm{i} f^{-1} \bar{\Lambda}^{\dot\alpha} \partial_{\alpha\dot\alpha} \Lambda_\beta \,, \qquad \bar{D}_{\dot\alpha} \Lambda_\beta = -\mathrm{i} f^{-1} \Lambda^\alpha \partial_{\alpha\dot\alpha} \Lambda_\beta \,.$$

- Chiral realisation S. Samuel and J. Wess (1983)

$$D_\alpha \Xi_\beta = -f\varepsilon_{\alpha\beta} \,, \qquad \bar{D}_{\dot\alpha} \Xi_\beta = -2\mathrm{i} f^{-1} \Xi^\alpha \partial_{\alpha\dot\alpha} \Xi_\beta \,.$$

Spinor Goldstino superfields are less convenient to deal with, than the scalar ones, in supergravity.

Irreducible Goldstino superfields

- All irreducible Goldstino superfields are equivalent
  Uniqueness of the Goldstino

  D.Volkov & V. Akulov(1972)
  E. Ivanov & A. Kapustnikov (1978)

- All irreducible Goldstino superfields can be realised as descendants of one of them.

$$
\begin{aligned}
f\phi &= -\frac{1}{4}\bar{D}^2(\bar{\Sigma}\Sigma) , \\
f\mathcal{V} &= \bar{\Sigma}\Sigma , \\
\Gamma &= \bar{\Sigma} - \frac{1}{4f}(\bar{D}_{\dot{\alpha}}\Sigma)\bar{D}^{\dot{\alpha}}\bar{\Sigma} , \\
\Xi_\alpha &= \frac{1}{2}D_\alpha\bar{\Sigma}
\end{aligned}
$$

Reducible Goldstino superfields

In addition to the nilpotent chiral scalar X, there also exists nilpotent real scalar

SMK, I. McArthur & G. Tartaglino-Mazzucchelli (2017)

$$V^2 = 0 \ , \quad V D_A D_B V = 0 \ , \quad V D_A D_B D_C V = 0$$

These nilpotency constraints have to be supplemented with the requirement that $D^\alpha W_\alpha \equiv \bar{D}_{\dot\alpha} \bar{W}^{\dot\alpha} \neq 0$, where

$$W_\alpha = -\frac{1}{4} \bar{D}^2 D_\alpha V \ .$$

V has two independent component fields:
(i) Goldstino $\propto W_\alpha|_{\theta=0}$; and (ii) auxiliary D-field $\propto D^\alpha W_\alpha|_{\theta=0}$.
All other component fields of V are composite ones, in particular

$$V = -4 \frac{W^2 \bar{W}^2}{(D^\alpha W_\alpha)^3} \ , \qquad W^2 = W^\alpha W_\alpha$$

Dynamics is governed by

$$S = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta \left\{ \frac{1}{16} V D^\alpha \bar{D}^2 D_\alpha V - 2fV \right\}$$

Reducible Goldstino superfields

Nilpotency constraints

$$V^2 = 0 \ , \quad V D_A D_B V = 0 \ , \quad V D_A D_B D_C V = 0$$

are invariant under local re-scalings of V,

$$V \ \to \ \mathrm{e}^{\rho} V \ ,$$

where ρ is an arbitrary real scalar superfield. Requiring the action

$$S = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \left\{ \frac{1}{16} V D^\alpha \bar{D}^2 D_\alpha V - 2fV \right\}$$

to be stationary under such rescalings leads to the constraint

$$fV = \frac{1}{16} V D^\alpha \bar{D}^2 D_\alpha V \ ,$$

which expresses the auxiliary field of V in terms of the Goldstino. Reducible Goldstino superfield V turns into $\mathcal{V}$, which is irreducible.

Reducible & irreducible Goldstino superfields

- Irreducible complex linear Goldstino superfield

$$-\frac{1}{4}\bar{D}^2\Sigma = f\,, \qquad \Sigma^2 = 0\,, \qquad fD_\alpha\Sigma = -\frac{1}{4}\Sigma\bar{D}^2 D_\alpha\Sigma$$

can be realised as a descendant of V

$$\Sigma = -4f\,\frac{D^2 V}{\bar{D}^2 D^2 V}$$

- Remarkable feature of this representation is that Σ is invariant under local re-scalings of V,

$$\delta_\rho V = \rho V \quad \Longrightarrow \quad \delta_\rho\Sigma = 0\;, \qquad \bar{\rho} = \rho\;.$$

- Since every irreducible Goldstino superfield is a descendant of Σ and $\bar{\Sigma}$, all irreducible Goldstino superfields are invariant under re-scalings of V.

E. Buchbinder & SMK (2017)

Reducible & irreducible Goldstino superfields

- Irreducible complex linear Goldstino superfield

$$-\frac{1}{4}\bar{D}^2\Sigma = f\,, \qquad \Sigma^2 = 0\,, \qquad fD_\alpha\Sigma = -\frac{1}{4}\Sigma\bar{D}^2 D_\alpha\Sigma$$

can be realised as a descendant of $\bar{X}$

$$\Sigma = -4f\,\frac{\bar{X}}{\bar{D}^2\bar{X}}\,,$$

- Remarkable feature of this representation is that Σ and $\bar{\Sigma}$ are invariant under local re-scalings of X,

$$\delta_\tau X = \tau X \quad \Longrightarrow \quad \delta_\tau\Sigma = 0\,, \qquad \bar{D}_{\dot\alpha}\tau = 0\,.$$

- Since every irreducible Goldstino superfield is a descendant of Σ and $\bar{\Sigma}$, all irreducible Goldstino superfields are invariant under re-scalings of X.

E. Buchbinder & SMK (2017)

Reducible & irreducible Goldstino superfields

- Every reducible Goldstino superfield can be represented as an irreducible one plus a "matter" superfield, which contains all the auxiliary component fields.

Reducible Goldstino superfield V can be realised as

$$V = \mathcal{V} + U \ , \qquad \mathcal{V} = \frac{1}{f}\bar{\Sigma}\Sigma \ , \qquad \Sigma = -4f \frac{D^2 V}{\bar{D}^2 D^2 V}$$

"Matter" superfield U obeys the generalised nilpotency condition

$$U^2 + 2\mathcal{V}U = 0$$

Reducible Goldstino superfields: from X to V

Nilpotency constraints

$$V^2 = 0 \ , \quad V D_A D_B V = 0 \ , \quad V D_A D_B D_C V = 0$$

are identically satisfied if V is given by

$$fV = \bar{X} X \ , \qquad \bar{D}_{\dot\alpha} X = 0 \ , \quad X^2 = 0 \ ,$$

compare with the irreducible case:

$$f\mathcal{V} = \bar{\phi}\phi \ .$$

V-action

$$S = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta \left\{ \frac{1}{16} V D^\alpha \bar{D}^2 D_\alpha V - 2fV \right\}$$

turns into higher-derivative action

$$S = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta\, E \left\{ \frac{1}{16 f^2} \mathcal{D}^\alpha X \mathcal{D}_\alpha X \bar{\mathcal{D}}_\beta \bar{X} \bar{\mathcal{D}}^\beta \bar{X} - 2\bar{X} X \right\} .$$

Reducible Goldstino superfields

Constrained three-form multiplet

E. Buchbinder & SMK (2017)

$$\bar{D}_{\dot{\alpha}}\Psi = 0 \ , \qquad \Psi^2 = 0 \ , \qquad \Psi = 2\bar{D}^2 \frac{\bar{\Psi}\Psi}{D^2\Psi + \bar{D}^2\bar{\Psi}}$$

It is equivalent to the real Goldstino superfield V

$$\begin{aligned} \Psi &= -\frac{1}{4}\bar{D}^2 V \ , \\ V &= -4\frac{\bar{\Psi}\Psi}{\bar{D}^2\bar{\Psi}} = -4\frac{\bar{\Psi}\Psi}{D^2\Psi} = -8\frac{\bar{\Psi}\Psi}{D^2\Psi + \bar{D}^2\bar{\Psi}} \ . \end{aligned}$$

Three-form multiplet

Three-form multiplet as a variant scalar multiplet J. Gates (1981)

$$\mathcal{Y} = -\frac{1}{4}\bar{D}^2\mathcal{U} , \qquad \bar{\mathcal{U}} = \mathcal{U} ,$$

where real prepotential $\mathcal{U}$ is *unconstrained*. Its specific feature is

$$D^2\mathcal{Y} - \bar{D}^2\bar{\mathcal{Y}} = \mathrm{i}\partial^{\alpha\dot\alpha}u_{\alpha\dot\alpha} , \qquad u_{\alpha\dot\alpha} = [D_\alpha, \bar{D}_{\dot\alpha}]\mathcal{U} ,$$

which means that the auxiliary F-field of $\mathcal{Y}$ is

$$-\frac{1}{4}D^2\mathcal{Y}| = F = H + \mathrm{i}G , \qquad G = \partial_a C^a$$

Gauge symmetry: $\delta\mathcal{U} = L , \qquad \bar{L} = L , \qquad \bar{D}^2 L = 0 ,$

for any linear multiplet L. Reducible gauge theory

Quantisation of the three-form multiplet coupled to supergravity:

I. Buchbinder & SMK (1985,1988)

Nilpotent three-form multiplet

Infrared limit of nonlinear σ-model E. Buchbinder & SMK (2017)

$$S = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, K(\bar{\mathcal{Y}}, \mathcal{Y}) + \Big\{ \int \mathrm{d}^4x\mathrm{d}^2\theta\, W(\mathcal{Y}) + \mathrm{c.c.}\Big\}$$

leads to nilpotent three-form multiplet

$$\mathcal{Y} = -\frac{1}{4}\bar{D}^2\mathcal{U}\,, \qquad \bar{\mathcal{U}} = \mathcal{U}$$
$$\mathcal{Y}^2 = 0$$

described by action

$$S = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, \bar{\mathcal{Y}}\mathcal{Y} - \Big\{ h \int \mathrm{d}^4x\mathrm{d}^2\theta\, \mathcal{Y} + \mathrm{c.c.}\Big\}\,, \qquad h = \bar{h}$$

The same Goldstino superfield was introduced by
F. Farakos, A. Kehagias, D. Racco & A. Riotto (2016)
as a variant formulation of the nilpotent chiral multiplet X.

Goldstino superfields and cosmological constant

- All irreducible Goldstino superfields, as well as the reducible Goldstino superfields X and V, produce a universal positive contribution, f^2, to the cosmological constant,

$$\Lambda = f^2 + \Lambda_{\rm AdS} ,$$

  where $\Lambda_{\rm AdS} = -3\frac{|\mu|^2}{\kappa^2}$ comes from a supersymmetric cosmological term. The latter exists only for (i) old minimal supergravity (and its variant versions); and (ii) $n = -1$ non-minimal supergravity.
- Nilpotent three-form multiplet $\mathcal{Y}$ is the only known Goldstino superfield, which produces two separate positive contributions to the cosmological constant coming from its auxiliary fields, $F = H + {\rm i}G$, of which H is a scalar and G is the field strength of a gauge three-form.
- While the contribution from H is uniquely determined by the parameter h, the contribution from G is dynamical.
  The latter may be used to cancel the contribution from $\Lambda_{\rm AdS}$.

Gauge three-form and cosmological constant

- Idea to use massless gauge three-forms to generate a cosmological constant dynamically.

V. Ogievetsky & E. Sokatchev (1980)
M. Duff & P. van Nieuwenhuizen (1980)
A. Aurilia, H. Nicolai & P. Townsend (1980)

- Further developments

S. Hawking (1984)
M. Duff (1989)
M. Duncan & L. Jensen (1990)
R. Bousso & J. Polchinski (2000)

Subtle feature of gauge three-form

M. Duff, "The cosmological constant is possibly zero, but the proof is probably wrong," Phys. Lett. B **226**, 36 (1989)

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\, e\, R - \Lambda \int \mathrm{d}^4x\, e + \int \mathrm{d}^4x\, e\, (\nabla_a C^a)^2$$

Equation of motion for the three-form

$$\nabla_a(\nabla \cdot C) = 0 \quad \Longrightarrow \quad \nabla_a C^a = c = \mathrm{const}$$

Equation of motion for the gravitational field

$$\frac{1}{\kappa^2}(R_{mn} - \frac{1}{2}g_{mn}R) + \Lambda g_{mn} = T_{mn}~, \quad T_{mn} = -g_{mn}(\nabla \cdot C)^2 = -c^2 g_{mn}$$

Correct effective cosmological constant: $\Lambda + c^2$.
However, plugging the solution for C^a back in S would give

$$\tilde{S} = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\, e\, R - (\Lambda - c^2)\int \mathrm{d}^4x\, e$$

$\mathcal{N} = 2$ Goldstino superfields

New feature of $\mathcal{N} = 2$ supersymmetry:

- One can consistently define nilpotent Goldstino superfields that contain either a gauge one-form or a gauge two-form in addition to spin-$1/2$ Goldstone fermions and auxiliary fields.

$\mathcal{N} = 2$ Goldstino superfields

Deformed reduced chiral superfield

SMK (2013)
SMK & G. Tartaglino-Mazzucchelli (2016)

$$\bar{\mathcal{D}}^i_{\dot\alpha} \mathcal{Z} = 0 \ , \qquad \big(\mathcal{D}^{ij} + 4S^{ij}\big)\mathcal{Z} - \big(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij}\big)\bar{\mathcal{Z}} = 4\mathrm{i}\, G^{ij} \ ,$$

where $\mathcal{D}^{ij} = \mathcal{D}^{\alpha(i}\mathcal{D}^{j)}_\alpha$ and $\bar{\mathcal{D}}^{ij} = \bar{\mathcal{D}}_{\dot\alpha}{}^{(i}\bar{\mathcal{D}}^{j)\dot\alpha}$.
• S^{ij} and $\bar{S}^{ij}$ are dimension-1 torsion superfields
(analogues of the chiral scalar torsion R in $\mathcal{N} = 1$ supergravity).
• G^{ij} is a linear multiplet which obeys the constraints

$$\mathcal{D}^{(i}_\alpha G^{jk)} = \bar{\mathcal{D}}^{(i}_{\dot\alpha} G^{jk)} = 0 \ .$$

and, in addition, is required to be nowhere vanishing, $G^{ij}G_{ij} \neq 0$.
• We identify G^{ij} with one of the two conformal compensators of the minimal formulation for $\mathcal{N} = 2$ supergravity proposed in
B. de Wit, R. Philippe & A. Van Proeyen (1983)

Deformed reduced chiral superfield in Minkowski superspace

$$\bar{D}^i_{\dot\alpha}\mathcal{Z} = 0~, \quad D^{ij}\mathcal{Z} - \bar{D}^{ij}\bar{\mathcal{Z}} = 4\mathrm{i}G^{ij}~, \qquad G^{ij} = \text{const}$$

I. Antoniadis, H. Partouche & T. R. Taylor (1996)
E. Ivanov & B. Zupnik (1998)

$\mathcal{N} = 2$ Goldstino superfields

Nilpotent deformed reduced chiral superfield

- Partial $\mathcal{N} = 2 \to \mathcal{N} = 1$ breaking of rigid supersymmetry in curved maximally supersymmetric spacetimes:
  (i) $\mathbb{R} \times S^3$; (ii) $\mathrm{AdS}_3 \times \mathbb{R}$; and (iii) supersymmetric plane wave isometric to the Nappi-Witten group.
  SMK & G. Tartaglino-Mazzucchelli (2016)

$$\mathcal{Z}^2 = 0$$

  Maxwell-Goldstone multiplet for partial $\mathcal{N} = 2 \to \mathcal{N} = 1$ breaking. Generalisation of the 1996 Bagger-Galperin construction for $\mathbb{M}^4$.

- $\mathcal{N} = 2 \to \mathcal{N} = 0$ breaking of local supersymmetry.
  SMK & G. Tartaglino-Mazzucchelli, arXiv:1707.07390

$$\mathcal{Z}^3 = 0$$

In the super-Poincaré case, constraint $\mathcal{Z}^3 = 0$ was studied by
E. Dudas, S. Ferrara & A. Sagnotti, arXiv:1707.03414

$\mathcal{N} = 2$ Goldstino superfields

Nilpotent linear superfield

SMK & G. Tartaglino-Mazzucchelli, arXiv:1707.07390

$\mathcal{N} = 2 \to \mathcal{N} = 0$ breaking of local supersymmetry can be described using a linear superfield $\mathcal{H}^{ij}$

$$\mathcal{D}_\alpha^{(i}\mathcal{H}^{jk)} = \bar{\mathcal{D}}_{\dot\alpha}^{(i}\mathcal{H}^{jk)} = 0 ,$$

which is subject to the following cubic nilpotency condition

$$\mathcal{H}^{(i_1 i_2}\mathcal{H}^{i_3 i_4}\mathcal{H}^{i_5 i_6)} = 0 .$$

This algebraic constraint is one of the several nonlinear constraints, which define the irreducible linear Goldstino superfield introduced in

SMK, I. McArthur & G. Tartaglino-Mazzucchelli (2017)

Outlook

$\mathcal{N} = 2$ story is not yet complete.