UNIVERSAL DEFORMATIONS OF CLASSICAL POISSON STRUCTURES.

Arthemy Kiselev (JBI GRONINGEN) & $\left\{\begin{array}{l}\text{R. BURING}\\ \text{N. RUTTEN}\end{array}\right.$ (JBI).

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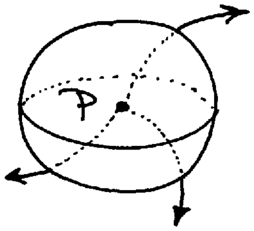
① AFFINE POISSON MANIFOLD $(N^{n<\infty}, \mathcal{P})$.

Ⓟ: $\mathcal{P} \longmapsto \mathcal{P} + \varepsilon Q(\mathcal{P}) + \bar{o}(\varepsilon)$ POISSON $(\mathrm{mod}\ \bar{o}(\varepsilon))$: $Q(\mathcal{P}) = ?$ $\underline{\underline{\exists}}\,!$



$\# \geqslant |\mathbb{N}|$. (2013-15)

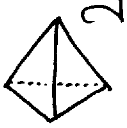
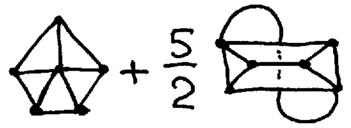
$$[\![\mathcal{P} + \varepsilon Q(\mathcal{P}) + \bar{o}(\varepsilon), \mathcal{P} + \varepsilon Q(\mathcal{P}) + \bar{o}(\varepsilon)]\!] = \bar{o}(\varepsilon)$$

ε^0: $[\![\mathcal{P},\mathcal{P}]\!] = 0$ ✓.

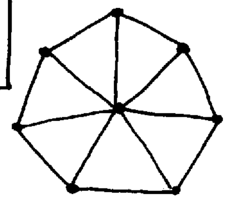
ε^1: $2\underbrace{[\![\mathcal{P}, Q(\mathcal{P})]\!]}_{} \doteq 0$ VIA $[\![\mathcal{P},\mathcal{P}]\!] = 0$.

$\partial_{\mathcal{P}} := [\![\mathcal{P}, \cdot]\!]$ – cocycle.

JACOBI IDENTITY.

② M. KONTSEVICH (1996, 2017):  $\overset{10^2}{\Rightarrow}$  ,

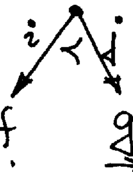
- T. WILLWACHER (2013-14)
- A.K. et al. (2016)
- R. BURING (2017)
- N. RUTTEN

 $+ \langle 45$ GRAPHS$\rangle$.

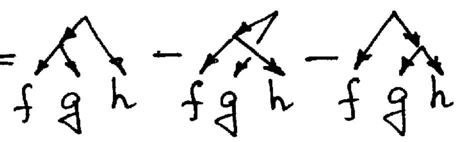
(now)

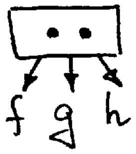
Ex. $\dot{\mathcal{P}} = \mathcal{P}$: $\mathcal{P} \longmapsto e^{\varepsilon}\mathcal{P}$; (elementary) Ex. $\dot{\mathcal{P}} = [\![\mathcal{P}, \mathfrak{X}]\!]\ \forall \vec{\mathfrak{X}}$ (trivial) Ex. $\dot{\mathcal{P}} = \nabla(\mathcal{P}, \mathrm{Jac}(\mathcal{P})) \equiv 0$. (improper).

③ $\bullet \overset{\text{def}}{=} \mathcal{P}^{ij}(x)$ $\qquad \sum_{i=1}^{n} \frac{\partial}{\partial x^i} \overset{\text{def}}{=} \bullet\xrightarrow{i}$ $\qquad$ GRAPH $\overset{\text{def}}{=} \sum_{\{i\}} \prod_{\{\text{VERTEX}\}}$.

$\{f, g\}_{\mathcal{P}} =$ 

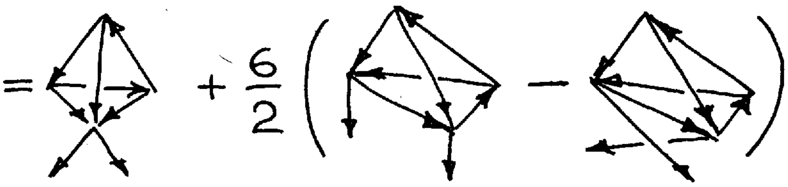
$(f)\frac{\overleftarrow{\partial}}{\partial x^i} \cdot \mathcal{P}^{ij}(x) \cdot \frac{\overrightarrow{\partial}}{\partial x^j}(g)$

$\mathrm{Jac}(\mathcal{P})(f,g,h) =$  $= 0$.

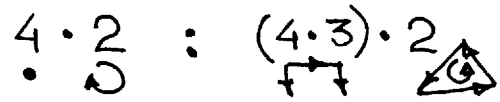
 $= 0$. JACOBI.

④ TH. (2015) $\left\{\begin{array}{l}\text{M.K.}\\ \text{T.W.}\end{array}\right\}$. $\begin{pmatrix}\text{NON-ORIENTED}\\ d_\Gamma^2 = 0\end{pmatrix} \xrightarrow[\text{QUASI}]{\sim} \begin{pmatrix}\text{ORIENTED}\\ [\![\mathcal{P}, \cdot]\!]^2 = 0\end{pmatrix}$: COCYCLES $\to$ $\to$ COCYCLES.

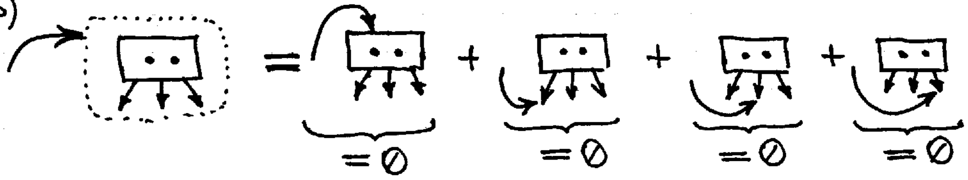
Ex.

$Q_{1:\frac{6}{2}}(\mathcal{P}) =$ 

- M.K. (1996)
- A.B., A.K. (2016)
- A.B., R.B., A.K. (2017)

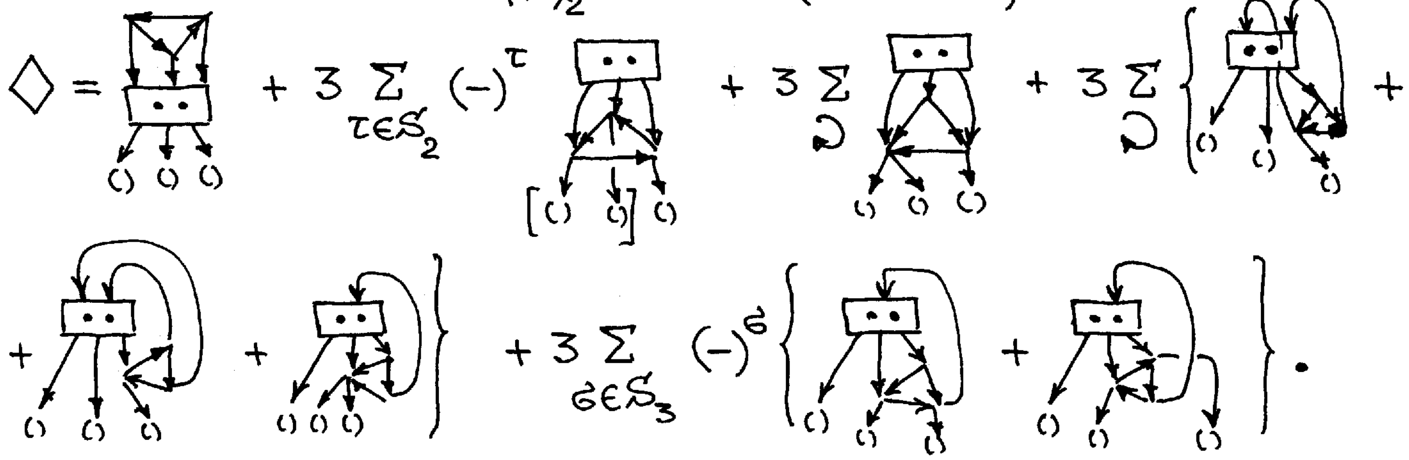
 $= 1 : {}^{6}\!/_{2}$. $\}$ necessary & sufficient.

(5) TH. (2016) [R.B. & A.K.]. MECHANISM $[\![\mathcal{P},\mathcal{Q}]\!] = \Diamond(\mathcal{P}, \mathcal{J}ac(\mathcal{P}))$:



{LEIBNIZ GRAPHS.

Ex. (JGP(2017)). $[\![\mathcal{P}, \mathcal{Q}_{1:6/2}(\mathcal{P})]\!] = \Diamond(\mathcal{P}, \mathcal{J}ac(\mathcal{P}))$.



8 SKEW LEIBNIZ GRAPHS ⇒ 27 LEIBNIZ GRAPHS ⇒ 39 KONTSEVICH GRAPHS.

(6) TH. (2017) [R.B., A.K., N.R.]. $k=1$: $Q(\mathcal{P})=\mathcal{P}$. $k=2$: $\nexists$ $k=3$: $\nexists$ $k=4$: 

NB. WE ALLOW 

$k=5$: $\nexists$ $k=6$: $Q(\mathcal{P})$ = 5-wheel  $+\frac{5}{2}$ 

$k=7$: $\nexists$ $k=8$: $Q(\mathcal{P})$ = 7-wheel + <45 terms>.

{ $|Q| \approx 100$ OR. GRAPHS
$|[\![\mathcal{P},Q]\!]| = 3411$.
$\Diamond$ { 250,000 UNKNOWNS
500,000 EQNS.

TH. (2015) [DOLGUSHEV, ROGERS, WILLWACHER]: $\forall k \geqslant 1$ (∃) $Q(\mathcal{P}) = (2k+1)$-wheel + ?.

(?) HOW BIG IS THE LIE ALGEBRA $(Q(\mathcal{P}), [\cdot,\cdot])$ mod $\nabla(\mathcal{P}, \mathcal{J}ac(\mathcal{P}))$?

(7) TRIVIAL $Q(\mathcal{P}) := [\![\mathcal{P}, \vec{\mathfrak{X}}]\!]$ $\forall \vec{\mathfrak{X}}$ ON N^n. Ex.  $= \vec{\mathfrak{X}}(\mathcal{P})$.

LEMMA. $[\![\cdot, \vec{\mathfrak{X}}]\!]$ *imitates* SMOOTH DIFFEO $\exp(\varepsilon\vec{\mathfrak{X}}): N^n \to N^n$ ON AFFINE N^n (!).

(?) DYN. SYSTEMS.

REFS.

- [M.K.: ASCONA'96]
- [A.B., A.K.: 1609.06677]
- [R.B., A.K.: 1602.09036]
- [A.B., R.B., A.K.: 1608.01710]
- [R.B., A.K.: 1702.00681]
- [A.B.: 1702.06044]
- [A.K.: 1705.01777]