Local Current Interactions from Nonlinear Higher-Spin Equations

O.A.Gelfond

NIISI & Lebedev Institute , Moscow

SQS, Dubna 2017

Main results

Reconstruction of local current interactions in the gauge sector from nonlinear HS equations

Resulting gauge invariant interactions reproduce the two types of $4d$ cubic vertices found by Metsaev (2005)

OG, M. Vasiliev, 1706.0318

AdS_4 background connections

In two-component spinor notations flat $sp(4)$ connection

$w = (\omega^L{}_{\alpha\beta}, \overline{\omega}^L{}_{\dot\alpha\dot\beta}, h_{\alpha\dot\beta})$:

Lorentz connection $\omega^L{}_{\alpha\beta}$, $\overline{\omega}^L{}_{\dot\alpha\dot\beta}$ + vierbein $h_{\alpha\dot\beta}$ $\qquad \alpha = 1,2\,, \quad \dot\alpha = 1,2$

$H^{\alpha\beta} = H^{\beta\alpha} := h^{\alpha\dot\alpha} h^{\beta}{}_{\dot\alpha}\,, \qquad \overline{H}^{\dot\alpha\dot\beta} = \overline{H}^{\dot\beta\dot\alpha} := h^{\alpha\dot\alpha} h_{\alpha}{}^{\dot\beta}$ the basis 2-forms

Space time coordinates $x^{\alpha\dot\beta} = x^{\underline{n}} \sigma_{\underline{n}}^{\alpha\dot\beta}$, σ - Hermitian 2×2 matrices

Spinorial (twistor) variables

$Y^A = (y^\alpha, \bar{y}^{\dot\alpha}),\; Z^A = (z^\alpha, \bar{z}^{\dot\alpha})$

Fields of the theory: $B(Z;Y;K|x) = C(Y;K|x) + Z(\dots)$ and

$W(Z;Y;K|x) = \omega(Y;K|x)\, d\,x + Z(\dots) + dZ(\dots)\,, \qquad K = (k, \bar{k})$

Spin s dynamical field $C(Y;K|x) = C^{1,0}(Y|x)k + C^{0,1}(Y|x)\bar{k}$

$C^{kj}(y,\bar{y}|x) = \frac{1}{2i} \sum_{|m-n|=2s} \frac{1}{m!n!} C^{kj}{}_{\alpha_1\dots\alpha_n,\dot\beta_1\dots\dot\beta_m}(x) y^{\alpha_1}\dots y^{\alpha_n} \bar{y}^{\dot\beta_1}\dots\bar{y}^{\dot\beta_m}$

Spin s dynamical field ω (for $s \geq 1$) even degree in $k, \bar{k}$

$\omega(y,\bar{y};K|x) = \frac{1}{2i} \sum_{n+m=2(s-1)} \frac{1}{m!n!} \omega_{\alpha_1\dots\alpha_n,\dot\beta_1\dots\dot\beta_m}(K|x) y^{\alpha_1}\dots y^{\alpha_n} \bar{y}^{\dot\beta_1}\dots\bar{y}^{\dot\beta_m}$

Nonlinear higher-spin equations in AdS_4

$$\mathrm{d}W + W * \wedge W = -i\theta_\alpha \wedge \theta^\alpha \left(1 + \eta B * \kappa * k\right) - i\bar{\theta}_{\dot\alpha} \wedge \bar{\theta}^{\dot\alpha} \left(1 + \bar\eta B * \bar\kappa * \bar{k}\right)$$

$$\mathrm{d}B + W * B - B * W = 0$$

$\mathrm{d} = dx^m \frac{\partial}{\partial x^m}$ **space-time de Rham differential** , $\quad \theta = dz,\ \bar\theta = d\bar{z}$

$$\left(f * g\right)\left(Z;Y\right) = \int d^4U d^4V f\left(Z+U;Y+U\right) g\left(Z-V;Y+V\right) \mathrm{e}^{iU_A V^A},$$

$U_A V^A = U^A V^B \epsilon_{AB} = u^\alpha v^\beta \epsilon_{\alpha\beta} + \bar{u}^{\dot\alpha} \bar{v}^{\dot\beta} \epsilon_{\dot\alpha\dot\beta}$

$sp\left(4\right)$**-invariant symplectic form** $\epsilon_{AB} = \left(\epsilon_{\alpha\beta}, \epsilon_{\dot\alpha\dot\beta}\right)$. $\ f * 1 = f$

κ **and** $\bar\kappa$ **-inner Klein operators**

$\kappa := \exp\left(iz_\alpha y^\alpha\right), \qquad \kappa * \kappa = 1, \qquad \kappa * f\left(z^\alpha; y^\alpha; dz^\alpha\right) = f\left(-z^\alpha; -y^\alpha; dz^\alpha\right) * \kappa$

k **and** $\bar{k}$ **-outer Klein operators**

$$k * k = 1, \qquad k * f\left(z^\alpha; y^\alpha; dz^\alpha\right) = f\left(-z^\alpha; -y^\alpha; -dz^\alpha\right) * k\,.$$

Eqs. can be solved perturbativly

Central on-shell theorem

Free unfolded equations Vasiliev (1989)

rank-one equtions

$$\begin{cases} D^{ad}\omega(y,\bar{y}|x) = \frac{i}{4}\Big(\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\overline{y}^{\dot{\alpha}}\partial\overline{y}^{\dot{\beta}}}\ C(0,\overline{y};K|x)k + \bar{\eta}H^{\alpha\beta}\frac{\partial^2}{\partial y^{\alpha}\partial y^{\beta}}\ C(y,0;K|x)\bar{k}\Big) \\ D^{tw}C(y,\bar{y}|x) = 0 \end{cases}$$

$$D^{ad}\omega(y,\bar{y}|x) := D^{L}\omega(y,\bar{y}|x) + \lambda h^{\alpha\dot{\beta}}\Big(y_{\alpha}\frac{\partial}{\partial\bar{y}^{\dot{\beta}}} + \frac{\partial}{\partial y^{\alpha}}\bar{y}_{\dot{\beta}}\Big)\omega(y,\bar{y}|x)\,,$$

$$D^{tw}C(y,\bar{y}|x) := D^{L}C(y,\bar{y}|x) - i\lambda h^{\alpha\dot{\beta}}\Big(y_{\alpha}\bar{y}_{\dot{\beta}} - \frac{\partial^2}{\partial y^{\alpha}\partial\bar{y}^{\dot{\beta}}}\Big)C(y,\bar{y}|x)\,,$$

$$D^{L}f(y,\bar{y}|x) := \mathrm{d}f(y,\bar{y}|x) + \Big(\omega^{L\,\alpha\beta}y_{\alpha}\frac{\partial}{\partial y^{\beta}} + \overline{\omega}^{L\,\dot{\alpha}\dot{\beta}}\bar{y}_{\dot{\alpha}}\frac{\partial}{\partial\bar{y}^{\dot{\beta}}}\Big)f(y,\bar{y}|x)\,.$$

η and $\bar{\eta}$ complex conjugated free parameters, $\lambda^{-1} = \rho$ **radius of** AdS_4

Current equations and current deformations

Rank-two unfolded equations in AdS_4 = current equations

$$D^{tw}_{cur}\,\mathcal{J}(y,\bar{y}|x) = 0$$

OG, Vasiliev (2003)

$$D^{tw}_{cur} = D^L + \lambda e^{\alpha\dot{\beta}}\Big(y^1{}_\alpha\,\bar{y}^1{}_{\dot{\beta}} - y^2{}_\alpha\,\bar{y}^2{}_{\dot{\beta}} - \frac{\partial^2}{\partial y^{1\alpha}\partial\bar{y}^{1\dot{\beta}}} + \frac{\partial^2}{\partial y^{2\alpha}\partial\bar{y}^{2\dot{\beta}}}\Big)$$

Solved by $\mathcal{J}(y_1,y_2,\bar{y}_1,\bar{y}_2;K|x) = C_1(y_1,\bar{y}_1;K|x)C_2(y_2,\bar{y}_2;K|x)$

Schematically for the flat connection $D = \mathrm{d} + w$

$$\begin{cases} D\omega + L(C,\overline{C},w) = 0 \\ DC = 0 \\ D_2\mathcal{J} = 0 \end{cases} \Rightarrow \begin{cases} D\omega + L(C,\overline{C},w) + \Gamma_{cur}(w,\mathcal{J}) = 0 \\ DC + \mathcal{H}_{cur}(w,\mathcal{J}) = 0 \\ D_2\mathcal{J} = 0 \end{cases}$$

$\Gamma_{cur}(w,\mathcal{J})$ **and** $\mathcal{H}_{cur}(w,\mathcal{J})$ **glue rank-one and rank-two modules**

Using consistency conditions OG, M. Vasiliev (2010)

In the unfolded dynamics approach current interactions result from a nontrivial mixing between fields of ranks one and two

Quadratic corrections from nonlinear equations

Quadratic corrections in the zero-form sector

$$D^{tw}C + [\omega\,, C]_* + \mathcal{H}_\eta(w,\mathcal{J}) + \mathcal{H}_{\bar\eta}(w,\mathcal{J}) \;= 0$$

$\mathcal{H}_\eta$ is non-local contains arbitrary degrees of $\partial_{1\alpha}\partial_2{}^\alpha\bar\partial_{1\dot\alpha}\bar\partial_2{}^{\dot\alpha}$

Non straightforward, but effective way to obtain local solution :

field redefinition $C := C + \Phi_\eta(\mathcal{J}) + \bar\Phi_{\bar\eta}(\mathcal{J})$ M. Vasiliev (2015)

$$\widetilde{\mathcal{H}}_\eta(w,\mathcal{J}) \;=\; \mathcal{H}_\eta(w,\mathcal{J}) + D^{tw}\Phi_\eta(\mathcal{J})$$

$$\Phi_\eta(\mathcal{J}) \;=\; \frac{1}{2}\eta\int\frac{dSdT}{2\pi^4}\exp iS_AT^A\int d\tau_i\prod_{i=1}^{3}\theta(\tau_i)\delta'\Big(1-\sum_{i=1}^{3}\tau_i\Big)$$
$$\mathcal{J}(\tau_3 s+\tau_1 y, t-\tau_2 y; \bar y+\bar s, \bar y+\bar t; K)*k$$

$\widetilde{\mathcal{H}}_\eta(w,\mathcal{J})$ **is local, cc is analogous.**

Uniqueness M. Vasiliev (2017)

Quadratic corrections in the one-form sector

$$\mathcal{D}_{ad}\omega + [\omega\,,\omega]_* - L(C - \Phi_\eta(\mathcal{J}) - \bar{\Phi}_{\bar{\eta}}(\mathcal{J})) = \Gamma(w,\mathcal{J}) + Q(C,\omega),$$

$$\Gamma(w,\mathcal{J}) = \Gamma_{\eta\eta}(w,\mathcal{J}) + \Gamma_{\bar{\eta}\bar{\eta}}(w,\mathcal{J})\ , \qquad \Gamma_{\eta\bar{\eta}}(w,\mathcal{J}) = 0$$

$$\Gamma_{\eta\eta} = \ \mathcal{D}_{ad}\Psi \quad -\frac{i\eta^2}{8}\bar{H}^{\dot{\alpha}\,\dot{\beta}}\int_0^1 d\tau \int dS\, dT \int \exp(is_\alpha t^\alpha + i\bar{s}_{\dot{\gamma}}\bar{t}^{\dot{\gamma}})$$
$$(\bar{t}-\bar{s})_{\dot{\alpha}}(\bar{t}-\bar{s})_{\dot{\beta}}\mathcal{J}(-\tau s,\, t,\, \bar{y}+\bar{s},\, \bar{y}+\bar{t})$$

for some Ψ

OG, Vasiliev (2016)

In different form $\Gamma(w,\mathcal{J})$ was obtained by Boulanger, Kessel, Skvortsov and Taronna (2015)

η^2,$\bar{\eta}^2$-independence

Field redefinition $\omega \to \omega - \Psi$

$$\widetilde{\Gamma}_{\eta\eta}(\mathcal{J}) =: \Gamma_{\eta\eta} - \mathcal{D}_{ad}\Psi$$

cancels

$$i\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial\overline{y}^{\dot{\alpha}}\partial\overline{y}^{\dot{\beta}}}\ \Phi_{\eta}(\mathcal{J})(0,\overline{y}|x)$$

resulting from the field redefinition in the zero-form sector via Central on-shell Theorem. cc is analogous

η^2,$\bar{\eta}^2$-independence is in accordance with the result obtained for lower-spin currents from analysis in the zero-form sector

Is in agreement with the old conjecture on self-dual HS theory

$\eta\bar{\eta}$- dependence

Resulting quadratic correction in the one-form sector $\sim \eta\bar{\eta}$:

$$\Gamma_{\eta\bar{\eta}} =: i\bar{\eta} H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\Phi_\eta(\mathcal{J})(y,0|x) + i\eta\overline{H}^{\dot{\alpha}\dot{\beta}}\frac{\partial^2}{\partial \overline{y}^{\dot{\alpha}} \partial \overline{y}^{\dot{\beta}}}\bar{\Phi}_{\bar{\eta}}(\mathcal{J})(0,\overline{y}|x)$$

from the field redefinition in the zero-form sector

As a result

$$\Gamma_{\eta\bar{\eta}} = -\frac{i}{8}\eta\bar{\eta}H^{\alpha\beta}\frac{\partial^2}{\partial y^\alpha \partial y^\beta}\int dS dT \exp i[s_\beta t^\beta + \bar{s}_{\dot{\beta}}\bar{t}^{\dot{\beta}}]\int d\tau_i \prod_{i=1}^{3}\theta(\tau_i)$$
$$\delta'\Big(\Big(1-\sum_{i=1}^{3}\tau_i\Big)\Big)\mathcal{J}(\tau_3 s + \tau_1 y, t - \tau_2 y; \bar{\tau}_3\bar{s} + \bar{\tau}_1\bar{y}, \bar{t} - \bar{\tau}_2\bar{y}; K)\Big|_{y=0} + cc$$

Nonlocal

Nonlocal deformation should be shifted to a local one modulo exact forms

Nonlocal $\to$ Local

$$X(\mathcal{J}) = \frac{i}{8}\eta\bar{\eta}\int d^3\tau d^3\bar{\tau}\Upsilon\delta(1-\tau_3-\tau_2)\delta(1-\bar{\tau}_3-\bar{\tau}_2)\delta'(1-\tau_1-\bar{\tau}_1)\,h(\partial,\bar{\partial})\frac{(1-\tau_3\bar{\tau}_3)}{\tau_2\bar{\tau}_2}$$

$$\exp i\Big(\tau_3\partial_{1\alpha}\partial_2{}^{\alpha}+\bar{\tau}_3\bar{\partial}_{1\dot{\alpha}}\bar{\partial}_2{}^{\dot{\alpha}}\Big)\mathcal{J}(\tau_2\tau_1 y,-\tau_2\bar{\tau}_1 y,\bar{\tau}_2\bar{\tau}_1\bar{y},-\bar{\tau}_2\tau_1\bar{y};K|x)\,,$$

$$\Upsilon = \prod_{i=1,2,3}\theta(\tau_i)\bar{\theta}(\tau_i)$$

$$\mathcal{D}_{ad}X(\mathcal{J}) = \Gamma_{\eta\bar{\eta}} - \Gamma^{loc}_{\eta\bar{\eta}}(\mathcal{J})$$

with

$$\Gamma^{loc}_{\eta\bar{\eta}}(\mathcal{J}) = \frac{i}{8}\eta\bar{\eta}\int dSdT\exp iS_A T^A\int d^3\bar{\tau}d^3\tau\theta(\tau_3)\theta(\bar{\tau}_3)$$

$$\theta(\bar{\tau}_1)\theta(\bar{\tau}_2)\delta(1-\sum_i\tau_i)\delta'(1-\sum_i\bar{\tau}_i)\delta(\tau_1)\delta(\tau_2)\bar{H}^{\dot{\alpha}\dot{\beta}}\bar{\partial}_{\dot{\alpha}}\bar{\partial}_{\dot{\beta}}$$

$$\mathcal{J}(\tau_3 s+\tau_1 y;t-\tau_2 y,\bar{\tau}_3\bar{s}+\bar{\tau}_1\bar{y};\bar{t}-\bar{\tau}_2\bar{y};K)+cc$$

Contains torsion, does not admit flat limit $\Rightarrow$ gauge invariant part should be shifted to a canonical one modulo local exact forms

Gauge invariant current interactions

Let $\mathbf{s}$ - spin of resulting field C,

s_1 and s_2 spins of the constituent fields of $\mathcal{J} \sim C_1 C_2$.

ω -dependent terms can be non-zero at $\mathbf{s} < s_1 + s_2$.

$\Rightarrow$ ω-independent current interactions are in the region

$$\mathbf{s} \geq s_1 + s_2 \, .$$

Local gauge invariant $\rightarrow$ Canonical

Decomposing

$$\Gamma^{loc}_{\eta\bar{\eta}} = \Gamma^{\geq} + \Gamma^{<}$$

$$\Gamma^{\geq}(\mathcal{J}) = \Gamma^{loc}_{\eta\bar{\eta}}(\mathcal{J})\Big|_{\mathrm{s}\geq s_1+s_2}\,, \qquad \Gamma^{<}(\mathcal{J}) = \Gamma^{loc}_{\eta\bar{\eta}}(\mathcal{J})\Big|_{\mathrm{s}<s_1+s_2}$$

There exists local zero-form $\Lambda(f_-, f_+, \mathcal{J})$ such that Vasiliev, OG (2017)

$$\mathcal{D}_{ad}\Lambda = \Gamma^{can} + \Gamma^{\geq} + B$$

where B does not contribute to the dynamical equations if $\mathrm{s} > 1$

reproduce our results of 2010 in the one-form sector

$f_+ = y^{1\nu}y^2{}_\nu - \frac{\partial^2}{\partial\bar{y}^{1\dot{\nu}}\partial\bar{y}^2{}_{\dot{\nu}}}$, $f_- = \overline{f_+}$, generate Howe dual algebra acting on solutions of current equations, *i.e.*, on a space of conserved currents

Current contribution to dynamical equations from the nonlinear HS equations

$$\mathcal{A}_{n,m}(y,\bar{y}) =: \mathcal{A}_{\alpha(n),\dot{\alpha}(m)}\,(y^{\alpha})^{n}\left(\bar{y}^{\dot{\alpha}}\right)^{m}$$

Fronsdal fields are $\omega_{n,m}(x)$ **with** $n=m$ **for bosons and** $|n-m|=1$ **for fermions , spin** $s=\frac{1}{2}(m+n)+1$. **For integer spin** $s\geq 2$

$$\begin{aligned} D^{L}\omega_{\mathrm{s}-1\,,\mathrm{s}-1} &= h(\partial,\bar{y})\omega_{\mathrm{s}\,,\mathrm{s}-2}+h(y,\bar{\partial})\omega_{\mathrm{s}-2\,,\mathrm{s}} \\ D^{L}_{\alpha\dot{\gamma}}\omega_{\mathrm{s}-2\,,\mathrm{s}\alpha}{}^{\dot{\gamma}} &= -\bar{y}_{\dot{\beta}}\partial_{\alpha}\omega_{\mathrm{s}-1\,,\mathrm{s}-1\alpha}{}^{\dot{\beta}}-y_{\alpha}\bar{\partial}_{\dot{\beta}}\omega_{\mathrm{s}-3\,,\mathrm{s}+1\alpha}{}^{\dot{\beta}}+\partial_{\alpha}\partial_{\alpha}\widetilde{\mathcal{J}}_{\mathrm{s},\mathrm{s}} \end{aligned}$$

$$\widetilde{\mathcal{J}}_{\mathrm{s},\mathrm{s}} = i\eta\bar{\eta}\frac{(\mathrm{s}-2)!}{8(2\mathrm{s})!}\sum_{k,m\in[0,\mathrm{s}]}\frac{(m+k)!(2\mathrm{s}-m-k)!}{(\mathrm{s}-k)!k!(\mathrm{s}-m)!m!}\Big(\mathcal{N}_1\Big)^{m}\Big(-\mathcal{N}_2\Big)^{\mathrm{s}-m}\Big(-\overline{\mathcal{N}}_2\Big)^{k}\Big(\overline{\mathcal{N}}_1\Big)^{\mathrm{s}-k}$$

$$\Big\{\sum_{0\leq\mathbf{n}\leq\mathrm{s}}\frac{1}{(\mathrm{s}+\mathbf{n}-1)!}(i\partial_{1\gamma}\partial_{2}{}^{\gamma})^{\mathbf{n}}\sum_{j,l=0,1}C^{j,1-j}(Y^1|x)k^{j}\bar{k}^{1-j}C^{l,1-l}(Y^2|x)k^{l}\bar{k}^{1-l}$$

$$+\sum_{0<\mathbf{n}\leq\mathrm{s}}\frac{1}{(\mathrm{s}+\mathbf{n}-1)!}(i\bar{\partial}_{1\dot{\gamma}}\bar{\partial}_{2}{}^{\dot{\gamma}})^{\mathbf{n}}\sum_{j,l=0,1}C^{j,1-j}(Y^1|x)k^{j}\bar{k}^{1-j}C^{l,1-l}(Y^2|x)k^{l}\bar{k}^{1-l}\Big\}\Big|_{Y^j=0}$$

$$\mathcal{N}_j = y^{\alpha}\partial_{j\alpha}\,,\qquad\qquad \overline{\mathcal{N}}_j=\bar{y}^{\dot{\alpha}}\bar{\partial}_{j\dot{\alpha}}$$

Although $\widetilde{\mathcal{J}}_{\mathrm{s},\mathrm{s}}\sim\eta\bar{\eta}$ **, the current contribution to Fronsdal equtions depends on the phase of** η, **via Central on-shell theorem.**

Number of derivatives

Let helicities of constituent fields be h_1 and h_2.

The number of space-time derivatives in the respective vertices by virtue of unfolded equations is

$$\sharp\partial_x = \mathrm{s} + |h_1 + h_2|$$

Maximal number of derivatives $\sharp\partial_x = \mathrm{s} + s_1 + s_2$ if $h_1 h_2 > 0$

Minimal number of derivatives $\sharp\partial_x = \mathrm{s} + s_1 + s_2 - 2min(s_1, s_2)$ if $h_1 h_2 < 0$

just reproducing the results of Metsaev of 2005 since $\mathrm{s} \geq s_1 + s_2$

Recently the explicit form of corrections to bosonic Fronsdal equations that are generated by canonical currents was obtained and shown to be in full agreement N.Misuna, 2017

with the HS cubic couplings that were found in flat space in lightcone formulation C. Sleight and M. Taronna, 2016

Conclusion

Quadratic corrections in nonlinear equations in the one-form sector are independent of η^2 and $\bar{\eta}^2$

Canonical quadratic corrections do not contribute to torsion-like terms

All improvements are removed, allowing a flat limit.

Our result reproduces all types of vertices with fixed coefficients.