On elementary ***integrating*** and ***approximating*** some `*non-integrable'* models of very early Universe with ***scalar*** or ***vector*** ***`inflatons'*** (results and program).

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The dynamics of any spherical cosmology with a scalar field **(`scalaron')** coupling to gravity is described by the nonlinear second-order differential equations for depending on `time' two metric functions and the scalaron. The equations depend on the scalaron potential and arbitrary gauge function but can be reduced to gauge invariant ones.

Replacing `time' by `metric' allows to **explicitly integrate general isotropic flat model in any gauge and with arbitrary potentials depending on metric** and to derive asymptotically small anisotropic corrections in general scalaron theory.

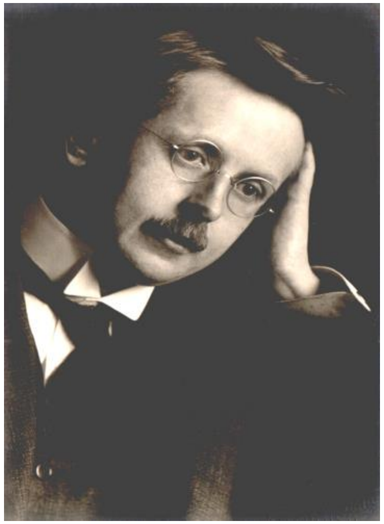
Restrictions on the potentials arise mainly from requirement of **positivity** of the expressions for solutions (essentially, ***canonical momenta squared***).

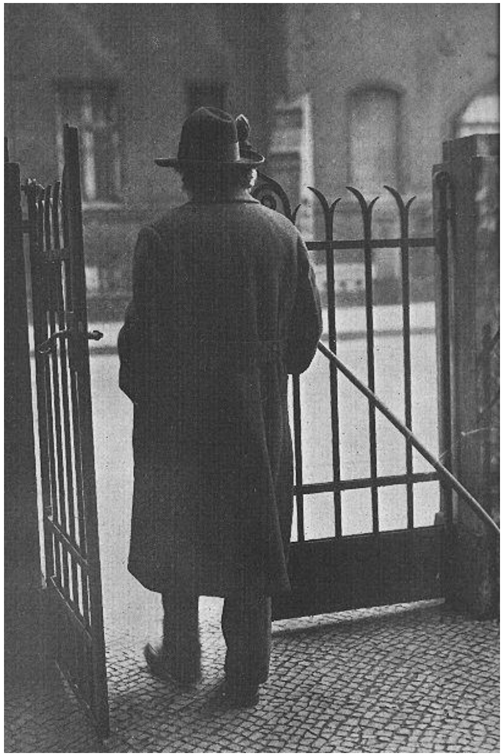
This approach is presently being applied to ***anisotropic models*** with a neutral massive vector field **(`vecton'** **-** ***a Dark Matter candidate*)** based on a remake of Einstein's attempts to construct a simplest affine gravity that in fact **predicts a dynamical effective** ***Dark Energy candidate theory***.

These works related to ideas of **Weyl and Eddington** inspired a ***`revision' of formal cosmology*** in the present report

Ideas of Weyl, Eddington and Einstein

1919 - 1923





arXiv:1011.2445 v1 (gr-qc) vecton as a relativistic particle in gravity field

arXiv:0812.2616 v2 (gr-qc) the first paper on new interpretation of **Einstein's** 3 papers of **1923**; simplified model, static solutions, existence of horizons, non-ntegrability, approximate solutions by various power series expansions.

see also:1003.0782, 1008.2333

Main principles (suggested by Einstein's approach)

1. **Geometry:** **dimensionless** *`action'* constructed of a ***scalar*** *density*; its variations give the geometry and main equations ***without complete specification of the analytic form of the Lagrangian .***
2. **Dynamics:** a concrete Lagrangian constructed of the ***geometric variables*** - homogeneous function of order *D* (e.g. , the square root of the determinant of the curvature) produces a physical **effective Lagrangian**.
3. **Duality** between the geometrical and physical variables and Lagrangians.

NB: This looks more artificial than the first two principles and works for rather special models (and may give *exotic fields*, *tachyons* etc.) (Einstein. did not know this! He was looking for unified theory of EM and Gravity.)

The generalized Einstein (Eddington-Weyl) model in dimension D

The last two terms – pure geometry. The first – a generalization of Einstein's Lagrangian
When D>4, the additional vector components produce scalar fields by dim. red.

$$\mathcal{L}_{eff} = \sqrt{-g}\left[-2\Lambda\left[\det(\delta_i^j + \lambda f_i^j)\right]^{1/(D-2)} + R(g) + c_a\, g^{ij} a_i a_j\right]$$

Restoring the dimensions and expanding the root term up to the second order in the vector and scalar fields.

$$\mathcal{L}_{eff} \cong \sqrt{-g}\left[R[g] - 2\Lambda - \kappa\left(\frac{1}{2}F_{ij}F^{ij} + \mu^2 A_i A^i + g^{ij}\partial_i\psi\,\partial_j\psi + m^2\psi^2\right)\right]$$

$$A_i \sim a_i,\; F_{ij} \sim f_{ij},\; \kappa \equiv G/c^4$$

NB: $\partial_i\psi$ Is proportional to F_{ij} for i < 4, j=4

The **original Einstein square-root Lagrangian** coincides the so-called **DBI** one.
DBI either did not read E-1923 paper or forgot it. Anyway, the author declared it wrong!

Spherical symmetry

$$ds_4^2 = e^{2\alpha} dr^2 + e^{2\beta} d\Omega^2(\theta,\phi) - e^{2\gamma} dt^2 + 2e^{2\delta} dr dt$$

1+1 dimensional Lagrangian for the EW linear model (plus scalar)

$$e^{2\beta}[e^{-\alpha-\gamma}(\dot{A}_1 - A_0')^2 - e^{-\alpha+\gamma}(\psi'^2 + \mu^2 A_1^2) + e^{\alpha-\gamma}(\dot{\psi}^2 + \mu^2 A_0^2) - e^{\alpha+\gamma}(V + 2\Lambda)] + \mathcal{L}_{gr}$$

To get the affine E-type model replace the first term by the Eddington-Einstein (BI) term

$$\mathcal{L}_{gr} \equiv e^{-\alpha+2\beta+\gamma}(2\beta'^2 + 4\beta'\gamma') - e^{\alpha+2\beta-\gamma}(2\dot{\beta}^2 + 4\dot{\beta}\dot{\alpha}) + 2ke^{\alpha+\gamma}$$

Reduction to **cosmological**, **static** (or wave) solutions

$$-\dot{\beta}' - \dot{\beta}\beta' + \dot{\alpha}\beta' + \dot{\beta}\gamma' = \frac{1}{2}[\dot{\psi}\psi' + A_0 A_1]$$

This is one of Einstein's equations corresponding to delta-variations

Separation of variables (dim. red. to static & cosmological states)

$$\alpha = \alpha_0(t) + \alpha_1(r), \quad \beta = \beta_0(t) + \beta_1(r),$$

For waves one uses instead variables like *ar+bt, cr+dt*

NB: To get FRW cosmology we usually take $\dot{\alpha} = \dot{\beta}\,, \quad \gamma' = 0$

Cosmological Lagrangian with scalaron

$$6\bar{k}e^{\alpha+\gamma} - e^{2\beta}[e^{\alpha+\gamma}(V + 2\Lambda) - e^{\alpha-\gamma}(2\dot{\beta}^2 + 4\dot{\beta}\dot{\alpha} - \dot{\psi}^2)]$$

$$-\dot{\beta}' - \dot{\beta}\beta' + \dot{\alpha}\beta' + \dot{\beta}\gamma' = \frac{1}{2}(\dot{\psi}\psi' + A_0 A_1)$$

The momentum constraint severely restricts possible cosmological models that can be derived from the two-dimensional Lagrangian. It is not difficult to find that there are four types of cosmological and static solutions. Two main solutions are: $\beta' = \gamma' = 0$ (‘*general anisotropic*’ cosmology)[4] and $\dot{\alpha} = \dot{\beta}, \gamma' = 0$ (general isotropic, or, FLRW-cosmology). For other solutions of (5) there emerge strong restrictions on $v(\psi)$,

Correction: there are altogether 7 types: 3 cosmologies + 3 dual static sol. + 1 self dual

We call the ‘*special*’ *anisotropic* the cosmology with $\beta' = \gamma'$ and $\dot{\alpha} = 0$, which is dual to FRLW. The *flat isotropic* cosmology is obtained from the general anisotropic one if in addition $\bar{k} = k = 0$ and $\dot{\alpha} = \dot{\beta}$.

Cosmology = grav. , scalaron, vecton: $\mathcal{L}_c = \mathcal{L}_g + \mathcal{L}_s + \mathcal{L}_v$,

$$\mathcal{L}_g \equiv -e^{2\beta+\alpha-\gamma}(2\dot\beta^2 + 4\dot\beta\dot\alpha) - 6k\, e^{\alpha+\gamma} \,.$$

$$\mathcal{L}_s = e^{2\beta+\alpha}\left[e^{-\gamma}\dot\psi^2 - e^{\gamma}v(\psi)\right]$$

\`Linearized' vecton:

$$\mathcal{L}_v = -2\Lambda\, e^{2\beta+\alpha+\gamma} + e^{2\beta-\alpha}\left[e^{-\gamma}\dot A^2 - M^2 e^{\gamma} A^2\right]$$

Remake of Einstein's (1923) effective cosmology: $\mathcal{L}_{ca} = \mathcal{L}_g + \mathcal{L}_a \equiv$

$$\equiv \mathcal{L}_g - e^{2\beta+\alpha+\gamma}\left[2\Lambda\sqrt{1 - \lambda^2\,\dot a^2\, e^{-2(\alpha+\gamma)}} + m^2 a^2 e^{-2\alpha}\right]$$

Renormalization of the a field to ***A.*** Only **2 free parameters remain for vecton**

$$\mathcal{L}_a \equiv -e^{2\beta+\alpha+\gamma}\left[2\Lambda\sqrt{1 - \Lambda^{-1}\dot A^2\, e^{-2(\alpha+\gamma)}} + M^2 A^2 e^{-2\alpha}\right]$$

Anisotropic variables and vecton contribution, *notation*

$$3\,\rho \equiv (\alpha + 2\beta)\,, \quad 3\,\sigma \equiv (\beta - \alpha)\,, \quad \alpha = \rho - 2\sigma$$

$$3\,A_{\pm} = e^{-2\rho+4\sigma}(\dot{A}^2 \pm m^2 e^{2\gamma} A^2)$$

$$\mathcal{L}_c = e^{3\rho-\gamma}(-6\dot{\rho}^2 + 6\dot{\sigma}^2 + \dot{\psi}^2) - 6k\, e^{\rho-2\sigma+\gamma} -$$

$$- e^{3\rho+\gamma}\, v(\psi) + e^{3\rho-\gamma}\, 3A_{-}$$

Anisotropic vecton plus scalaron Hamiltonian constraint

$$\mathcal{H}_c \equiv -6\dot{\rho}^2 + 6\dot{\sigma}^2 + \dot{\psi}^2 + 6k\, e^{2\gamma-2(\rho+\sigma)} +$$

$$+ e^{2\gamma}\, v(\psi) + 3\,A_{+} = 0$$

E.O.M. for the **anisotropic scalaron *plus* linear vecton**

$$\ddot{\rho} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\rho} - e^{2\gamma}\,v(\psi)/2 =$$

$$= 2k\,e^{2\gamma-2(\rho+\sigma)} + (3A_+ - A_-)/4\,,$$

$$\ddot{\sigma} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\sigma} = k\,e^{2\gamma-2(\rho+\sigma)} + A_-\,,$$

$$\ddot{\psi} + (3\,\dot{\rho} - \dot{\gamma})\,\dot{\psi} + e^{2\gamma}\,v'(\psi)/2 = 0\,,$$

$$\ddot{A} + (\dot{\rho} + 4\dot{\sigma} - \dot{\gamma})\dot{A} + e^{2\gamma}\,m^2 A = 0\,.$$

Note **simplicity** of scalaron and vecton equations and

$(3\,\dot{\rho} - \dot{\gamma})$ dependence on the gauge parameter

Important *equation of motion* **independent of scalar potential**

$$\ddot{\rho} - \dot{\rho}\dot{\gamma} + 3\,\dot{\sigma}^2 + \dot{\psi}^2/2 =$$

$$= -k\,e^{2\gamma-2(\rho+\sigma)} - (3A_+ + A_-)/4$$

Generalized Hubble function[6] $H(t) \equiv \dot{\rho}$

Strong restrictions on the generalized Hubble function

$$\dot{H}(t) \equiv \ddot{\rho}(t) \geq 0, \quad \text{if} \quad \gamma'(\rho) \geq 3,\ v \geq 0,\ k \geq 0;$$

$$\dot{H}(t) \leq 0, \quad \text{if} \quad \gamma = 0,\ k \geq 0.$$

Canonical momenta for *graviton, scalaron and vecton*

$$(p_\rho,\, p_\psi,\, p_\sigma) = 2\,e^{3\rho-\gamma}(-6\dot{\rho},\, \dot{\psi},\, 6\,\dot{\sigma})\,, \quad p_A = 2\,e^{\rho+4\sigma-\gamma}\dot{A}$$

1605.03948 hep-th

see also: 1506.01664

A fresh view of cosmological models describing very early Universe: general solution of the dynamical equations.

$$(\dot\rho,\, \dot\psi,\, \dot\sigma) \equiv [\,\xi(\rho)\,, \eta(\rho)\,, \zeta(\rho)\,] = [\,\xi(\rho),\, \xi\,\psi'(\rho),\, \xi\,\sigma'(\rho)\,] \equiv \xi(\rho)[\,1,\, \chi(\rho),\, \omega(\rho)]$$

$\chi(\rho) \equiv \eta/\xi = \psi'(\rho)$ and $\omega(\rho) \equiv \zeta/\xi = \sigma'(\rho)$ are gauge invariant

$$v(\psi) = \bar v[\rho(\psi)] \quad \text{for arbitrary } \bar v(\rho)$$

$$v'(\psi) = \frac{dv}{d\psi} = \frac{dv}{d\rho}\frac{d\rho}{d\psi} = \bar v'(\rho)\frac{\xi}{\eta} = \bar v'(\rho)/\chi(\rho)$$

Gauge invariant Anzatz for solving all equations for vanishing anisotropy

$$[\,x(\rho),\, y(\rho),\, z(\rho)\,] \equiv \exp\,(6\rho - 2\gamma)\,[\,\xi^2(\rho)\,, \eta^2(\rho),\, \zeta^2(\rho)\,]$$

Equations of motion for the **positive square-momentum type variables**

$$y'(\rho) + V'(\rho) - 6V(\rho) = 0\,, \qquad V \equiv e^{6\rho}\,\bar{v}(\rho)$$

$$x'(\rho) - V(\rho) = 4k\,e^{4\rho-2\sigma}\,, \qquad z'(\rho) = 2k\,e^{4\rho-2\sigma}\,\sigma'(\rho).$$

The constraint eqn. $$6\,x(\rho) = y(\rho) + V(\rho) + 6z(\rho) + 6k\,e^{4\rho-2\sigma}\,.$$

$$y(\rho) = 6\left(C_y + \int V(\rho)\right) - V(\rho)$$

The solution of the *psi* equation

Solution of the *ksi* eq.

$$x(\rho) = \left(C_x + \int V(\rho)\right) + 4k\int e^{4\rho-2\sigma(\rho)}$$

Sigma equation

$$x(\rho)\,\sigma'^2(\rho) \equiv C_x - C_y + 2k\int \sigma'(\rho)\,e^{4\rho-2\sigma(\rho)}\,.$$

The fundamental expressions for the solution with vanishing anisotropy

and exact relation between the fundamental cosmolo0gical functions

$$\hat{r}(\rho) \equiv \dot{\psi}^2 e^{-2\gamma}/v(\psi) \;=\; \chi^2(1+6k\,e^{-2\rho}/\bar{v})\,[6(1-\omega^2)-\chi^2]^{-1}$$

$$\hat{r}(\rho) = \frac{6\,C_y}{V(\rho)} + \frac{6}{V}\int V(\rho) - 1 = \frac{6\,C_y}{V(\rho)} + \sum_1^{\infty}(-1)^n\,\frac{\bar{v}^{(n)}(\rho)}{6^n\,\bar{v}(\rho)}$$

$$\chi^2 = 6(1-\omega^2)\left[\sum_1^{\infty}(-1)^n\,\frac{\bar{v}^{(n)}(\rho)}{6^n\,\bar{v}(\rho)} + \frac{6\,C_y}{V}\right] \times \left[1+\sum_1^{\infty}(-1)^n\,\frac{\bar{v}^{(n)}(\rho)}{6^n\,\bar{v}(\rho)} + \frac{6}{V}\left(k\,e^{4\rho-2\sigma}+C_y\right)\right]^{-1}$$

All these formulas are exact, the second is independent on anisotropy

First terms of the exact expression for the transition function

$$\chi^2 = (1-\omega^2)\left[\left(-\bar{l}' + \mathrm{o}(\bar{l}')\right) + 36\, C_y\, \frac{e^{-6\rho}}{\bar{v}(\rho)}\right] \text{ x}$$

$$\left[\left(1 - \frac{1}{6}\bar{l}' + \mathrm{o}(\bar{l}')\right) + 6\,\frac{e^{-2\rho}}{\bar{v}(\rho)}\left(k\, e^{-2\sigma} + C_y\, e^{-4\rho}\right)\right]^{-1}$$

$$\chi^2 = -\bar{l}'(\rho) + \mathrm{o}(\bar{l}') = -\chi\, v'(\psi)/v(\psi) + \ldots$$

$$\chi = -v'(\psi)/v(\psi) + \ldots \equiv -l'(\psi) + \mathrm{o}(l')$$

This provides the relation to approximate standard formulas.

The small anisotropic corrections are derived in asymptotic domain of large ***\rho***

Approximate solution of the anisotropy equation

estimating $\sigma(\rho)$ in the weak anisotropy limit

by asymptotically solving** the **Sigma equation

$$\sigma(\rho) = -3k \int_{\rho}^{\infty} \frac{e^{-2\rho}[\bar{v}(\rho)]^{-1}}{1 + \Sigma_1(\rho) + O(e^{-2\rho})} + O(e^{-4\rho})$$

$$\Sigma_1(\rho) = \sum_{1}^{\infty} (-1)^n \frac{\bar{v}^{(n)}(\rho)}{6^n \, \bar{v}(\rho)} .$$

where **\sigma** defines the ***\x*** and ***\y*** functions of ***\rho***

Why gen. WEE-type generalized models properties are of interest for cosmology?

$$L = -2e^{2\beta}\left[e^{\alpha-\gamma}(\dot{\beta}^2 + 2\dot{\beta}\dot{\alpha}) + \Lambda\sqrt{e^{2(\alpha+\gamma)} - \lambda^2 \dot{A}^2} + \frac{1}{2}\mu^2 A^2 e^{-\alpha+\gamma}\right]$$

In the gauge $\gamma = -\alpha$ and with notation $\alpha = \rho - 2\sigma \text{ and } \beta = \rho + \sigma$

$$\mathcal{L}_c = -2e^{2\beta}\left[3e^{2\alpha}(\dot{\rho}^2 - \dot{\sigma}^2) + \Lambda\sqrt{1 - \lambda^2 \dot{A}^2} + \frac{1}{2}\mu^2 A^2 e^{-2\alpha}\right]$$

$$\mathcal{H} = \bar{c}\sqrt{p_A^2 + M_A^2\,\bar{c}^2} + \mu^2 A^2 e^{2(\beta-\alpha)} + \frac{1}{24}\,e^{2(\beta+\alpha)}\,(p_\sigma^2 - p_\rho^2)$$

Which is zero if there are no other fields. $M_A \equiv 2\lambda^2\Lambda e^{2\beta}$ $\lambda^{-1} \equiv \bar{c}$

In a sense, the WEE vecton looks like a massive particle in a **giant gravitational accelerator**. Just for fan, let us call it **GGA**

Remarks on the vecton theory:

1. The structure of the linearized theory is similar to the scalaron case but anisotropy requires additional efforts.

2. With zero anisotropy, the equations of the linearized theory can be solved, otherwise they give a sort of very useful `sum rules'. Asymptotically small anisotropy approximation can be as effective as in the scalaron case.

3. The most difficult 'large vecton momentum' case for nonlinear vecton may also be treated asymptotically. It is very interesting for transition from inflation to particle production processes, or else, for description of the so called bouncing phenomena.

THE

END

THANK YOU FOR ATTENTION!