Bifurcation sets in non-minimal supersymmetry

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Outline

1. Introduction

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In the simple isoscalar model the standard-like Higgs potential

$$U(\varphi) = -\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4.$$

Two solutions

$$v(0) = 0 \ \text{ and } \ v^2(T) = \frac{\mu^2}{\lambda} - \frac{T^2}{4},$$

demonstrate the second order phase transition at the critical temperature

$$T_c = \frac{2\mu}{\sqrt{\lambda}} = 2v(0),$$

The thermal Higgs boson mass

$$m_h^2 = -\mu^2 + \lambda\frac{T^2}{4}.$$

Introduction

Temperature loop corrections from the stop and other additional scalar states could be large and lead to the first order phase transition, the intensity of the latter depends on

$$\xi = \frac{v(T_c)}{T_c}, \text{ where } v(T_c) = \sqrt{v_1^2(T_c) + v_2^2(T_c)}$$

is the vacuum expectation value at the critical temperature T_c. The electroweak baryogenesis could be explained if

$$\frac{v(T_c)}{T_c} > 1,$$

the case of strong first order phase transition.

Introduction

In a number of analyses the MSSM finite-temperature effective potential is taken in the representation

$$V_{eff}(v,T) = V_0(v_1,v_2,0) + V_1(m(v),0) + V_1(T) + V_{ring}(T), \quad (1)$$

- V_0 **is the tree-level MSSM two-doublet potential at the SUSY scale**
- V_1 **is the (non-temperature) one-loop resumed Coleman-Weinberg term, dominated by stop and sbottom contributions**
- $V_1(T)$ **is the one-loop temperature term**
- V_{ring} **is the correction of re-summed leading infrared contribution from multi-loop ring (or daisy) diagrams**

Effective potential of MSSM

In two-doublet model there are two identical $SU(2)$ doublets of complex scalar fields Φ_1 and Φ_2

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix}$$

with nonzero vacuum expectation values

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}.$$

Neutral components of doublets

$$\phi_1^0(x) = \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1), \quad \phi_2^0(x) = \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2).$$

Effective potential of MSSM

The most general renormalizable hermitian $SU(2) \times U(1)$ invariant potential: [Akhmetzyanova E.N., M.V.D, Dubinin M.N. Soft SUSY Breaking and Explicit CP Violation in the THDM // SQS03 Proc.]

$$U(\Phi_1, \Phi_2) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \overset{*}{\mu_{12}^2}(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\overset{*}{\lambda}_5}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) +$$

$$+\overset{*}{\lambda}_6(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda}_7(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

with effective real parameters μ_1^2, μ_2^2, $\lambda_1, \ldots, \lambda_4$ and complex parameters μ_{12}^2, λ_5, λ_6, λ_7.

Effective potential of MSSM

In the tree approximation on the energy scale M_{SUSY}, the parameters λ_{1-7} are real and are expressed using the coupling constants g_1 and g_2 of electroweak group of the gauge symmetry $SU(2) \otimes U(1)$ as follows:

$$\lambda_1(M_{SUSY}) = \lambda_2(M_{SUSY}) = \frac{1}{4}\left(g_2^2(M_{SUSY}) + g_1^2(M_{SUSY})\right),$$

$$\lambda_3(M_{SUSY}) = \frac{1}{4}\left(g_2^2(M_{SUSY}) - g_1^2(M_{SUSY})\right),$$

$$\lambda_4(M_{SUSY}) = -\frac{1}{2}g_2^2(M_{SUSY}),$$

$$\lambda_5(M_{SUSY}) = \lambda_6(M_{SUSY}) = \lambda_7(M_{SUSY}) = 0.$$

Effective potential of MSSM

The supersymmetric scalar potential of interaction of Higgs bosons with the third generation quark superpartners on the tree level has the form

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\widetilde{Q}}\,,$$

$$\mathcal{V}_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\widetilde{Q}}^2 \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + M_{\widetilde{U}}^2 \widetilde{U}^* \widetilde{U} + M_{\widetilde{D}}^2 \widetilde{D}^* \widetilde{D}\,,$$

$$\mathcal{V}_\Gamma = \Gamma_i^D \left(\Phi_i^\dagger \widetilde{Q}\right) \widetilde{D} + \Gamma_i^U \left(i\Phi_i^T \sigma_2 \widetilde{Q}\right) \widetilde{U} + \overset{*}{\Gamma}{}_i^D \left(\widetilde{Q}^\dagger \Phi_i\right) \widetilde{D}^* - \overset{*}{\Gamma}{}_i^U \left(i\widetilde{Q}^\dagger \sigma_2 \Phi_i^*\right) \widetilde{U}^*\,,$$

$$\mathcal{V}_\Lambda = \Lambda_{ik}^{jl} \left(\Phi_i^\dagger \Phi_j\right) \left(\Phi_k^\dagger \Phi_l\right) + \left(\Phi_i^\dagger \Phi_j\right) \left[\Lambda_{ij}^Q \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + \Lambda_{ij}^U \widetilde{U}^* \widetilde{U} + \Lambda_{ij}^D \widetilde{D}^* \widetilde{D}\right] +$$

$$+ \overline{\Lambda}_{ij}^Q \left(\Phi_i^\dagger \widetilde{Q}\right) \left(\widetilde{Q}^\dagger \Phi_j\right) + \frac{1}{2} \left[\Lambda \epsilon_{ij} \left(i\Phi_i^T \sigma_2 \Phi_j\right) \widetilde{D}^* \widetilde{U} + \text{э.c.}\right]\,, i, j,\, k, l = 1, 2\,,$$

$\mathcal{V}_{\widetilde{Q}}$ denotes the terms of interaction of four scalar quarks.

Bifurcation sets

N	Solutions	Hessian $H(\overline{v}_1, \overline{v}_2)$	local minimum conditions
1	$\overline{v}_1 = 0, \quad \overline{v}_2 = 0$	$-\begin{pmatrix} \overline{\mu}_1^2 & 0 \\ 0 & \overline{\mu}_2^2 \end{pmatrix}$	$\overline{\mu}_1^2 + \overline{\mu}_2^2 < 0, \quad \overline{\mu}_1^2 \cdot \overline{\mu}_2^2 \geq 0$
2	$\overline{v}_1 = 0, \quad \lambda_2 \overline{v}_2^2 - \overline{\mu}_2^2 = 0$	$\begin{pmatrix} -\overline{\mu}_1^2 + \frac{\lambda_{345}}{2}\overline{v}_2^2 & 0 \\ 0 & 2\lambda_2\overline{v}_2^2 \end{pmatrix}$	$-\overline{\mu}_1^2 + \overline{v}_2^2(2\lambda_2 + \frac{1}{2}\lambda_{345}) > 0$ $(-\overline{\mu}_1^2 + \frac{1}{2}\lambda_{345}\overline{v}_2^2)\lambda_2\overline{v}_2^2 \geq 0$
3	$\overline{v}_2 = 0, \quad \lambda_1 \overline{v}_2^2 - \overline{\mu}_1^2 = 0$	$\begin{pmatrix} 2\lambda_1\overline{v}_1^2 & 0 \\ 0 & -\overline{\mu}_2^2 + \frac{\lambda_{345}}{2}\overline{v}_1^2 \end{pmatrix}$	$-\overline{\mu}_2^2 + \overline{v}_1^2(2\lambda_1 + \frac{1}{2}\lambda_{345}) > 0$ $(-\overline{\mu}_2^2 + \frac{1}{2}\lambda_{345}\overline{v}_1^2)\lambda_1\overline{v}_1^2 \geq 0$
4	$\lambda_1\overline{v}_1^2 + \frac{\lambda_{435}}{2}\overline{v}_2^2 - \overline{\mu}_1^2 = 0,$ $\lambda_2\overline{v}_2^2 + \frac{\lambda_{435}}{2}\overline{v}_1^2 - \overline{\mu}_2^2 = 0$	$\begin{pmatrix} 2\lambda_1\overline{v}_1^2 & \lambda_{345}\overline{v}_1\overline{v}_2 \\ \lambda_{345}\overline{v}_1\overline{v}_2 & 2\lambda_2\overline{v}_2^2 \end{pmatrix}$	$\lambda_1\overline{v}_1^2 + \lambda_2\overline{v}_2^2 > 0$ $\overline{v}_1^2\overline{v}_2^2(4\lambda_1\lambda_2 - \lambda_{345}^2) \geq 0$

Finite temperature corrections of squarks

In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies $\omega_n = 2\pi n T$ $(n = 0, \pm 1, \pm 2, ...)$, lead to structures of the form

$$I[m_1, m_2, ..., m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \prod_{i=1}^{b} \frac{(-1)^b}{(\mathbf{k}^2 + \omega_n^2 + m_j^2)}, \qquad (2)$$

$\mathbf{k}$ is the three-dimensional momentum in a system with the temperature T.

Finite temperature corrections of squarks

At $n \neq 0$ the result is

$$I[m_1, m_2, ..., m_b] = 2T\,(2\pi T)^{3-2b}\,\frac{(-1)^b \pi^{3/2}}{(2\pi)^3}\,\frac{\Gamma(b-3/2)}{\Gamma(b)}\,S(M, b{-}3/2), \quad (3)$$

where

$$S(M, b{-}3/2) = \int \{dx\} \sum_{n=1}^{\infty} \frac{1}{(n^2 + M^2)^{b-3/2}}, \qquad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2.$$

Finite temperature corrections of squarks

We calculate the integral

$$J_0[a_1, a_2] = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)(\mathbf{k}^2 + a_2^2)} = \frac{1}{4\pi(a_1 + a_2)},$$

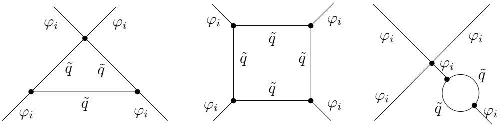
taking a residue in the spherical coordinate system.
$a_{1;2}^2$ are the sums of squared frequency and squared mass.
Derivatives of J_0 with respect to a_1 and a_2 can be used for calculation of integrals

$$J_1[a_1, a_2] = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)} = \frac{1}{8\pi a_1(a_1 + a_2)^2},$$

$$J_2[a_1, a_2] = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)^2} = \frac{1}{8\pi a_1 a_2(a_1 + a_2)^3}.$$

Squarks corrections

Threshold corrections (left and central diagram) and diagram contributing to the wave-function renormalization (right): [E. N. Akhmetzyanova, M. V. D., M. N. Dubinin Supersymmetric Threshold Corrections to the Higgs Sector in the Minimal Supersymmetric Model with Explicit CP Violation // The International Workshop "Supersymmetries and Quantum Symmetries" - (SQS'05 Proc.)]



Finite temperature corrections of squarks

Calculation of the finite-temperature diagrams for the general case of complex-valued μ and $A_{t,b}$ gives the result

$$\Delta\lambda_1^{thr} = 3h_t^4|\mu|^4 I_2[m_Q, m_U] + 3h_b^4|A_b|^4 I_2[m_Q, m_D] +$$
$$+h_t^2|\mu|^2\left(-\frac{g_1^2 - 3g_2^2}{2} I_1[m_Q, m_U] + 2g_1^2 I_1[m_U, m_Q]\right)$$
$$+h_b^2|A_b|^2\left(\frac{12h_b^2 - g_1^2 - 3g_2^2}{2} I_1[m_Q, m_D] + (6h_b^2 - g_1^2) I_1[m_D, m_Q]\right)$$

$$\Delta\lambda_2^{thr} = 3h_t^4|A_t|^4 I_2[m_Q, m_U] + 3h_b^4|\mu|^4 I_2[m_Q, m_D] +$$
$$+h_b^2|\mu|^2\left(\frac{g_1^2 + 3g_2^2}{2} I_1[m_Q, m_D] + g_1^2 I_1[m_D, m_Q]\right) +$$
$$+h_t^2|A_t|^2\left(\frac{12h_t^2 + g_1^2 - 3g_2^2}{2} I_1[m_Q, m_U] + (6h_t^2 - 2g_1^2) I_1[m_U, m_Q]\right)$$

Effective potential of NMSSM

In the NMSSM two identical scalar $SU(2)$ doublets of the complex scalar fields Φ_1 and Φ_2 are introduced

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix} = \begin{pmatrix} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix},$$

$$\Phi_2 = \begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix} = \begin{pmatrix} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{pmatrix}$$

Singlet superfield S:

$$S = \frac{1}{\sqrt{2}}(v_3 + s_1 + is_2).$$

Effective potential of NMSSM

The most general Hermitian form of the renormalized $SU(2) \times U(1)$ invariant potential for system of fields has the form:

$$U(\Phi_1, \Phi_2, S) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_3^2 S^* S - (\mu_{12}^2(\Phi_1^\dagger\Phi_2) + h.c.)$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\overset{*}{\lambda}_5}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + h.c.$$

$$+k_1(\Phi_1^\dagger\Phi_1)S^* S + k_2(\Phi_2^\dagger\Phi_2)S^* S + (k_3(\Phi_1^\dagger\Phi_2)S^* S + h.c.)k_4(S^* S)^2 +$$

$$+k_5(\Phi_1^\dagger\Phi_1)S + k_6(\Phi_2^\dagger\Phi_2)S + k_7(\Phi_1^\dagger\Phi_2)S + \overset{*}{k}_7\,(\Phi_2^\dagger\Phi_1)S^* + k_8 S^3.$$

Parameters of Effective Potential of NMSSM

In the tree approximation on the M_{SUSY} mass-energy scale parameters λ_i, κ_j expressed as:

$$\lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{8}, \ \lambda_3 = \frac{g_2^2 - g_1^2}{4}, \ \lambda_4 = -\frac{g_2^2}{2}, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0 \quad (4)$$

$$k_1 = |\lambda|^2, \ \ k_2 = |\lambda|^2, \ \ k_3 = \lambda k^*, \ \ k_4 = |k|^2, \ \ k_5 = \lambda A_\lambda, \ \ k_6 = \frac{1}{3} k A_k, \quad (5)$$

Free parameters of the model are chosen in the range possible values:

$$1.0 < tg\beta \leq 60, \quad M_1 = M_2, \quad 100\,\text{GeV} \leq M_2 \leq 2000\,\text{GeV},$$

$$0.0001 \leq \lambda \leq 0.7, \quad 0 \leq \kappa \leq 0.65.$$

$$0\,\text{GeV} \leq A_\lambda \leq 1000\,\text{GeV}, \quad -100\,\text{GeV} \leq A_\kappa \leq -10\,\text{GeV}$$

Parameters of Effective Potential of NMSSM

The supersymmetric scalar potential of interaction of Higgs bosons with the third generation quark superpartners on the tree level has the form $V =$

$$|y_u(\widetilde{Q}\epsilon H_u)|^2+|y_d(\widetilde{Q}\epsilon H_d)|^2+|y_u\widetilde{u}_R^* H_u^0-y_d\widetilde{d}_R^* H_d^-|^2+|y_d\widetilde{d}_R^* H_d^0-y_d\widetilde{u}_R^* H_u^+|^2-$$

$$-y_u(\widetilde{u}_R\widetilde{u}_L^*\lambda S H_d^0+\widetilde{u}_R\widetilde{d}_L^*\lambda S H_d^-+c.c.)-y_d(\widetilde{d}_R\widetilde{d}_L^*\lambda S H_u^0+\widetilde{d}_R\widetilde{d}_L^*\lambda S H_u^++c.c.)+$$

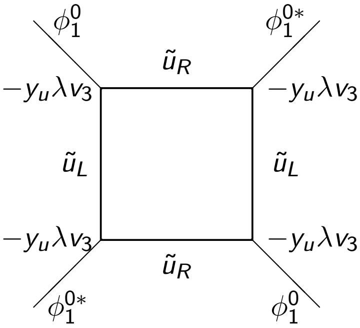
$$+\frac{g_2^2}{8}(4|H_d^\dagger\widetilde{Q}|^2-2(H_d^\dagger H_d)(\widetilde{Q}^\dagger\widetilde{Q})+4|H_u^\dagger\widetilde{Q}|^2-2(H_u^\dagger H_u)(\widetilde{Q}^\dagger\widetilde{Q}))+$$

$$+\frac{g_1^2}{2}(\frac{1}{6}(\widetilde{Q}^\dagger\widetilde{Q})-\frac{2}{3}\widetilde{u}_R^*\widetilde{u}_R+\frac{1}{3}\widetilde{d}_R^*\widetilde{d}_R+\frac{1}{2}(H_u^\dagger H_u)-\frac{1}{2}(H_d^\dagger H_d))^2+$$

$$+(\widetilde{u}_R^* y_u A_u(\widetilde{Q}^T\epsilon H_u)-\widetilde{d}_R y_d A_d(\widetilde{Q}^T\epsilon H_d)+c.c.)$$

Parameters of Effective Potential of NMSSM

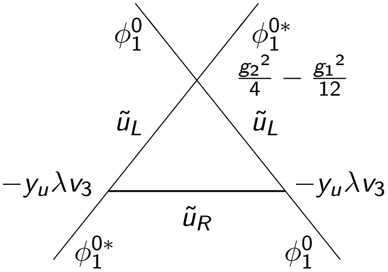
The oneloop corrections to the effective potential parameters



$$(y_u\lambda v_3)^4 I_2[m_Q, m_U]$$

Parameters of Effective Potential of NMSSM

The one-loop corrections to the effective potential parameters



$$(-y_u\lambda v_3)^2\left(\frac{{g_2}^2}{4} - \frac{{g_1}^2}{12}\right)I_1[m_Q, m_U]$$

Effective Potential Parameters in NMSSM

One-loop corrections to the effective potential parameters

$$\Delta\lambda_1 = h_u^4\lambda^4 v_3^4 I_2[m_Q, m_U] + h_d^4 A_d^4 I_2[m_Q, m_D] +$$
$$+ h_u^2\lambda^2 v_3^2 \left(\left(\frac{g_2^2}{4} - \frac{g_1^2}{12} \right) I_1[m_Q, m_U] + \frac{1}{3} g_1^2 I_1[m_U, m_Q] \right) +$$
$$+ h_d^2 A_d^2 \left(\left(h_d^2 - \frac{g_1^2}{12} - \frac{g_2^2}{4} \right) I_1[m_Q, m_D] + \left(h_d^2 - \frac{g_1^2}{6} \right) I_1[m_D, m_Q] \right)$$

$$\Delta\lambda_2 = h_u^4 A_u^4 I_2[m_Q, m_U] + h_d^4\lambda^4 v_3^4 I_2[m_Q, m_D] +$$
$$+ h_u^2 A_u^2 \left(\left(\frac{g_1^2}{12} - \frac{g_2^2}{4} \right) I_1[m_Q, m_U] + (h_u^2 - \frac{1}{3} g_1^2 I_1[m_U, m_Q] \right) +$$
$$+ h_d^2\lambda^2 v_3^2 \left(\left(\frac{g_1^2}{12} + \frac{g_2^2}{4} \right) I_1[m_Q, m_D] + \frac{g_1^2}{6} I_1[m_D, m_Q] \right)$$

Higgs bosons masses in NMSSM

Mass matrix of the Higgs bosons:

$$m^2 = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} \end{pmatrix},$$

and its elements are defined as the second derivatives of the Higgs potential $m_{ij} = \frac{\partial^2 V}{\partial\phi_i \partial\phi_j}$

$$m_{11} = \frac{1}{4}v^2(\lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + 2\sin^2\beta\cos^2\beta(-Im\lambda_5 \sin(2\theta) + \lambda_3 +$$

$$+\lambda_4 - Re\lambda_5 \cos(2\theta)) + 4\sin\beta\cos^3\beta(Re\lambda_6 \sin\theta - Im\lambda_6 \cos\theta) +$$

$$+4\sin^3\beta\cos\beta(Re\lambda_7 \sin\theta - Im\lambda_7 \cos\theta))$$

solutions	Hessian $H(v_1, v_2, v_3)$	local minimum conditions
$v_1 = 0$, $v_2 = 0$, $v_3 = 0$	$\begin{pmatrix} -\mu_1^2 & 0 & 0 \\ 0 & -\mu_2^2 & 0 \\ 0 & 0 & -2\mu_3^2 \end{pmatrix}$	$-\mu_1^2 - \mu_2^2 - 2\mu_3^2 > 0,$ $\mu_1^2 \cdot \mu_2^2 \cdot \mu_3^2 < 0.$
$v_1 \neq 0$, $v_2 = 0$, $v_3 = 0$	$\begin{pmatrix} v_1^2\lambda_1 & 0 & 0 \\ 0 & \frac{1}{2}v_1^2\lambda_{34} - \mu_2^2 & k_5 v_1 \\ 0 & k_5 v_1 & k_1 v_1^2 - 2\mu_3^2 \end{pmatrix}$	$k_1 v_1^2 - \mu_2^2 - 2\mu_3^2 + \lambda_1 v_1^2 + \frac{1}{2}(\lambda_3 + \lambda_4)v_1^2 > 0,$ $\lambda_1 v_1^2\{(\left(k_1 v_1^2 - 2\mu_3^2\right) \left(\frac{1}{2}\lambda_{34} v_1^2 - \mu_2^2\right) - k_5^2 v_1^2)\} > 0.$
$v_1 = 0$, $v_2 \neq 0$, $v_3 = 0$	$\begin{pmatrix} \frac{1}{2}v_2^2\lambda_{34} - \mu_1^2 & 0 & k_5 v_2 \\ 0 & v_2^2\lambda_2 & 0 \\ k_5 v_2 & 0 & k_2 v_2^2 - 2\mu_3^2 \end{pmatrix}$	$k_2 v_2^2 - \mu_1^2 - 2\mu_3^2 + \lambda_2 v_2^2 + \frac{1}{2}(\lambda_3 + \lambda_4)v_2^2 > 0,$ $\lambda_2 v_2^2\{\left(k_2 v_2^2 - 2\mu_3^2\right) \left(\frac{1}{2}\lambda_{34} v_2^2 - \mu_1^2\right) - k_5^2 v_2^2\} > 0.$

Bifurcation sets

Solutions	Hessian $H(v_1, v_2, v_3)$	Local minimum conditions
$v_1 \neq 0$ $v_2 \neq 0$ $v_3 = 0$	$\begin{pmatrix} v_1^2\lambda_1 & v_1 v_2 \lambda_{34} & 0 \\ v_1 v_2(\lambda_3 + \lambda_4) & v_2^2 \lambda_2 & 0 \\ 0 & 0 & k_1 v_1^2 + 2k_3 v_2 v_1 + k_2 v_2^2 - 2\mu_3^2 \end{pmatrix}$	$Det > 0, \ \ Tr > 0$
$v_1 \neq 0$ $v_2 = 0$ $v_3 \neq 0$	$\begin{pmatrix} v_1^2\lambda_1 & 0 & 2k_1 v_1 v_3 \\ 0 & \frac{1}{2}\left(\lambda_{34} v_1^2 + 2k_2 v_3^2 - 2\mu_2^2\right) & k_3 v_1 v_3 \\ 2k_1 v_1 v_3 & k_3 v_1 v_3 & 2v_3(3k_6 + 4k_4 v_3) \end{pmatrix}$	$Det > 0, \ \ Tr > 0$
$v_1 = 0$ $v_2 \neq 0$ $v_3 \neq 0$	$\begin{pmatrix} \frac{1}{2}\left(\lambda_{34} v_2^2 + 2k_1 v_3^2 - 2\mu_1^2\right) & 0 & k_3 v_2 v_3 \\ 0 & v_2^2\lambda_2 & 2k_2 v_2 v_3 \\ k_3 v_2 v_3 & 2k_2 v_2 v_3 & 2v_3(3k_6 + 4k_4 v_3) \end{pmatrix}$	$Det > 0, \ \ Tr > 0$
$v_1 \neq 0$ $v_2 \neq 0$ $v_3 \neq 0$	$\begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{pmatrix}$ $H_{11} = v_1^2\lambda_1 - v_2 v_3(k_5 + k_3 v_3)/v_1,$ $H_{12} = H_{21} = v_3(k_5 + k_3 v_3) + v_1 v_2(\lambda_3 + \lambda_4),$ $H_{13} = H_{31} = k_5 v_2 + 2(k_1 v_1 + k_3 v_2) v_3,$	$* * *$

$* * *$ The last case $(v_1 \neq 0,\ v_2 \neq 0,\ v_3 \neq 0)$:

$$-\frac{k_5 v_1 v_2}{v_3} + 8k_4 v_3^2 + 6k_6 v_3 - v_3(k_3 v_3 + k_5)\frac{v_1^2 + v_2^2}{v_1 v_2} + \lambda_1 v_1^2 + \lambda_2 v_2^2 > 0,$$

$$\frac{1}{v_1 v_2 v_3} \cdot \Bigg(v_3\Big(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3\Big)\Big(v_1 v_2\Big(k_5 v_1 + 2k_3 v_1 v_3 + 2k_2 v_2 v_3\Big)\times$$

$$\times\Big(k_3 v_3^2 + k_5 v_3 + v_1 v_2(\lambda_3 + \lambda_4)\Big) - v_1\Big(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3\Big)\Big(- k_3 v_1 v_3^2 -$$

$$-k_5 v_1 v_3 + \lambda_2 v_2^3\Big)\Big) - v_3\Big(k_5 v_1 + 2k_3 v_1 v_3 + 2k_2 v_2 v_3\Big)\Big(v_2\Big(k_5 v_1 + 2k_3 v_1 v_3 +$$

$$+2k_2 v_2 v_3\Big)\Big(- k_3 v_2 v_3^2 - k_5 v_2 v_3 + \lambda_1 v_1^3\Big) - v_1 v_2\Big(k_5 v_2 + 2k_1 v_1 v_3 + 2k_3 v_2 v_3\Big)\times$$

$$\times\Big(k_3 v_3^2 + k_5 v_3 + v_1 v_2(\lambda_3 + \lambda_4)\Big)\Big) + \Big(8k_4 v_3^3 + 6k_6 v_3^2 - k_5 v_1 v_2\Big)\Big(\Big(- k_3 v_2 v_3^2 -$$

$$-k_5 v_2 v_3 + \lambda_1 v_1^3\Big)\Big(- k_3 v_1 v_3^2 - k_5 v_1 v_3 + \lambda_2 v_2^3\Big) - v_1 v_2\Big(k_3 v_3^2 + k_5 v_3 +$$

$$+v_1 v_2(\lambda_3 + \lambda_4)\Big)^2\Big)\Bigg) > 0.$$

Conclusion

- Our analysis of the effective MSSM and NMSSM finite-temperature potentials is based on the calculation of various one-loop temperature corrections from the squark-Higgs boson sector for the case of nonzero trilinear parameters A_t, A_b and Higgs superfield parameter μ.
- Quantum corrections are incorporated in control parameters $\lambda_{1,...7...}(T)$ of the effective two-doublet (+singlet) potential, which is then explicitly rewritten in terms of Higgs boson mass eigenstates.
- Bifurcation sets types for the two-Higgs-doublet(+singlet) potential $U_{eff}(v_1, v_2)$ are determined.

Summary

1. Bifurcation sets for Higgs potential at the case of Peccei–Quinn symmetry are obtain. These sets always describe system in the local minimum with the critical morse point. ———
2. Constrains on MSSM and NMSSM allowed parameter space are evaluated
   at the presence of the effective potential local minimum. ———
3. Higgs prepotential as canonical morse form and non-morse term (catastrophe function at critical temperature) are reconstructed. ———