

ONE-LOOP DIVERGENCES IN SIX DIMENSIONAL $\mathcal{N} = (1, 0)$ AND $\mathcal{N} = (1, 1)$ SUPERSYMMETRIC GAUGE THEORIES

I.L. Buchbinder

Tomsk

International Workshop "SUPERSYMMETRY AND QUANTUM
SYMMETRIES - SQS'2017", JINR, Dubna, July 31-August 5, 2017

- Construction of the background field methods and getting the manifestly supersymmetric and gauge invariant effective action for $6D, \mathcal{N} = (1, 0)$ non-Abelian vector multiplet coupled to hypermultiplet
- Calculation of the one-loop off-shell divergences in vector multiplet and hypermultiplet sectors for arbitrary $\mathcal{N} = (1, 0)$ gauge theory
- Analysis of divergences in the $\mathcal{N} = (1, 1)$ SYM theory and prove that the one-loop off-shell divergence are completely absent

Based on:

I.L.B, E.A. Ivanov, K.V. Stepanayantz, B.M. Merzlikin, Phys.Lett, B763, 375, 2016, arXiv:1609.00975[hep-th];

I.L.B, E.A. Ivanov, K.V. Stepanayantz, B.M. Merzlikin, JHEP, 01 (2017) 128, arXiv:1612.03190[hep-th].

I.L.B, E.A. Ivanov, K.V. Stepanayantz, B.M. Merzlikin, Nucl. Phys., B921 (2017) 127, arXiv:1704.02530[hep-th]

The modern interest to $6D$ supersymmetric gauge theories is stipulated by the following problems:

1. Problem of describing the quantum structure of six-dimensional supersymmetric gauge theories dimensionally reduced from superstrings (N. Seiberg, E. Witten, 1996; N. Seiberg, 1997).
2. Problem of field description of the interacting multiple $M5$ -branes (see e.g. review J. Bagger, N. Lambert, S. Mikhu, C. Papageorgakis, Phys.Repts. 527 (2013) 1).
 - Hypothetic M -theory is characterized by two extended objects: $M2$ -brane and $M5$ -brane.
 - The field description of interacting multiple $M2$ -branes is given by Bagger-Lambert-Gustavsson (BGL) theory which is $3D$, $\mathcal{N} = 8$ supersymmetric gauge theory.
 - Lagrangian description of the interacting multiple $M5$ -branes is not constructed.

3. Problem of miraculous cancellation of some on-shell divergences in higher dimensional maximally supersymmetric gauge theories (theories with 16 supercharges). All these theories are non-renormalizable by power counting.

- Field limit of superstring amplitude shows that $6D, \mathcal{N} = (1, 1)$ SYM theory is on-shell finite at one-loop (M.B. Green, J.H. Schwarz, L. Brink, 1982).
- Analysis based on on-shell supersymmetries, gauge invariance and field redefinitions (P.S. Howe, K.S. Stelle, 1984, 2003; G. Bossard, P.S. Howe, K.S. Stelle, 2009).
- Direct one-loop and two-loop component calculations (mainly in bosonic sector and mainly on-shell) (E.S. Fradkin, A.A. Tseytlin, 1983; N. Marcus, A. Sagnotti, 1984, 1985.)
- Direct calculations of scattering amplitudes in $6D$ theory up to five loops and in $D8, 10$ theories up to four loops (L.V. Bork, D.I. Kazakov, M.V. Kompaniets, D.M. Tolkachev, D.E. Vlasenko, 2015).

Results: On-shell divergences in $6D$ theory start at three loops. One-shell divergences in $8D$ and $10D$ theories start at one loop.

Purpose: to show that the $\mathcal{N} = (1, 1)$ SYM theory is off-shell finite at one-loop in spite of it is non-renormalizable on power counting

$6D, \mathcal{N} = (1, 1)$ SYM theory possesses some properties close or analogous to $4D, \mathcal{N} = 4$ SYM theory.

- The $6D, \mathcal{N} = (1, 1)$ SYM theory can be formulated in harmonic superspace as well as the $4D, \mathcal{N} = 4$ SYM theory
- The $6D, \mathcal{N} = (1, 1)$ SYM theory possesses the manifest $\mathcal{N} = (1, 0)$ supersymmetry and additional hidden $\mathcal{N} = (0, 1)$ supersymmetry analogous to $4D, \mathcal{N} = 4$ SYM theory where there is the manifest $\mathcal{N} = 2$ supersymmetry and additional hidden $\mathcal{N} = 2$ supersymmetry
- The $6D, \mathcal{N} = (1, 1)$ SYM theory as well as the $4D, \mathcal{N} = 4$ SYM theory is characterized by the non-trivial moduli space
- The $6D, \mathcal{N} = (1, 1)$ SYM theory is anomaly free as well as the $4D, \mathcal{N} = 4$ SYM theory and satisfies some non-renormalization theorems

Reminder of the notions

- Supersymmetry is extension of special relativity symmetry by fermionic generators.
- Supersymmetry unifies the bosonic and fermionic fields into one supermultiplet.
- Supersymmetric field models can be formulated in terms of conventional bosonic and fermionic fields. Component approach.
- In some cases the supersymmetric field models can be formulated in terms of superfields. A superfield depends on space-time coordinates x and some number of anticommuting (Grassmann) coordinates θ . The coefficients of expansion of superfield in anticommuting coordinates are the conventional bosonic and fermionic fields of supermultiplet.
- Advantage of component formulation: close relation with conventional field theory, convenient in classical field theory and to calculate the scattering amplitudes. Disadvantage: supersymmetry is not manifest.
- Advantage of superfield formulation: manifest supersymmetry, convenient in quantum field theory to study off-shell effects, simple proof of the non-renormalization theorems.
- Problem of unconstrained formulation. Harmonic superspace approach.

- Review part
 1. $6D$ supersymmetry
 2. $6D, \mathcal{N} = (1, 0)$ harmonic superspace
 3. $6D, \mathcal{N} = (1, 0)$ hypermultiplet
 4. $6D, \mathcal{N} = (1, 0)$ vector multiplet
 5. Action for vector multiplet coupled to hypermultiplet
 6. $\mathcal{N} = (1, 1)$ SYM theory in terms of $\mathcal{N} = (1, 0)$ harmonic supersfields
- Background field method
- Structure of one-loop counterterms
- Divergent part of one-loop effective action
- Summary

P.S. Howe, G. Sierra, P.K. Townsend, 1983.

6D Minkowski space

- Coordinates x^M , $M = 0, 1, 2, 3, 4, 5$
- Metric $\eta_{MN} = \text{diag}(1, -1, -1, -1, -1, -1)$
- Proper Lorentz group $SO(1, 5)$

Two types of 6D Spinors

- Left $(1, 0)$ spinors ψ_a , $a = 1, 2, 3, 4$
- Right $(0, 1)$ spinors ϕ^a , $a = 1, 2, 3, 4$

Dirac matrices

- 8×8 Dirac matrices Γ_M ,

$$\Gamma_M \Gamma_N + \Gamma_N \Gamma_M = 2\eta_{MN}$$

- Representation of the Dirac matrices

$$\Gamma_M = \begin{pmatrix} 0 & \tilde{\gamma}_M \\ -\gamma_M & 0 \end{pmatrix},$$

- Antisymmetric 4×4 Pauli-type matrices γ_M and $\tilde{\gamma}_M$,

$$\gamma_M \tilde{\gamma}_N + \gamma_N \tilde{\gamma}_M = -2\eta_{MN}$$

$$(\tilde{\gamma}_M)^{ab} = \frac{1}{2} \epsilon^{abcd} (\gamma_M)_{cd}$$

- Spinor representation of the vectors, $V_{ab} = \frac{1}{2} (\gamma^M)_{ab} V_M$

6D superalgebra

- Two types of independent supercharges
 $Q_a^I, Q_J^a, I = 1, \dots, 2m; J = 1, \dots, 2n$
- Anticommutational relations for supercharges

$$\{Q_a^I, Q_b^K\} = 2\Omega^{IK} P_{ab}$$

$$\{Q_J^a, Q_L^b\} = 2\Omega_{JL} P^{ab}$$

Matrix Ω_{IK} belongs to $USp(2n)$ group (R-symmetry group), $\Omega_{IK}\Omega^{KJ} = \delta_I^J$

- $\mathcal{N} = (m, n)$ supersymmetry
- $\mathcal{N} = (1, 0)$ superspace
 Coordinates $z = (x^M, \theta_i^a), i = 1, 2$
- Basic spinor derivatives

$$D_a^i = \frac{\partial}{\partial \theta_i^a} - i\theta^{ib} \partial_{ab}, \quad \{D_a^i, D_b^j\} = -2i\Omega^{ij} \partial_{ab}$$

Harmonic superspace

Basic references:

4D

A.Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky, E. Sokatchev, 1985.

A.Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic Superspace, 2001.

General purpose: to formulate $\mathcal{N} = 2$ models in terms of unconstrained $\mathcal{N} = 2$ superfields. General idea: to use the parameters $u^{\pm i} (i = 1, 2)$ (harmonics) related to $SU(2)$ automorphism group of the $\mathcal{N} = 2$ superalgebra and parameterizing the 2-sphere,

$$u^{+i}u_i^- = 1$$

It allows to introduce the $\mathcal{N} = 2$ superfields with the same number of anticommuting coordinates as in case of the $\mathcal{N} = 1$ supersymmetry. Prices for this are the extra bosonic variables, harmonics $u^{\pm i}$.

6D

P.S. Howe, K.S. Stelle, P.C. West, 1985.

B.M. Zupnik, 1986; 1999.

G. Bossard, E. Ivanov, A. Smilga, JHEP, 2015.

(1, 0) harmonic superspace

- $USp(2) \sim SU(2)$, $I = i$ The same harmonics $u^{\pm i}$ as in 4D, $\mathcal{N} = 2$ supersymmetry
- Harmonic 6D, (1, 0) superspace with coordinates $Z = (x^M, \theta_i^a, u_i^{\pm})$
- Analytic basis $Z_{(an)} = (x_{(an)}^M, \theta^{\pm a}, u_i^{\pm})$,
 $x_{(an)}^M = x^M + \frac{i}{2} \theta^{-a} (\gamma^M)_{ab} \theta^{+b}$, $\theta^{\pm a} = u_i^{\pm} \theta^{ai}$
 The coordinates $\zeta = (x_{(an)}^M, \theta^{\pm a}, u_i^{\pm})$ form a subspace closed under (1, 0) supersymmetry
- The harmonic derivatives

$$D^{++} = u^{+i} \frac{\partial}{\partial u^{-i}} + i\theta^+ \not{\partial} \theta^+ + \theta^{+a} \frac{\partial}{\partial \theta^{-a}},$$

$$D^{--} = u^{-i} \frac{\partial}{\partial u^{+i}} + i\theta^- \not{\partial} \theta^- + \theta^{-a} \frac{\partial}{\partial \theta^{+a}},$$

$$D^0 = u^{+i} \frac{\partial}{\partial u^{+i}} - u^{-i} \frac{\partial}{\partial u^{-i}} + \theta^{+a} \frac{\partial}{\partial \theta^{+a}} - \theta^{-a} \frac{\partial}{\partial \theta^{-a}}$$

- Spinor derivatives in the analytic basis

$$D_a^+ = \frac{\partial}{\partial \theta^{-a}}, \quad D_a^- = -\frac{\partial}{\partial \theta^{+a}} - 2i\partial_{ab}\theta^{-b}, \quad \{D_a^+, D_b^-\} = 2i\partial_{ab}$$

Hypermultiplet in conventional $6D$ superspace

- The $\mathcal{N} = (1, 0)$ hypermultiplet is described in conventional $6D$, $\mathcal{N} = (1, 0)$ superspace by the superfields $q^i(x, \theta)$ and their conjugate $\bar{q}_i(x, \theta)$, where $\bar{q}_i = (q^i)^+$ under the constraint

$$D_a^{(i} q^{j)}(x, \theta) = 0$$

- On-shell component form of the hypermultiplet

$$q^i(z) = f^i(x) + \theta^{ai} \psi_a(x)$$

where the scalar field $f^i(x)$ and the spinor field $\psi_a(x)$ satisfy the equations $\square f^i = 0, \partial^{ab} \psi_b = 0$

- The on-shell $\mathcal{N} = (1, 0)$ hypermultiplet in six dimensions has 2 bosonic+2 fermionic complex degrees of freedom.

Hypermultiplet in harmonic superspace: off-shell Lagrangian formulation

- Off-shell hypermultiplet is described by the analytic superfield $q_A^+(\zeta, u)$, $D_a^+ q_A^+(\zeta, u) = 0$, satisfying the reality condition $\widetilde{(q^{+A})} \equiv q_A^+ = \varepsilon_{AB} q^{+B}$. Pauli-Gürsey indices $A, B = 1, 2$
- Off-shell hypermultiplet harmonic superfield contains infinite set of auxiliary fields which vanish on-shell due to the equations of motion

$$D^{++} q^+(\zeta, u) = 0$$

- The equations of motion follow from the action

$$S_{HYPER} = -\frac{1}{2} \int d\zeta^{(-4)} du q^{+A} D^{++} q_A^+$$

Here $d\zeta^{(-4)} = d^6 x d^4 \theta^+$.

The $\mathcal{N} = (1, 0)$ non-Abelian vector multiplet in $6D$ conventional superspace

- Gauge covariant derivatives

$$\nabla_{\mathcal{M}} = D_{\mathcal{M}} + \mathcal{A}_{\mathcal{M}}, \quad [\nabla_{\mathcal{M}}, \nabla_{\mathcal{N}}] = T_{\mathcal{M}\mathcal{N}}{}^{\mathcal{L}} \nabla_{\mathcal{L}} + F_{\mathcal{M}\mathcal{N}}$$

with $D_{\mathcal{M}} = \{\partial_M, D_a^i\}$ being the flat covariant derivatives and \mathcal{A}_M being the gauge connection taking the values in the Lie algebra of the gauge group.

- The constraints

$$F_{ab}^{ij} = 0, \quad \{\nabla_a^i, \nabla_b^j\} = -2i\varepsilon^{ij} \nabla_{ab}, \quad [\nabla_c^i, \nabla_{ab}] = -2i\varepsilon_{abcd} W^{id}$$

Here W^{ia} is the superfield strength obeying the Bianchi identities.

The constraints are solved in the framework of the harmonic superspace

The $\mathcal{N} = (1, 0)$ non-Abelian vector multiplet in $6D$, $\mathcal{N} = (1, 0)$ harmonic superspace

- Harmonic covariant derivative

$$\nabla^{++} = D^{++} + V^{++}$$

Connection V^{++} , takes the values in the Lie algebra of the gauge group, this is an unconstrained analytic potential of the $6D, \mathcal{N} = (1, 0)$ SYM theory.

- On-shell contents: $V^{++} = \theta^{+a}\theta^{+b}A_{ab} + 2(\theta^+)_a\lambda^{-a}$, A_{ab} is a vector field, $\lambda^{-a} = \lambda^{ai}u_i^-$, λ^{ai} is a spinor field.
- The superfield action of $6D, \mathcal{N} = (1, 0)$ SYM theory is written in the form

$$S_{SYM} = \frac{1}{f^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \text{tr} \int d^{14}z du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u_1^+ u_2^+) \dots (u_n^+ u_1^+)}$$

Here f is the dimensional coupling constant ($[f] = -1$)

- Gauge transformations

$$V^{++'} = -ie^{i\lambda}D^{++}e^{-i\lambda} + e^{i\lambda}V^{++}e^{-i\lambda}, \quad q^{+'} = e^{i\lambda}q^+$$

Theory of $\mathcal{N} = (1, 0)$ non-Abelian vector multiplet coupled to hypermultiplet

- Action

$$S[V^{++}, q^+] = \frac{1}{f^2} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{tr} \int d^{14}z du_1 \dots du_n \frac{V^{++}(z, u_1) \dots V^{++}(z, u_n)}{(u_1^+ u_2^+) \dots (u_n^+ u_1^+)} - \int d\zeta^{-4} du \tilde{q}^+ \nabla^{++} q^+$$

- Harmonic covariant derivative

$$\nabla^{++} = D^{++} + iV^{++}$$

- Equations of motion

$$\frac{1}{2f^2} F^{++} - i\tilde{q}^+ q^+ = 0, \quad \nabla^{++} q^+ = 0.$$

$$F^{++} = (D^+)^4 V^{--}, \quad D^{++} V^{--} - D^{--} V^{++} + i[V^{++}, V^{--}] = 0$$

$\mathcal{N} = (1, 1)$ SYM theory can be formulated in terms of $\mathcal{N} = (1, 0)$ harmonic superfields as the $\mathcal{N} = (1, 0)$ vector multiplet coupled to hypermultiplet in adjoint representation. The theory is manifestly $\mathcal{N} = (1, 0)$ supersymmetric and possesses the extra hidden $\mathcal{N} = (0, 1)$ supersymmetry.

- Action

$$S[V^{++}, q^+] = S_{SYM}[V^{++}] + S_{HYPER}[q^+, V^{++}]$$

- The action is manifestly $\mathcal{N} = (1, 0)$ supersymmetric.
- The action is invariant under the transformations of extra hidden $\mathcal{N} = (0, 1)$ supersymmetry

$$\delta V^{++} = \epsilon^+ q^+, \quad \delta q^+ = -(D^+)^4 (\epsilon^- V^{--})$$

where the transformation parameter $\epsilon_A^\pm = \epsilon_{aA} \theta^{\pm A}$.

- We start with harmonic superfield formulations of vector multiplet coupled to hypermultiplet.
- Effective action is formulated in the framework of the harmonic superfield background field method. It provides manifest $\mathcal{N} = (1, 0)$ supersymmetry and gauge invariance of effective action under the classical gauge transformations.
- Effective action can be calculated on the base of superfield proper-time technique. It provides preservation of manifest $\mathcal{N} = (1, 0)$ supersymmetry and manifest gauge invariance at all steps of calculations.
- The effective action can also be calculated perturbatively on the base of Feynman diagrams in superspace (supergraph technique).
- We study the model where the $\mathcal{N} = (1, 0)$ vector multiplet interacts with hypermultiplet in the arbitrary representation of the gauge group. Then, we assume in the final result for one-loop divergences, that this representation is adjoint what corresponds to $\mathcal{N} = (1, 1)$ SYM theory.

Aim: construction of gauge invariant effective action

Realization

- The superfields V^{++}, q^+ are splitting into the sum of the background superfields V^{++}, Q^+ and the quantum superfields v^{++}, q^+

$$V^{++} \rightarrow V^{++} + f v^{++}, \quad q^+ \rightarrow Q^+ + q^+$$

- The action is expanding in a power series in quantum fields. As a result, we obtain the initial action $S[V^{++}, q^+]$ as a functional $\tilde{S}[v^{++}, q^+; V^{++}, Q^+]$ of background superfields and quantum superfields.
- The gauge-fixing function are imposed only on quantum superfiled

$$\mathcal{F}_\tau^{(+4)} = D^{++} v_\tau^{++} = e^{-ib} (\nabla^{++} v^{++}) e^{ib} = e^{-ib} \mathcal{F}^{(+4)} e^{ib},$$

where $b(z)$ is a background-dependent gauge bridge superfield and τ means τ -frame. In the non-Abelian gauge theory, the gauge-fixing function is background-dependent.

- Faddev-Popov procedure is used. Ones obtain the effective action $\Gamma[V^{++}, Q^+]$ which is gauge invariant under the classical gauge transformations. Background field construction in the case under consideration is analogous to one in $4D, \mathcal{N} = 2$ SYM theory (I.L.B, E.I. Buchbinder, S.M. Kuzenko, B.A. Ovrut, 1998).

- The effective action $\Gamma[V^{++}, Q^+]$ is written in terms of path integral

$$e^{i\Gamma[V^{++}, Q^+]} = \text{Det}^{1/2} \widehat{\square} \int \mathcal{D}v^{++} \mathcal{D}q^+ \mathcal{D}\mathbf{b} \mathcal{D}\mathbf{c} \mathcal{D}\varphi e^{iS_{quant}[v^{++}, q^+, \mathbf{b}, \mathbf{c}, \varphi, V^{++}, Q^+]}$$

- The quantum action S_{quant} has the structure

$$S_{quant} = S[V^{++} + fv^{++}, Q^+ + q^+] + \\ + S_{GF}[v^{++}, V^{++}] + S_{FP}[\mathbf{b}, \mathbf{c}, v^{++}, V^{++}] + S_{NK}[\varphi, V^{++}].$$

- Gauge fixing term $S_{GF}[v^{++}, V^{++}]$, Faddeed-Popov ghost action $S_{FP}[\mathbf{b}, \mathbf{c}, v^{++}, V^{++}]$, Nelson-Kalosh ghost action $S_{NK}[\varphi, V^{++}]$
- Operator $\widehat{\square}$

$$\widehat{\square} = \eta^{MN} \nabla_M \nabla_N + W^{+a} \nabla_a^- + F^{++} \nabla^{--} - \frac{1}{2} (\nabla^{--} F^{++})$$

- All ghosts are the analytic superfields

One-loop approximation. Only quadratic in quantum fields and ghosts terms are taken into account in the path integral for effective action. It gives after some transformation the one-loop contribution $\Gamma^{(1)}[V^{++}, Q^+]$ to effective action in terms of formal functional determinants in analytic subspace of harmonic superspace

$$\Gamma^{(1)}[V^{++}, Q] = \frac{i}{2} Tr_{(2,2)} \ln[\delta^{(2,2)} \widehat{\square}^{AB} - 2f^2 Q^{+m} (T^A G_{(1,1)} T^B)_m{}^n Q_n^+] -$$

$$- \frac{i}{2} Tr_{(4,0)} \ln \widehat{\square} - i Tr \ln (\nabla^{++})_{\text{Adj}}^2 + \frac{i}{2} Tr \ln (\nabla^{++})_{\text{Adj}}^2 + i Tr \ln \nabla_{\text{R}}^{++}$$

As usual, $Tr \ln O \sim Det O$, Tr means the functional trace in analytic subspace and matrix trace.

$(T^A)_m{}^n$ are generators of the representation for the hypermultiplet.

The $G_{(1,1)}$ is the Green function for the operator ∇^{++} .

Index A numerates the generators, $V^{++} = V^{++A} T^A$. Operator $\widehat{\square}$ acts on the components V^{++A} as $(\widehat{\square} V^{++})^A = \widehat{\square}^{AB} V^{++B}$

Adj and R mean that the corresponding operators are taken in the adjoint representation and in the representation for hypermultiplet.

Superfield Feynman diagrams (supergraphs)

- Perturbation theory can be given in terms of Feynman diagrams formulated in superspace
- Vector multiplet propagator

$$G^{(2,2)}(1|2) = -2 \frac{(D_1^+)^4}{\square_1} \delta^{14}(z_1 - z_2) \delta^{(-2,2)}(u_1, u_2)$$

- Hypermultiplet propagator

$$G^{(1,1)}(1|2) = \frac{(D_1^+)^4 (D_2^+)^4}{\square_1} \frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}$$

- Ghost propagators have the analogous structure
- Superspace delta-function

$$\delta^{14}(z_1 - z_2) = \delta^6(x_1 - x_2) \delta^8(\theta_1 - \theta_2)$$

- The vertices are taken from the superfield action as usual

Superficial degree of divergence ω -total degree in momenta in loop integral.

- Consider the L loop supergraph G with P propagators, V vertices, N_Q external hypermultiplet legs, and an arbitrary vector multiplet external legs.
- One can prove that due to structure of the propagators and the Grassmann delta-functions in the propagators, any supergraph for effective action can be written through the integrals over full $\mathcal{N} = (1, 0)$ superspace and contains only a single integral over $d^8\theta$ (non-renormalization theorem).
- Mass dimensions: $[x] = -1$, $[p] = 1$, $[\int d^6p] = 6$, $[\theta] = -\frac{1}{2}$, $[\int d^8\theta] = 4$, $[q^+] = 1$, $[V^{++}] = 0$.
- After summing all dimensions and using some identities, power counting gives $\omega(G) = 2L - N_Q - \frac{1}{2}N_D$
- N_D is a number of spinor derivatives acting on external lines
- A number of space-time derivatives in the counterterms increases with L . The theory is multiplicatively non-renormalizable.
- One loop approximation $\omega_{1-loop}(G) = 2 - N_Q$
- The possible divergences correspond to $\omega_{1-loop} = 2$ and $\omega_{1-loop} = 0$

Calculations of ω are analogous to ones in $4D, \mathcal{N} = 2$ gauge theory (ILB, S.M. Kuzenko, B.A. Ovrut, 1998).

Structure of one-loop counterterm

Possible candidate for one-loop divergences can be constructed on the basis of dimensions, gauge invariance and $\mathcal{N} = (1, 0)$ supersymmetry in the form (G. Bossard, E. Ivanov, A. Smilga, 2015)

$$\Gamma_{div}^{(1)} = \int d\zeta^{(-4)} du \left[c_1 (F^{++A})^2 + i c_2 F^{++A} (\tilde{q}^+)^m (T^A)_m{}^n (q^+)_n + c_3 \left((\tilde{q}^+)^m (q^+)_m \right)^2 \right]$$

Where c_1, c_2, c_3 are arbitrary dimensionless real numbers.

We consider a possible form of one-loop counterterms on the basis of superficial degree of divergences in quadratic approximation in fields and then restore a complete result of the basis of gauge invariance.

- Let $N_Q = 0$, $N_D = 0$, so that $\omega = 2$, and we use the dimensional regularization. The corresponding counterterm has to be quadratic in momenta and given by the full $\mathcal{N} = (1, 0)$ superspace integral. The only admissible possibility is

$$\Gamma_1^{(1)} \sim \int d^{14}z du V^{--} \square V^{++}$$

After some transformation it coincides with the first term in $\Gamma_{div}^{(1)}$, with dimensionless divergent coefficient c_1 . Being dimensionless, this coefficient must be proportional to $1/\varepsilon$, where $\varepsilon = d - 6$ is a regularization parameter.

- Let $N_Q = 2$, $N_D = 0$ so that $\omega = 0$ and we use the dimensional regularization. The corresponding counterterm has to be momentum independent and given by the full $\mathcal{N} = (1, 0)$ superspace integral. The only admissible possibility is

$$\Gamma_2^{(1)} \sim \int d^{14}z du \tilde{q}^+ V^{--} q^+$$

After some transformation it coincides with second term in $\Gamma_{div}^{(1)}$, with dimensionless divergent coefficient c_2 . Being dimensionless, this coefficient must be proportional to $1/\varepsilon$, where $\varepsilon = d - 6$ is a regularization parameter.

- Let $N_Q = 4$, $N_D = 0$ so that $\omega = -2$ and the corresponding supergraph is convergent. It means that $c_3 = 0$ in $\Gamma_{div}^{(1)}$. As a result, all one-loop $(q^+)^4$ possible contributions to effective action are finite.

Manifestly covariant calculation

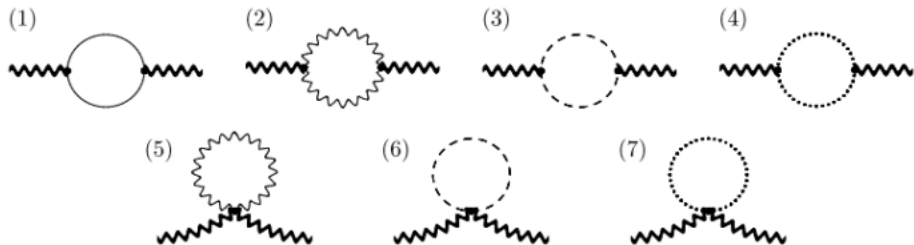
Calculating the one-loop divergences of superfield functional determinants is carried out in the framework of proper-time technique (superfield version of Schwinger-De Witt technique).. Such technique allows us to preserve the manifest gauge invariance and manifest $\mathcal{N} = (1, 0)$ supersymmetry at all steps of calculations.

General scheme of calculations

- Proper-time representation

$$\text{Tr} \ln O \sim \text{Tr} \int_0^\infty \frac{d(is)}{(is)^{1+\varepsilon}} e^{isO_1} \delta(1, 2)|_{2=1}$$

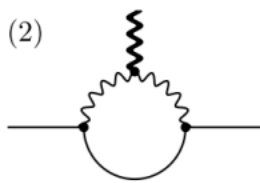
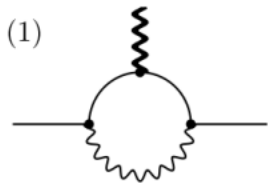
- Here s is the proper-time parameter and ε is a parameter of dimensional regularization.
- Typically the $\delta(1, 2)$ contains $\delta^8(\theta_1 - \theta_2)$, which vanishes at $\theta_1 = \theta_2$
- Typically the operator O contains some number of spinor derivatives D_a^+, D_a^- which act on the Grassmann delta-functions $\delta^8(\theta_1 - \theta_2)$ and can kill them. Non-zero result will be only if all these δ -functions are killed.
- Only these terms are taking into account which have the pole $\frac{1}{\varepsilon}$ after integration over proper-time.



Supergraphs for two-point function of vector multiplet



Supergraph for two-point function of hypermultiplet



Supergraphs for three-point vector multiplet - hypermultiplet function

Results of calculations

$$\Gamma_{div}^{(1)}[V^{++}, Q^+] = \frac{C_2 - T(R)}{3(4\pi)^3 \varepsilon} \text{tr} \int d\zeta^{(-4)} du (F^{++})^2 -$$
$$- \frac{2if^2}{(4\pi)^3 \varepsilon} \int d\zeta^{(-4)} du \tilde{Q}^{+m} (C_2 \delta_m^n - C(R)_m^n) F^{++} Q^+_n.$$

- The quantities $C_2, T(R), C(R)$ are defined as follows

$$\text{tr}(T^A T^B) = T(R) \delta^{AB}$$

$$\text{tr}(T_{Adj}^A T_{Adj}^B) = f^{ACD} f^{BCD} = C_2 \delta^{AB}$$

$$(T^A T^A)_m^n = C(R)_m^n.$$

- Results of calculations correspond to analysis done on the base of power counting. The coefficients c_1, c_2 are found. The coefficient $c_3 = 0$ as we expected.
- In $\mathcal{N} = (1, 1)$ SYM theory, the hypermultiplet is in the same representation as the vector multiplet. Then $C_2 = T(R) = C(R)$. Then $\Gamma_{div}^{(1)}[V^{++}, Q^+] = 0!$

- The six-dimensional $\mathcal{N} = (1, 0)$ supersymmetric theory of the non-Abelian vector multiplet coupled to hypermultiplet in the $6D$, $\mathcal{N} = (1, 0)$ harmonic superspace was considered.
- Background field method in harmonic superspace was constructed .
- Manifestly supersymmetric and gauge invariant effective action, depending both on vector multiplet and hypermultiplet superfields, was defined.
- Superficial degree of divergence is evaluated and structure of one-loop counterterms was studied.
- An efficient manifestly gauge invariant and $\mathcal{N} = (1, 0)$ supersymmetric technique to calculate the one-loop effective action was developed. As an application of this technique, we found the one-loop divergences of the theory under consideration.
- The same one-loop divergences have been calculated independently with help of $\mathcal{N} = (1, 0)$ supergraphs.
- It is proved that $\mathcal{N} = (1, 1)$ SYM theory is one-loop off-shell finite. There is no need to use the equations of motion to prove this property.

- Gauge dependence?
- Two-loop off-shell finiteness?
- Low-energy effective action?

THANK YOU VERY MUCH!