Magnetized AdS black holes and solitons in Einstein-Maxwell-Chern-Simons theory

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1. Introduction

2. Magnetized and asymptotically AdS solutions

3. Magnetized and squashed solutions

4. Conclusions

1. Introduction

1. Introduction

5 dimensional black holes and solitons in
Einstein-Maxwell-Chern-Simons theory
with AdS asymptotics

Interesting in the context of the AdS/CFT correspondence:

Gravitating fields propagating in a
5 dimensional asymptotically AdS spacetime

⇕

Fields propagating in a conformal field theory in 4 dimensions

Interesting in the context of supergravity:

Bosonic sector of the minimally gauged 5D supergravity

1. Introduction

5D minimal gauged supergravity (SUGRA)

- Compactification of Type IIb string theories (bosonic sector)

$$I = \frac{1}{16\pi}\int_{\mathcal{M}} d^5x\left[\sqrt{-g}\left(R + \frac{12}{L^2} - F_{\mu\nu}F^{\mu\nu}\right) + \frac{2\lambda}{3\sqrt{3}}\varepsilon^{\mu\nu\alpha\beta\gamma}A_\mu F_{\nu\alpha}F_{\beta\gamma}\right] + I_b$$

$$I_b = -\frac{1}{8\pi}\int_{\partial\mathcal{M}} d^4x\sqrt{-h}\left[\mathrm{K} - \frac{3}{\mathrm{L}}\left(1 + \frac{\mathrm{L}^2}{12}\mathrm{R}\right) - \frac{\mathrm{L}}{2}\log\left(\frac{\mathrm{L}}{\mathrm{r}}\right)\left\{\mathrm{F_{ab}F^{ab}}\right\}\right]$$

Field equations:

$$G_{\mu\nu} = \frac{6}{L^2}g_{\mu\nu} + 2\left(F_{\mu\rho}F^{\rho}{}_{\nu} - \frac{1}{4}F^2\right)$$

$$\nabla_\nu F^{\mu\nu} + \frac{\lambda}{2\sqrt{3}}\varepsilon^{\mu\nu\alpha\beta\gamma}F_{\nu\alpha}F_{\beta\gamma} = 0$$

1. Introduction

Known black hole solutions in SUGRA:

- Static vacuum (Schwarzchild-Tangherlini solution)
- Stationary vacuum (Myers-Perry-AdS)
- Static electrically charged (Reissner-Nordström-AdS)

1. Introduction

Known black hole solutions in SUGRA:

- Stationary and electrically charged (Chong-Cvetic-Lü-Pope solution)

$$ds^2 = -\frac{\Delta_\theta[(1+g^2r^2)\rho^2 dt + 2q\nu]dt}{\Xi_a\Xi_b\rho^2} + \frac{2q\nu\omega}{\rho^2} + \frac{f}{\rho^4}\left(\frac{\Delta_\theta dt}{\Xi_a\Xi_b} - \omega\right)^2 + \frac{\rho^2 dr^2}{\Delta_r} + \frac{\rho^2 d\theta^2}{\Delta_\theta} + \frac{r^2+a^2}{\Xi_a}\sin^2\theta d\phi^2 + \frac{r^2+b^2}{\Xi_b}\cos^2\theta d\psi^2$$

$$A = \frac{\sqrt{3}q}{\rho^2}\left(\frac{\Delta_\theta dt}{\Xi_a\Xi_b} - \omega\right)$$

$$\nu = b\sin^2\theta d\phi + a\cos^2\theta d\psi$$

$$\omega = a\sin^2\theta\frac{d\phi}{\Xi_a} + b\cos^2\theta\frac{d\psi}{\Xi_b}$$

$$\Delta_\theta = 1 - a^2g^2\cos^2\theta - b^2g^2\sin^2\theta$$

$$\Delta_r = \frac{(r^2+a^2)(r^2+b^2)(1+g^2r^2) + q^2 + 2abq}{r^2} - 2m$$

$$\rho^2 = r^2 + a^2\cos^2\theta + b^2\sin^2\theta$$

$$\Xi_a = 1 - a^2g^2, \quad \Xi_b = 1 - b^2g^2$$

$$f = 2m\rho^2 - q^2 + 2abqg^2\rho^2$$

Two independent angular momenta

[PRL 95 (2005) 161301]

$g=1/L$

1. Introduction

Known black hole solutions in SUGRA:

- Extremal limit of the Chong-Cvetic-Lü-Pope solution

Supersymetric black holes [Gutowski-Reall JHEP 02 (2004) 006]

One parameter family of extremal solutions

1. Introduction

Recent new class of AdS solutions:
[Radu-Herdeiro PLB (2015) 749]
[R-H PLB (2016) 757]
[R-H PRL 221102 (2017)]

Regular electromagnetic solitons in Einstein-Maxwell with global AdS_4

Box-like behaviour of AdS space-time
+
“Multipolar” behavior of the electromagnetic field
=
Electric and magnetic regular solitons

Similar construction for black holes

1. Introduction

Einstein-Maxwell with global AdS_5

Black holes with solitonic limit
[JLBS, Kunz, Navarro-Lérida, Radu Entropy 18 (2016) 438]

Static configurations with purely magnetic gauge field

Box-like behaviour of AdS space-time
+
"Multipolar" behavior of the electromagnetic field
=
Electric and magnetic regular solitons

Similar construction for black holes

2. Magnetized Solutions

2. Magnetized Solutions

2.1 Cohomogeneity-1 Ansatz

2. Magnetized solutions || 2.1 Cohomogeneity-1 Ansatz

Metric:

$$ds^2 = -f(r)N(r)dt^2 + \frac{1}{f(r)}\left[\frac{m(r)}{N(r)}dr^2 + \frac{1}{4}r^2\left(m(r)(\sigma_1^2+\sigma_2^2) + n(r)(\sigma_3 - \frac{2\omega(r)}{r}dt)^2\right)\right]$$

Gauge field:

$$A = a_0(r)dt + a_\varphi(r)\frac{1}{2}\sigma_3$$

$$\sigma_1 = \cos\psi d\theta + \sin\psi \sin\theta d\phi$$
$$\sigma_2 = -\sin\psi d\theta + \cos\psi \sin\theta d\phi$$
$$\sigma_3 = d\psi + \cos\theta d\phi$$
$$N(r) = 1 + \frac{r^2}{L^2}$$

- Lewis-Papapetrou coordinates: $\theta \in [0,\pi]$ $\Phi \in [0,2\pi]$ $\Psi \in [0,4\pi]$
- Quasi-isotropic coordinatae radius: $r \in [r_H,\infty)$

$r = r_H$ outer horizon

$r_H = 0$ extremality

2. Magnetized solutions || 2.1 Cohomogeneity-1 Ansatz

Properties of the Ansatz:

- Cohomogeneity-1

$$|J_1|=|J_2|=J$$

- Outer horizon of spherical topology

- Adapted to the conformal boundary (asymptotically AdS_5)

2. Magnetized solutions || 2.1 Cohomogeneity-1 Ansatz

Field Equations

$$G_{\mu\nu} = \frac{6}{L^2} g_{\mu\nu} + 2\left(F_{\mu\rho}F^{\rho}{}_{\nu} - \frac{1}{4}F^2\right)$$

$$\nabla_\nu F^{\mu\nu} + \frac{\lambda}{2\sqrt{3}} \varepsilon^{\mu\nu\alpha\beta\gamma} F_{\nu\alpha} F_{\beta\gamma} = 0$$

Reduce to second order ordinary differential equations in r

First integrals:

$$a_0' + \frac{\omega}{r} a_\varphi' - \frac{4\lambda}{\sqrt{3}} \frac{f^{3/2} a_\varphi^2}{r^3 \sqrt{mn}} = \frac{2\, f^{3/2}}{\pi \sqrt{mn} r^3} c_1$$

$$\frac{16\lambda}{3\sqrt{3}} a_\varphi^3 - \frac{n^{3/2}\sqrt{m} r^3}{f^{5/2}} (r\omega' - \omega) = c_2 - \frac{8\, c_1}{\pi} a_\varphi$$

2. Magnetized Solutions

2.2 Far-field Asymptotics and perturbative solutions

Magnetized AdS_5 solutions:

$$r \longrightarrow \infty$$

AdS_5:

$$ds^2 = -N(r)dt^2 + \frac{dr^2}{N(r)} + \frac{1}{4}r^2(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)$$

Non-vanishing gauge field:

$$A = \frac{c_m}{2}\sigma_3$$

Magnetic flux:

$$\Phi_m = \frac{1}{4\pi}\int_{S^2_\infty} F = -\frac{1}{2}c_m$$

2. Magnetized solutions || 2.2 Asymptotics and perturbative solutions

Far-field asymptotics compatible with field equations:

$$f(r) = 1 + \left(\hat{\alpha} + \frac{12}{5} c_m^2 L^2 \log(\frac{L}{r})\right) \frac{1}{r^4} + \dots$$

$$m(r) = 1 + \left(\hat{\beta} + \frac{4}{5} c_m^2 L^2 \log(\frac{L}{r})\right) \frac{1}{r^4} + \dots$$

$$a_\varphi(r) = c_m + \left(\hat{\mu} + 2 c_m L^2 \log(\frac{L}{r})\right) \frac{1}{r^2} + \dots$$

$$\omega(r) = \frac{\hat{J}}{r^3} - \frac{4q}{3} \left(\hat{\mu} - \frac{1}{3} c_m L^2 (1 - 6 \log(\frac{L}{r}))\right) \frac{1}{r^5} + \dots$$

$$n(r) = 1 + \left(3(\hat{\alpha} - \hat{\beta}) + \frac{4}{15} c_m^2 L^2 + \frac{24}{5} c_m^2 L^2 \log(\frac{L}{r})\right) \frac{1}{r^4} + \dots$$

$$a_0(r) = -\frac{q}{r^2} + \frac{c_m \lambda}{\sqrt{3}} \left(2\hat{\mu} + c_m L^2 \left(-1 + 4 \log(\frac{L}{r})\right)\right) \frac{1}{r^4} + \dots$$

2. Magnetized solutions || 2.2 Asymptotics and perturbative solutions

Solitonic solutions: Small-r perturbative solution

$$f(r) = f_0 + \left(\frac{m_0 - f_0}{L^2} + \frac{4u^2 f_0^2}{3m_0}\right) r^2 + \dots, \quad m(r) = m_0 + m_2 r^2 + \dots,$$

$$n(r) = m_0 + \left(\frac{3m_0(m_0 - f_0)}{f_0 L^2} - m_2 + \frac{4u^2 f_0}{3}\right) r^2 + \dots,$$

$$a_\varphi(r) = u r^2 + \frac{u}{9 f_0 L^2 m_0}\Big(4u^2 f_0^2 L^2 (1 + 2\lambda^2) + 3(4m_0^2 - 3f_0(2m_0 + L^2 M_2))\Big) r^4 + \dots,$$

$$\omega(r) = w_1 r - \frac{8u^3 f_0^{5/2}\lambda}{3\sqrt{3}m_0^2} r^2 + \dots,$$

$$a_0(r) = v_0 - \left(\frac{2u^2 f_0^{3/2}\lambda}{\sqrt{3}m_0} + u w_1\right) r^2 + \dots,$$

2. Magnetized solutions || 2.2 Asymptotics and perturbative solutions

Black holes: perturbative solution around r_H

$$f(r) = f_2(r-r_H)^2 + O\left(r-r_H\right)^3, \quad m(r) = m_2(r-r_H)^2 + O\left(r-r_H\right)^3,$$
$$n(r) = n_2(r-r_H)^2 + O\left(r-r_H\right)^3, \quad \omega(r) = \omega_0 + O\left(r-r_H\right),$$
$$a_0(r) = a_0^{(0)} + O\left(r-r_H\right)^2, \quad a_\varphi(r) = a_\varphi^{(0)} + O\left(r-r_H\right)^2,$$

Angular velocity $$\Omega_H = \frac{\omega_0}{r_H}$$

Area $$A_H = 2\pi^2 r_H^3 \frac{m_2}{f_2}\sqrt{\frac{n_2}{f_2}}$$

Hawking temperature $$T_H = \frac{1}{2\pi}\left(1+\frac{r_H^2}{L^2}\right)\frac{f_2}{\sqrt{m_2}}$$

Deformation $$\varepsilon = \left.\frac{n(r)}{m(r)}\right|_{r=r_H} = \frac{n_2}{m_2}$$

2. Magnetized Solutions

2.3 Global charges

Electric charge

$$Q = -\frac{1}{2}\int_{S^3_\infty} \tilde{F} = \pi q$$

Page charge

$$Q^{(P)} = -\frac{1}{2}\int_{S^3_\infty} \left(\tilde{F} + \frac{\lambda}{\sqrt{3}} A \wedge F\right) = Q - \frac{2\pi}{\sqrt{3}}\lambda c_m^2 \equiv c_1$$

Noether R-charge

$$Q^{(R)} = -\frac{1}{2}\int_{S^3_\infty} \left(\tilde{F} + \frac{2\lambda}{3\sqrt{3}} A \wedge F\right) = Q - \frac{4\pi}{3\sqrt{3}}\lambda c_m^2$$

2. Magnetized solutions || 2.3 Global charges

Boundary stress-energy tensor

$$I_b = -\frac{1}{8\pi}\int_{\partial\mathcal{M}} d^4x\sqrt{-h}\Big[\mathrm{K} - \frac{3}{\mathrm{L}}(1+\frac{\mathrm{L}^2}{12}\mathrm{R}) - \frac{\mathrm{L}}{2}\log(\frac{\mathrm{L}}{\mathrm{r}})\,\{\mathrm{F_{ab}F^{ab}}\}\Big]$$

$$\mathrm{T}_{ab} = -\frac{2}{\sqrt{-h}}\frac{\delta I_b}{\delta h^{ab}} = -\frac{1}{8\pi G}\Big\{\Big[K_{ab} - Kh_{ab} + \frac{3}{L}h_{ab} - \frac{L}{2}E_{ab}\Big] + \log\left(\frac{L}{r}\right)\frac{L^3}{2}\Big[\frac{1}{12}h_{ab}\mathrm{R}^2 - \frac{1}{4}h_{ab}\mathrm{R}_{cd}\mathrm{R}^{cd} - \frac{1}{3}\mathrm{R}\mathrm{R}_{ab} + \mathrm{R}_{acbd}\mathrm{R}^{cd} + \frac{1}{2}\nabla^2 R_{ab} - \frac{1}{12}(h_{ab}\nabla^2 + 2\nabla_a\nabla_b)\mathrm{R} - \frac{4}{L^2}(\mathrm{F}_{ac}\mathrm{F}_b{}^c - \frac{1}{4}h_{ab}\mathrm{F}_{cd}\mathrm{F}^{cd})\Big]\Big\}$$

Angular momentum $J = \frac{\pi}{4}\hat{J}$

Mass $M = -\frac{\pi}{8}\frac{(3\hat{\alpha}+\hat{\beta})}{L^2} + \frac{c_m^2\pi}{30} + \frac{3\pi}{32}L^2$

Analytical relations for solitons: $c_1 = c_2 = 0$

$$J = -\frac{\lambda\pi}{3\sqrt{3}} c_m^3$$

$$Q = 3Q^{(R)} = \frac{2\lambda\pi}{\sqrt{3}} c_m^2$$

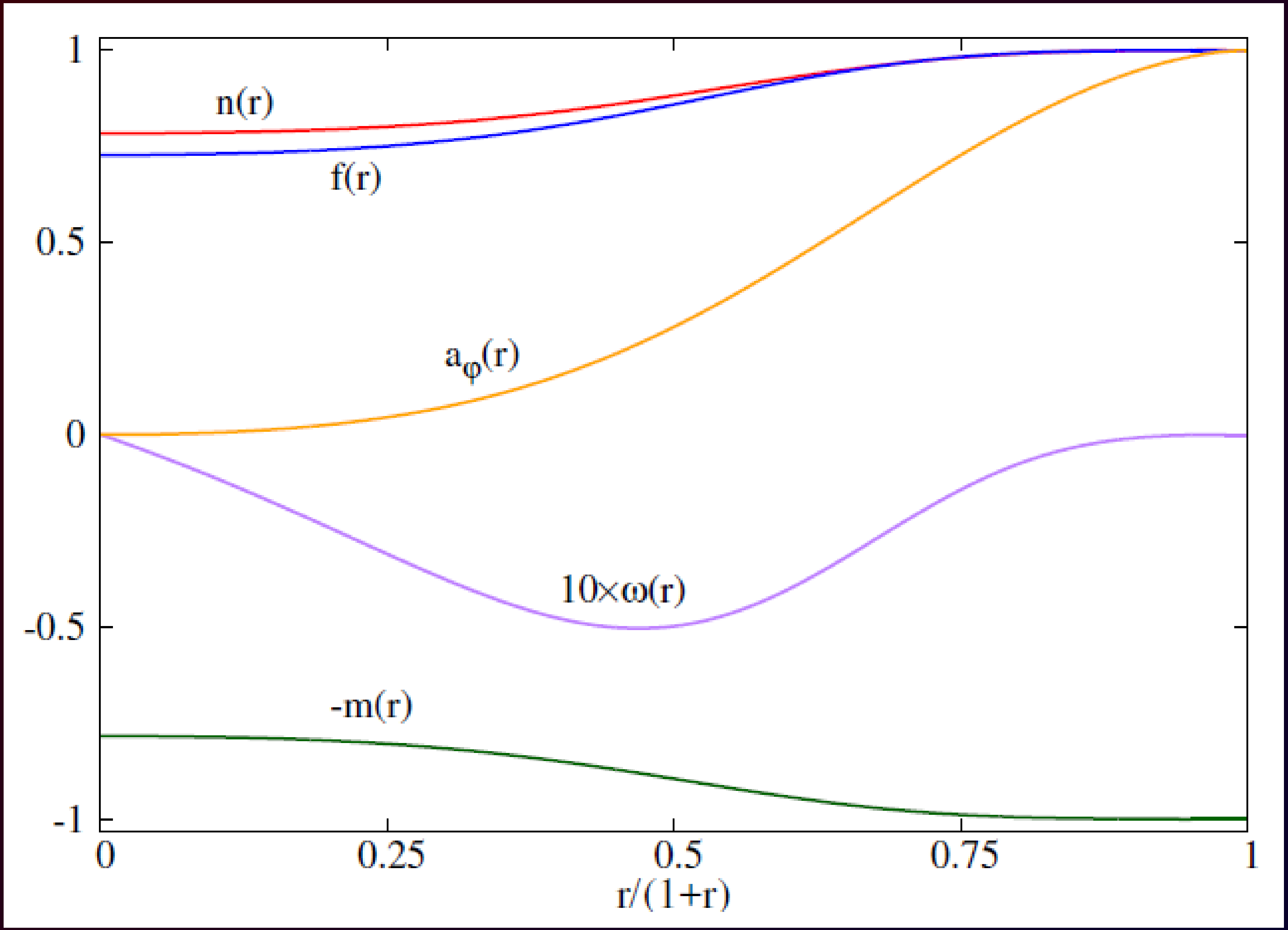
$$J = \Phi_m Q^{(R)}$$

2. Magnetized Solutions

2.4 Properties of the global solutions

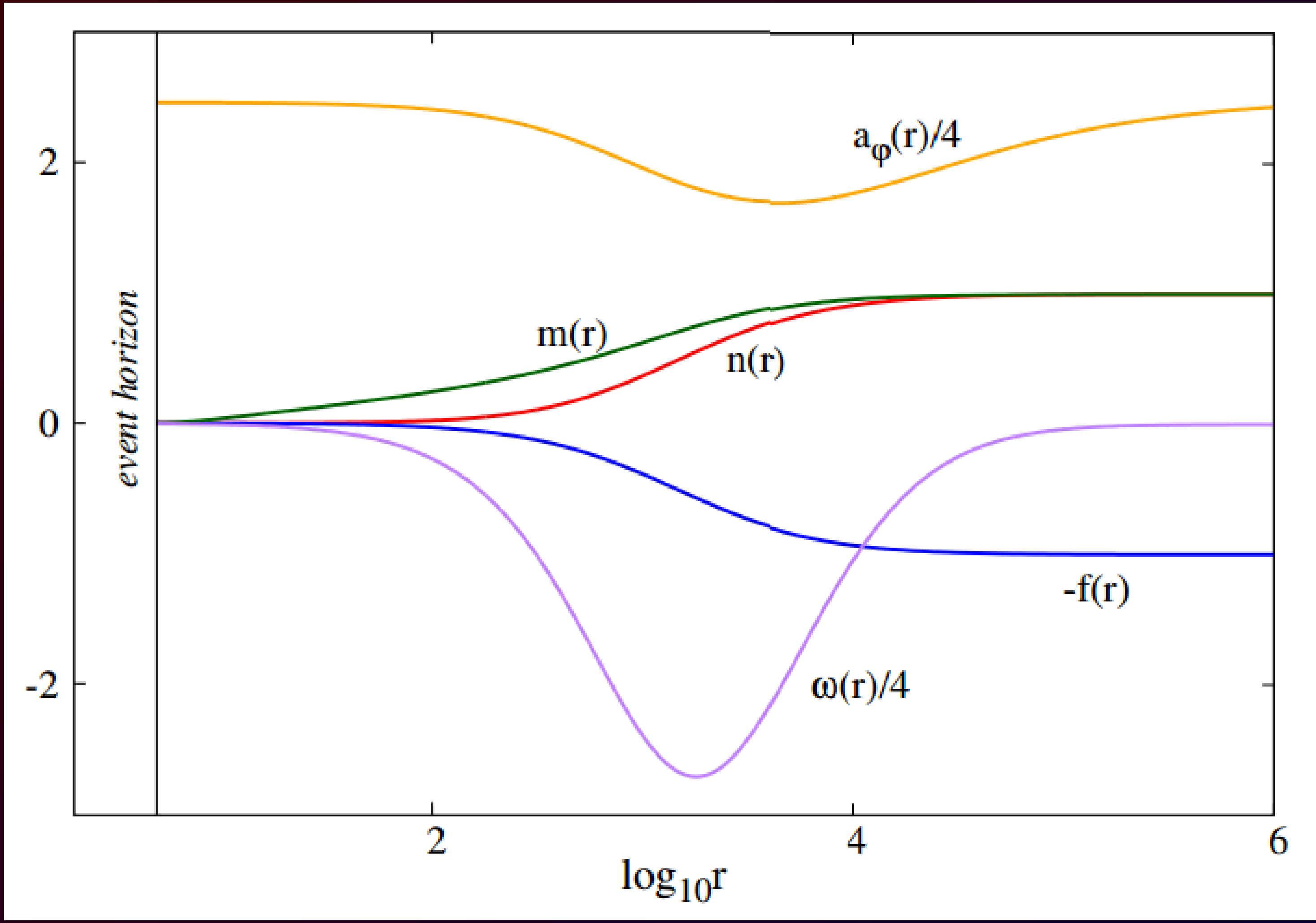
2. Magnetized solutions || 2.4 Properties of the global solutions

Typical profile of a soliton



2. Magnetized solutions || 2.4 Properties of the global solutions

Typical profile of a black hole



2. Magnetized solutions || 2.4 Properties of the global solutions

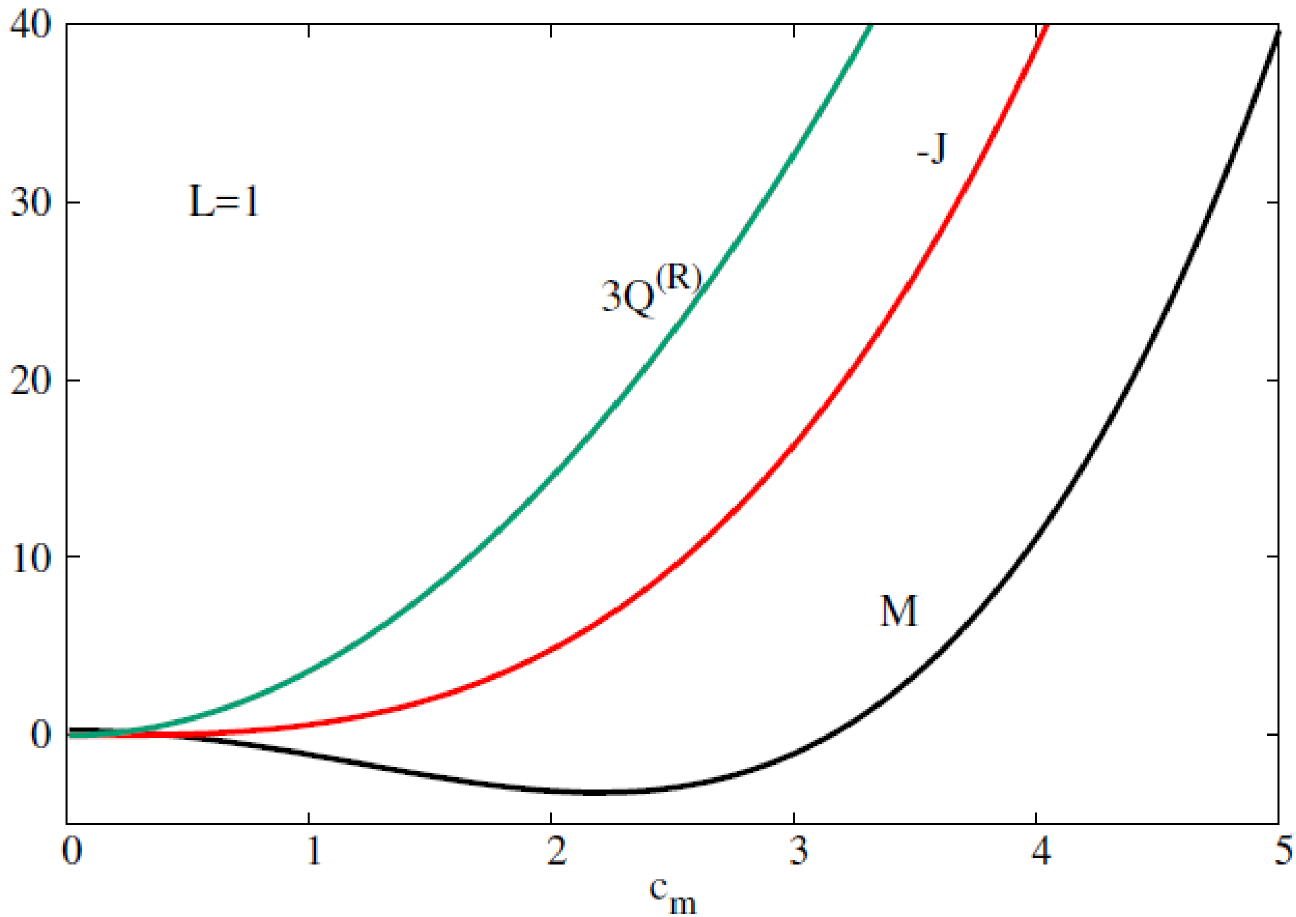
Solitons:

$$\frac{M}{L^2} = \frac{3\pi}{32} + a_2 \frac{c_m^2}{L^2} + a_4 \frac{c_m^4}{L^4} + a_6 \frac{c_m^6}{L^6}$$

a_2=-1.52

a_4=0.175

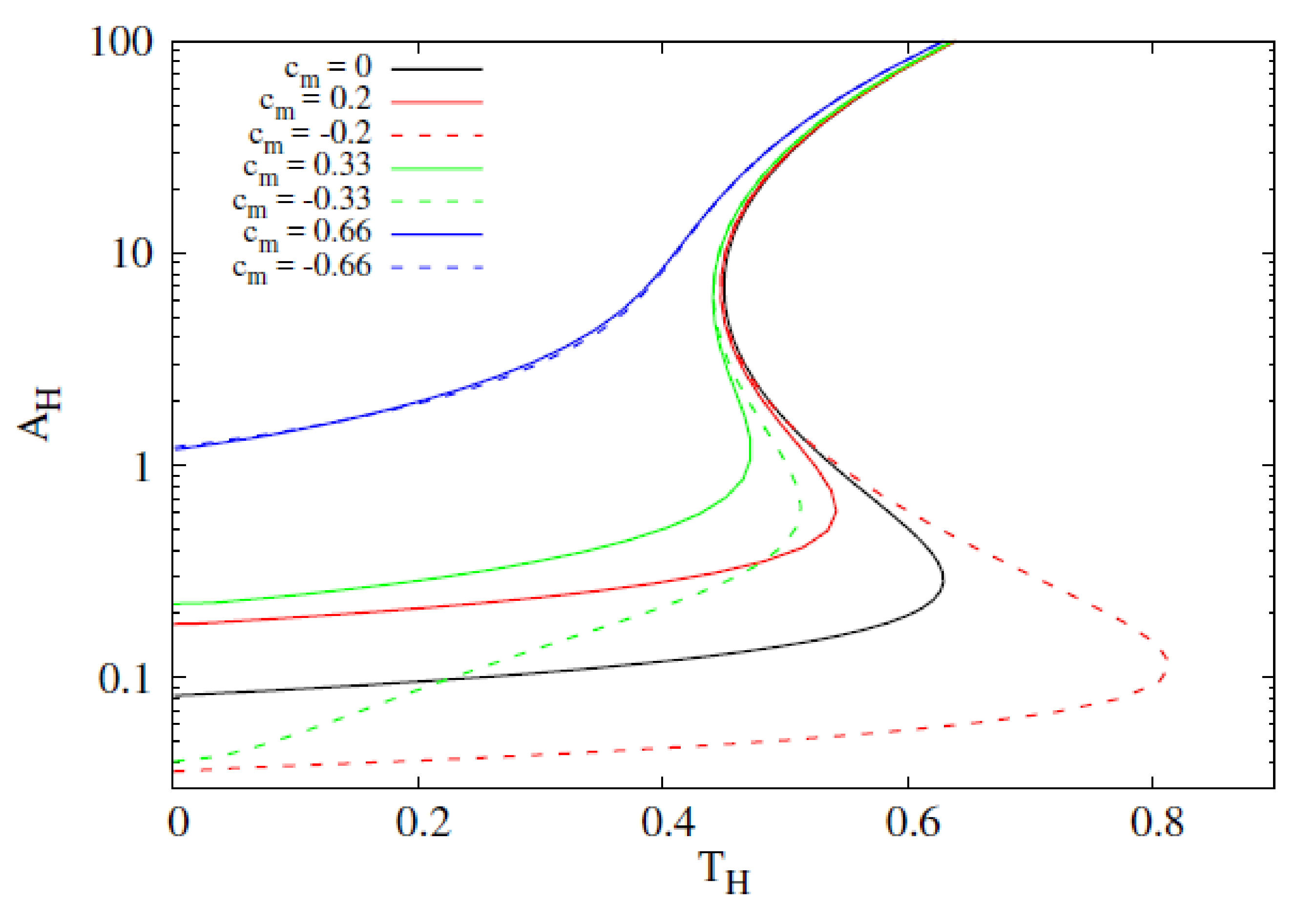
a_6=0.0025



2. Magnetized solutions || 2.4 Properties of the global solutions

Black holes: $Q^{(R)} = 0.044$, $J = 0.003$, $L = 1$

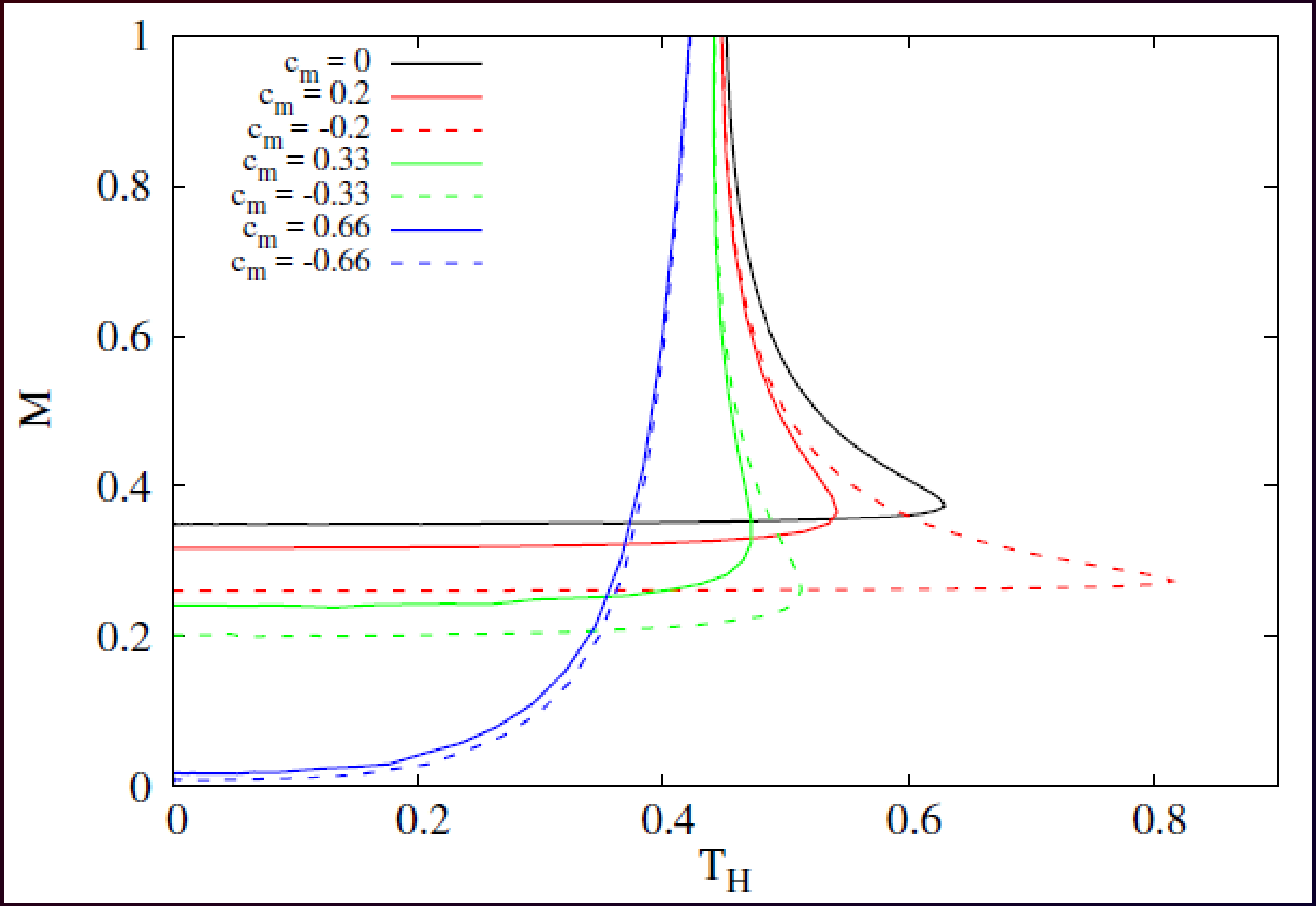
Area vs temperature



2. Magnetized solutions || 2.4 Properties of the global solutions

Black holes: $Q^{(R)} = 0.044$, $J = 0.003$, $L = 1$

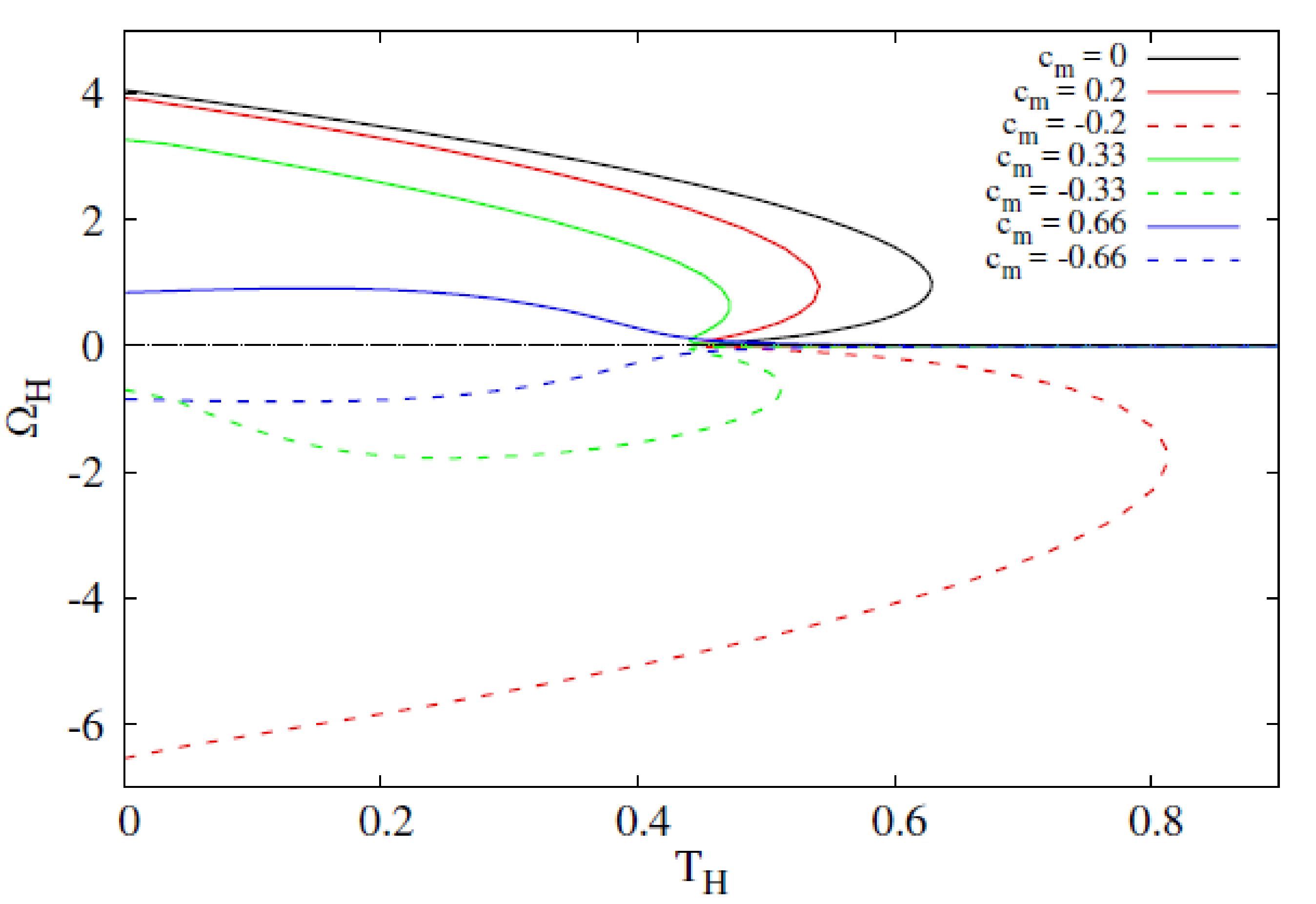
Mass vs temperature



2. Magnetized solutions || 2.4 Properties of the global solutions

Black holes: $Q^{(R)} = 0.044$, $J = 0.003$, $L = 1$

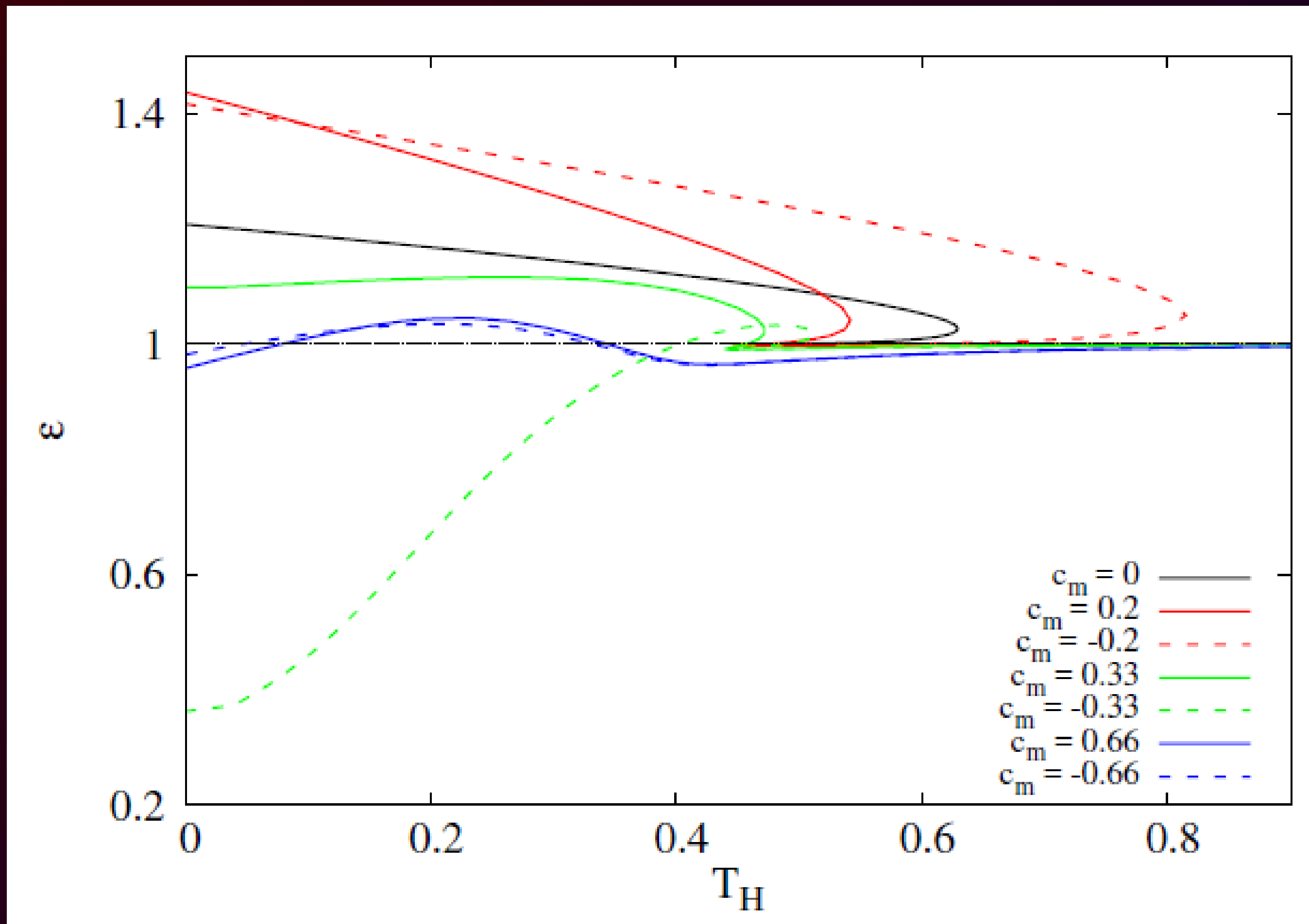
Angular velocity vs temperature



2. Magnetized solutions || 2.4 Properties of the global solutions

Black holes: $Q^{(R)} = 0.044$, $J = 0.003$, $L = 1$

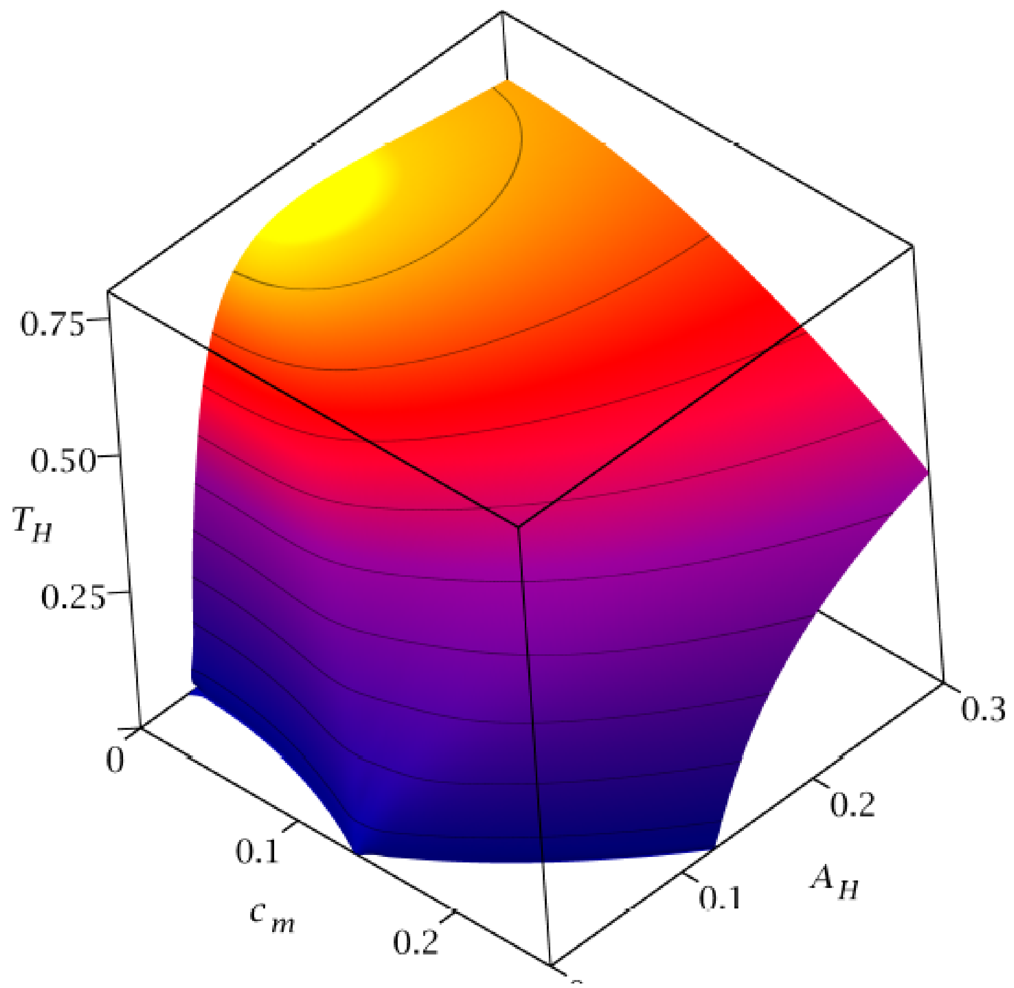
Deformation vs temperature



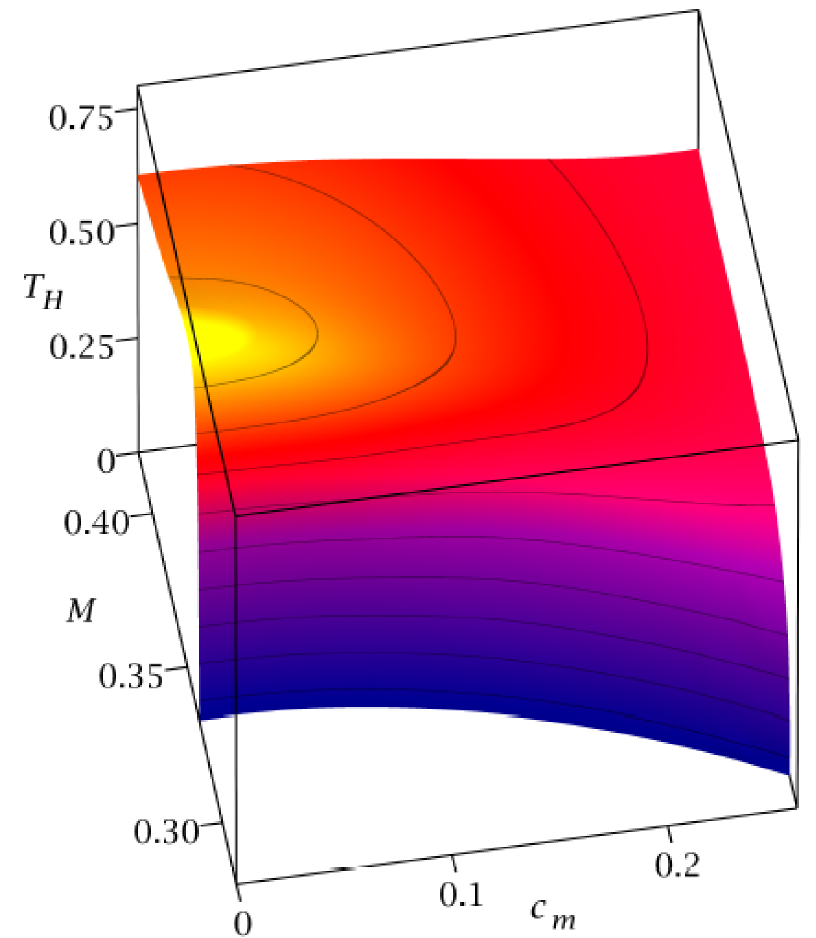
2. Magnetized solutions || 2.4 Properties of the global solutions

Black holes: $Q^{(R)} = -0.044$, $J = 0$, $L = 1$

Area - c_m - temperature



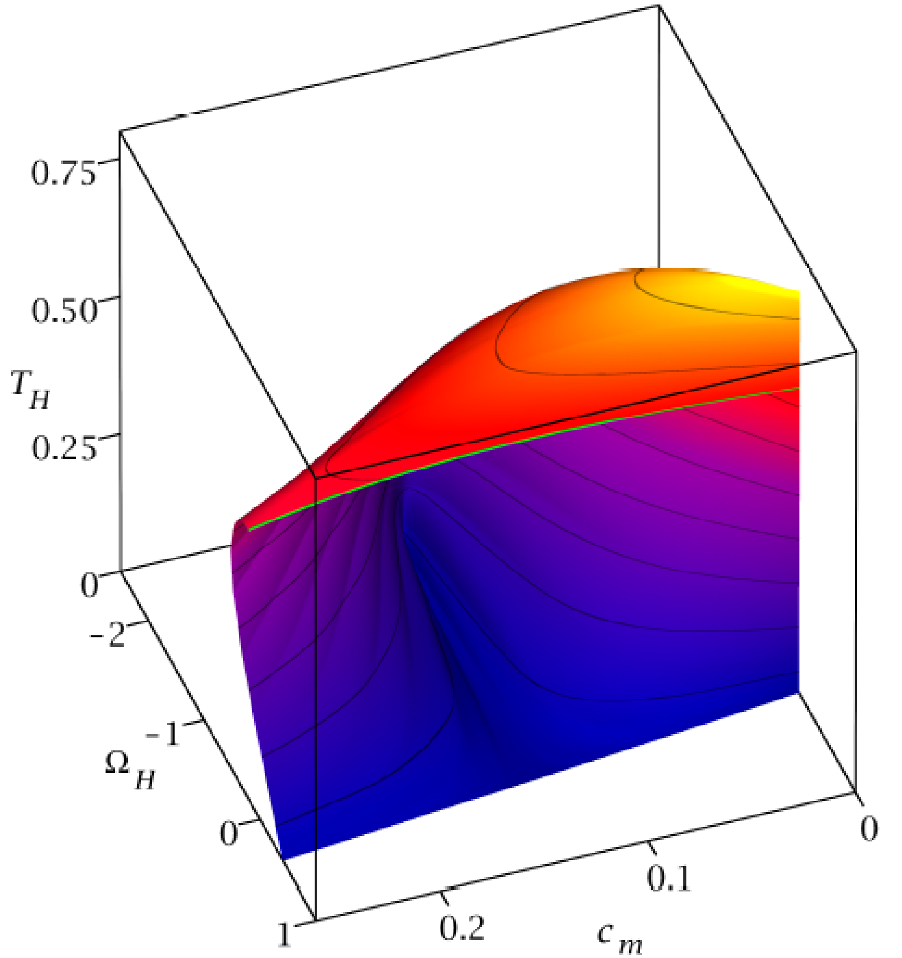
Mass - c_m - temperature



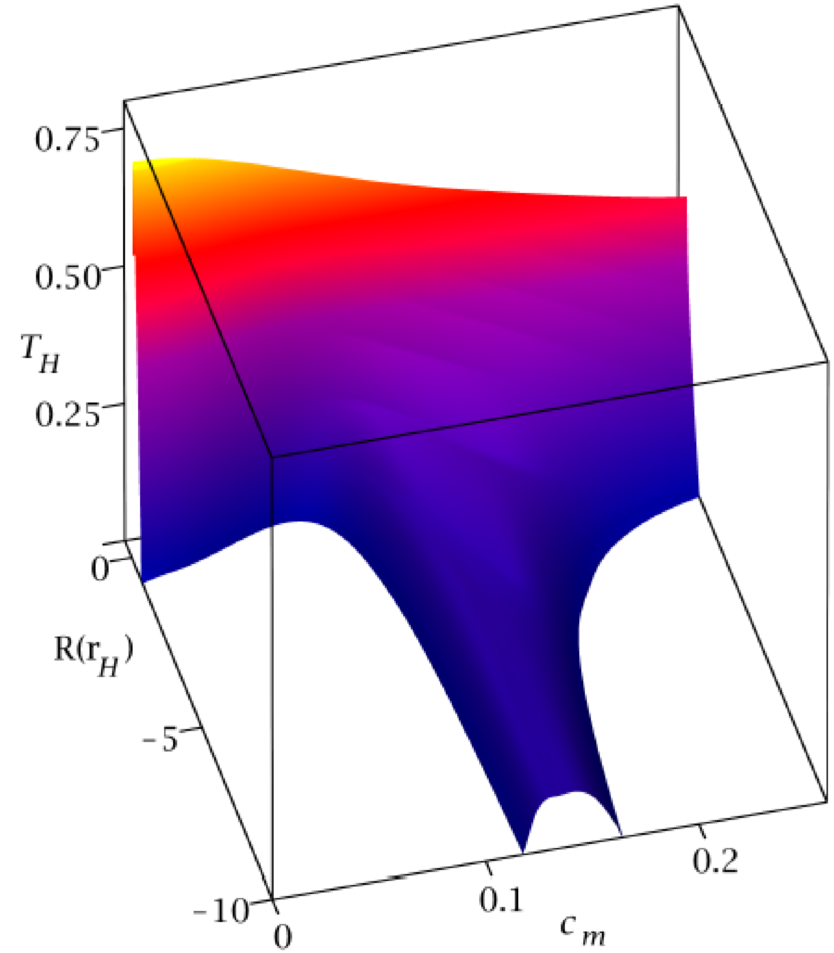
2. Magnetized solutions || 2.4 Properties of the global solutions

Black holes: $Q^{(R)} = -0.044$, $J = 0$, $L = 1$

Angular velocity - c_m - temperature



Ricci - c_m - temperature



3. Magnetized and Squashed Solutions

3. Magnetized and Squashed solutions

Squashing the AdS boundary metric

$$ds^2_{(bdry)} = -\, dt^2 \; + L^2 d\Omega^2_{(v)}$$

$$d\Omega^2_{(v)} = \frac{1}{4}\left(d\theta^2 + \sin^2\theta d\phi^2 + (d\psi + v\cos\theta d\phi)^2\right)$$

Squashed S^3

$$f(r) = 1 + \frac{4}{9}(1-v^2)\left(\frac{L}{r}\right)^2 + \left[\frac{\hat{\alpha}}{L^4} + \frac{4}{15}\left(\frac{9c_m^2 v^2}{L^2} + (1-v^2)(4v^2-3)\log(\frac{r}{L})\right)\right]\left(\frac{L}{r}\right)^4 + \dots,$$

$$m(r) = 1 - \frac{1}{9}(1-v^2)\left(\frac{L}{r}\right)^2 + \left[\frac{\hat{\beta}}{L^4} + \frac{4}{15}\left(\frac{3c_m^2 v^2}{L^2} - (1-v^2)(2v^2+1)\log(\frac{r}{L})\right)\right]\left(\frac{L}{r}\right)^4 + \dots,$$

$$n(r) = v^2\left(1 + \frac{17}{9}(1-v^2)\left(\frac{L}{r}\right)^2 + \left[\frac{3(\hat{\alpha}-\hat{\beta})}{L^4} + \frac{4c_m^2 v^2}{15L^2} + \frac{1}{405}(389-497v^2)(1-v^2)\right.\right.$$
$$\left.\left. + \frac{8}{5}\left(\frac{3c_m^2 v^2}{L^2} - (1-v^2)(1-3v^2)\log(\frac{r}{L})\right)\right]\left(\frac{L}{r}\right)^4\right) + \dots,$$

$$w(r) = \hat{j}\frac{1}{r^3} + \dots, \quad a_0(r) = -\frac{q}{r^2} + \dots, \quad a_\varphi(r) = c_m + (\mu - 2c_m L^2 v^2 \log(\frac{r}{L}))\frac{1}{r^2} + \dots,$$

3. Magnetized and Squashed solutions

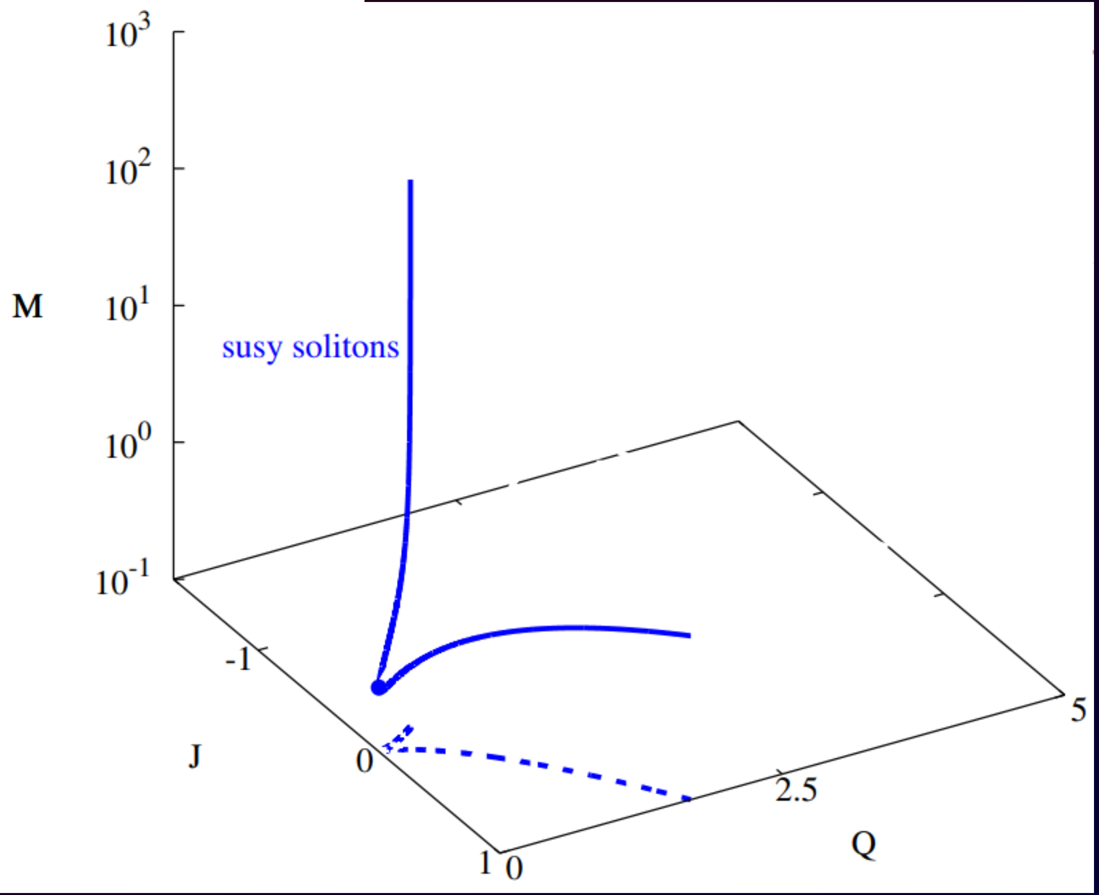
Supersymmetric squashed and magnetized solitons:

$$M = \pi L^2 \left(\frac{5}{288} + \frac{2}{27 v^2} - \frac{7}{36} v^2 + \frac{89}{864} v^4 \right)$$

$$J = -\frac{\pi L^3}{27} \left(v^2 - 1 \right)^3$$

$$Q = -\frac{2\pi L^2}{9\sqrt{3}} \left(v^2 - 1 \right)^2$$

$$c_{\mathrm{m}} = +\frac{L}{\sqrt{3}} \left(1 - v^2 \right)$$

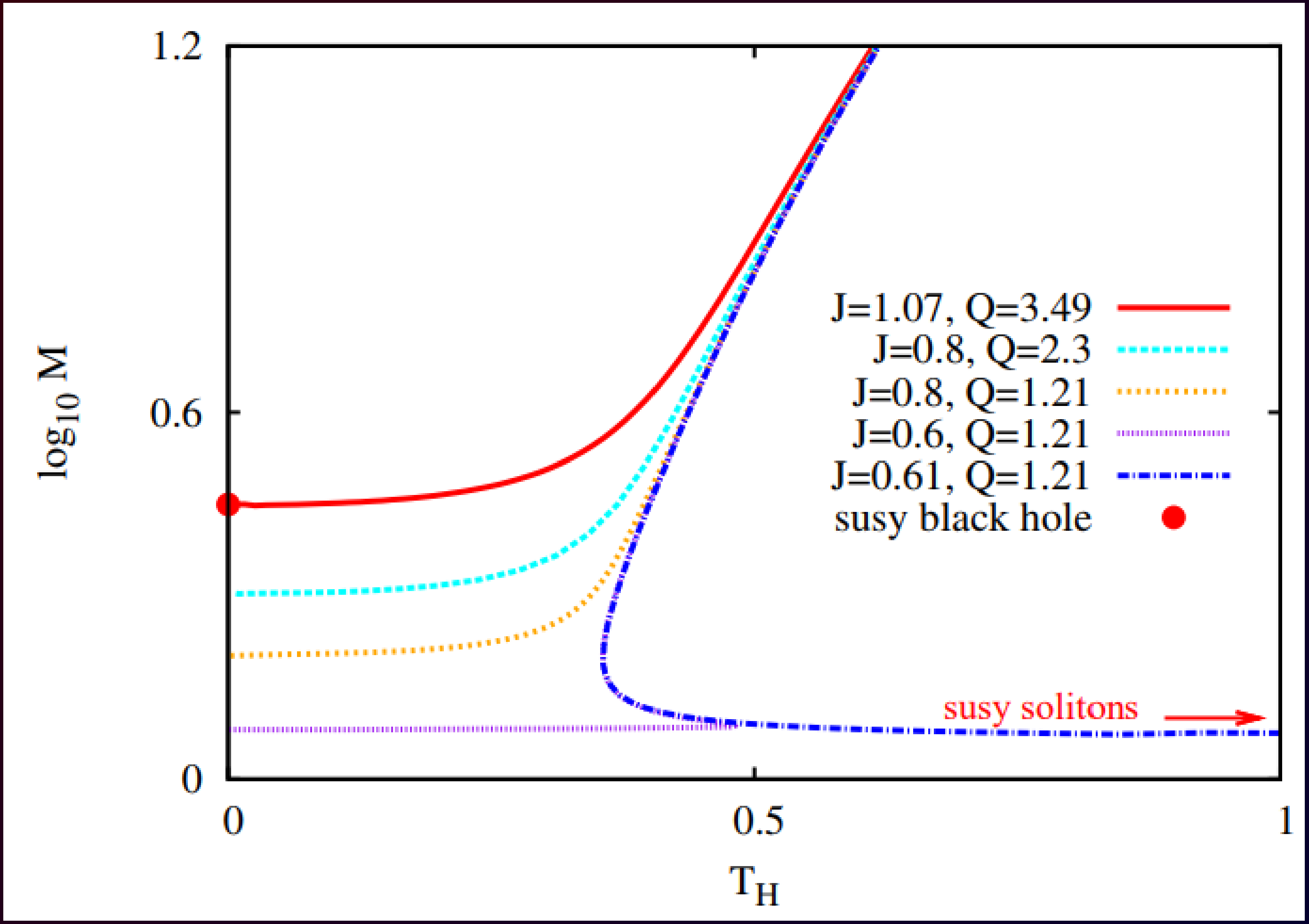


Cassani, Martelli
JHEP 08 044 (2014)

3. Magnetized and Squashed solutions

Squashed and magnetized non-extremal black holes (work in progress):

$v=1.65$
$c_m=-1$
$L=1$



Two limits: extremal black holes or solitons (shrinking horizon)

3. Magnetized and Squashed solutions

Squashed and magnetized non-extremal black holes (work in progress):

$v=1.65$
$c_m=-1$
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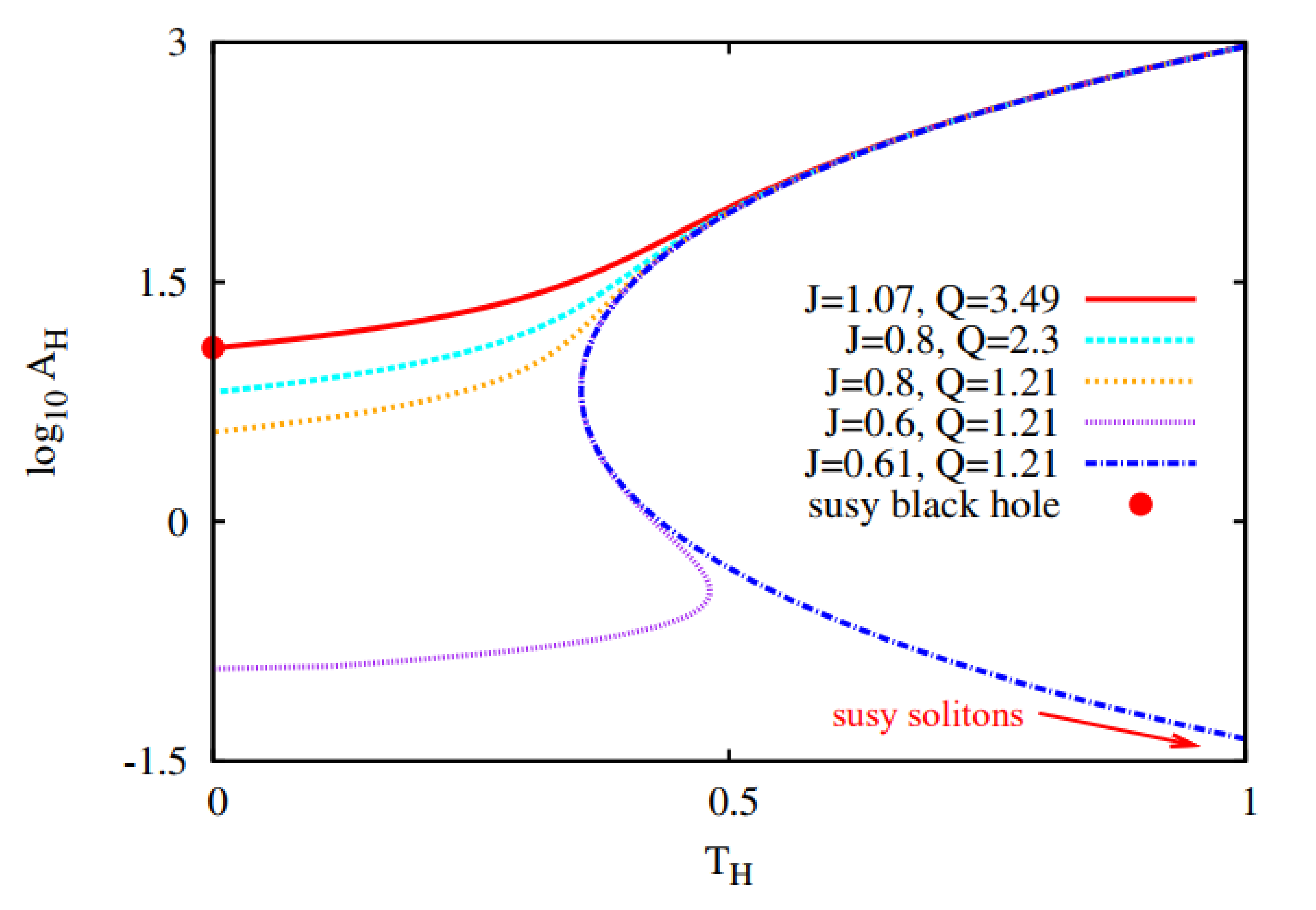


Two limits: extremal black holes or solitons (shrinking horizon)

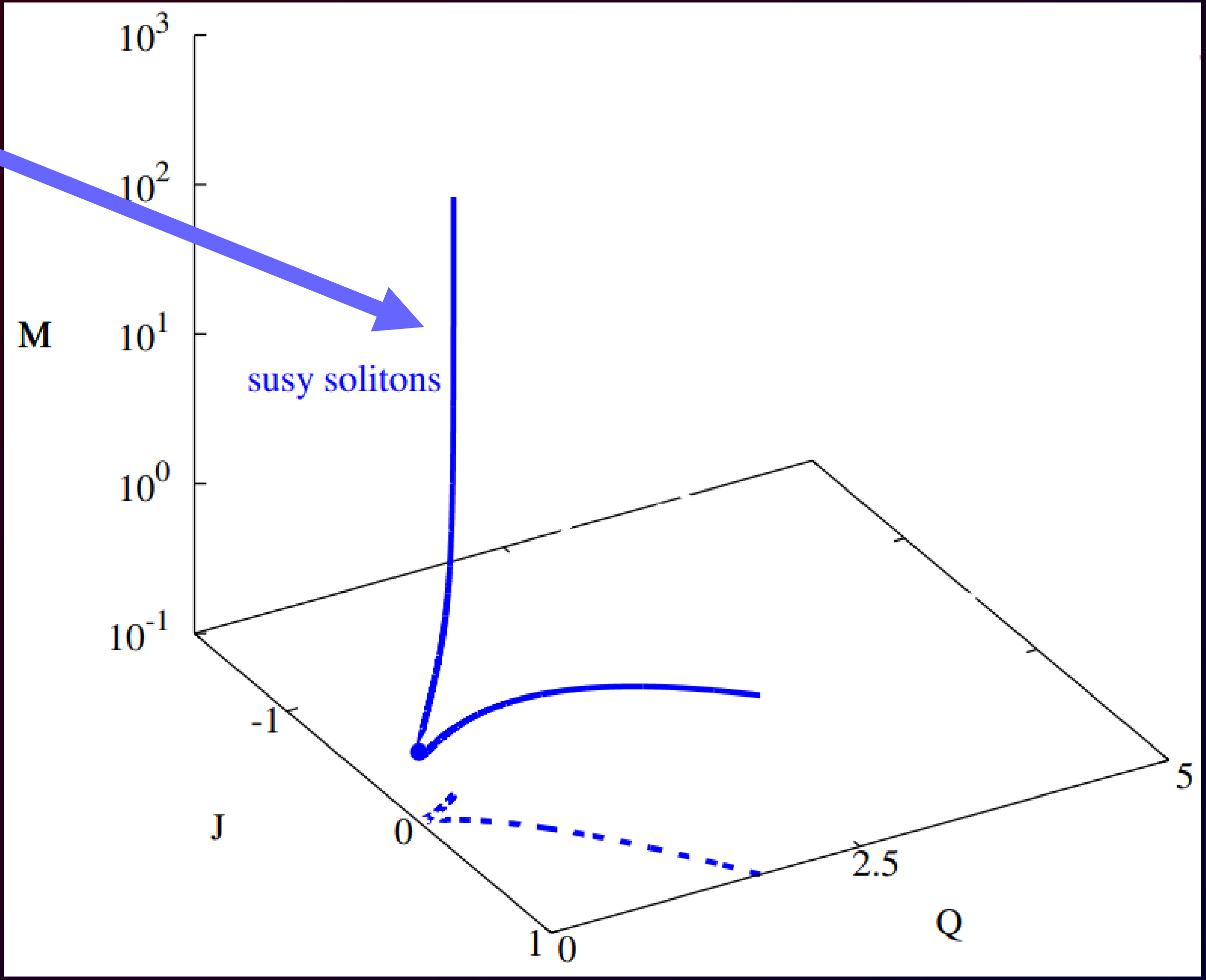
3. Magnetized and Squashed solutions

Supersymetric solutions:

3. Magnetized and Squashed solutions

Supersymetric solutions:

- susy solitons:
  Magnetized and squashed
  (Cassani, Martelli)

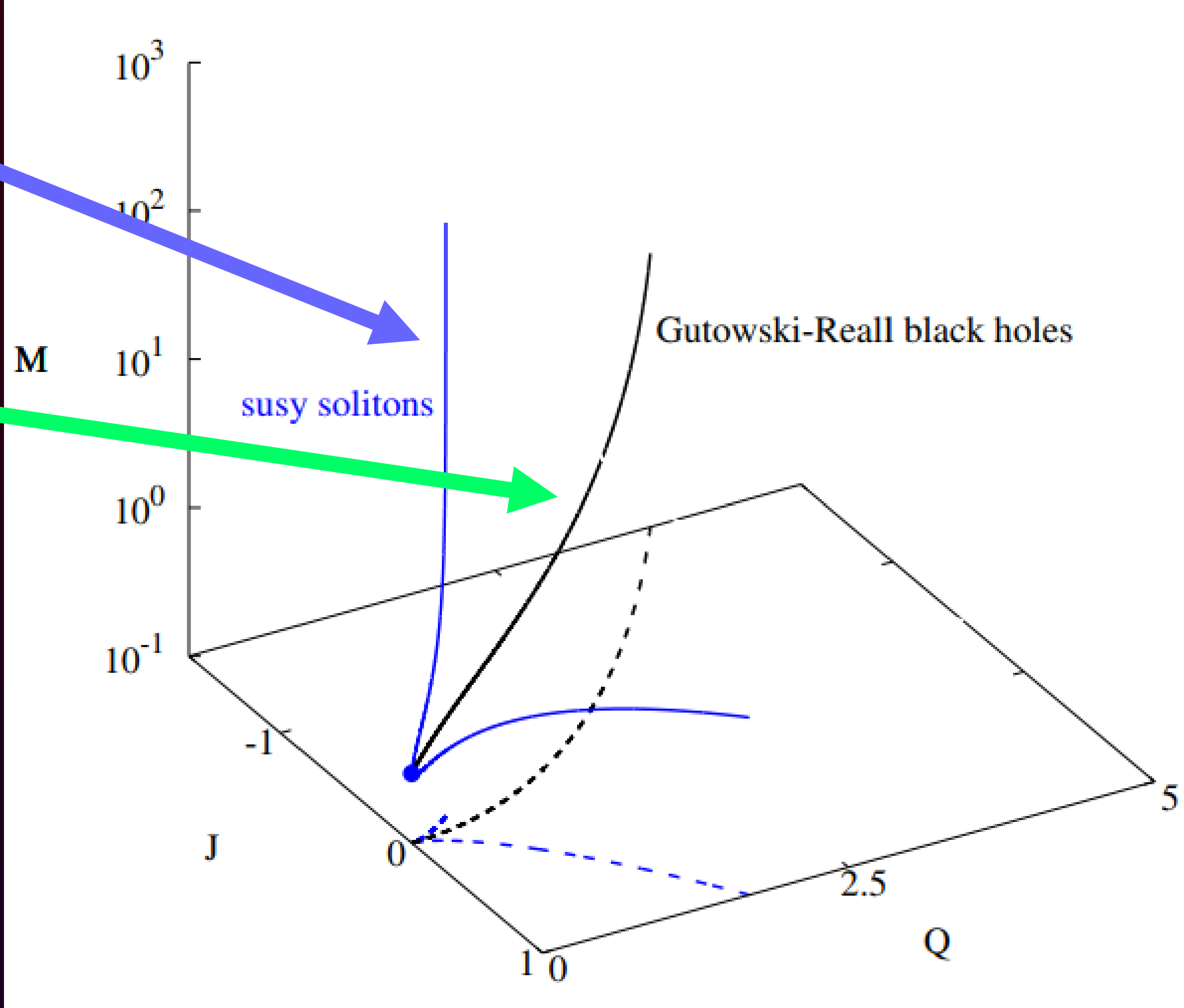


3. Magnetized and Squashed solutions

Supersymetric solutions:

- susy solitons:
  Magnetized and squashed
  (Cassani, Martelli)

- susy black holes:
  Unmagnetized and AdS_5
  (Gutowski, Reall)



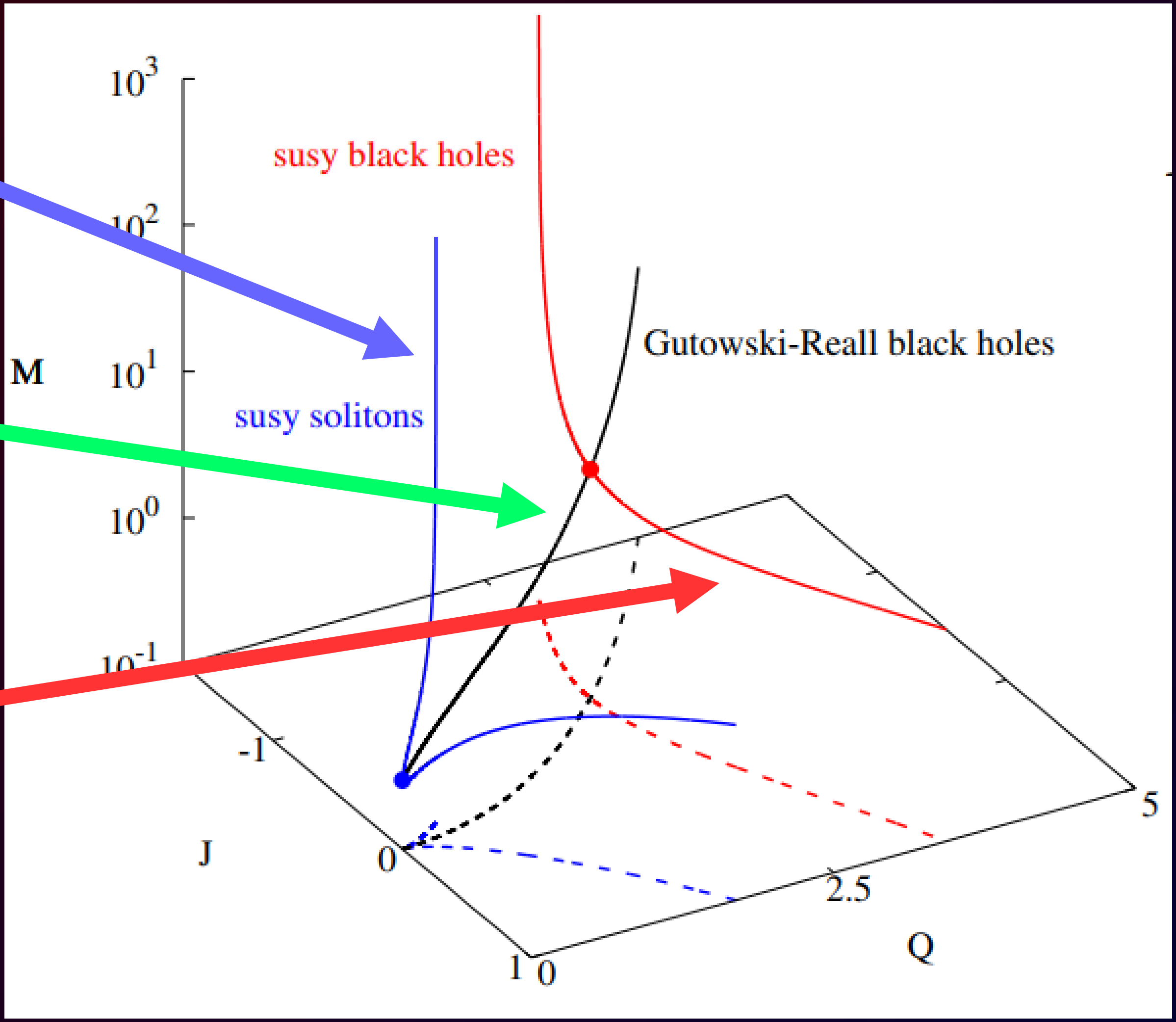
3. Magnetized and Squashed solutions

Supersymetric solutions:

- susy solitons:
  Magnetized and squashed
  (Cassani, Martelli)

- susy black holes:
  Unmagnetized and AdS_5
  (Gutowski, Reall)

- squashed susy black holes:
  Magnetized and squashed



3. Magnetized and Squashed solutions

New squashed and magnetized susy black holes:

$$M = \pi L^2 \left(\frac{7913}{34848} + \frac{33280}{35937 v^2} - \frac{7}{36} v^2 + \frac{89}{864} v^4 \right)$$

$$J = \pi L^3 \left(\frac{16640}{35937} - \frac{2795}{8712} v^2 + \frac{1}{9} v^4 - \frac{1}{27} v^6 \right)$$

$$Q = \pi \sqrt{3} L^2 \frac{1}{13068} \left(6449 - 1936\, v^2 + 968\, v^4 \right)$$

$$A_{\mathrm{H}} = 7 \pi^2 L^3 \frac{\sqrt{455}}{121}$$

$$c_{\mathrm{m}} = \pm \frac{L}{\sqrt{3}} \left(1 - v^2 \right)$$

4. Conclusions

4. Conclusions

- New class of magnetized black holes and solitons in EMCS theory (SUGRA)

- New class of squashed and magnetized black holes in SUGRA

Perturbative solutions in the far-field region / horizon / origin
Global charges and thermodynamic properties

Non-extremal black holes can be continuously deformed to solitons (susy)

Extremal black holes include a new class of
squashed and magnetized susy black holes

4. Conclusions

Thank you for your attention!

Entropy 2016, 18(12), 438
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arxiv:1708.xxxxx