Non-relativistic Limits and Massive Gravity

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Motivation

special feature FQH Effect: existence of a gapped collective non-relativistic massive spin-2 excitation, known as the GMP mode

Girvin, MacDonald and Platzman (1985)

recent proposal for a non-relativistic spatially covariant bimetric EFT describing non-linear dynamics of this massive spin-2 GMP mode

Gromov and Son (2017)

proposal is reminiscent to non-relativistic and massive gravity

Cartan (1923); de Rham, Gabadadze, Tolley (2011)

Can EFT be obtained by some limit of Relativistic Massive Graviy?

Outline

Newton-Cartan Gravity

Outline

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Massive Gravity

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Newton-Cartan Gravity

Massive Gravity

Non-relativistic Limits

Conclusions

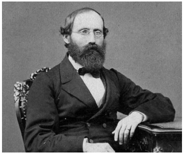
NC Gravity in a Nutshell

- Inertial frames: Galilean symmetries

- Constant acceleration: Newtonian gravity/Newton potential $\Phi(x)$

- no frame-independent formulation

  (needs geometry!)



Riemann (1867)

Galilei Symmetries

- time translations: $\delta t = \xi^0$ <u>but not</u> $\delta t = \lambda^i x^i$!
- space translations: $\delta x^i = \xi^i \qquad i = 1, 2, 3$
- spatial rotations: $\delta x^i = \lambda^i{}_j \, x^j$
- Galilean boosts: $\delta x^i = \lambda^i t$

$$[J_{ab}, P_c] = -2\delta_{c[a}P_{b]}\,, \qquad [J_{ab}, G_c] = -2\delta_{c[a}G_{b]}\,,$$

$$[G_a, H] = -P_a\,, \qquad [J_{ab}, J_{cd}] = \delta_{c[a}J_{b]d} - \delta_{a[c}J_{d]b}\,, \qquad a = 1, 2, 3$$

'Gauging' Galilei

symmetry	generators	gauge field	curvatures
time translations	H	τ_μ	$\mathcal{R}_{\mu\nu}(H)$
space translations	P^a	$e_\mu{}^a$	$\mathcal{R}_{\mu\nu}{}^a(P)$
Galilean boosts	G^a	$\omega_\mu{}^a$	$\mathcal{R}_{\mu\nu}{}^a(G)$
spatial rotations	J^{ab}	$\omega_\mu{}^{ab}$	$\mathcal{R}_{\mu\nu}{}^{ab}(J)$

Imposing Constraints

$\mathcal{R}_{\mu\nu}{}^a(P) = 0$: does only solve for part of $\omega_\mu{}^{ab}$

$\mathcal{R}_{\mu\nu}(H) = \partial_{[\mu}\tau_{\nu]} = 0 \quad \rightarrow$ absolute time $T = \int \tau_\mu \, dx^\mu$

'Gauging' Bargmann

symmetry	generators	gauge field	curvatures
time translations	H	τ_μ	$\mathcal{R}_{\mu\nu}(H)$
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spatial rotations	J^{ab}	${\omega_\mu}^{ab}$	${\mathcal{R}_{\mu\nu}}^{ab}(J)$
central charge transf.	Z	m_μ	$\mathcal{R}_{\mu\nu}(Z)$

Imposing Constraints

$${\mathcal{R}_{\mu\nu}}^a(P) = 0\,, \qquad \mathcal{R}_{\mu\nu}(Z) = 0\ :$$ solve for spin-connection fields

$$\mathcal{R}_{\mu\nu}(H) = \partial_{[\mu}\tau_{\nu]} = 0 \quad \rightarrow$$ absolute time ('zero torsion')

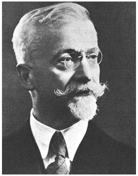
The NC Transformation Rules

The independent NC fields $\{\tau_\mu, e_\mu{}^a, m_\mu\}$ transform as follows:

$$\begin{aligned}\delta\tau_\mu &= 0\,,\\ \delta e_\mu{}^a &= \lambda^a{}_b\, e_\mu{}^b + \lambda^a\tau_\mu\,,\\ \delta m_\mu &= \partial_\mu\sigma + \lambda_a\, e_\mu{}^a\end{aligned}$$

The spin-connection fields $\omega_\mu{}^{ab}$ and $\omega_\mu{}^a$ are functions of e, τ and m

The NC Equations of Motion



Élie Cartan 1923

The NC equations of motion are given by

$$\tau^\mu e^\nu{}_a \mathcal{R}_{\mu\nu}{}^a(G) = 0 \qquad \mathbf{1}$$

$$e^\nu{}_a \mathcal{R}_{\mu\nu}{}^{ab}(J) = 0 \qquad \mathbf{a} + (\mathbf{ab})$$

- after gauge-fixing and assuming flat space the first NC e.o.m. becomes $\triangle\Phi = 0$
- there is no known action that gives rise to these equations of motion

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What is Massive Gravity ?

Massive Gravity is the name we have given to the attempt to understand what the gravitational force would be like if the graviton, the carrier of the gravitational force, has a small, but non-zero, mass

1939: Fierz-Pauli

- the free massive graviton, with mass m, is a spin-2 particle described by a symmetric tensor field $h_{\mu\nu}(x)$

- for $m = 0$ this tensor can be viewed as the linearized approximation to a metric tensor $g_{\mu\nu}(x)$:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x) + O(h^2)$$

- the Fierz-Pauli kinetic term is the linearization of the Einstein-Hilbert term of general relativity

The Fierz-Pauli mass term

$$\mathcal{L}_{\text{FP}}(\text{mass}) \sim m^2\left(h^{\mu\nu}h_{\mu\nu} - h^2\right) \qquad h \equiv \eta^{\mu\nu}h_{\mu\nu}$$

The Fierz-Pauli mass term

- breaks the linearized g.c.t. of the kinetic term:

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

- contains a reference metric $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$

- requires fine-tuning

New Developments

First proposals for a non-linear mass term date back to

Salam, Strathdee (1969); Isham, Salam, Strathdee (1970); Zumino (1970)

- recent proposal in 4D by

  de Rham, Gabadadze, Tolley (2011)

- use Vierbeins ${e_\mu}^a$ instead of metric $g_{\mu\nu}$

  Hinterbichler, Rosen (2010)

- 3D: Chern-Simons formulation in terms of $({e_\mu}^a, {\omega_\mu}^a)$

  Deser, Jackiw, 't Hooft (1984)

  Achucarro, Townsend (1986); Witten (1988)

The 3D dRGT Chern Simons Model

$$\mathcal{L}_{\mathrm{dRGT}}(e,\omega;\bar{e}) \;=\; -M_{\mathrm{P}}\Big\{ e_a\wedge R^a(\omega) + \epsilon^{abc}\big(\alpha_1\, e_a\wedge e_b\wedge e_c + \beta_1\, e_a\wedge e_b\wedge \bar{e}_c + \beta_2\, e_a\wedge \bar{e}_b\wedge \bar{e}_c\big)\Big\}$$

$$R^a(\omega) = d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b\wedge\omega_c \;: \qquad \text{curvature tensor}$$

- reduces to Fierz-Pauli in linearized approximation

The 3D dRGT CS model can be obtained as a scaling limit of an underlying 3D "ZDG model" containing zwei Dreibeine

Zwei-Dreibein Gravity

4D : Hassan, Rosen (2012); de Haan, Hohm, Merbis, Townsend (2013)

we introduce two ("zwei") Dreibeine ${e_I}_\mu{}^a$ and two independent spin-connections ${\omega_I}_\mu{}^a$ $\quad (I = 1, 2)$

$$
\begin{aligned}
\mathcal{L}_{\text{ZDG}}({e_I}^a, {\omega_I}^a) \;=\; -M_{\mathrm{P}}\Big\{ & e_{1a} \wedge R_1^a + e_{2a} \wedge R_2^a \\
& +\epsilon^{abc}\Big(\alpha_1\, e_{1a} \wedge e_{1b} \wedge e_{1c} + \alpha_2\, e_{2a} \wedge e_{2b} \wedge e_{2c} + \\
& +\beta_1\, e_{1a} \wedge e_{1b} \wedge e_{2c} + \beta_2\, e_{1a} \wedge e_{2b} \wedge e_{2c}\Big)\Big\}
\end{aligned}
$$

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From General Relativity to NC gravity

Poincare $\otimes$ U(1) $\stackrel{\text{'gauging'}}{\Longrightarrow}$ GR plus $\partial_\mu M_\nu - \partial_\nu M_\mu = 0$

contraction $\Downarrow$ $\Downarrow$ non-relativistic limit

Bargmann $\stackrel{\text{'gauging'}}{\Longrightarrow}$ Newton-Cartan gravity

The NC Limit I

Dautcourt (1964); Rosseel, Zojer + E.B. (2015)

STEP I: express relativistic fields $\{E_\mu{}^A, M_\mu\}$ in terms of non-relativistic fields $\{\tau_\mu, e_\mu{}^a, m_\mu\}$

$$E_\mu{}^0 = \omega\,\tau_\mu + \frac{1}{2\omega}\,m_\mu\,, \quad M_\mu = \omega\,\tau_\mu - \frac{1}{2\omega}\,m_\mu\,, \quad E_\mu{}^a = e_\mu{}^a \quad \Rightarrow$$

$$E^\mu{}_a = e^\mu{}_a - \frac{1}{2\omega^2}\,\tau^\mu e^\rho{}_a m_\rho + \mathcal{O}\left(\omega^{-4}\right) \text{ and similar for } E^\mu{}_0$$

The NC Limit II

STEP II: take the limit $\omega \to \infty$ in e.o.m. $\Rightarrow$

- the NC transformation rules are obtained
- the NC equations of motion are obtained (but no action!)

Note: the standard textbook limit gives Newton gravity

What about Matter?

Real Representations

consider a real scalar field with mass M

$$E^{-1}\,\mathcal{L}_{\rm rel} = -\frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\Phi\partial_\nu\Phi - \frac{M^2}{2}\,\Phi^2$$

Rescale $\Phi = \frac{1}{\sqrt{\omega}}\phi$ and take non-relativistic limit $\omega \to \infty \quad \to$

$$e^{-1}\mathcal{L}_{\rm non-rel.} = -\frac{1}{2}\left(\partial_a\phi\right)^2 - \frac{M^2}{2}\,\phi^2$$

From Klein-Gordon to Schrödinger

Jensen, Karch (2014), Rosseel, Zojer + E.B. (2015), Fuini, Karch, Uhlemann (2015)

Klein-Gordon + GR $\overset{\text{'limit'}}{\Longrightarrow}$ Schrödinger + NC

general frames $\Uparrow$ $\Downarrow$ inertial frames

Klein-Gordon $\overset{?}{\Longrightarrow}$ Schrödinger

The Schrödinger Limit I

consider a complex scalar field with mass M

Lévy Leblond (1963,1967)

$$E^{-1}\,\mathcal{L}_{\rm rel} = -\frac{1}{2}\,g^{\mu\nu}\,D_\mu\Phi^*D_\nu\Phi - \frac{M^2}{2}\,\Phi^*\Phi \quad \text{with}$$

$$D_\mu\Phi = \partial_\mu\Phi - \mathrm{i}\,M\,M_\mu\,\Phi\,, \qquad \delta\Phi = \mathrm{i}\,M\,\Lambda\,\Phi$$

- M_μ is not an electromagnetic field ($M \neq q$)!
- M_μ couples to the current that expresses conservation of

  # particles – # antiparticles

The Schrödinger Limit II

Take non-relativistic limit extended with $M = \omega m \, , \Phi = \sqrt{\frac{\omega}{m}}\phi \quad \rightarrow$

$$e^{-1}\mathcal{L}_{\text{Schroedinger}} = \Big[\, \frac{\mathrm{i}}{2}\left(\phi^*\mathcal{D}_0\phi - \phi\mathcal{D}_0\phi^*\right) - \frac{1}{2m}\left|\mathcal{D}_a\phi\right|^2 \Big] \quad \text{with}$$

$$\mathcal{D}_\mu\phi = \partial_\mu\phi + \mathrm{i}\, m\, m_\mu\, \phi\,, \qquad \delta\phi = \xi^\mu\partial_\mu\phi - \mathrm{i}\, m\,\sigma\,\phi$$

- m_μ couples to the current that expresses conservation of # particles
- going to inertial frames gives Schrödinger equation

The Massive Spin 1 Case

consider a complex Proca field with mass M

$$E^{-1}\mathcal{L}_{\rm rel} = -\frac{1}{4}\, g^{\mu\rho}g^{\nu\sigma}F^*_{\mu\nu}F_{\rho\sigma} - \frac{1}{2}\, M^2\, g^{\mu\nu}A^*_\mu A_\nu \quad \text{with}$$

$$F_{\mu\nu} = 2\, D_{[\mu}A_{\nu]} = 2\,\partial_{[\mu}A_{\nu]} - 2\,\mathrm{i}\, M\, M_{[\mu}A_{\nu]}\,, \qquad \delta A_\mu = \mathrm{i}\, M\, \Lambda\, A_\mu$$

Take non-rel. limit extended with $M = \omega m$ and go to inertial frames $\rightarrow$

$$e^{-1}\mathcal{L}_{\rm non-rel} = -\frac{1}{4}\, F^*_{ab}F^{ab} - \frac{1}{2}\,\mathrm{i}\, m\, A^*_a F_{a0} + \frac{1}{2}\,\mathrm{i}\, m\, A_a\, F^*_{a0} + \frac{1}{2}\, m^2\, \left|A_0\right|^2$$

The 3D Case is Reducible

$$Q = A_1 + iA_2\,, \qquad \tilde{Q} = A_1 - iA_2 \qquad \Rightarrow$$

$$mA_0 = i\partial Q + i\bar{\partial}\tilde{Q} \qquad \Rightarrow$$

$$\dot{Q} + \tfrac{i}{2m}\bar{\partial}\partial Q = 0 \quad \text{and} \quad \dot{\tilde{Q}} + \tfrac{i}{2m}\bar{\partial}\partial\tilde{Q} = 0$$

- spin determines transformation of Q under spatial rotations
- $Q = A_1 + iA_2$ does not give Schrödinger for real (A_0, A_1, A_2)

What is the Non-relativistic Limit of TME?

$$\mathcal{L}_{\mathrm{TME}} = -\tfrac{1}{4} F^{\mu\nu}(A) F_{\mu\nu}(A) + \tfrac{1}{2} m \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

$$\begin{array}{ccc} \text{Proca} & \overset{\text{non rel. limit}}{\Longrightarrow} & Q \; + \; \tilde{Q} \\ \text{'truncation'} \Downarrow & & \Downarrow \text{truncation} \\ \text{TME} & \overset{?}{\Longrightarrow} & Q \end{array}$$

A Road Map for Non-relativistic Massive Spin-2

- check non-relativistic limit of Fierz-Pauli

- Take non-relativistic limit of Zwei-Dreibein Gravity
  - no action!

- Check linearized E.O.M. for flat background $\Rightarrow \quad Q \; + \; \tilde{Q}$?

3D Gravity is not unique!

- 3D Galilei Algebra allows two central extensions. The second central extension is related to anyons

Jackiw, Nair (2000)

- This so-called Extended Bargmann Algebra has a non-degenerate bilinear form allowing a CS action. This EBG CS action can be obtained from non-relativistic limit of relativistic CS action plus term of the form $\epsilon^{\mu\nu\rho} M_\mu \partial_\nu S_\rho$

- consider limit of Zwei-Dreibein Gravity with extra vector fields $\Rightarrow$
    - Bimetric Extended Bargmann Gravity
    - EFT describing non-linear dynamics of massive spin 2?

Outline

Open questions

- Does a particular non-relativistic limit of relativistic Zwei-Dreibein Gravity plus auxiliary gauge fields give the fully-covariant completion of the proposal of the GMP mode in FQE Effect?

  Gromov and Son (2017)

- If so, can you use this limiting procedure to construct extensions involving higher derivatives and/or higher spins?

- New Massive Gravity?

Take-Home Message

Taking non-relativistic limits is non-trivial!