Supersymmetries & Quantum Symmetries – SQS'2017

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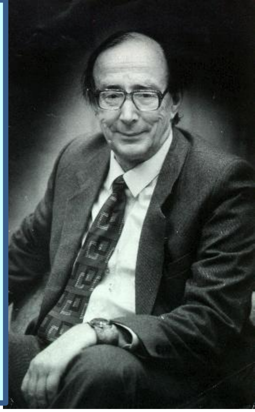

BRST structure of renormalization in the background field approach

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based on works with:
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Introduction

Consider a quantum gauge field theory

Assume existence of a gauge invariant regularized path integral measure (no anomalies)

Task: `Prove that gauge invariance is preserved by renormalization`

40+ year old topic. What news can there be ?

- simplified treatment for standard effective theories (YM with higher dimensional operators, gravity)
- extension to effective theories (derivative expansion)
- extension to a very broad class (e.g. theories without Lorentz invariance)

What about gauge invariance ?

this is tricky ...

GI is explicitly broken by the gauge-fixing. Instead, we have to rely on the Slavnov-Taylor identities $\Leftrightarrow$ BRST symmetry.

The latter gets deformed at each loop order and requires non-linear field renormalization to restore ...

Use the background-field method

quantum fields
(integration variables)

φ^a

background fields
(external)

ϕ^a

Gauge-fixing term invariant under <u>simultaneous</u> gauge transformations of φ and ϕ

Very efficient at 1 loop:

$$\Gamma^{1-\mathrm{loop}}\Big|_{\langle\varphi\rangle=\phi,\ \langle\omega\rangle=0} = \Gamma^{1-\mathrm{loop}}[\phi]$$

gauge invariant

At 2 loop and higher need $\langle\varphi\rangle \neq \phi$ to subtract subdivergences
= need to classify the BRST cohomology.

Otherwise non-local divergences at intermediate steps

Kallosh (1974); Arefeva, Faddeev, Slavnov (1974); Abbott (1981); Ichinose, Omote (1982); Barvinsky, Vilkovisky (1987)

Still, presence of background gauge transformations greatly simplifies the task !

Background field extension of the BRST operator (Grassi, 1996)

Inclusion of generating functional sources into the gauge fermion

BRST structure of renormalization via decoupling of the background field

Quantum corrected gauge fermion is a generating functional of the field reparameterization

No power counting or use of field dimensionalities

Extension to (nonrenormalizable) effective field theories

Plan

Introduction

BRST formalism

Assumptions on the class of gauge theories

Background field gauge fixing method

Introduction of sources

Main result: BRST structure of renormalization and field reparametrization

Slavnov-Taylor and Ward identities

Decoupling the background field and field redifinition

Example: *2D O(N)* sigma model

Summary and outlook: the power and beauty of the nilpotent BRST operator

BRST formalism

Gauge theory:

$$\varphi = \varphi^a, \qquad S = S[\varphi]$$

$$\frac{\delta S}{\delta \varphi^a} R^a_\alpha = 0$$

Generators of gauge transformations:

$$R^a_\alpha = R^a_\alpha(\varphi), \qquad \delta_\epsilon \varphi^a = R^a_\alpha \epsilon^\alpha,$$

$$R^a_\alpha \frac{\delta R^b_\beta}{\delta \varphi^a} - R^a_\beta \frac{\delta R^b_\alpha}{\delta \varphi^a} = C^\gamma_{\alpha\beta} R^b_\gamma$$

structure constants

DeWitt summation rule: $a = (A, x), \qquad F^a \Psi_a \equiv \int dx\, F^A(x) \Psi_A(x)$

Feynman-DeWitt-Faddeev-Popov functional integral

$$e^{-W[\,J\,]} = \int d\varphi\, e^{-S[\,\varphi\,]\underbrace{-\frac{1}{2}\chi^\alpha O_{\alpha\beta}\chi^\beta}_{\text{gauge-breaking term}}-J\varphi}\,\underbrace{\left(\det O_{\alpha\beta}\right)}_{\text{measure}}{}^{1/2}\det\underbrace{\left(\frac{\delta\chi^\alpha}{\delta\varphi^a}\,R^a_\beta\right)}_{\text{Faddeev-Popov operator}}$$

$$S[\,\varphi\,] \to \Sigma[\,\Phi\,] = S[\,\varphi\,] + \overbrace{b_\alpha\chi^\alpha(\varphi) - \frac{1}{2}O^{\alpha\beta}\,b_\alpha b_\beta}\, - \bar\omega_\alpha\left(\frac{\delta\chi^\alpha}{\delta\varphi^a}\,R^a_\beta\right)\omega^\beta$$

$$\varphi^a \to \Phi = \varphi^a, \underbrace{\omega^\alpha, \bar\omega_\alpha, b_\alpha}$$

BRST functional integral

$$e^{-W[\,J\,]} = \int d\Phi\, e^{-\Sigma[\,\Phi\,]-J\varphi}$$

$$\Sigma[\Phi] = S[\varphi] + \boldsymbol{s}\,\Psi[\Phi]$$

BRST action

$$\boldsymbol{s} = (\boldsymbol{s}\Phi)\frac{\delta}{\delta\Phi}, \qquad \boldsymbol{s}^2 = 0$$

nilpotent BRST operator

BRST transformations of Φ

$$\boldsymbol{s}\Phi : \underbrace{\boldsymbol{s}\varphi^a = R^a_\alpha(\varphi)\,\omega^\alpha\,, \quad \boldsymbol{s}\omega^\alpha = \frac{1}{2}C^\alpha_{\beta\gamma}\,\omega^\beta\omega^\gamma,}_{\text{bilinear in } \Phi} \quad \boldsymbol{s}\bar{\omega}_\alpha = b_\alpha, \quad \boldsymbol{s}b_\alpha = 0\,.$$

bilinear in Φ

$$\Psi[\Phi] = \bar{\omega}_\alpha\Big(\chi^\alpha(\varphi) - \frac{1}{2}\,O^{\alpha\beta}\,b_\beta\Big)$$

gauge fermion

gauge conditions

gauge-fixing matrix

Local BRST cohomology

gauge-fixed action

$$\Sigma_{\rm ren}^{L-{\rm loop}} = S^{L-{\rm loop}}[\varphi] + \boldsymbol{s}\,\Psi^{L-{\rm loop}}[\varphi,\omega,\ldots]$$

gauge-invariant   BRST operator   gauge fermion

$\boldsymbol{s} \Leftrightarrow \boldsymbol{s}_+$ differ in the sector of antighost – see below

- $\Gamma^{L-{\rm loop}}$ satisfies ST identities
- $\boldsymbol{s}_+\Gamma_{\rm div}^{L-{\rm loop}} = 0$
- $\Gamma_{\rm div}^{L-{\rm loop}} = \boldsymbol{S}_{L+1}[\varphi] + \boldsymbol{s}_+\Upsilon_{L+1}[\varphi,\omega,\ldots]$

must be local !

for relativistic YM and GR:

Barnich, Brandt, Henneaux (1994)

- subtraction and field redefinition

$$\varphi \mapsto \varphi'(\varphi,\omega,\ldots)\,, \quad \omega \mapsto \omega'(\varphi,\omega,\ldots)$$

Voronov,Tyutin(1982)
Anselmi (1994)
Gomis,Weinberg (1996)

- $\Sigma_{\rm ren}^{(L+1)-{\rm loop}}$ has the BRST structure with Q

Assumptions on the class of theories

i) linear gauge generators: $\delta_\varepsilon \varphi^a = \underbrace{(R^a{}_{b\alpha}\varphi^b + P^a{}_\alpha)}_{R^a{}_\alpha(\varphi)}\varepsilon^\alpha$

ii) off-shell closure: $[\delta_\varepsilon, \delta_\eta]\varphi^a = \delta_\varsigma \varphi^a\,, \quad \varsigma^\alpha = C^\alpha{}_{\beta\gamma}\varepsilon^\beta\eta^\gamma$

iii) <u>local</u> completeness:

$$\frac{\delta S}{\delta\varphi^a}X^a{}_\alpha(\varphi) = 0 \quad \Longrightarrow \quad X^a{}_\alpha(\varphi) = R^a{}_\beta Y^\beta_\alpha + \frac{\delta S}{\delta\varphi^b}I^{[ba]}_\alpha$$

local

iv) irreducibility:

$$R^a{}_\alpha(\varphi_0)\varepsilon^\alpha = 0 \quad \Longrightarrow \quad \varepsilon^\alpha = 0$$

v) absence of anomalies

vi) locality of divergences after removal of subdivergences

(nontrivial in Lorentz-violating theories, like Horava gravity, and should be verified in each model)

A.B, D. Blas, M. Herrero-Valea, S.Sibiryakov and C. Steinwachs (2016)

Examples:

YM: $\delta_\varepsilon A^i_\mu = f^{ijk} A^j_\mu \varepsilon^k + \partial_\mu \varepsilon^i$

GR: $\delta_\varepsilon g_{\mu\nu} = \varepsilon^\lambda \partial_\lambda g_{\mu\nu} + g_{\mu\lambda}\partial_\nu \varepsilon^\lambda + g_{\nu\lambda}\partial_\mu \varepsilon^\lambda$

Also higher-derivative gravity, also non-relativistic (Lifshitz) theories

Counterexample:

Supergravity (the algebra does not close off-shell)

Background gauge-fixing

- choose g.f. function $\chi^\alpha(\varphi,\phi) = \chi^\alpha_a(\phi)(\varphi-\phi)^a$ to be

  invariant under BGT: $\delta_\varepsilon \varphi^a = R^a{}_\alpha(\varphi)\varepsilon^\alpha \qquad \delta_\varepsilon \phi^a = R^a{}_\alpha(\phi)\varepsilon^\alpha$

  the same

- promote $\boldsymbol{s} \mapsto \boldsymbol{Q} = \boldsymbol{s} + \Omega^a \dfrac{\delta}{\delta\phi^a}$

  anticommuting auxiliary field,
  controls dependence of g.f. on background

g.f. term at tree level: $\boldsymbol{Q}\Psi_0$

auxiliary (anti-)fields coupled to $\boldsymbol{s}\varphi^a$, $\boldsymbol{s}\omega^\alpha$

antighost

Lagrange multiplier

$$\Psi_0 = -\big(\underbrace{\gamma_a - \bar\omega_\alpha \chi^\alpha_a(\phi)}_{\hat\gamma_a}\big)(\varphi-\phi)^a + \zeta_\alpha\omega^\alpha - \frac{1}{2}\bar\omega_\alpha O^{\alpha\beta}(\phi) b_\beta$$

Introduction of sources

backgrpound source

$$\mathcal{J} = J_a,\ \bar{\xi}_\alpha,\ \xi^\alpha,\ y^\alpha,\ \gamma_a,\ \zeta_\alpha,\ \Omega^a$$

sources for $\Phi = \varphi^a, \omega^\alpha, \bar{\omega}_\alpha, b_\alpha$

sources for $\boldsymbol{s}\varphi^a, \boldsymbol{s}\omega^\alpha$

$$\exp\left\{-\frac{1}{\hbar}W_0[\mathcal{J}]\right\} = \int d\Phi \exp\left\{-\frac{1}{\hbar}\Big(\Sigma_0 + J_a(\varphi^a - \phi^a) + \bar{\xi}_\alpha\omega^\alpha + \xi^\alpha\bar{\omega}_\alpha + y^\alpha b_\alpha\Big)\right\}$$

$$\Sigma_0[\Phi, \mathcal{J}] = S[\varphi] + \boldsymbol{Q}\Psi_0[\Phi, \mathcal{J}]$$

$$\Psi_0[\varphi, \omega, \bar{\omega}, b, \phi, \gamma, \zeta] = \bar{\omega}_\alpha\Big(\chi^\alpha_a(\phi)(\varphi^a - \phi^a) - \frac{1}{2}O^{\alpha\beta}(\phi)\, b_\beta\Big) - \gamma_a(\varphi^a - \phi^a) + \zeta_\alpha\omega^\alpha$$

More on background gauge transformations – linear representation of the gauge group

$$\delta_\varepsilon \varphi^a = R^a{}_\alpha(\varphi)\,\varepsilon^\alpha \ , \qquad \delta_\varepsilon \phi^a = R^a{}_\alpha(\phi)\,\varepsilon^\alpha$$

$$\delta_\varepsilon(\varphi^a - \phi^a) = \frac{\delta R^a{}_\alpha}{\delta \varphi^b}(\varphi^b - \phi^b)\varepsilon^\alpha$$

fundamental representation

$$\delta_\varepsilon \chi^\alpha \equiv \frac{\delta \chi^\alpha}{\delta \varphi^a}\delta_\varepsilon \varphi^a + \frac{\delta \chi^\alpha}{\delta \phi^a}\delta_\varepsilon \phi^a = -C^\alpha{}_{\beta\gamma}\chi^\beta \varepsilon^\gamma$$

adjoint representation

B.g.t. of sources:

$$\delta_\varepsilon \gamma_a = -\gamma_b R^b{}_{a\alpha}\varepsilon^\alpha \ , \qquad \delta_\varepsilon \omega^\alpha = -C^\alpha{}_{\beta\gamma}\,\omega^\beta \varepsilon^\gamma$$

$$\delta_\varepsilon \zeta_\alpha = \zeta_\beta C^\beta{}_{\alpha\gamma}\varepsilon^\gamma, \qquad \delta_\varepsilon \Omega^\alpha = R^a{}_{b\alpha}\Omega^b \varepsilon^\alpha$$

$$\Downarrow$$

$$\delta_\varepsilon \Psi_0 = 0, \quad \delta_\varepsilon \Sigma_0 = 0$$

Renormalization at a glance

generating functional $$W[J] = -\hbar \log \int d\Phi \exp\left[-\frac{1}{\hbar}(S[\Phi] + J\Phi)\right]$$

mean fields, effective action $$\langle\Phi\rangle = \frac{\delta W}{\delta J}\ , \qquad \Gamma[\langle\Phi\rangle] = W - J\langle\Phi\rangle$$

subtraction $$S_{\rm ren} = S_0[\Phi] - \hbar\,\Gamma^{1-{\rm loop}}_{\rm div}[\Phi] - \hbar^2\Gamma^{2-{\rm loop}}_{\rm div}[\Phi] - \ldots$$

local

Linearly realized global symmetries of S ➡ symmetries of Γ

But gauge symmetries must be broken by the gauge fixing

Remaining BRST invariance is non-linear:

$\delta\varphi = \varphi\,\omega$, but $\delta\langle\varphi\rangle \neq \langle\varphi\rangle\langle\omega\rangle$ ➡ not a symmetry of Γ

Apply to a gauge theory:

counterterms

$$\Sigma_{L-1} = \Sigma_0 + \sum_{l=1}^{L-1} \hbar^l \Sigma_l^C$$

renormalized ***to the order $\hbar^{L-1}$***

Effective action:

$$W[\mathcal{J}] \to \Gamma\Big[\langle\Phi\rangle, \phi, \gamma, \zeta, \Omega\Big] = W - J_a(\langle\varphi^a\rangle - \phi^a) - \bar{\xi}_\alpha\langle\omega^\alpha\rangle - \xi^\alpha\langle\bar{\omega}_\alpha\rangle - y^\alpha\langle b_\alpha\rangle$$

mean field

$$\Gamma_{L-1} = \Sigma_0 + \sum_{l=1}^{\infty} \hbar^l \Gamma_{L-1}^{(l)}$$

$$\Gamma_{L-1}^{(l)} < \infty, \quad l \le L-1$$ ***finite***

$$\Gamma_{L-1,\infty}^{(L)} \equiv \Gamma_{L,\infty}[\langle\Phi\rangle, \phi, \gamma, \zeta, \Omega]$$ ***local – BPHZ technique***

L-th order renormalization:

quantum field

$$\Sigma_L[\Phi, \phi, \gamma, \zeta, \Omega] = \Sigma_{L-1} - \hbar^L \Gamma_{L,\infty}[\Phi, \phi, \gamma, \zeta, \Omega]$$

Main result

L-th order generating functional:

reparameterized fields

$$\exp\left\{-\frac{1}{\hbar}W_L[\mathcal{J}]\right\} = \int d\Phi \exp\left\{-\frac{1}{\hbar}\Big(\Sigma_L + J_a(\tilde{\varphi}^a_L - \phi^a) + \bar{\xi}_\alpha\tilde{\omega}^\alpha_L + \xi^\alpha\bar{\omega}_\alpha + y^\alpha b_\alpha\Big)\right\}$$

BRST structure of the renormalized action

$$\Sigma_L[\Phi,\phi,\gamma,\zeta,\Omega] = S_L[\varphi] + \boldsymbol{Q}\,\Psi_L[\Phi,\phi,\gamma,\zeta,\Omega]$$

only original gauge field

Renormalized gauge fermion

$$\Psi_L = \hat{\Psi}_L[\varphi,\omega,\phi,\hat{\gamma},\zeta,\Omega] - \frac{1}{2}\bar{\omega}_\alpha O^{\alpha\beta}(\phi)b_\beta$$ *L-th loop order*

$$\hat{\Psi}_0 = -\hat{\gamma}_a(\varphi^a - \phi^a) + \zeta_\alpha\omega^\alpha$$ *tree level*

Local reparameterization of quantum fields to composite operators including external sources

$$\tilde{\varphi}^a_L = \tilde{\varphi}^a_L(\varphi, \omega, \phi, \hat{\gamma}, \zeta, \Omega) \qquad \tilde{\omega}^\alpha_L = \tilde{\omega}^\alpha_L(\varphi, \omega, \phi, \hat{\gamma}, \zeta, \Omega)$$

Gauge fermion is a generating function of the field redefinition

$$\tilde{\varphi}^a_L - \phi^a = -\frac{\delta \Psi_L}{\delta \gamma_a}\,, \qquad \tilde{\omega}^\alpha_L = \frac{\delta \Psi_L}{\delta \zeta_\alpha}$$

Applies to (non-renormalizable) EFT --nonlinear dependence on sources ξ and ζ

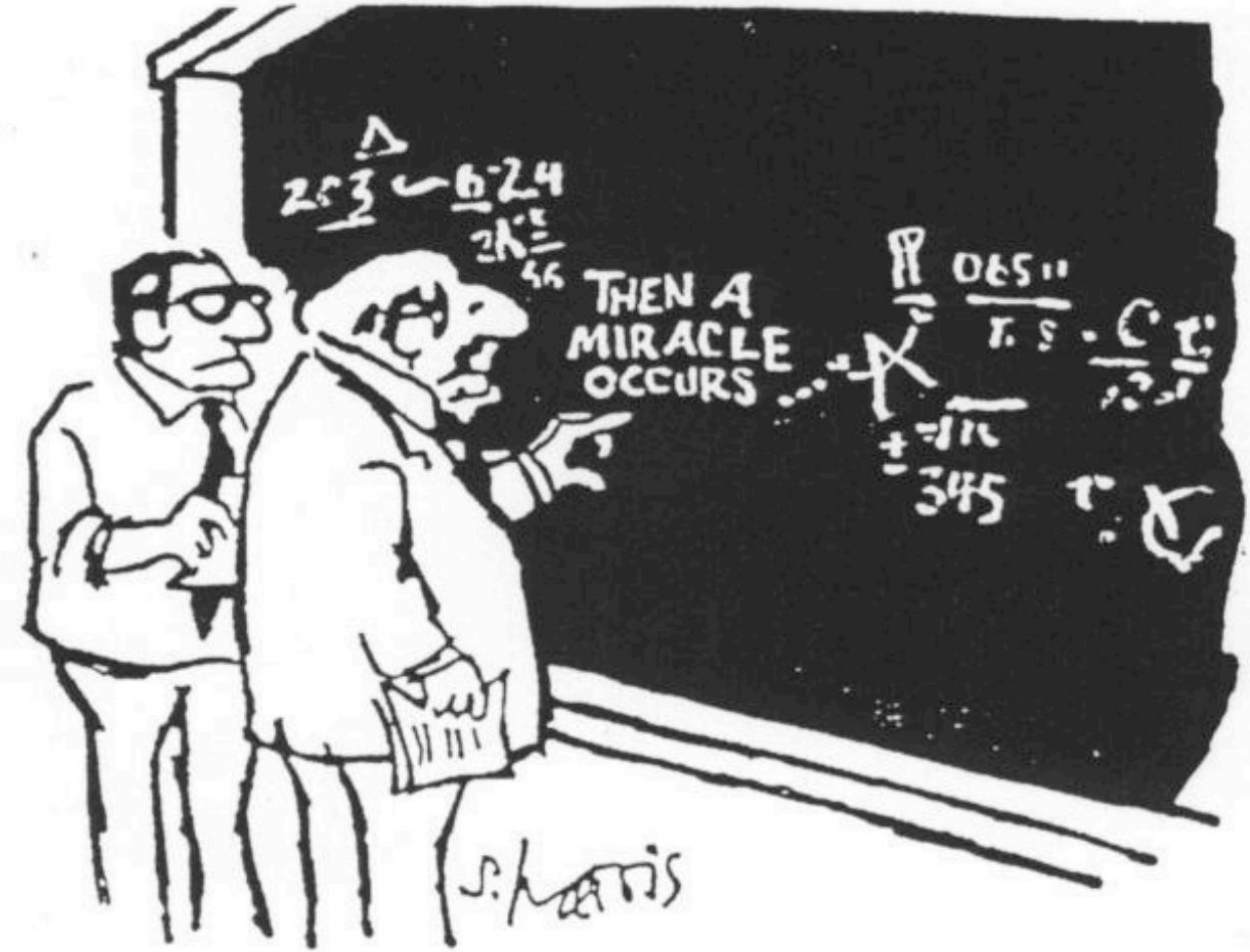
For renormalizable theories

$$\hat{\Psi}_L = -\hat{\gamma}_a {U_L}^a(\varphi, \phi) + \zeta_\alpha \omega^\beta {V_L}^\alpha_\beta(\varphi, \phi)$$

linear in ξ and ζ

$$\tilde{\varphi}^a_L = \phi^a + {U_L}^a(\varphi, \phi)\;, \quad \tilde{\omega}^\alpha_L = {V_L}^\alpha_\beta(\varphi, \phi)\,\omega^\beta$$

independent of other sources



I think you should be a little more specific, here in Step 2

Slavnov-Taylor and Ward identities

BRST $\rightarrow$ $$\left[-J\frac{\delta}{\delta\gamma}+\bar{\xi}\frac{\delta}{\delta\zeta}+\xi\frac{\delta}{\delta y}+\Omega\frac{\delta}{\delta\phi}\right]W=0$$

b.g.t, $\rightarrow$ $$\left[\delta_\varepsilon\phi\frac{\delta}{\delta\phi}+\delta_\varepsilon\mathcal{J}\frac{\delta}{\delta\mathcal{J}}\right]W=0$$

all sources

$$\mathcal{J}=J,\,\bar{\xi},\,\xi,\,y,\,\gamma,\,\zeta,\,\Omega$$

reduced action

$$W\rightarrow\Gamma\rightarrow\widehat{\Gamma}=\Gamma-\left(b_\alpha-\Omega^a\bar{\omega}_\alpha\frac{\delta}{\delta\phi^a}\right)\left(\chi_a^\alpha(\varphi-\phi)^a-\frac{1}{2}O^{\alpha\beta}b_\beta\right)$$

$$\widehat{\Gamma}=\widehat{\Gamma}[\varphi,\omega,\phi,\widehat{\gamma},\zeta,\Omega],\quad \widehat{\gamma}_a=\gamma_a-\bar{\omega}^\alpha\chi_\alpha^a,\quad \frac{\delta\widehat{\Gamma}}{\delta b}=0$$

Slavnov-Taylor and Ward identities for $\widehat{\Gamma}$

Decoupling of background fields in Γ_{div}

L-th order renormalization:

ST identity for $\Gamma_{L,\infty} = \widehat{\Gamma}_{L,\infty} \equiv \widehat{\Gamma}_{\infty}[\varphi, \omega, \phi, \widehat{\gamma}, \zeta, \Omega]$

$$\boldsymbol{Q}_{+}\widehat{\Gamma}_{\infty} = \left[\frac{\delta\widehat{\Gamma}_0}{\delta\widehat{\gamma}}\frac{\delta}{\delta\varphi} + \frac{\delta\widehat{\Gamma}_0}{\delta\varphi}\frac{\delta}{\delta\widehat{\gamma}} + \frac{\delta\widehat{\Gamma}_0}{\delta\zeta}\frac{\delta}{\delta\omega} + \frac{\delta\widehat{\Gamma}_0}{\delta\omega}\frac{\delta}{\delta\zeta} + \Omega\frac{\delta}{\delta\phi}\right]\widehat{\Gamma}_{\infty}$$

$$\equiv (\widehat{\Gamma}_0, \widehat{\Gamma}_{\infty}) + \Omega\frac{\delta\widehat{\Gamma}_{\infty}}{\delta\phi} = 0$$

nilpotent antibracket

$(\widehat{\Gamma}_0, (\widehat{\Gamma}_0, X)) = 0$

$$\widehat{\Gamma}_{\infty} = \sum_{k}^{K} \Omega^{a_1} \ldots \Omega^{a_k} \widehat{\Gamma}_{\infty,\{k\},[a_1,\ldots,a_k]}$$

***K* finite for any *L*<∞ !**

*Applies to nonrenormalizable EFT within **gradient** expansion*

Truncate in number of derivatives + locality + fermionic statistics of Ω ➡ $0 \leq k \leq K$

trivial cohomology of $\Omega\frac{\delta}{\delta\phi}$ Batalin, Vilkovisky (1985)

$$\widehat{\Gamma}_\infty = \boldsymbol{S}[\varphi] + \Lambda[\varphi,\omega,\widehat{\gamma},\zeta] + \boldsymbol{Q}_+\Upsilon$$

gauge invariant single field counterterm

ghost term

BRST exact term

$$\frac{\delta \boldsymbol{S}}{\delta\varphi^a}R^a{}_\alpha(\varphi) = 0 \qquad \Lambda\Big|_{\omega=0} = 0$$

Structure of Λ

ST+W identities: $$(\boldsymbol{q}_0+\boldsymbol{q}_1)\,\Lambda=0$$

$$(\boldsymbol{q}_0)^2=(\boldsymbol{q}_1)^2=\boldsymbol{q}_0\boldsymbol{q}_1+\boldsymbol{q}_1\boldsymbol{q}_0=0$$

$$\underbrace{\boldsymbol{q}_0=\frac{\delta S}{\delta\varphi^a}\frac{\delta}{\delta\widehat{\gamma}_a}-\widehat{\gamma}_a R^a{}_\alpha(\varphi)\frac{\delta}{\delta\zeta_\alpha}}, \qquad \boldsymbol{q}_1=-\frac{1}{2}C^\gamma{}_{\alpha\beta}\,\omega^\alpha\omega^\beta\frac{\delta}{\delta\omega^\gamma}$$

Kozhul-Tate differential *has a trivial cohomology under the assumption of* ***local completeness*** *and* ***irreducibility*** *of gauge generators for* $\Lambda\Big|_{\omega=\widehat{\gamma}=\zeta=0}=0$

Batalin, Vilkovisky (1985)
Henneaux (1991)
Vandoren, Van Proeyen (1994)

$$\boldsymbol{q}_0X=0,\ \ X\Big|_{\omega=\widehat{\gamma}=\zeta=0}=0$$
$$\Rightarrow X=\boldsymbol{q}_0Y$$

local

$$\Lambda=\sum_{k=1}^{\infty}\omega^{\alpha_1}\dots\omega^{\alpha_k}\Lambda^{\{k\}}_{[\alpha_1,\dots,\alpha_k]}$$

$$\Downarrow$$

$$\Gamma_{L,\infty}=\boldsymbol{S}_L[\varphi]+\boldsymbol{Q}_+\Upsilon_L$$

L-th order subtraction and $Q_+ \to Q$ transition via field redefinition

$$\Sigma_L[\Phi,\phi,\gamma,\zeta,\Omega] = \Sigma_{L-1} - \hbar^L \Gamma_{L,\infty} + \mathcal{O}(\hbar^{L+1})$$

field redefinition $\Phi \to \Phi'$:

gauge invariant counterterm

$$\Sigma_L[\Phi,\phi,\gamma,\zeta,\Omega] = \Big[\, S_{L-1} - \hbar^L \boldsymbol{S}_L + \boldsymbol{Q}\,\Psi_L \,\Big]_{\Phi\to\Phi'}$$

gauge fermion renormalization at L-th order

$$\Psi_L = \Psi_{L-1} - \hbar^L \boldsymbol{\Upsilon}_L$$

$$\varphi^a - \phi^a = -\frac{\delta\Psi_L(\Phi')}{\delta\gamma_a}\,, \quad \omega^\alpha = \frac{\delta\Psi_L(\Phi')}{\delta\zeta_\alpha}$$

Gauge fermion is a generating function of the field redefinition

Example: 2D O(N) gauge sigma model

$$S[\varphi] = \frac{1}{2g^2}\int \mathrm{d}^2x \left\{\frac{1}{\varphi^2}\left[\delta_{ij} - \frac{\varphi_i\varphi_j}{\varphi^2}\right] \partial_\mu\varphi^i\partial^\mu\varphi^j\right\}, \quad i = 1, \ldots N$$

Abelian gauge symmetry $\quad \delta_\varepsilon\varphi^i(x) = \varphi^i(x)\,\varepsilon(x)$

One-loop renormalization

$$S_1[\varphi] = \left(\frac{1}{2g^2} + \hbar\frac{N-2}{4\pi(2-d)}\right)\int \mathrm{d}^2x \left\{\frac{1}{\varphi^2}\left[\delta_{ij} - \frac{\varphi_i\varphi_j}{\varphi^2}\right] \partial_\mu\varphi^i\partial^\mu\varphi^j\right\}$$

$$\varphi^i \mapsto \tilde{\varphi}_1^i = \varphi^i - \frac{\hbar}{4\pi(2-d)}\left[\frac{\phi^2(\varphi^2+\phi^2)}{(\varphi\cdot\phi)^2}\varphi^i - \frac{2\varphi^2}{(\varphi\cdot\phi)}\phi^i\right], \quad (\varphi\cdot\phi) \equiv \varphi_i\phi^i$$

essentially nonlinear

Conclusions and Outlook

- Background field method is not only a convenient calculational tool, but is also efficient for general analysis of the structure of renormalization

  cf. Grassi (1996), Anselmi (2014)

- BRST structure (gauge invariance) is preserved by renormalization for non-anomalous theories whose gauge algebra:

  i) has linear generators

  ii) closes off-shell — *can be relaxed (?)*

  iii) is locally complete

  iv) is irreducible — *can be relaxed*

- Generalizations: open algebras, supersymmetry, composite operators, anomalies

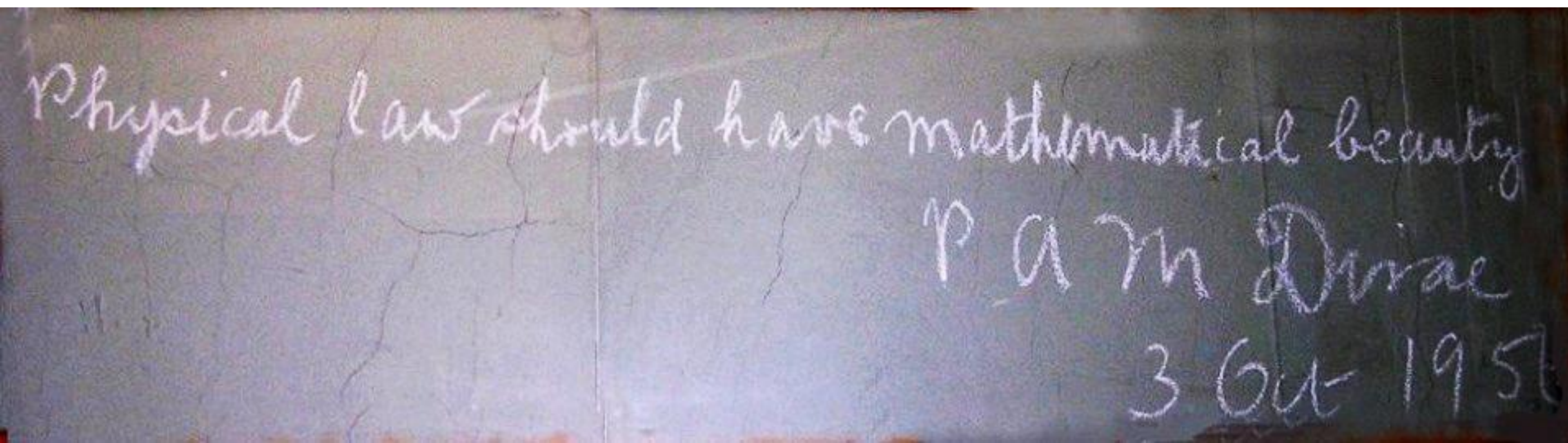
The power and beauty of nilpotent BRST charge

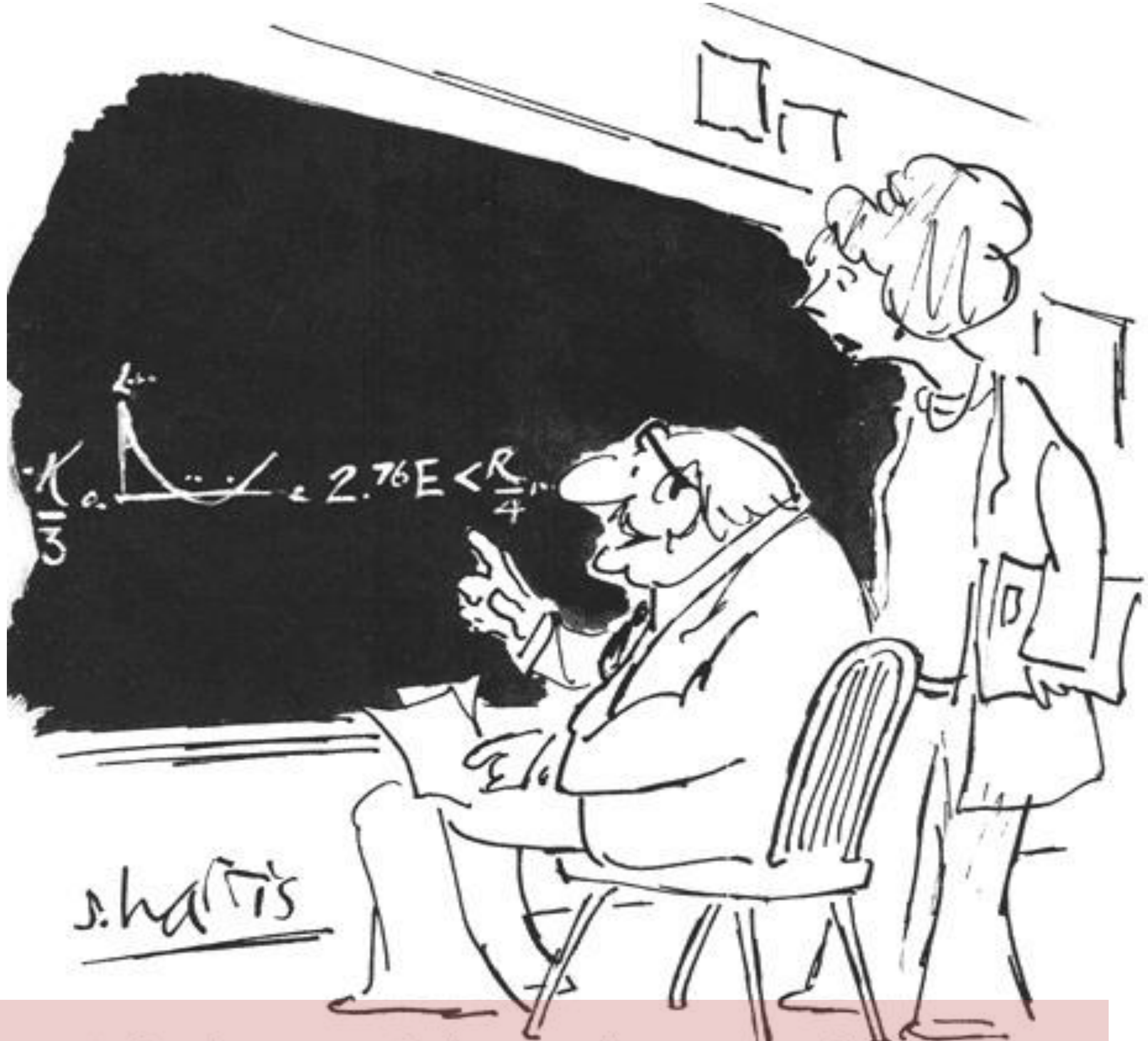
$$\boldsymbol{Q} \to \boldsymbol{Q}_{\rm ext} = \boldsymbol{s} + \Omega\frac{\delta}{\delta\phi} - J\frac{\delta}{\delta\gamma} + \bar{\xi}\frac{\delta}{\delta\zeta} + \xi\frac{\delta}{\delta y}, \qquad \boldsymbol{Q}^2_{\rm ext} = 0$$

$$\Psi \to \boldsymbol{\Psi}_{\rm ext} \equiv \Psi + y\bar{\omega}$$

$$\Sigma \to \Sigma_{\rm ext} = \Sigma - J\frac{\delta\Psi}{\delta\gamma} + \bar{\xi}\frac{\delta\Psi}{\delta\zeta} + \xi\bar{\omega} + yb = S + \boldsymbol{Q}_{\rm ext}\boldsymbol{\Psi}_{\rm ext}$$

$$e^{-W/\hbar} = \int d\Phi\, e^{-(S + \boldsymbol{Q}_{\rm ext}\boldsymbol{\Psi}_{\rm ext})/\hbar}$$





"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."