International Workshop

Supersymmetries & Quantum Symmetries

Joint Institute for Nuclear Research

Bogoliubov Laboratory of Theoretical Physics

Dubna, Russia, July 31 - August 5, 2017.

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NSVZ relation and the dimensional reduction in $\mathcal{N} = 1$ SQED

$\mathcal{N}=1$ Supersymmetric Quantum Electrodynamics with N_f flavors

Let us consider the $\mathcal{N}=1$ Supersymmetric Quantum Electrodynamics (**SQED**) with N_f flavors and massless matter superfields which in the superspace is described by the action

$$S=\frac{1}{4e_0^2}Re\int d^4x d^2\theta W^a W_a+\sum_{i=1}^{N_f}\frac{1}{4}\int d^4x d^4\theta(\phi_i^* e^{2V}\phi_i+\tilde{\phi}_i^* e^{-2V}\tilde{\phi}_i)$$

V — real abelian $U(1)$ gauge superfield (photon+superpartner);

ϕ_i, $\tilde{\phi}_i$ — chiral matter superfields with $i=1,...,N_f$;

W_a — supersymmetric gauge field stress tensor, which is defined by

$$W_a=\frac{1}{4}\bar{D}^2 D_a V.$$

NSVZ β-function in $\mathcal{N}=1$ SQED with N_f flavors

It was found that β-function in $\mathcal{N}=1$ SQED with N_f flavors is related with anomalous dimension of the matter superfields. Such relation is called the exact Novikov, Shifman, Vainshtein and Zakharov (NSVZ) β-function

$$\beta(\alpha_0) = \frac{\alpha_0^2 N_f}{\pi}(1-\gamma(\alpha_0))$$

V.Novikov, M.A.Shifman, A.Vainshtein, V.I.Zakharov, Nucl.Phys. B 229, (1983), 381; Phys.Lett. 166B, (1985), 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. B 277, (1986), 456.

This ratio can be obtained for all orders of perturbation theory if the theory is regularized by higher derivatives and RG functions are defined in terms of the bare coupling constant

$$\beta\Big(\alpha_0(\alpha,\Lambda/\mu)\Big) \equiv \frac{d\alpha_0(\alpha,\Lambda/\mu)}{d\ln\Lambda}\Big|_{\alpha=\text{const}}; \gamma\Big(\alpha_0(\alpha,\Lambda/\mu)\Big) \equiv -\frac{d\ln Z(\alpha,\Lambda/\mu)}{d\ln\Lambda}\Big|_{\alpha=\text{const}},$$

α - renormalized coupling constant and Z - renormalization constant for matter superfields

K. V. Stepanyantz, JHEP 1408 (2014) 096; K. V. Stepanyantz, Nucl.Phys.B 852 (2011) 71.

NSVZ β-function in $\mathcal{N}=1$ SQED with N_f flavors

Let us consider a part of the effective action which corresponds to the two-point Green functions of gauge and matter superfields

$$\Gamma^{(2)} - S_{gf} = -\frac{1}{16\pi}\int \frac{d^4p}{(2\pi)^4} d^4\theta V(-p,\theta)\partial^2\Pi_{1/2}V(p,\theta) d^{-1}(\alpha,\mu/p)$$

$$+\frac{1}{4}\sum_{i=1}^{N_f}\int \frac{d^4p}{(2\pi)^4} d^4\theta \Big(\phi_i^*(-p,\theta)\phi_i(p,\theta) + \tilde\phi_i^*(-p,\theta)\tilde\phi_i(p,\theta)\Big) G(\alpha,\mu/p),$$

μ — renormalization scale;

$\alpha = \alpha(\mu)$ — renormalized coupling constant;

$\partial^2\Pi_{1/2} = -D^a\bar D^2 D_a/8$ — supersymmetric transversal projection operator.

NSVZ β-function in $\mathcal{N}=1$ SQED with N_f flavors

If the theory is regularized by higher derivatives it has been proved in all loops that

$$\frac{d}{d\ln\Lambda}\Big(d^{-1}-\alpha_0^{-1}\Big)\Big|_{p=0}=\frac{d}{d\ln\Lambda}\Big(\text{One-loop}-16\pi^3 N_f\int\frac{d^4q}{(2\pi)^4}\delta^4(q)\ln G\Big),$$

where the derivative is calculated at the fixed coupling constant, Λ is the constant with the dimension of mass.

δ - function appears from the integrals of double total derivatives due to the identity:

$$\frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q_\mu}\frac{1}{q^2}=-4\pi^2\delta^4(q)$$

In the case of $N=1$ SQED with N_f flavors regularized by higher covariant derivatives NSVZ β – function can be obtained by summing supergraphs for the RG functions defined in terms of the bare coupling constant.

Formulation of the problem

Higher derivative

$$\int \frac{d^4q}{(2\pi)^4}\delta^4(q)$$

Dimensional reduction

$$\frac{1}{2\pi^2}\Lambda^{\varepsilon}\cdot\frac{(L-1)\varepsilon}{1-L\varepsilon/2}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2(q+p)^2},$$

L – number of loops.

1. Investigate the influence of the analogous of the structures of double total derivatives integrals on NSVZ relation.

2. Investigate the reasons of NSVZ ralation violation in the $\overline{\text{DR}}$-scheme.

I. Jack, D. R. T. Jones and C. G. North. Phys. Lett. B 386, 138 (1996).

3. Search the boundary conditions to the renormalization constants giving the NSVZ scheme with the dimensional reduction for the RG functions defined in terms of the renormalized coupling constant.

Dimensional technique in supersymmetric theories

- DREG breaks the supersymmetry

R. Delbourgo and V. B. Prasad, J. Phys. G **1** (1975) 377

- DRED manifestly preserves gauge invariance, unitarity and global supersymmetry

W. Siegel, Phys. Lett. B **84** (1979) 193.

- DRED is not self-consistent

W. Siegel, Phys. Lett. B **94** (1980) 37.

- Removing the theory inconsistencies the explicit supersymmetry is broken

L. V. Avdeev, G. A. Chochia and A. A. Vladimirov,
Phys. Lett. B **105** (1981) 272.

Structure of the scheme-dependent three-loop contribution to the β-function

The expression for three-loop d^{-1} function in $\mathcal{N}=1$ SQED with N_f flavors regularized by the dimensional reduction can be presented in the form:

$$d^{-1}(\alpha_0,\Lambda/p) = 1/\alpha_0 + I_1 + I_2 + I_3 + O(e_0^4 N_f) + O(e_0^6),$$

$\dfrac{1}{\alpha_0}$ — tree contribution;

I_1 and I_2 — one and two-loop contributions;

I_3 — scheme-dependent part of three-loop contribution, proportional $(N_f)^2$ (terms $\sim N_f$ scheme-independent).

Structure of the scheme-dependent three-loop contribution to the β-function

$$I_1 = 8\pi N_f (\Lambda)^{\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k+p)^2},$$

$$I_2 = 16\pi e_0^2 N_f (\Lambda)^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \Big(\frac{2}{k^2 (k+q)^2 q^2 (q+p)^2}$$

$$- \frac{1}{(k+q)^2 (k+q+p)^2 q^2 (q+p)^2} - \frac{p^2}{k^2 (k+q)^2 (k+p+q)^2 q^2 (q+p)^2} \Big),$$

$$I_3 = -32\pi e_0^4 N_f^2 (\Lambda)^{3\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{d^d t}{(2\pi)^d} \frac{1}{t^2 (t+k)^2} \cdot \Big(\frac{2}{k^2 (k+q)^2 q^2 (q+p)^2}$$

$$- \frac{1}{(k+q)^2 (k+q+p)^2 q^2 (q+p)^2} - \frac{p^2}{k^2 (k+q)^2 (k+p+q)^2 q^2 (q+p)^2} \Big),$$

where $\varepsilon \equiv 4 - d$.

$$I_n \sim (\Lambda/p)^{n\varepsilon}, \quad \text{where} \quad n \in \mathbb{Z}_+$$

Analysis of the two-loop contribution to the d^{-1}-function

Let us consider I_2- two-loop part of the expression $d^{-1}(\alpha_0, \Lambda/p)$ and add to it:

$$0 = 16\pi e_0^2 N_f (\Lambda)^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{\partial}{\partial q^\mu} \Big(\frac{q^\mu}{q^2 (q+p)^2 k^2 (q+k)^2} \Big(1 - \frac{p^2}{2(q+k+p)^2} \Big) \Big).$$

The result can be written in the form:

$$I_2 = 16\pi e_0^2 N_f (\Lambda)^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \Big(- \frac{\varepsilon}{k^2 (k+q)^2 q^2 (q+p)^2}$$
$$- 2 p^\mu \frac{\partial}{\partial p^\mu} \frac{1}{k^2 (k+q)^2 q^2 (q+p)^2} + \frac{1}{2} p^\mu \frac{\partial}{\partial p^\mu} \frac{1}{(k+q)^2 (k+q+p)^2 q^2 (q+p)^2}$$
$$+ \frac{\varepsilon p^2}{2k^2 q^2 (q+p)^2 (q+k)^2 (q+k+p)^2} + \frac{1}{2} p^2 p^\mu \frac{\partial}{\partial p^\mu} \frac{1}{k^2 q^2 (q+p)^2 (q+k)^2 (q+k+p)^2} \Big)$$

Analysis of the two-loop contribution to the d^{-1}-function

In the two-loop approximation all the integrals $\sim (\Lambda/p)^{2\varepsilon}$. Therefore,

$$I_2 = \varepsilon I_2 + 16\pi e_0^2 N_f (\Lambda)^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \Big(\frac{\varepsilon}{k^2 (k+q)^2 q^2 (q+p)^2} + \frac{p^2}{k^2 q^2 (q+p)^2 (q+k)^2 (q+k+p)^2} \Big(-1 + \frac{\varepsilon}{2} \Big) \Big)$$

Thus, we obtain

$$I_2 = 16\pi e_0^2 N_f (\Lambda)^{2\varepsilon} \Big(\frac{\varepsilon}{1-\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{1}{k^2 (k+q)^2 q^2 (q+p)^2} + \frac{\varepsilon - 2}{2(1-\varepsilon)} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{p^2}{k^2 q^2 (q+p)^2 (q+k)^2 (q+k+p)^2} \Big)$$

Analysis of the three-loop contribution to the d^{-1}-function

Let us consider I_3- three-loop part of the expression $d^{-1}(\alpha_0, \Lambda/p)$ and add to it

$$0 = -32\pi e_0^4 N_f^2 (\Lambda)^{3\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{d^d t}{(2\pi)^d} \frac{1}{t^2 (t+k)^2} \cdot \frac{\partial}{\partial q^\mu} \Big(\frac{q^\mu}{q^2 (q+p)^2 k^2 (q+k)^2} - \frac{p^2 q^\mu}{2k^2 q^2 (q+p)^2 (q+k)^2 (q+k+p)^2} \Big).$$

Making the transformations similar to the two-loop case, we get

$$I_3 = -32\pi e_0^4 N_f^2 \Lambda^{3\varepsilon} \Big(\frac{2\varepsilon}{1-3\varepsilon/2} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{d^d t}{(2\pi)^d} \frac{1}{t^2 (t+k)^2 k^2 (k+q)^2 q^2 (q+p)^2}$$
$$+ \frac{\varepsilon - 2}{2(1-3\varepsilon/2)} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{d^d t}{(2\pi)^d} \frac{p^2}{t^2 k^2 q^2 (t+k)^2 (q+p)^2 (q+k)^2 (q+k+p)^2} \Big)$$

Structure of loop contribution to the β-function

The expression for $d^{-1} - \alpha_0^{-1}$ function can be presented as:

$$d^{-1}(\alpha_0, \Lambda/p) - \alpha_0^{-1} = 8\pi N_f \Lambda^{\varepsilon} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2(k+p)^2}$$

$$+ 64\pi^2 \alpha_0 N_f \Lambda^{2\varepsilon} \frac{\varepsilon}{1-\varepsilon} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2(q+p)^2} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2(k+q)^2}$$

$$- 512\pi^3 \alpha_0^2 (N_f)^2 \Lambda^{3\varepsilon} \frac{2\varepsilon}{1-3\varepsilon/2} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2(q+p)^2} \int \frac{d^d k d^d t}{(2\pi)^{2d}} \frac{1}{k^2(k+q)^2 t^2 (t+k)^2}$$

$$+ \text{finite terms} + O(\alpha_0^2 N_f) + O(\alpha_0^3)$$

In this expression we do not write explicitly terms which are finite functions of the renormalized coupling constant related to the bare coupling constant by the equality

$$\frac{1}{\alpha_0} = \frac{1}{\alpha} - \frac{N_f}{\pi}\Big(\frac{1}{\varepsilon} + \ln\frac{\Lambda}{\mu} + b_1\Big) + O(\alpha^2),$$

b_1 – a finite constant depending on the renormalization scheme.

Structure of loop contribution to the β-function

Comparing our result with similar terms in the logarithm of the two-point Green function of the matter superfields

$$\ln G = -8\pi\alpha_0\Lambda^\varepsilon \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2(k+q)^2}$$
$$+ 64\pi^2\alpha_0^2 N_f \Lambda^{2\varepsilon}\int \frac{d^dk}{(2\pi)^d}\frac{d^dt}{(2\pi)^d}\frac{1}{k^2(k+q)^2t^2(t+k)^2} + O(\alpha_0^2(N_f)^0) + O(\alpha_0^3),$$

we obtain:

$$d^{-1} - \alpha_0^{-1} = 8\pi N_f\Lambda^\varepsilon \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2(k+p)^2}$$
$$- 8\pi N_f \Lambda^\varepsilon \frac{\varepsilon}{1-\varepsilon}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2(k+p)^2}(\ln G)_{\text{1-loop}}$$
$$- 8\pi N_f \Lambda^\varepsilon \frac{2\varepsilon}{1-3\varepsilon/2}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2(k+p)^2}(\ln G)_{\text{2-loops},N_f}$$
$$+ \text{finite terms} + O(\alpha_0^2 N_f) + O(\alpha_0^3)$$

S. A., A. L. Kataev and K. V. Stepanyantz, JETP Lett. **103** (2016) no.2, 77.

RG functions defined in terms of the bare coupling constant

$$\beta(\alpha_0) \equiv \frac{d\alpha_0}{d\ln\Lambda}\Big|_{\alpha=\text{const}}; \qquad \gamma(\alpha_0) \equiv -\frac{d\ln Z}{d\ln\Lambda}\Big|_{\alpha=\text{const}}, \tag{1}$$

where $\alpha = e^2/4\pi = \alpha(\alpha_0, \varepsilon, \Lambda/\mu)$ is the renormalized coupling constant. $\ln G$-function can be represented in the following form:

$$\ln G = \sum_{n=1}^{\infty} (\alpha_0)^n \Big(\frac{4\pi\Lambda^2}{q^2}\Big)^{\varepsilon n/2} g_n(\varepsilon), \tag{2}$$

here only the coefficients

$$g_1(\varepsilon) = -\frac{1}{2\pi}\, G(1,1);$$

$$g_2(\varepsilon) = \frac{1}{4\pi^2}\,(1+N_f) G(1,1) G(1,1+\varepsilon/2) - \frac{1}{8\pi^2}\, G(1,1)^2 + \text{finite terms},$$

are essential in the two-loop approximation.

$$G(\alpha,\beta) \equiv \frac{\Gamma(\alpha+\beta-2+\varepsilon/2)}{\Gamma(\alpha)\Gamma(\beta)} B(2-\alpha-\varepsilon/2, 2-\beta-\varepsilon/2).$$

RG functions defined in terms of the bare coupling constant

$$\gamma(\alpha_0) = -\frac{\alpha_0}{\pi} + \frac{\alpha_0^2}{2\pi^2}(1 + N_f) + O(\alpha_0^3). \qquad (3)$$

This anomalous dimension coincides with the one defined in terms of the renormalized coupling constant in the $\overline{\text{DR}}$-scheme.

Using found connection between two-point Green functions of gauge and matter superfields we get that the β-function can be represented as:

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{N_f}{\pi}\Big(1 - \gamma(\alpha_0)\Big) + \frac{\Delta\beta(\alpha_0)}{\alpha_0^2} + O(\alpha_0^2 N_f) + O(\alpha_0^3), \qquad (4)$$

where

$$\Delta\beta(\alpha_0) = \frac{\alpha_0^4 (N_f)^2}{\pi^3} \cdot \lim_{\varepsilon\to 0} \Bigg(\frac{2}{\varepsilon}\Big[\frac{\varepsilon}{1-\varepsilon}\Big(\frac{4\pi\Lambda^2}{p^2}\Big)^{\varepsilon/2} G(1, 1+\varepsilon/2) - 1\Big] - \frac{\varepsilon}{4(1-\varepsilon)}$$
$$\times \Big(\frac{4\pi\Lambda^2}{p^2}\Big)^{\varepsilon} G(1,1)G(1,1+\varepsilon/2) - \frac{1}{\varepsilon}\Big[\frac{3\varepsilon/2}{1-3\varepsilon/2}\Big(\frac{4\pi\Lambda^2}{p^2}\Big)^{\varepsilon/2} G(1,1+\varepsilon) - 1\Big]\Bigg).$$

Modification of NSVZ relation in the dimensional technique on three-loop level

$$\Delta\beta = \frac{\alpha_0^4 N_f^2}{\pi^3}\Big(-\frac{1}{2\varepsilon} - \frac{1}{4}\Big)$$

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{N_f}{\pi}\Big(1-\gamma(\alpha_0)\Big) + \frac{\alpha_0^2 (N_f)^2}{\pi^3}\Big(-\frac{1}{2\varepsilon} - \frac{1}{4}\Big) + O(\alpha_0^2 N_f) + O(\alpha_0^3).$$

- The RG-functions defined in terms of the bare coupling constant do not satisfy the NSVZ relation;
- β-function definitely depends on ε.

S. A., I. O. Goriachuk, A. L. Kataev and K. V. Stepanyantz,
Phys. Lett. B **764** (2017) 222.

The RG functions defined in terms of the renormalized coupling constant

RG functions defined in terms of the renormalized coupling constant:

$$\widetilde{\beta}(\alpha(\alpha_0,\Lambda/\mu)) \equiv \frac{d\alpha(\alpha_0,\Lambda/\mu)}{d\ln\mu}\Big|_{\alpha_0=\text{const}};$$

$$\widetilde{\gamma}(\alpha(\alpha_0,\Lambda/\mu)) \equiv \frac{d\ln Z(\alpha(\alpha_0,\Lambda/\mu),\Lambda/\mu)}{d\ln\mu}\Big|_{\alpha_0=\text{const}},$$

where μ is the normalization point. For the theory regularized by higher derivatives the RG functions coincide with the RG functions defined in terms of the bare coupling constant, if the boundary conditions

$$\widetilde{\beta}(\alpha) = \beta(\alpha), \quad \widetilde{\gamma}(\alpha) = \gamma(\alpha). \quad \Longleftrightarrow \quad \left[\begin{array}{ll} Z_3(\alpha,x_0) = 1, & Z(\alpha,x_0) = 1; \\ \alpha_0(\alpha,x_0) = \alpha, & Z(\alpha,x_0) = 1. \end{array}\right. \qquad (5)$$

are imposed on the renormalization constants. Here x_0 is a fixed value of $\ln\Lambda_{HD}/\mu$.

A. L. Kataev and K. V. Stepanyantz Nucl.Phys.B 875 (2013) 459; Phys.Lett.B 730 (2014) 184; Theor.Math.Phys.181 (2014) 1531.

$\overline{\text{DR}}$-scheme

$\overline{\text{DR}}$-scheme is defined by the equations

$$\alpha_0(\alpha, \varepsilon \to \infty, x = 0) = \alpha; \qquad Z(\alpha, \varepsilon \to \infty, x = 0) = 1. \qquad (6)$$

$$\widetilde{\beta}_{\overline{\text{DR}}}(\alpha) = \frac{\alpha^2 N_f}{\pi}\Big(1 + \frac{\alpha}{\pi} - \frac{3\alpha^2 N_f}{4\pi^2}\Big) + O(\alpha^4 N_f) + O(\alpha^5) = \lim_{\varepsilon\to\infty} \beta(\alpha_0, \varepsilon)\Big|_{\alpha_0=\alpha};$$

$$\widetilde{\gamma}_{\overline{\text{DR}}}(\alpha) = -\frac{\alpha}{\pi} + \frac{\alpha^2}{2\pi^2}(1 + N_f) + O(\alpha^3) = \lim_{\varepsilon\to\infty} \gamma(\alpha_0, \varepsilon)\Big|_{\alpha_0=\alpha}.$$

After eliminating ε-poles, the RG functions defined in terms of the bare coupling constant coincide with the RG functions in $\overline{\text{DR}}$-scheme.

$$\frac{\widetilde{\beta}_{\overline{\text{DR}}}(\alpha)}{\alpha^2} = \frac{N_f}{\pi}\Big(1 - \gamma_{\overline{\text{DR}}}(\alpha)\Big) - \frac{\alpha^2 (N_f)^2}{4\pi^3} + O(\alpha^3).$$

NSVZ scheme in the three-loop approximation

NSVZ scheme and $\overline{\text{DR}}$ scheme can be related by the finite renormalization:

$$\alpha' = \alpha'(\alpha); \qquad \phi'_R = z(\alpha)^{-1/2}\phi_R, \tag{7}$$

where $\alpha'(\alpha)$ and $z(\alpha)$ are finite functions, then

$$\widetilde{\beta}'(\alpha') = \frac{d\alpha'}{d\alpha}\cdot\widetilde{\beta}(\alpha); \qquad \widetilde{\gamma}'(\alpha') = \frac{d\ln z}{d\alpha}\cdot\widetilde{\beta}(\alpha) + \widetilde{\gamma}(\alpha). \tag{8}$$

Let us assume that $\alpha = \alpha_{\overline{\text{DR}}}$, $\alpha' = \alpha_{\text{NSVZ}}$ and $z(\alpha) = 1$, then

$$\widetilde{\beta}(\alpha) = \frac{d\alpha}{d\alpha'}\cdot\frac{\alpha'^2 N_f}{\pi}\Big(1-\widetilde{\gamma}(\alpha)\Big)\Big|_{\alpha'=\alpha'(\alpha)}; \qquad \widetilde{\gamma}(\alpha) = \widetilde{\gamma}'(\alpha'). \tag{9}$$

$$\alpha' = \alpha + \frac{\alpha^3 N_f}{4\pi^2} + O(\alpha^4). \tag{10}$$

NSVZ scheme in the three-loop approximation

Higher derivative

$$\widetilde{\beta}(\alpha) = \beta(\alpha);\ \widetilde{\gamma}(\alpha) = \gamma(\alpha)$$

$$\alpha_0(\alpha, x_0) = \alpha;$$

$$Z(\alpha, x_0) = 1,$$

where $x_0 = \ln \Lambda_{\mathsf{HD}}/\mu$

Dimensional reduction

$$\lim_{\varepsilon\to\infty} \alpha_0(\alpha', \varepsilon, x_0 = 0)$$

$$= \alpha' - \frac{\alpha'^3 N_f}{4\pi^2} + O(\alpha'^4);$$

$$\lim_{\varepsilon\to\infty} Z'(\alpha', \varepsilon, x_0 = 0) = 1,$$

where $x_0 = \ln \overline{\Lambda}/\mu$

Conclusion

- Using the analogues of the integral of the integrals of the δ-function found for the theory regularized by the dimensional reduction the connection of 3-loop β-function with 2-loop anomalous dimension of matter superfield was built.
- It was shown that unlike the case of using the higher derivatives, in the theory regularized by the dimensional reduction the RG functions defined in terms of the bare coupling constant do not satisfy the NSVZ relation.
- In the considered approximation the boundary conditions to the renormalization constants giving the NSVZ scheme with the dimensional reduction for the RG functions defined in terms of the renormalized coupling constant was found.

Thank you for the attention!