Interactions for massive mixed symmetry field

Yu. M. Zinoviev

Institute for High Energy Physics, Protvino, Russia

18.07.2011

Outlook

1. Massive fields interactions a la Fradkin-Vasiliev
2. Electromagnetic interactions for massive hook
3. Gravitational interactions for massive hook

Massless fields in a flat space

- Any attempt to switch on minimal gravitational interaction spoils gauge invariance

$$e_\mu{}^a \Rightarrow e_\mu{}^a + h_\mu{}^a, \quad D_\mu \Rightarrow D_\mu - \omega_\mu \quad \Longrightarrow \quad \delta S \sim R$$

  where R stands for Riemann tensor and for massless fields in a flat space it can not be restored by non-minimal corrections to Lagrangian and/or gauge transformations.

- Similarly, any attempt to switch on minimal electromagnetic interactions spoils gauge invariance

$$D_\mu \Rightarrow D_\mu + e_0 A_\mu \quad \Longrightarrow \quad \delta S \sim F$$

  where F stands for electromagnetic field strength and for massless fields in a flat space it also can not be restored by non-minimal corrections to Lagrangian and/or gauge transformations.

Massless fields in a flat space

- Any attempt to switch on minimal gravitational interaction spoils gauge invariance

  $$e_\mu{}^a \Rightarrow e_\mu{}^a + h_\mu{}^a, \quad D_\mu \Rightarrow D_\mu - \omega_\mu \quad \Longrightarrow \quad \delta S \sim R$$

  where R stands for Riemann tensor and for massless fields in a flat space it can not be restored by non-minimal corrections to Lagrangian and/or gauge transformations.

- Similarly, any attempt to switch on minimal electromagnetic interactions spoils gauge invariance

  $$D_\mu \Rightarrow D_\mu + e_0 A_\mu \quad \Longrightarrow \quad \delta S \sim F$$

  where F stands for electromagnetic field strength and for massless fields in a flat space it also can not be restored by non-minimal corrections to Lagrangian and/or gauge transformations.

Massless fields in anti de Sitter space

- Fradkin and Vasiliev showed that in *AdS* space it is possible to restore broken invariance with the non-minimal corrections

$$\delta S \sim \sum \frac{1}{\Lambda^n} R^n$$

so that the flat limit $\Lambda \to 0$ is impossible.

- Similarly, for the e/m interactions it is possible to restore broken invariance with the non-minimal corrections

$$\delta S \sim \sum \frac{1}{\Lambda^n} F^n$$

- The procedure can be reversed: starting with massless particle in flat space and non-minimal interactions with highest number of derivatives one can reproduce minimal gravitational or electromagnetic interactions by *AdS* deformation.

Massless fields in anti de Sitter space

- Fradkin and Vasiliev showed that in *AdS* space it is possible to restore broken invariance with the non-minimal corrections

$$\delta S \sim \sum \frac{1}{\Lambda^n} R^n$$

so that the flat limit $\Lambda \rightarrow 0$ is impossible.

- Similarly, for the e/m interactions it is possible to restore broken invariance with the non-minimal corrections

$$\delta S \sim \sum \frac{1}{\Lambda^n} F^n$$

- The procedure can be reversed: starting with massless particle in flat space and non-minimal interactions with highest number of derivatives one can reproduce minimal gravitational or electromagnetic interactions by *AdS* deformation.

Massless fields in anti de Sitter space

- Fradkin and Vasiliev showed that in *AdS* space it is possible to restore broken invariance with the non-minimal corrections

$$\delta S \sim \sum \frac{1}{\Lambda^n} R^n$$

so that the flat limit $\Lambda \to 0$ is impossible.

- Similarly, for the e/m interactions it is possible to restore broken invariance with the non-minimal corrections

$$\delta S \sim \sum \frac{1}{\Lambda^n} F^n$$

- The procedure can be reversed: starting with massless particle in flat space and non-minimal interactions with highest number of derivatives one can reproduce minimal gravitational or electromagnetic interactions by *AdS* deformation.

Frame-like formalism

- Frame-like formalism for spin 2:

$$h_{\mu\nu} \Rightarrow h_\mu{}^a \oplus \omega_\mu{}^{ab}, \qquad \xi_\mu \Rightarrow \xi^a \oplus \eta^{ab}$$

  On shell: $\omega \sim Dh$, $\eta \sim D\xi$

- Spin 3:

$$\begin{array}{rcl} \Phi_{\mu\nu\alpha} & \Rightarrow & \Phi_\mu{}^{ab} \oplus \Omega_\mu{}^{ab,c} \oplus \Sigma_\mu{}^{ab,cd} \\ \xi_{\mu\nu} & \Rightarrow & \xi^{ab} \oplus \eta^{ab,c} \oplus \zeta^{ab,cd} \end{array}$$

  On shell: $\Omega \sim D\Phi$, $\Sigma \sim D^2\Phi$, $\eta \sim D\xi$, $\zeta \sim D^2\xi$

- Arbitrary spin:

$$\begin{array}{rcl} \Phi & \Rightarrow & \Phi \oplus \Omega \oplus \Sigma_1 \oplus \cdots \oplus \Sigma_{s-2} \\ \xi & \Rightarrow & \xi \oplus \eta \oplus \zeta_1 \oplus \cdots \oplus \zeta_{s-2} \end{array}$$

Fradkin-Vasiliev approach — spin 2 example

- Free Lagrangian for massless spin 2 in *AdS*:

$$\mathcal{L}_0 = \frac{1}{2} \left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^{ac} \omega_\nu{}^{bc} - \frac{1}{2} \left\{ {}^{\mu\nu\alpha}_{abc} \right\} \omega_\mu{}^{ab} D_\nu h_\alpha{}^c - \frac{\Lambda(d-2)}{2} \left\{ {}^{\mu\nu}_{ab} \right\} h_\mu{}^a h_\nu{}^b$$

- It is invariant under the following gauge transformations:

$$\delta_0 h_\mu{}^a = D_\mu \xi^a + \eta_\mu{}^a, \qquad \delta_0 \omega_\mu{}^{ab} = D_\mu \eta^{ab} + \Lambda e_\mu{}^{[a} \xi^{b]}$$

- Gauge invariant objects (linearized curvature and torsion):

$$\begin{aligned} R_{\mu\nu}{}^{ab} &= D_{[\mu} \omega_{\nu]}{}^{ab} + \Lambda e_{[\mu}{}^{[a} h_{\nu]}{}^{b]} \\ T_{\mu\nu}{}^a &= D_{[\mu} h_{\nu]}{}^a - \omega_{[\mu,\nu]}{}^a \end{aligned}$$

- Free Lagrangian in terms of curvatures:

$$\mathcal{L}_0 = -\frac{1}{32\Lambda(d-3)} \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} R_{\mu\nu}{}^{ab} R_{\alpha\beta}{}^{cd}$$

Spin 2 example (cont.)

- Deformations for curvatures:

$$\hat{R}_{\mu\nu}{}^{ab} = D_{[\mu}\omega_{\nu]}{}^{ab} + \Lambda e_{[\mu}{}^{[a}h_{\nu]}{}^{b]} + \omega_{[\mu}{}^{ac}\omega_{\nu]}{}^{bc} - \Lambda h_{[\mu}{}^{a}h_{\nu]}{}^{b}$$
$$\hat{T}_{\mu\nu}{}^{a} = D_{[\mu}h_{\nu]}{}^{a} - \omega_{[\mu,\nu]}{}^{a} - \omega_{[\mu}{}^{ab}h_{\nu]}{}^{b}$$

- Corrections to gauge transformations:

$$\delta\omega_\mu{}^{ab} = \eta^{c[a}\omega_\mu{}^{b]c} - \Lambda h_\mu{}^{[a}\xi^{b]}, \qquad \delta h_\mu{}^{a} = \eta^{ab}h_\mu{}^{b} - \omega_\mu{}^{ab}\xi^{b}$$

- Transformations for deformed curvatures:

$$\delta\hat{R}_{\mu\nu}{}^{ab} = \eta^{c[a}R_{\mu\nu}{}^{b]c} - \Lambda T_{\mu\nu}{}^{[a}\xi^{b]}, \qquad \delta\hat{T}_{\mu\nu}{}^{a} = \eta^{ab}T_{\mu\nu}{}^{b} - R_{\mu\nu}{}^{ab}\xi^{b}$$

- Interacting Lagrangian:

$$\mathcal{L}_0 = -\frac{1}{32\Lambda(d-3)}\left\{ \begin{matrix} \mu\nu\alpha\beta \\ abcd \end{matrix} \right\} \hat{R}_{\mu\nu}{}^{ab}\hat{R}_{\alpha\beta}{}^{cd}$$

Fradkin-Vasiliev approach for massive fields

- Two main ingredients — gauge invariance and frame-like formalism.
- There exists frame-like gauge invariant description for massive fields (both symmetric and mixed symmetry ones) that nicely works in $(A)dS$ spaces and flat Minkowski space including all possible massless and partially massless limits.
- It seems natural to apply Fradkin-Vasiliev approach to such frame-like gauge invariant formalism for investigation of possible interactions for any combination of massive and/or massless particles.

Fradkin-Vasiliev approach for massive fields

- Two main ingredients — gauge invariance and frame-like formalism.
- There exists frame-like gauge invariant description for massive fields (both symmetric and mixed symmetry ones) that nicely works in $(A)dS$ spaces and flat Minkowski space including all possible massless and partially massless limits.
- It seems natural to apply Fradkin-Vasiliev approach to such frame-like gauge invariant formalism for investigation of possible interactions for any combination of massive and/or massless particles.

Fradkin-Vasiliev approach for massive fields

- Two main ingredients — gauge invariance and frame-like formalism.
- There exists frame-like gauge invariant description for massive fields (both symmetric and mixed symmetry ones) that nicely works in $(A)dS$ spaces and flat Minkowski space including all possible massless and partially massless limits.
- It seems natural to apply Fradkin-Vasiliev approach to such frame-like gauge invariant formalism for investigation of possible interactions for any combination of massive and/or massless particles.

Frame-like formalism for massive hook

- Free Lagrangian for massive hook in *AdS*:

$$\begin{aligned} \mathcal{L}_0 &= -\frac{3}{4}\left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^{acd}\Omega_\nu{}^{bcd} - \frac{3}{8}\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \Omega_\mu{}^{abd} D_\nu \Omega_\alpha{}^{cd} - \\ & -\frac{1}{6} C_{abc}{}^2 - \frac{1}{4} e^\mu{}_a C^{abc} D_\mu B^{bc} - \\ & -m_2 e^\mu{}_a [\frac{3}{2}\Omega_\mu{}^{abc} B^{bc} + C^{abc}\Omega_\mu{}^{bc}] - \\ & -\frac{m_1{}^2}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^{ac}\Omega_\nu{}^{bc} - \frac{\tilde{m}_1{}^2}{4} B_{ab}{}^2 \end{aligned}$$

  where $3m_2{}^2 - m_1{}^2 \sim \Lambda$

- It is invariant under the following gauge transformations:

$$\begin{aligned} \delta\Omega_\mu{}^{abc} &= D_\mu \eta^{abc} + \frac{4m_1{}^2}{3(d-3)} e_\mu{}^{[a}\eta^{bc]}, \qquad \delta\Omega_\mu{}^{ab} = D_\mu\eta^{ab} - 2\eta_\mu{}^{ab} \\ \delta C^{abc} &= 6m_2\eta^{abc}, \qquad \delta B^{ab} = -4m_2\eta^{ab} \end{aligned}$$

Gauge invariant objects and free Lagrangian

- Gauge invariant objects (curvatures):

$$
\begin{aligned}
\mathcal{R}_{\mu\nu}{}^{abc} &= D_{[\mu}\Omega_{\nu]}{}^{abc} + \frac{4m_2}{3(d-3)} e_{[\mu}{}^{[a} C_{\nu]}{}^{bc]} + \frac{4m_1{}^2}{3(d-3)} e_{[\mu}{}^{[a} \Omega_{\nu]}{}^{bc]} \\
\mathcal{F}_{\mu\nu}{}^{ab} &= D_{[\mu}\Omega_{\nu]}{}^{ab} + 2\Omega_{[\mu,\nu]}{}^{ab} - \frac{2m_2}{(d-3)} e_{[\mu}{}^{[a} B_{\nu]}{}^{b]} \\
\mathcal{C}_{\mu}{}^{abc} &= D_{\mu} C^{abc} - 6m_2 \Omega_{\mu}{}^{abc} - \frac{2m_1{}^2}{(d-3)} e_{\mu}{}^{[a} B^{bc]} \\
\mathcal{B}_{\mu}{}^{ab} &= D_{\mu} B^{ab} + \frac{4}{3} C_{\mu}{}^{ab} + 4m_2 \Omega_{\mu}{}^{ab}
\end{aligned}
$$

- Free Lagrangian in terms of curvatures:

$$
\begin{aligned}
\mathcal{L}_0 &= \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} [a_1 \mathcal{R}_{\mu\nu}{}^{abe} \mathcal{R}_{\alpha\beta}{}^{cde} + a_2 \mathcal{F}_{\mu\nu}{}^{ab} \mathcal{F}_{\alpha\beta}{}^{cd}] + \\
&\quad + \left\{ {}^{\mu\nu}_{ab} \right\} [a_3 \mathcal{C}_{\mu}{}^{acd} \mathcal{C}_{\nu}{}^{bcd} + a_4 \mathcal{B}_{\mu}{}^{ac} \mathcal{B}_{\nu}{}^{bc}]
\end{aligned}
$$

where $a_1, a_3 \sim \frac{1}{m_1{}^2}$, $a_2, a_4 \sim 1$.

Deformations for curvatures

- Deformations for hook's curvatures correspond to standard minimal substitution $D_\mu \Rightarrow D_\mu + e_0 A_\mu$. After that they transform as follows:

$$\delta \hat{R}_{\mu\nu}{}^{abc,i} = e_0 \varepsilon^{ij} F_{\mu\nu} \eta^{abc,j}, \qquad \delta \hat{\mathcal{F}}_{\mu\nu}{}^{ab,i} = e_0 \varepsilon^{ij} F_{\mu\nu} \eta^{ab,j}$$

- Deformations fro e/m field strength turns out to be:

$$\begin{aligned} \hat{F}_{\mu\nu} &= F_{\mu\nu} + a_0 \varepsilon^{ij} [\Omega_{[\mu}{}^{abc,i} \Omega_{\nu]}{}^{abc,j} - \frac{2}{3(d-3)} C_{[\mu}{}^{ab,i} C_{\nu]}{}^{ab,j} + \\ & + \frac{2{m_1}^2}{(d-3)} \Omega_{[\mu}{}^{ab,i} \Omega_{\nu]}{}^{ab,j} - \frac{2{m_1}^2}{(d-3)^2} B_{[\mu}{}^{a,i} B_{\nu]}{}^{a,j}] \end{aligned}$$

- Transformations for deformed field strength:

$$\delta \hat{F}_{\mu\nu} = 2 a_0 \varepsilon^{ij} [\mathcal{R}_{\mu\nu}{}^{abc,i} + \frac{2{m_1}^2}{(d-3)} \mathcal{F}_{\mu\nu}{}^{ab,i} \eta^{ab,j}]$$

Interaction

- Interacting Lagrangian:

$$\mathcal{L} = \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} [a_1 \hat{\mathcal{R}}_{\mu\nu}{}^{abe} \hat{\mathcal{R}}_{\alpha\beta}{}^{cde} + a_2 \hat{\mathcal{F}}_{\mu\nu}{}^{ab} \hat{\mathcal{F}}_{\alpha\beta}{}^{cd}] + + \left\{ {}^{\mu\nu}_{ab} \right\} [a_3 \hat{\mathcal{C}}_{\mu}{}^{acd} \hat{\mathcal{C}}_{\nu}{}^{bcd} + a_4 \hat{\mathcal{B}}_{\mu}{}^{ac} \hat{\mathcal{B}}_{\nu}{}^{bc}] - \frac{1}{4} \hat{F}_{\mu\nu}{}^2$$

- Gauge invariance requires that

$$e_0 \sim a_0 m_1{}^2$$

- This includes in particular:
  - massless limit in AdS: $m_2 \to 0$, $m_1{}^2 \sim -\Lambda$
  - flat limit: $m_2{}^2 \to m_1{}^2/3$, $\Lambda \to 0$

  but it is impossible to take massless limit in dS $m_1 \to 0$ without switching off e/m interaction

Interaction

- Interacting Lagrangian:

$$\mathcal{L} = \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} [a_1 \hat{\mathcal{R}}_{\mu\nu}{}^{abe} \hat{\mathcal{R}}_{\alpha\beta}{}^{cde} + a_2 \hat{\mathcal{F}}_{\mu\nu}{}^{ab} \hat{\mathcal{F}}_{\alpha\beta}{}^{cd}] + + \left\{ {}^{\mu\nu}_{ab} \right\} [a_3 \hat{\mathcal{C}}_{\mu}{}^{acd} \hat{\mathcal{C}}_{\nu}{}^{bcd} + a_4 \hat{\mathcal{B}}_{\mu}{}^{ac} \hat{\mathcal{B}}_{\nu}{}^{bc}] - \frac{1}{4} \hat{F}_{\mu\nu}{}^2$$

- Gauge invariance requires that

$$e_0 \sim a_0 {m_1}^2$$

- This includes in particular:
  - massless limit in *AdS*: $m_2 \to 0$, ${m_1}^2 \sim -\Lambda$
  - flat limit: ${m_2}^2 \to {m_1}^2/3$, $\Lambda \to 0$

  but it is impossible to take massless limit in *dS* $m_1 \to 0$ without switching off e/m interaction

Deformations for hook's curvatures

- Correspond to standard minimal substitution rules:

$$e_\mu{}^a \Rightarrow e_\mu{}^a + h_\mu{}^a, \qquad D_\mu \Rightarrow D_\mu - \omega_\mu$$

  both in curvatures as well as gauge transformations

- Transformations for deformed curvatures:

$$\begin{aligned}
\delta\hat{\mathcal{R}}_{\mu\nu}{}^{abc} &= R_{\mu\nu}{}^{d[a}\eta^{bc]d} - \frac{4m_1{}^2}{3(d-3)} T_{\mu\nu}{}^{[a}\eta^{bc]} \\
\delta\hat{\mathcal{F}}_{\mu\nu}{}^{ab} &= 2\eta^{abc} T_{\mu\nu}{}^c - R_{\mu\nu}{}^{c[a}\eta^{b]c} \\
\delta\hat{\mathcal{C}}_\mu{}^{abc} &= 0, \qquad \delta\hat{\mathcal{B}}_\mu{}^{ab} = 0
\end{aligned}$$

Deformations for gravitational curvatures

- Deformed Riemann tensor:

$$\hat{R}_{\mu\nu}{}^{ab} = R_{\mu\nu}{}^{ab} + a_0[\Omega_{[\mu}{}^{acd}\Omega_{\nu]}{}^{bcd} + \frac{4}{9(d-3)}C_{[\mu}{}^{ca}C_{\nu]}{}^{bc} - \\ - \frac{4{m_1}^2}{3(d-3)}\Omega_{[\mu}{}^{ca}\Omega_{\nu]}{}^{bc} - \frac{{m_1}^2}{3(d-3)^2}B_{[\mu}{}^{a}B_{\nu]}{}^{b}]$$

- Its transformations:

$$\delta\hat{R}_{\mu\nu}{}^{ab} = -a_0\eta^{cd[a}\mathcal{R}_{\mu\nu}{}^{b]cd} + \frac{4{m_1}^2 a_0}{3(d-3)}\eta^{c[a}\mathcal{F}_{\mu\nu}{}^{b]c}$$

Interaction

- Interacting Lagrangian:

$$\begin{aligned} \mathcal{L}_0 &= \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} [a_1 \hat{\mathcal{R}}_{\mu\nu}{}^{abe} \hat{\mathcal{R}}_{\alpha\beta}{}^{cde} + a_2 \hat{\mathcal{F}}_{\mu\nu}{}^{ab} \hat{\mathcal{F}}_{\alpha\beta}{}^{cd}] + \\ & + \left\{ {}^{\mu\nu}_{ab} \right\} [a_3 \hat{\mathcal{C}}_\mu{}^{acd} \hat{\mathcal{C}}_\nu{}^{bcd} + a_4 \hat{\mathcal{B}}_\mu{}^{ac} \hat{\mathcal{B}}_\nu{}^{bc}] - \\ & - \frac{1}{32\Lambda(d-3)} \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \hat{R}_{\mu\nu}{}^{ab} \hat{R}_{\alpha\beta}{}^{cd} \end{aligned}$$

- Gauge invariance requires that

$$a_0 = \frac{9(d-3)\Lambda}{32 {m_1}^2}$$

Particular limits

- Massless limit in AdS space ($m_2 \to 0$) is non-singular. Stueckelberg fields completely decouple and we obtain massless hook in ASV formalism interacting with gravity.
- Flat limit is non-trivial due to massless graviton:

$$\mathcal{L}_g \sim \frac{1}{\Lambda}\hat{R}^2, \qquad \delta\hat{R} \sim \frac{\Lambda}{{m_1}^2}\mathcal{R}(\mathcal{F})$$

  but at least for cubic interactions it is non-singular.
- Similarly to electromagnetic case, it is impossible to take massless limit in dS space ($m_1 \to 0$) without switching off minimal interactions.

Particular limits

- Massless limit in *AdS* space ($m_2 \to 0$) is non-singular. Stueckelberg fields completely decouple and we obtain massless hook in ASV formalism interacting with gravity.
- Flat limit is non-trivial due to massless graviton:

$$\mathcal{L}_g \sim \frac{1}{\Lambda}\hat{R}^2, \qquad \delta\hat{R} \sim \frac{\Lambda}{{m_1}^2}\mathcal{R}(\mathcal{F})$$

  but at least for cubic interactions it is non-singular.
- Similarly to electromagnetic case, it is impossible to take massless limit in *dS* space ($m_1 \to 0$) without switching off minimal interactions.

Particular limits

- Massless limit in *AdS* space ($m_2 \to 0$) is non-singular. Stueckelberg fields completely decouple and we obtain massless hook in ASV formalism interacting with gravity.
- Flat limit is non-trivial due to massless graviton:

$$\mathcal{L}_g \sim \frac{1}{\Lambda}\hat{R}^2, \qquad \delta\hat{R} \sim \frac{\Lambda}{{m_1}^2}\mathcal{R}(\mathcal{F})$$

  but at least for cubic interactions it is non-singular.
- Similarly to electromagnetic case, it is impossible to take massless limit in *dS* space ($m_1 \to 0$) without switching off minimal interactions.

Conclusion

- Fradkin-Vasiliev approach with frame-like gauge invariant formalism allows one effectively investigate possible interactions for any set of massive and/or massless fields both in *AdS* as well as flat Minkowski space.
- One of the questions that deserves further study — flat limit for interactions of massive higher spin particles with massless gravity.
- Also it is would be interesting to understand striking difference between massless limits in *AdS* and *dS* spaces as far as possibility to switch on interactions is concerned.

Conclusion

- Fradkin-Vasiliev approach with frame-like gauge invariant formalism allows one effectively investigate possible interactions for any set of massive and/or massless fields both in *AdS* as well as flat Minkowski space.
- One of the questions that deserves further study — flat limit for interactions of massive higher spin particles with massless gravity.
- Also it is would be interesting to understand striking difference between massless limits in *AdS* and *dS* spaces as far as possibility to switch on interactions is concerned.

Conclusion

- Fradkin-Vasiliev approach with frame-like gauge invariant formalism allows one effectively investigate possible interactions for any set of massive and/or massless fields both in *AdS* as well as flat Minkowski space.
- One of the questions that deserves further study — flat limit for interactions of massive higher spin particles with massless gravity.
- Also it is would be interesting to understand striking difference between massless limits in *AdS* and *dS* spaces as far as possibility to switch on interactions is concerned.