AdS_4/CFT_3 correspondence: $3d$ conformal symmetry perspective

D.V. Uvarov

Based on ArXiv:0811.2813; 0906.4699; 0912.1044; 1011.5457

Outline:

- Aharony-Bergman-Jafferis-Maldacena duality as a realization of AdS_4/CFT_3 correspondence;
- $osp(4|6)$ superalgebra as $D=3$ $\mathcal{N}=6$ sconf algebra;
- $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset space: parametrization and $D=3$ $\mathcal{N}=6$ sconf symmetry;
- $OSp(4|6)/(SO(1,3)\times U(3))$ sigma-model action in conformal basis for Cartan forms;
- $AdS_4\times CP^3$ superstring beyond the $OSp(4|6)/(SO(1,3)\times U(3))$ sigma-model approach;
- AdS light-cone gauge action for $AdS_4\times CP^3$ superstring;
- summary.

AdS/CFT correspondence is the simplest realization of the gauge fields/strings duality idea. One of the maxsusy examples of AdS/CFT correspondence originally proposed by Maldacena (1997) is the AdS_4/CFT_3 correspondence conjecturing that

- M-theory on $AdS_4 \times S^7$ background is dual to a $D = 3$ $\mathcal{N} = 8$ superconformal field theory (SCFT). Both theories have the same $OSp(4|8)$ symmetry realized as $D = 3$ $\mathcal{N} = 8$ superconformal one in SCFT.

Hard to verify/prove because dual theories are not explored enough.

The reformulation proposed by Aharony, Bergman, Jafferis and Maldacena (2008) makes the problem tractable:

- M-theory on $AdS_4 \times (S^7/Z_k)$ background is dual to $D = 3$ $\mathcal{N} = 6$ superconformal field theory (SCFT).

When $k > 2$ orbifolding S^7 reduces background isometry supergroup to $OSp(4|6)$ isomorphic to $D = 3$ $\mathcal{N} = 6$ superconformal symmetry. ABJM succeeded in constructing non-Abelian gauge theory invariant (on-shell) under the requisite $D = 3$ $\mathcal{N} = 6$ superconformal symmetry (and not only classically!).

ABJM is the Chern-Simons-matter theory with $U(N)_k \times U(N)_{-k}$ gauge symmetry and the inverse Chern-Simons level $k \in \mathbb{N}$ playing the role of the coupling constant (Bandres, Lipstein and Schwarz, 2008)

$$\begin{aligned}
S &= S_{kin} + S_{CS} + S_4 + S_6 : \\
S_{kin} &= \frac{k}{2\pi}\int d^3x Tr(-D^\mu X^A D_\mu X_A + i\bar{\Psi}_A \gamma^\mu D_\mu \Psi^A), \\
S_{CS} &= \frac{k}{2\pi}\int d^3x Tr(\frac{1}{2}AdA + \frac{i}{3}A^3 - \frac{1}{2}\tilde{A}d\tilde{A} - \frac{i}{3}\tilde{A}^3), \\
S_4 &= i\int d^3x Tr(\varepsilon^{ABCD}\bar{\Psi}_A X_B \Psi_C X_D - \varepsilon_{ABCD}\bar{\Psi}^A X^B \Psi^C X^D \\
&+ \bar{\Psi}^A \Psi_A X_B X^B - \bar{\Psi}_A \Psi^A X^B X_B + 2\bar{\Psi}_A \Psi^B X^a X_b - 2\bar{\Psi}^B \Psi_A X_B X^A), \\
S_6 &= \frac{1}{3}\int d^3x Tr(X^A X_A X^B X_B X^C X_C + X_A X^A X_B X^B X_C X^C \\
&+ 4X_A X^B X_C X^A X_B X^C - 6X^A X_B X^B X_A X^C X_C), \;\; A,B,C,D = 1,...,4,
\end{aligned}$$

where the gauge fields are $(A_\mu)^t{}_s$ and $(\tilde{A}_\mu)^{\hat{t}}{}_{\hat{s}}$; $(\Psi^A)^t_{\hat{s}} \in (\bar{N}, N)$ are w/volume Dirac spinors, and $(X_A)^t{}_{\hat{s}} \in (\bar{N}, N)$ are complex w/volume scalars.

On the M-theory side of duality $AdS_4 \times (S^7/Z_k)$ background supported by 4-form flux $F_4 \sim Nk\varepsilon_4$ is the solution of $D = 11$ supergravity equations that preserves 24 of 32 space-time supersymmetries.
Z_k orbifold projection can be chosen to act on S^1 within S^7 using *Hopf fibration* realization $S^7 = CP^3 \times S^1$ (Nilsson and Pope, 1984, and also Sorokin, Tkach and Volkov, 1984-85).
When $N, k \gg 1$ with $\lambda = N/k$ fixed and $k^5 \gg N$ dual description of the 't Hooft limit of ABJM gauge theory is given by the Type IIA string theory on $AdS_4 \times CP^3$ background with nonzero fluxes $F_4 \sim \varepsilon_4$ and $F_2 \sim J$, where J is the Kähler 2-form of CP^3 manifold. It is characterized by the same $OSp(4|6)$ isometry supergroup.

$OSp(4|6)$ supergroup has bosonic subgroup

$$Sp(4) \times SO(6) \sim SO(2,3) \times SU(4),$$

where $SO(2,3)$ is the isometry group of $AdS_4 = SO(2,3)/SO(1,3)$ and is locally isomorphic to $conf_3$ group, and $SU(4)$ is the isometry group of $CP^3 = SU(4)/U(3)$ manifold. 24 fermionic generators of $OSp(4|6)$ are in $1 \leftrightarrow 1$ correspondence with susys preserved by $AdS_4 \times CP^3$ background.
$osp(4|6)$ superalgebra relations can be written in concise manner

$$\{g_k, g_l] \sim g_{k+l mod 4}$$

taking into $\mathbb{Z}_4$ automorphism Υ under which the generators split into 4 eigenspaces $g_0 \oplus g_1 \oplus g_2 \oplus g_3$

$$\begin{aligned} g_0 = so(1,3) \oplus u(3) = (M_{m'n'}, V_a{}^b) \quad &: \quad \Upsilon(g_0) = g_0, \\ g_2 = \tfrac{so(2,3)}{so(1,3)} \oplus \tfrac{su(4)}{u(3)} = (M_{0'm'}, V_a{}^4, V_4{}^a) \quad &: \quad \Upsilon(g_2) = -g_2, \\ g_1 = Q^a_\mu + iS^a_\mu \quad &: \quad \Upsilon(g_1) = ig_1, \\ g_3 = Q^a_\mu - iS^a_\mu \quad &: \quad \Upsilon(g_3) = -ig_3 \end{aligned}$$

$so(2,3)$ algebra relations

$$[M^{\underline{kl}}, M^{\underline{mn}}] = \eta^{\underline{kn}} M^{\underline{lm}} - \eta^{\underline{km}} M^{\underline{ln}} - \eta^{\underline{ln}} M^{\underline{km}} + \eta^{\underline{lm}} M^{\underline{kn}}, \ \ \underline{k} = 0', 0, 1, 2, 3$$

can be presented in the form of ads_4 algebra with manifest $so(1,3)$ covariance

$$[M^{0'm'}, M^{0'n'}] = M^{m'n'}, \quad [M^{0'k'}, M^{m'n'}] = \eta^{k'm'} M^{0'n'} - \eta^{k'n'} M^{0'm'},$$
$$[M^{k'l'}, M^{m'n'}] = \eta^{k'n'} M^{l'm'} - \eta^{k'm'} M^{l'n'} - \eta^{l'n'} M^{k'm'} + \eta^{l'm'} M^{k'n'}$$

and, introducing $3d$ conformal group generators

$$D = 2M^{0'3}, \quad P^m = -(M^{0'm} + M^{3m}), \quad K^m = M^{3m} - M^{0'm},$$

as $conf_3$ algebra relations

$$[P^m, D] = -2P^m, \quad [K^m, D] = 2K^m, \quad [P^m, K^n] = \eta^{mn} D + 2M^{mn},$$
$$[P^l, M^{mn}] = \eta^{lm} P^n - \eta^{ln} P^m, \quad [K^l, M^{mn}] = \eta^{lm} K^n - \eta^{ln} K^m,$$
$$[M^{kl}, M^{mn}] = \eta^{kn} M^{lm} - \eta^{km} M^{ln} - \eta^{ln} M^{km} + \eta^{lm} M^{kn}.$$

Similarly $su(4)$ algebra relations

$$[V_A{}^B, V_C{}^D] = i\left(\delta^B_C V_A{}^D - \delta^D_A V_C{}^B\right)$$

can be decomposed preserving manifest $u(3)$ covariance and exhibiting $\mathbb{Z}_4-$invariance

$$[V_a{}^4, V_4{}^b] = i(V_a{}^b + \delta^b_a V_c{}^c), \quad [V_a{}^4, V_b{}^c] = -i\delta^c_a V_b{}^4, \quad [V_4{}^a, V_b{}^c] = i\delta^a_b V_4{}^c,$$
$$[V_a{}^b, V_c{}^d] = i(\delta^b_c V_a{}^d - \delta^d_a V_c{}^b),$$

$a, b, c = 1, 2, 3 \in \mathbf{3}_{SU(3)}, \bar{\mathbf{3}}_{SU(3)}$.

Fermionic generators of $osp(4|6)$ superalgebra can be split into the supertranslation $(Q^a_\mu, \bar{Q}_{\mu a})$ and conformal supersymmetry $(S^{\mu a}, \bar{S}^\mu_a)$ ones satisfying the following relations

$$
\begin{gathered}
\{Q^a_\mu, \bar{Q}_{\nu b}\} = 2i\delta^a_b \sigma^m_{\mu\nu} P_m, \quad \{S^{\mu a}, \bar{S}^\nu_b\} = 2i\delta^a_b \tilde{\sigma}^{m\mu\nu} K_m, \\
\{Q^a_\mu, S^{\nu b}\} = 2\delta^\nu_\mu \varepsilon^{abc} V_c{}^4, \quad \{\bar{Q}_{\mu a}, \bar{S}^\nu_b\} = -2\delta^\nu_\mu \varepsilon_{abc} V_4{}^c, \\
\{Q^a_\mu, \bar{S}^\nu_b\} = -i\delta^a_b \delta^\nu_\mu D + i\delta^a_b \sigma^{mn}{}_\mu{}^\nu M_{mn} - 2\delta^\nu_\mu (V_b{}^a - \delta^a_b V_c{}^c), \\
\{\bar{Q}_{\mu a}, S^{\nu b}\} = -i\delta^b_a \delta^\nu_\mu D + i\delta^b_a \sigma^{mn}{}_\mu{}^\nu M_{mn} + 2\delta^\nu_\mu (V_a{}^b - \delta^b_a V_c{}^c).
\end{gathered}
$$

'Off-diagonal' commutators of $osp(4|6)$ superalgebra include $[conf_3, Fermi]$

$$[D, Q^a_\mu] = Q^a_\mu, \quad [D, \bar{Q}_{\mu a}] = \bar{Q}_{\mu a},$$
$$[M^{mn}, Q^a_\mu] = \tfrac{1}{2}\sigma^{mn}{}_\mu{}^\nu Q^a_\nu, \quad [M^{mn}, \bar{Q}_{\mu a}] = \tfrac{1}{2}\sigma^{mn}{}_\mu{}^\nu \bar{Q}_{\nu a},$$
$$[K^m, Q^a_\mu] = \sigma^m_{\mu\nu} S^{\nu a}, \quad [K^m, \bar{Q}_{\mu a}] = \sigma^m_{\mu\nu} \bar{S}^\nu_a,$$
$$[D, S^{\mu a}] = -S^{\mu a}, \quad [D, \bar{S}^\mu_a] = -\bar{S}^\mu_a,$$
$$[M^{mn}, S^{\mu a}] = -\tfrac{1}{2} S^{\nu a} \sigma^{mn}{}_\nu{}^\mu, \quad [M^{mn}, \bar{S}^\mu_a] = -\tfrac{1}{2}\bar{S}^\nu_a \sigma^{mn}{}_\nu{}^\mu,$$
$$[P^m, S^{\mu a}] = -\tilde{\sigma}^{m\mu\nu} Q^a_\nu, \quad [P^m, \bar{S}^{\mu a}] = -\tilde{\sigma}^{m\mu\nu}\bar{Q}_{\nu a};$$

and $[su(4), Fermi]$ relations

$$[V_a{}^b, Q^c_\mu] = \tfrac{i}{2}\delta^b_a Q^c_\mu - i\delta^c_a Q^b_\mu, \quad [V_4{}^a, Q^b_\mu] = i\varepsilon^{abc}\bar{Q}_{\mu c},$$
$$[V_a{}^b, \bar{Q}_{\mu c}] = -\tfrac{i}{2}\delta^b_a \bar{Q}_{\mu c} + i\delta^b_c \bar{Q}_{\mu a}, \quad [V_a{}^4, \bar{Q}_{\mu b}] = -i\varepsilon_{abc} Q^c_\mu,$$
$$[V_a{}^b, S^{\mu c}] = \tfrac{i}{2}\delta^b_a S^{\mu c} - i\delta^c_a S^{\mu b}, \quad [V_4{}^a, S^{\mu b}] = i\varepsilon^{abc}\bar{S}^\mu_c,$$
$$[V_a{}^b, \bar{S}^\mu_c] = -\tfrac{i}{2}\delta^b_a \bar{S}^\mu_c + i\delta^b_c \bar{S}^\mu_a, \quad [V_a{}^4, \bar{S}^\mu_b] = -i\varepsilon_{abc} S^{\mu c}.$$

$D = 3\,\mathcal{N} = 6$ sconf symmetry can be realized on the $(10|24)-$dimensional subspace of $(10|32)-$dimensional $AdS_4 \times CP^3$ superspace isomorphic to $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset manifold

$$\dim[OSp(4|6)/(SO(1,3)\times U(3))] = (10|24).$$

It is invariant under global $OSp(4|6)$ symmetry modulo compensating $SO(1,3)\times U(3)$ tranformations

$$\mathscr{G}'H = G\mathscr{G},\ G \in OSp(4|6),\ H \in SO(1,3)\times U(3)$$

or in the infinitesimal form

$$\delta\mathscr{G} = g\mathscr{G} - \mathscr{G}h,\ g \in osp(4|6),\ h \in so(1,3)\oplus u(3),$$

where $\mathscr{G} \in OSp(4|6)/(SO(1,3)\times U(3))$ is a supercoset representative.

Using the realization of $osp(4|6)$ superalgebra as $D = 3$ $\mathcal{N} = 6$ sconf algebra infinitesimal tranformation parameter can be brought to the form

$$\begin{aligned} g = \; & a^m P_m + b_m K^m + f D + \tfrac{1}{2} l^{mn} M_{mn} + y^a V_a{}^4 + \bar{y}_a V_4{}^a \\ + \; & w_a{}^b V_b{}^a + w_4{}^4 V_4{}^4 + \varepsilon_a^\mu Q_\mu^a + \bar{\varepsilon}^{\mu a} \bar{Q}_{\mu a} + \xi_{\mu a} S^{\mu a} + \bar{\xi}_\mu^a \bar{S}_a^\mu , \end{aligned}$$

where

a^m, b_m, f and l^{mn} are the parameters of 3d space-time translations, conformal boosts, dilatations and Lorentz rotations;

$w_a{}^b$ and $(y^a, \bar{y}_a)$ parametrize $U(3)$ rotations and $SU(4)/U(3)$ boosts;

$(\varepsilon_a^\mu, \bar{\varepsilon}^{\mu a})$ and $(\xi_{\mu a}, \bar{\xi}_\mu^a)$ correspond to Poincare and conformal susy transformations.

Left-invariant Cartan 1-forms in conformal basis read

$$\begin{aligned}\mathscr{C}(d) = \mathscr{G}^{-1}d\mathscr{G} = \; & \widehat{\omega}^m(d)P_m + \widehat{c}^m(d)K_m + \Delta(d)D + G^{mn}(d)M_{mn} \\ + \; & \Omega_a{}^4(d)V_4{}^a + \Omega_4{}^a(d)V_a{}^4 + \Omega_a{}^b(d)V_b{}^a + \Omega_4{}^4(d)V_4{}^4 \\ + \; & \widehat{\omega}_a^\mu(d)Q_\mu^a + \widehat{\bar{\omega}}^{\mu a}(d)\bar{Q}_{\mu a} + \widehat{\chi}_{\mu a}(d)S^{\mu a} + \widehat{\bar{\chi}}_\mu^a(d)\bar{S}_a^\mu\end{aligned}$$

or manifesting the $\mathbb{Z}_4-$grading

$$\mathscr{C}(d) = \mathscr{C}_0(d) + \mathscr{C}_2(d) + \mathscr{C}_1(d) + \mathscr{C}_3(d),$$

where

$$\begin{aligned}\mathscr{C}_0(d) = \; & \tfrac{1}{2}(\widehat{\omega}^m(d) - \widehat{c}^m(d))(P_m - K_m) + G^{mn}(d)M_{mn} + \Omega_a{}^b(d)V_b{}^a + \Omega_4{}^4(d)V_4{}^4, \\ \mathscr{C}_2(d) = \; & \tfrac{1}{2}(\widehat{\omega}^m(d) + \widehat{c}^m(d))(P_m + K_m) + \Delta(d)D + \Omega_a{}^4(d)V_4{}^a + \Omega_4{}^a(d)V_a{}^4, \\ \mathscr{C}_1(d) = \; & \tfrac{1}{2}(\widehat{\omega}_a^\mu(d) + i\widehat{\chi}_a^\mu(d))(Q_\mu^a + iS_\mu^a) + \tfrac{1}{2}(\widehat{\bar{\omega}}^{\mu a}(d) - i\widehat{\bar{\chi}}^{\mu a}(d))(\bar{Q}_{\mu a} - i\bar{S}_{\mu a}), \\ \mathscr{C}_3(d) = \; & \tfrac{1}{2}(\widehat{\omega}_a^\mu(d) - i\widehat{\chi}_a^\mu(d))(Q_\mu^a - iS_\mu^a) + \tfrac{1}{2}(\widehat{\bar{\omega}}^{\mu a}(d) + i\widehat{\bar{\chi}}^{\mu a}(d))(\bar{Q}_{\mu a} + i\bar{S}_{\mu a}).\end{aligned}$$

Using the transformation properties of the $OSp(4|6)/(SO(1,3)\times U(3))$ representative under global $OSp(4|6)$ symmetry yields that

$$\mathscr{C}(\delta) = \mathscr{G}^{-1}\delta\mathscr{G} = \mathscr{G}^{-1}g\mathscr{G} - h$$

or explicitly in conformal basis

$$\begin{aligned}
\mathscr{C}(\delta) = \; & (\widehat{\omega}^m(\delta) - \widehat{b}^m)P_m + (\widehat{c}^m(\delta) + \widehat{b}^m)K_m \\
+ \; & \Delta(\delta)D + (G^{mn}(\delta) + \tfrac{1}{2}\widehat{l}^{mn})M_{mn} \\
+ \; & \Omega_4{}^a(\delta)V_a{}^4 + \Omega_a{}^4(\delta)V_4{}^a \\
+ \; & (\Omega_a{}^b(\delta) + \widehat{w}_a{}^b)V_b{}^a + (\Omega_4{}^4(\delta) + \widehat{w}_4{}^4)V_4{}^4 \\
+ \; & \widehat{\omega}_a^\mu(\delta)Q_\mu^a + \widehat{\bar{\omega}}^{\mu a}(\delta)Q_{\mu a} + \widehat{\chi}_{\mu a}(\delta)S^{\mu a} + \widehat{\bar{\chi}}_\mu^a(\delta)\bar{S}_a^\mu
\end{aligned}$$

with $\widehat{l}^{mn}$, $\widehat{b}^m$ being the parameters of compensating $SO(1,3)$ Lorentz transformations and $\widehat{w}_a{}^b$ corresponding to compensating $U(3)$ rotation.

The following $OSp(4|6)/(SO(1,3)\times U(3))$ representative parametrization is one of the possible choices compatible with conformal structure

$$\mathscr{G} = e^{x^m P_m + \theta^\mu_a Q^a_\mu + \bar\theta^{\mu a}\bar Q_{\mu a}} e^{\eta_{\mu a}S^{\mu a} + \bar\eta^a_\mu \bar S^\mu_a} e^{z^a V_a{}^4 + \bar z_a V_4{}^a} e^{\varphi D}.$$

(x^m, φ) are the Poincare coordinates for AdS_4;
$(z^a, \bar z_a)$ parametrize CP^3 manifold;
$(\theta^\mu_a, \bar\theta^{\mu a})$ and $(\eta_{\mu a}, \bar\eta^a_\mu)$ are associated with Poincare and conformal supersymmetries.
Similar parametrizations were considered previously when studying branes on $AdS \times S$ backgrounds (Dall'Agata et.al.; Kallosh; Pasti, Sorokin and Tonin, 1998) and $AdS_5 \times S^5$ superstring (Metsaev and Tseytlin, 2000).

For the above chosen supercoset representative Cartan forms associated with the $so(2,3)/so(1,3)$ generators acquire the form

$$
\begin{aligned}
\widehat{\omega}^m(d) = & \; e^{-2\varphi}\omega^m(d), \quad \omega^m(d) = dx^m - id\theta^\mu_a \sigma^m_{\mu\nu}\bar{\theta}^{\nu a} + i\theta^\mu_a \sigma^m_{\mu\nu} d\bar{\theta}^{\nu a}, \\
\widehat{c}^m(d) = & \; e^{2\varphi}c^m(d), \quad c^m(d) = -id\eta_{\mu a}\tilde{\sigma}^{m\mu\nu}\bar{\eta}^a_\nu + i\eta_{\mu a}\tilde{\sigma}^{m\mu\nu} d\bar{\eta}^a_\nu \\
+ & \; 2(\bar{\eta}\eta)\left[\eta_{\mu a}\tilde{\sigma}^{m\mu\nu}(d\bar{\theta}^a_\nu + \tfrac{1}{4}\bar{\zeta}^a_\nu(d)) - (d\theta_{\mu a} + \tfrac{1}{4}\zeta_{\mu a}(d))\tilde{\sigma}^{m\mu\nu}\bar{\eta}^a_\nu\right], \\
\Delta(d) = & \; d\varphi + i(d\theta^\mu_a \bar{\eta}^a_\mu + d\bar{\theta}^{\mu a}\eta_{\mu a}),
\end{aligned}
$$

where

$$
\zeta^\mu_a(d) = -\tilde{\sigma}^{m\mu\nu}\omega_m(d)\eta_{\nu a} = -\tilde{\omega}^{\mu\nu}(d)\eta_{\nu a}
$$

and c.c.

$SU(4)/U(3)$ Cartan forms equal

$$\widehat{\Omega}_a{}^4(d) = \Omega_a{}^4(d) + \widehat{\Psi}_a{}^4(d), \quad \widehat{\Omega}_4{}^a(d) = \Omega_4{}^a(d) + \widehat{\Psi}_4{}^a(d),$$

where

$$\begin{aligned} \Omega_a{}^4(d) &= d\bar{z}_a \frac{\sin|z|}{|z|} + \bar{z}_a \frac{\sin|z|(1-\cos|z|)}{2|z|^3}(dz^c\bar{z}_c - z^c d\bar{z}_c) + \bar{z}_a\left(\frac{1}{|z|} - \frac{\sin|z|}{|z|^2}\right) d|z|, \\ \widehat{\Psi}_a{}^4(d) &= 2\varepsilon_{abc}(\widehat{d}\widehat{\bar{\theta}}^{\mu b} + \tfrac{1}{2}\widehat{\bar{\zeta}}^{\mu b}(d))\widehat{\bar{\eta}}^c_\mu \end{aligned}$$

and c.c. Grassmann coordiates with hats are defined by means of the $SU(4)/U(3)$ transformation

$$\widehat{\eta}_{\mu\widehat{a}} = \begin{pmatrix} \widehat{\eta}_{\mu a} \\ \widehat{\bar{\eta}}^a_\mu \end{pmatrix} = T_{\widehat{a}}{}^{\widehat{b}}\eta_{\mu\widehat{b}}, \quad T_{\widehat{a}}{}^{\widehat{b}}(z,\bar{z}) \in SU(4)/U(3)$$

etc.

Fermionic Cartan forms related to Poincare susy generators $(Q^a_\mu, \bar{Q}_{\mu a})$ equal

$$\widehat{\omega}^\mu_{\widehat{a}}(d) = \begin{pmatrix} \widehat{\omega}^\mu_a \\ \widehat{\bar{\omega}}^{\mu a} \end{pmatrix} = e^{-\varphi} T_{\widehat{a}}{}^{\widehat{b}} \omega^\mu_{\widehat{b}}(d), \quad \omega^\mu_{\widehat{b}}(d) = d\theta^\mu_{\widehat{b}} + \zeta^\mu_{\widehat{b}}(d)$$

and those related to conformal susy generators $(S^{\mu a}, \bar{S}^\mu_a)$ equal

$$\widehat{\chi}_{\mu\widehat{a}}(d) = \begin{pmatrix} \widehat{\chi}_{\mu a} \\ \widehat{\bar{\chi}}^a_\mu \end{pmatrix} = e^{\varphi} T_{\widehat{a}}{}^{\widehat{b}} \chi_{\mu\widehat{b}}(d),$$

$$\chi_{\mu\widehat{a}}(d) = d\eta_{\mu\widehat{a}} + 2i\eta^{\widehat{b}}_\mu d\theta^\nu_{\widehat{b}} \eta_{\nu\widehat{a}} - i(\bar{\eta}\eta)\omega_{\mu\widehat{a}}(d).$$

Based on the above results it is possible to derive the transformation properties under global $OSp(4|6)$ symmetry of the coordinates parametrizing $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset.
For the AdS_4 Poincare coordinates one finds

$$\begin{aligned}
\delta x^m = & \; a^m + l^m{}_n x^n + 2f x^m + b^m(x^2 + (\bar\theta\theta)^2) - 2x^m b_n x^n \\
- & \; i\left[(\varepsilon_a\sigma^m\bar\theta^a) + (\bar\varepsilon^a\sigma^m\theta_a)\right] - i\left[(\xi_a\hat{\tilde{x}}\sigma^m\bar\theta^a) + (\bar\xi^a\hat{\tilde{x}}\sigma^m\theta_a)\right] \\
+ & \; e^{2\varphi}\left\{\hat b^m + i\left[(\eta_a\hat{\tilde{b}}\sigma^m\bar\theta^a) + (\bar\eta^a\hat{\tilde{b}}\sigma^m\theta_a)\right]\right\}, \\
\delta\varphi = & \; f - b_m x^m + i(\xi_{\mu a}\bar\theta^{\mu a} + \bar\xi^a_\mu\theta^\mu_a),
\end{aligned}$$

where $\hat{\tilde{x}}^{\mu\nu} = x^m\tilde\sigma_m^{\mu\nu} - i\varepsilon^{\mu\nu}(\bar\theta\theta)$.

Variations of the fermionic coordinates equal

$$
\begin{aligned}
\delta\theta_a^\mu &= \varepsilon_a^\mu + \tfrac{1}{4} l^{mn}\theta_a^\nu \sigma_{mn\nu}{}^\mu + f\theta_a^\mu + iw_b{}^b\theta_a^\mu - iw_a{}^b\theta_b^\mu - i\varepsilon_{abc}y^b\bar{\theta}^{\mu c} \\
&+ \hat{\bar{x}}^{\mu\nu} b_{\nu\lambda}\theta_a^\lambda + \hat{\bar{x}}^{\mu\nu}\xi_{\nu a} - 2i(\theta_b^\mu \bar{\xi}_\nu^b + \bar{\theta}^{\mu b}\xi_{\nu b})\theta_a^\nu + e^{2\varphi}\hat{\bar{b}}^{\mu\nu}\eta_{\nu a}, \\
\delta\eta_{\mu a} &= -\tfrac{1}{4} l^{mn}\sigma_{mn\mu}{}^\nu \eta_{\nu a} - f\eta_{\mu a} + iw_b{}^b\eta_{\mu a} - iw_a{}^b\eta_{\mu b} - i\varepsilon_{abc}y^b\bar{\eta}_\mu^c \\
&+ b_{\mu\nu}\theta_a^\nu - \eta_{\nu a}\hat{\bar{x}}^{\nu\lambda} b_{\lambda\mu} + 2i\left[(\theta_a b\theta_b)\bar{\eta}_\mu^b + (\theta_a b\bar{\theta}^b)\eta_{\mu b}\right] \\
&+ \xi_{\mu a} - 2i(\bar{\xi}_\mu^b \theta_b^\nu + \xi_{\mu b}\bar{\theta}^{\nu b})\eta_{\nu a} - 2i(\eta_{\mu b}\bar{\xi}_\nu^b + \bar{\eta}_\mu^b \xi_{\nu b})\theta_a^\nu \\
&+ 2i\xi_{\nu a}(\theta_b^\nu \bar{\eta}_\mu^b + \bar{\theta}^{\nu b}\eta_{\mu b}) + 2ie^{2\varphi}(\bar{\eta}\eta)\varepsilon_{\mu\lambda}\hat{\bar{b}}^{\lambda\nu}\eta_{\nu a}.
\end{aligned}
$$

Left-invariant Cartan forms associated with $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset generators transform under compensating $SO(1,3)\times U(3)$ rotations in accordance with their representation structure, e.g.

$$\begin{aligned}
\delta\widehat{\omega}^m(d)+\delta\widetilde{c}^m(d) = &\ \widehat{l}^{mn}(\widehat{\omega}_n(d)+\widehat{c}_n(d))+4\widehat{b}^m\Delta(d),\\
\delta\Delta(d) = &\ -\widehat{b}_m(\widehat{\omega}^m(d)+\widetilde{c}^m(d)),\\
\delta\widehat{\omega}^{\nu}_{\widehat{a}}(d) = &\ \tfrac{1}{4}\widehat{\omega}^{\lambda}_{\widehat{a}}(d)\widehat{l}_{\lambda}{}^{\nu}+\widehat{\widetilde{b}}^{\nu\lambda}\widehat{\chi}_{\lambda\widehat{a}}(d)-i\widehat{W}_{\widehat{a}}{}^{\widehat{b}}\widehat{\omega}^{\nu}_{\widehat{b}}(d).
\end{aligned}$$

Other Cartan forms exhibit connection-type transformation properties, e.g. $G^{\mu\nu}(d)=\varepsilon^{\mu\lambda}\sigma_{mn\lambda}{}^{\nu}G^{mn}(d)$

$$\begin{aligned}
\delta G^{\mu\nu}(d) = &\ \tfrac{1}{4}(G^{\mu\lambda}(d)\widehat{l}_{\lambda}{}^{\nu}+G^{\nu\lambda}(d)\widehat{l}_{\lambda}{}^{\mu})+\widehat{b}^{\mu}{}_{\lambda}(\widehat{\widetilde{\omega}}(d)-\widehat{\widetilde{c}}(d))^{\lambda\nu}\\
+ &\ \widehat{b}^{\nu}{}_{\lambda}(\widehat{\widetilde{\omega}}(d)-\widehat{\widetilde{c}}(d))^{\lambda\mu}-\tfrac{1}{2}d\widehat{l}^{\mu\nu}.
\end{aligned}$$

Above derived $OSp(4|6)/(SO(1,3)\times U(3))$ Cartan forms can be identified with the supervielbein and connection components of $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset manifold.
So the superstring action (Arutyunov and Frolov; Stefanski, 2008) on the $OSp(4|6)/(SO(1,3)\times U(3))$ subspace of $AdS_4\times CP^3$ superspace can be obtained by applying the approach elaborated for the costruction of $\sigma-$model actions on supercosets with isometry superalgebras invariant under Z_4 automorphism (Metsaev and Tseytlin, 1998; Berkovits et.al., 1999; Roiban and Siegel, 2000 ...)

$$S=\int d^2\xi\mathcal{L}(\xi),\quad \mathcal{L}=-\frac{1}{2}\gamma^{ij}\langle g_i^2 g_j^2\rangle+\varepsilon^{ij}\langle g_i^1 g_j^3\rangle.$$

In conformal basis for the $OSp(4|6)/(SO(1,3)\times U(3))$ Cartan forms explicit form of the action is (D.U., 2008)

$$S=\int d^2\xi(\mathcal{L}_{kin}+\mathcal{L}_{WZ}),$$

where

$$\begin{aligned}\mathcal{L}_{kin}= & -\tfrac{1}{2}\gamma^{ij}\left[\tfrac{1}{4}(\widehat{\omega}_i^m+\widehat{c}_i^m)(\widehat{\omega}_{jm}+\widehat{c}_{jm})+\Delta_i\Delta_j\right.\\ + & \left.\tfrac{1}{2}(\Omega_{ia}{}^4\Omega_{j4}{}^a+\Omega_{ja}{}^4\Omega_{i4}{}^a)\right],\\ \mathcal{L}_{WZ}= & -\tfrac{1}{2}\varepsilon^{ij}\left(\widehat{\omega}_{ia}^{\mu}\varepsilon_{\mu\nu}\widehat{\bar{\omega}}_j^{\nu a}+\widehat{\chi}_{i\mu a}\varepsilon^{\mu\nu}\widehat{\bar{\chi}}_{j\nu}^a\right).\end{aligned}$$

It is by construction $OSp(4|6)$ invariant, describes correct number of bosonic and fermionic physical variables due to 8-parameter $\kappa-$symmetry, and is classically integrable.

WZ Lagrangian can be written as

$$\mathcal{L}_{WZ} = -\tfrac{i}{8}\varepsilon^{ij}\mathfrak{J}_{\hat{a}}{}^{\hat{b}}\left(\hat{\omega}_i^{\mu\hat{a}}\varepsilon_{\mu\nu}\hat{\omega}^{\nu}_{j\hat{b}} + \hat{\chi}^{\hat{a}}_{i\mu}\varepsilon^{\mu\nu}\hat{\chi}_{j\nu\hat{b}}\right),$$

where $\mathfrak{J}_{\hat{a}}{}^{\hat{b}}$ is the Kähler 2-form of CP^3 in $\mathbf{3}\oplus\bar{\mathbf{3}}$ basis. One of its convenient realizations is

$$\mathfrak{J}^{IJ} = \frac{i}{2}(\rho^I_{4a}\tilde{\rho}^{J4a} - \rho^J_{4a}\tilde{\rho}^{I4a})$$

or in the $\mathbf{3}\oplus\bar{\mathbf{3}}$ basis

$$\mathfrak{J}_{\hat{a}}{}^{\hat{b}} = 2i\begin{pmatrix} \delta_a^b & 0 \\ 0 & -\delta_b^a \end{pmatrix}.$$

Contraction with the $SU(4)$ generators $\rho^{IJ}{}_A{}^B = \frac{1}{2}(\rho^I_{AC}\tilde{\rho}^{JCB} + (I \leftrightarrow J))$ yields diagonal matrix

$$\mathfrak{J}_A{}^B = \mathfrak{J}^{IJ}\rho^{IJ}{}_A{}^B = \begin{pmatrix} -2i\delta_a^b & 0 \\ 0 & 6i \end{pmatrix}$$

satisfying the defining relation $\mathfrak{J}_A{}^C\mathfrak{J}_C{}^B - 4i\mathfrak{J}_A{}^B + 12\delta_A^B = 0$.

The $\sigma-$model action on the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset of dim(10|24) corresponds to fixing 8 of 16 $\kappa-$symmetries in the full IIA superstring action on $AdS_4 \times CP^3$ superbackground by putting to zero 8 Grassmann coordinates

$$\theta^\mu = \bar{\theta}^\mu = \eta_\mu = \bar{\eta}_\mu = 0$$

associated with 8 broken by background susys.
So that not all string configurations can be described within the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset approach, e.g. supercoset Lagrangian degenerates when string moves solely within the AdS_4 part of background (Arutynov and Frolov, 2008; Gomis, Sorokin and Wulff, 2008).
Such string configurations should be treated using the full action (Gomis, Sorokin and Wulff, 2008). It cannot be obtained within the supercoset approach.

There are 2 ways to obtain the superstring action on $AdS_4 \times CP^3$ superspace beyond the $OSp(4|6)/(SO(1,3) \times U(3))$ subspace:

general approach pursued by Gomis, Sorokin and Wulff consists in finding expressions for $AdS_4 \times CP^3$ supergeometry and plugging them into the IIA superstring action on a curved superbackground (Grisaru et.al., 1985). Allows also to construct brane actions;

'shortcut' spproach (Arutyunov and Frolov, 2008; D.U.) is to perform the double dimensional reduction (Duff et.al., 1987; Howe and Sezgin, 2004) of the $D = 11$ supermembrane on max. susy $AdS_4 \times S^7$ superbackground using that $S^7 = CP^3 \times S^1$.

Since the $AdS_4 \times S^7$ superspace can be described as $OSp(4|8)/(SO(1,3) \times SO(7))$ manifold the supermembrane action can be obtained within the supercoset approach (de Wit et.al., 1998).
The $osp(4|8)$ isometry superalgebra of $AdS_4 \times S^7$ superspace can be realized as $D = 3$ $\mathcal{N} = 8$ sconf algebra similarly to $osp(4|6)$ case.
Suitable choice of $OSp(4|8)/(SO(1,3) \times SO(7))$ representative to derive explicit expression for the supermembrane action is

$$\begin{aligned} \mathscr{G}_{OSp(4|8)/(SO(1,3)\times SO(7))} = & \ \mathscr{G}_{OSp(4|6)/(SO(1,3)\times U(3))} \\ \times & \ e^{yH} e^{\theta^{\mu}_{4} Q^{4}_{\mu} + \bar\theta^{\mu 4} \bar Q_{\mu 4}} e^{\eta_{\mu 4} S^{\mu 4} + \bar\eta^{4}_{\mu} \bar S^{\mu}_{4}}, \end{aligned}$$

where $y \in [0, 2\pi)$ parametrizes S^1.
Resulting complete $AdS_4 \times CP^3$ superstring action is highly non-linear and hence hard to deal with.

As the consistency check it can be verified to match known quadratic in the fermions Lagrangian for superstrings on arbitrary superbackground (Cvetic et.al., 1999)

$$\mathcal{L}_{quad} = -\tfrac{1}{2}\gamma^{ij}e_i^{\hat{m}'}e_{j\hat{m}'} - \tfrac{i}{2}e_i^{\hat{m}'}\Theta^{\hat{\alpha}}(\gamma^{ij}\delta_{\hat{\alpha}}^{\hat{\beta}} - \varepsilon^{ij}\mathfrak{g}^{11}{}_{\hat{\alpha}}{}^{\hat{\beta}})\mathfrak{g}_{\hat{m}'\hat{\beta}}{}^{\hat{\gamma}}\mathscr{D}_j\Theta_{\hat{\gamma}}$$
$$- \tfrac{i}{16}e_i^{\hat{m}'}e_j^{\hat{n}'}\Theta^{\hat{\alpha}}(\gamma^{ij}\delta_{\hat{\alpha}}^{\hat{\beta}} - \varepsilon^{ij}\mathfrak{g}^{11}{}_{\hat{\alpha}}{}^{\hat{\beta}})[\underbrace{(\mathfrak{g}^{11}\mathfrak{g}_{\hat{m}'}J\mathfrak{g}_{\hat{n}'})_{\hat{\beta}}{}^{\hat{\gamma}}}_{F_2\,\text{contrib.}} + \underbrace{6(\mathfrak{g}_{\hat{m}'}\Gamma^5\mathfrak{g}_{\hat{n}'})_{\hat{\beta}}{}^{\hat{\gamma}}}_{F_4\,\text{contrib.}}]\Theta_{\hat{\gamma}},$$

where $e^{\hat{m}'} = (e^{m'}, e^I)$ is the $AdS_4 \times CP^3$ bosonic vielbein;
32-component Grasmann coordinates $\Theta^{\hat{\alpha}}$ have the following realization in terms of those associated with the supergenerators of $D = 3$ $\mathcal{N} = 8$ sconf algebra

$$\Theta^{\hat{\alpha}} = (\bar{\Theta}^{\alpha A}; \Theta_A^{\alpha}) = \left(e^{-\varphi}\hat{\bar{\theta}}^{\mu a}, \bar{\theta}^{\mu 4}, e^{\varphi}\hat{\bar{\eta}}_{\mu}^{a}, \bar{\eta}_{\mu}^{4}; e^{-\varphi}\hat{\theta}_a^{\mu}, \theta_4^{\mu}, e^{\varphi}\hat{\eta}_{\mu a}, \eta_{\mu 4}\right);$$

$(\mathfrak{g}^{\hat{m}'}, \mathfrak{g}^{11})$ are $D = 11$ $\gamma-$matrices; $J_{\hat{\alpha}}{}^{\hat{\beta}} = \mathfrak{J}^{IJ}\mathfrak{g}_{\hat{\alpha}}^{IJ\hat{\beta}}$, $\mathfrak{J}^{IJ}$ is the CP^3 Kähler 2-form; $\Gamma^5{}_{\alpha}{}^{\beta}$ is the product of $D = 1 + 3$ $\gamma-$matrices, and $\mathscr{D}\Theta_{\hat{\alpha}}$ is bosonic limit of the covariant derivative

$$\mathscr{D}\Theta_{\hat{\alpha}} = d\Theta_{\hat{\alpha}} + \frac{1}{2}\omega^{m'n'}\mathfrak{g}_{m'n'\hat{\alpha}}{}^{\hat{\beta}}\Theta_{\hat{\beta}} + \frac{1}{2}\Omega^{IJ}\mathfrak{g}^{IJ}{}_{\hat{\alpha}}{}^{\hat{\beta}}\Theta_{\hat{\beta}} + \frac{1}{4}\Omega^{KL}\mathfrak{J}^{KL}\mathfrak{J}^{IJ}\mathfrak{g}_{\hat{\alpha}}^{IJ\hat{\beta}}\Theta_{\hat{\beta}}.$$

Certain simplification of the superstring action is achieved by (partially) fixing the $\kappa-$symmetry gauge. The following gauge conditions have been considered:

- $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset sigma-model: 8 $\kappa-$symmetries fixed (Arutyunov and Frolov; Stefanski, 2008...);

- set =0 coordinates related to either to Poincare susy or conf. susy generators (Grassi, Sorokin and Wulff, 2009);

- AdS light-cone gauge, i.e. both light-like directions lie within Minkowski boundary of AdS_4 (D.U., 2009)

The AdS light-cone gauge is characterized by the conditions

$$\theta^+_A = \bar{\theta}^{+A} = \eta^+_A = \bar{\eta}^{+A} = 0, \quad A = 1, ..., 4 \in \mathbf{4}_{SU(4)}, \bar{\mathbf{4}}_{SU(4)}$$

similarly to $AdS_5 \times S^5$ superstring case (Metsaev and Tseytiln, 2000).
Remaining 16 Grassmann coordinates

$$\theta^-_A \equiv \theta_A, \ \bar{\theta}^{-A} \equiv \bar{\theta}^A, \ \eta^-_A \equiv \eta_A, \ \bar{\eta}^{-A} \equiv \bar{\eta}^A$$

become physical fermions.
Gauge fixed action equals

$$S_{l.c.} = \int d^2\xi (\mathcal{L}_{kin} + \mathcal{L}_{WZ})$$

with

$$\mathcal{L}_{kin} = -\frac{1}{2} \int d^2\xi \gamma^{ij} (g^{AdS}_{ij} + g^{CP}_{ij}),$$

where g^{AdS}_{ij} and g^{CP}_{ij} are the AdS_4 and CP^3 contributions to the induced w/sheet metric.

AdS_4 part of the induced world-sheet metric is given by

$$
\begin{aligned}
g_{ij}^{AdS} = & \ \tfrac{1}{2}(e_i^{+}e_j^{-} + e_j^{+}e_i^{-}) + e_i^{1}e_j^{1} + \partial_i\varphi\partial_j\varphi \\
+ & \ \tfrac{1}{2}e_i^{+}(\varpi_j + 4W\Omega_{ja}{}^{a}) + \tfrac{1}{2}e_j^{+}(\varpi_i + 4W\Omega_{ia}{}^{a}) \\
- & \ 16e_i^{+}e_j^{+}\theta_4\bar{\theta}^4\eta_4\bar{\eta}^4,
\end{aligned}
$$

where $e^m(d) = (e^+, e^-, e^1) = \frac{1}{2}e^{-2\varphi}dx^m$ and $d\varphi$ are AdS_4 bosonic veilbein components, $W = \theta_4\bar{\theta}^4 + \eta_4\bar{\eta}^4$,

$$
\begin{aligned}
\varpi(d) = & \ ie^{-2\varphi}(d\theta_a\bar{\theta}^a - \theta_a d\bar{\theta}^a) + i(d\theta_4\bar{\theta}^4 - \theta_4 d\bar{\theta}^4) \\
+ & \ ie^{2\varphi}(d\eta_a\bar{\eta}^a - \eta_a d\bar{\eta}^a) + i(d\eta_4\bar{\eta}^4 - \eta_4 d\bar{\eta}^4)
\end{aligned}
$$

and

$$
\Omega_a{}^a = i(dz^a\bar{z}_a - z^a d\bar{z}_a)\frac{\cos|z|(1-\cos|z|)}{|z|^2}
$$

is the bosonic part of RR 1-form potential.

CP^3 part of the induced metric equals

$$
\begin{aligned}
g_{ij}^{CP} = & \ \tfrac{1}{2}(\Omega_{ia}\Omega_j^a + \Omega_{ja}\Omega_i^a) + \{e_i^+[e^{2\varphi}(\varepsilon_{abc}\Omega_j{}^a\bar{\hat{\eta}}^b\bar{\hat{\eta}}^c - \varepsilon^{abc}\Omega_{ja}\hat{\eta}_b\hat{\eta}_c) \\
+ & \ 2e^{\varphi}(\Omega_{ja}\bar{\hat{\eta}}^a\bar{\eta}^4 - \Omega_j^a\hat{\eta}_a\eta_4)] + (i \leftrightarrow j)\} + 8e_i^+ e_j^+[e^{4\varphi}(\hat{\eta}_a\bar{\hat{\eta}}^a)^2 \\
+ & \ e^{3\varphi}(\varepsilon_{abc}\bar{\hat{\eta}}^a\bar{\hat{\eta}}^b\bar{\hat{\eta}}^c\bar{\eta} + \varepsilon^{abc}\hat{\eta}_a\hat{\eta}_b\hat{\eta}_c\eta) + 2e^{2\varphi}\hat{\eta}_a\bar{\hat{\eta}}^a\eta_4\bar{\eta}^4],
\end{aligned}
$$

where CP^3 bosonic dreibein components equal

$$
\begin{aligned}
\Omega_a = & \ d\bar{z}_a\frac{\sin|z|}{|z|} + \bar{z}_a\frac{\sin|z|(1-\cos|z|)}{2|z|^3}(dz^c\bar{z}_c - z^c d\bar{z}_c) \\
+ & \ \bar{z}_a\left(\frac{1}{|z|} - \frac{\sin|z|}{|z|^2}\right)d|z|, \\
\Omega^a = & \ dz^a\frac{\sin|z|}{|z|} + z^a\frac{\sin|z|(1-\cos|z|)}{2|z|^3}(z^c d\bar{z}_c - dz^c\bar{z}_c) \\
+ & \ z^a\left(\frac{1}{|z|} - \frac{\sin|z|}{|z|^2}\right)d|z|.
\end{aligned}
$$

WZ Lagrangian

$$\mathcal{L}_{WZ} = B_{(2)l.c.}$$

is determined by the NS-NS 2-form potential in the AdS light-cone gauge

$$\begin{aligned} B_{(2)l.c} = & \ 2(\theta_4\bar{\theta}^4 + \eta_4\bar{\eta}^4)e^1 \wedge e^+ \\ + & \ \frac{1}{2}(d\theta_4\bar{\eta}^4 - d\eta_4\bar{\theta}^4 + \eta_4 d\bar{\theta}^4 - \theta_4 d\bar{\eta}^4) \wedge e^+ \\ + & \ 2i(\theta_4\bar{\eta}^4 - \eta_4\bar{\theta}^4)e^+ \wedge \Omega_a{}^a + 2ie^{\varphi}\hat{\eta}_a\theta_4 e^+ \wedge \Omega^a \\ + & \ 2ie^{\varphi}\bar{\hat{\eta}}^a\bar{\theta}^4 e^+ \wedge \Omega_a + 4e^{2\varphi}\hat{\eta}_a\bar{\hat{\eta}}^a e^1 \wedge e^+ \\ + & \ (\hat{\eta}_a\hat{d}\bar{\theta}^a + \hat{d}\theta_a\bar{\hat{\eta}}^a) \wedge e^+. \end{aligned}$$

Summary

Superstring Lagrangian on the (10|24)−dimensional subspace of $AdS_4 \times CP^3$ superspace described by the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset sigma-model (Arutyunov and Frolov; Stefanski, 2008) has been presented in the conformal basis for $OSp(4|6)/(SO(1,3) \times U(3))$ Cartan forms using the isomorphism between $osp(4|6)$ and $D = 3$ $\mathcal{N} = 6$ sconf algebras.

The $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model action invariance under $D = 3$ $\mathcal{N} = 6$ sconf symmetry was verified.

Explicit form of the sigma-model Lagrangian has been found for the $OSp(4|6)/(SO(1,3) \times U(3))$ representative parametrized by the coordinates related to $D = 3$ $\mathcal{N} = 6$ sconf generators and their complete transformation rules under $D = 3$ $\mathcal{N} = 6$ sconf symmetry have been derived.

Full $AdS_4 \times CP^3$ superstring action (Gomis, Sorokin and Wulff, 2008) can be obtained by double-dimensional reduction (Duff et.al., 1987; Howe and Sezgin, 2004) of the $D = 11$ supermembrane on $AdS_4 \times S^7$ superbackground (de Witt et.al., 1998). In addition to (10|24) supercoordinates parametrizing $OSp(4|6)/(SO(1,3) \times U(3))$ manifold it depends also on other 8 Grassmann coordinates that can be attributed to the generators of Poincare and conf susys broken by $AdS_4 \times CP^3$ background following the realization of $osp(4|8)$ isometry superalgebra of $AdS_4 \times S^7$ superspace as $D = 3$ $\mathcal{N} = 8$ sconf algerba.

As a consistency chesk we have shown that the explicit form of the $AdS_4 \times CP^3$ superstring Lagrangian in the conformal basis found to quadratic order in the Grassmann coordinates matches the expression obtained from the known quadratic Lagrangian for a general super-background (Cvetic et.al., 1999).

The AdS light-cone gauge Lagrangian includes also contributions quartic in the fermions.

Non-linearity of the $AdS_4 \times CP^3$ superstring action even after fixing the gauge symmetries precludes from addressing directly the problem of spectrum identification unlike the case of superstring on flat background. The quantization can be performed in linearizing limits around particular string configurations solving classical e.o.m., in particular taking the Penrose limit (Berenstein, Maldacena and Nastace; Metsaev and Tseytlin).

We have shown that in the Penrose limit taken around null geodesic on Minkowski boundary of AdS_4 space the $AdS_4 \times CP^3$ superstring Lagrangian reduces to the quadratic one corresponding to the IIA superstring on flat background.

Promising approach to the spectrum identification is based on the classical integrability of superstring e.o.m. on $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset space (Arutyunov and Frolov; Stefanski, 2008) that may survive when extended to full $AdS_4 \times CP^3$ superspace (Sorokin and Wulff, 2010).