$D = 4$ Black Holes From Geodesics

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Outline

Seed Solutions

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Seed solution: Simplest solution with all duality invariant properties of the most general one

Static, Asymtotically Flat Black Holes in D=4 SUGRAS

Bosonic field content

- n_S scalar fields ϕ^r $\quad (r = 1, \ldots, n_S)$
- n_V vector fields A^Λ_μ $\quad (\Lambda = 0, \ldots, n_V - 1)$
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The ansatz

$$ds^2 = -e^{2\,U}\,dt^2 + e^{-2\,U}\left[\frac{c^4}{\sinh^4(c\,\tau)}\,d\tau^2 + \frac{c^2}{\sinh^2(c\,\tau)}\left(d\theta^2 + \sin(\theta)\,d\varphi^2\right)\right]$$

- $\phi^r = \phi^r(\tau)$, $U = U(\tau)$, $\quad \frac{d\tau}{dr} = \frac{\sinh^2(c\,\tau)}{c^2} = \frac{1}{(r-r_0)^2 - c^2}$;
- c extremality parameter, two horizons: $\quad r_\pm = r_0 \pm c$
- electric and magnetic charges e_Λ, m^Λ: $\quad \Gamma^M \equiv (m^\Lambda, e_\Lambda)$
- Extreme solutions $c = 0$: $\quad \lim_{\tau \to -\infty} e^{-2U} = \frac{A_H}{4\,\pi}\,\tau^2$

Seed solution in maximal SUGRA

- 70 scalar fields $\phi^r \in \mathscr{M}_{scal} = \frac{G_4}{H_4} = \frac{\mathrm{E}_{7(7)}}{\mathrm{SU}(8)}$
- 28 vector fields A^Λ_μ
- Duality group is $\mathrm{E}_{7(7)}$; $\Gamma^M = (m^\Lambda, e_\Lambda) \in$ **56** *symplectic* representation
- Parameters of a black-hole encoded in central charges computed at infinity:

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$$Z_{AB} \xrightarrow{\mathrm{SU}(8)} \begin{pmatrix} Z_1\,\epsilon & 0 & 0 & 0 \\ 0 & Z_2\,\epsilon & 0 & 0 \\ 0 & 0 & Z_3\,\epsilon & 0 \\ 0 & 0 & 0 & Z_4\,\epsilon \end{pmatrix}$$

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- (Z_k) can be identified with the charges $(Z, \overline{Z}_s, \overline{Z}_t, \overline{Z}_u)$ of a $\mathcal{N}=2$ STU truncation
- Five $\mathrm{SU}(8)$ invariants: $\rho_k = |Z_k|$, $\theta = \mathrm{Arg}(Z_1\,Z_2\,Z_3\,Z_4)$
- Seed solution, also solution to the STU truncation, has 5 parameters

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- Spherical symmetry: $\phi^I = \phi^I(\tau)$, solution is a geodesic on $\mathscr{M}^{(3)}_{scal}$
- Invariant measure along the geodesic coincides with the extremality parameter: $G_{IJ}(\phi)\,\dot{\phi}^I\,\dot{\phi}^J = 2\,c^2$

Mathematical description of the geodesic

Definitions...

- Let $\mathfrak{g}$, $\mathfrak{H}$ be the Lie algebras of G and H. Involution $\sigma(\mathfrak{H}) = -\eta\mathfrak{H}^T\eta = \mathfrak{H}$ induces the (pseudo-) Cartan decomposition:

$$\mathfrak{g} = \mathfrak{H} \bigoplus \mathfrak{K}$$

with $\sigma(\mathfrak{K}) = -\mathfrak{K}$

- Given coset representative $\mathcal{V}(\phi^I) \in e^{Solv}$ and geodesic $\phi^I(\tau)$, define $\mathcal{V}(\tau) \equiv \mathcal{V}(\phi^I(\tau))$:

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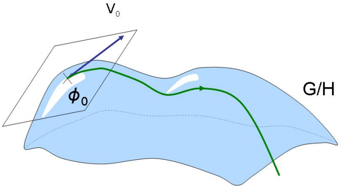

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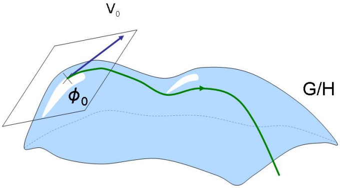

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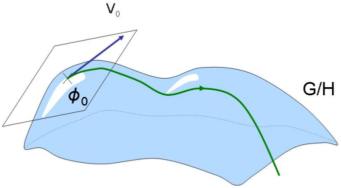

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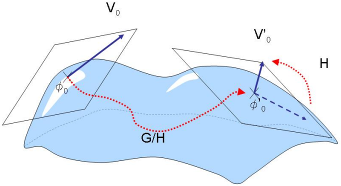

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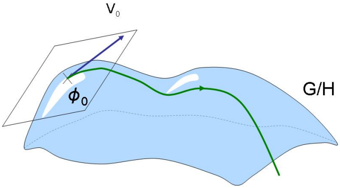

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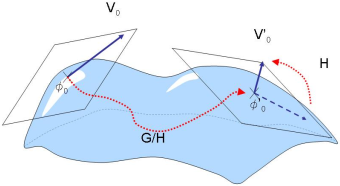

Some algebra....

- G is larger than G_4: $G_4 \times \mathrm{SL}(2,\mathbb{R})_E \subset G$. Its algebra decomposes as follows:

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- $\mathcal{N} = 8$ example:

$$G_4 = \mathrm{E}_{7(7)} \;, \;\; H_4 = \mathrm{SU}(8) \;, \;\; G = \mathrm{E}_{8(8)} \;, \;\; H = \mathrm{SO}^*(16)$$

$\mathbf{R} = \mathbf{56}$ of $\mathrm{E}_{7(7)}$ and $\hat{\mathbf{R}} = \mathbf{28}_{-1} + \overline{\mathbf{28}}_{+1}$ of $\mathrm{U}(1)_E \times H_4 = \mathrm{U}(8)$

Seed Geodesic in Universal Submanifold

- Any element of $\mathfrak{K}^{(\hat{R})}$ or $\mathfrak{H}^{(\hat{R})}$ can be rotated by $\mathrm{U}(1)_E \times H_4$ into minimal *abelian* subalgebras $\mathfrak{K}^{(N)} = \mathrm{Span}(k_k)$ and $\mathfrak{H}^{(N)} = \mathrm{Span}(J_k)$, where $k = 0, \ldots p-1$ and

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- Seed geodesic is a p-charge solution within (arXiv:0806.2310)

$$\mathcal{M}_N = \left(\frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(1,1)}\right)^p \times \mathrm{SO}(1,1)^{r-p} \subset \frac{G}{H}$$

Seed Geodesic in Universal Submanifold

- Any element of $\mathfrak{K}^{(\hat{R})}$ or $\mathfrak{H}^{(\hat{R})}$ can be rotated by $\mathrm{U}(1)_E \times H_4$ into minimal *abelian* subalgebras $\mathfrak{K}^{(N)} = \mathrm{Span}(k_k)$ and $\mathfrak{H}^{(N)} = \mathrm{Span}(J_k)$, where $k = 0, \ldots p-1$ and

$$p = \mathrm{rank}\left(\frac{H}{\mathrm{U}(1)_E \times H_4}\right)$$

  Together with $H_k = [J_k, k_k]$ they generate $\mathrm{SL}(2,\mathbb{R})^p \subset G$

- $\mathcal{N} = 8$ example: $p = \mathrm{rank}\left(\frac{\mathrm{SO}^*(16)}{\mathrm{U}(8)}\right) = 4$

$$V_0 \bigcap \mathfrak{K}^{(\hat{R})} = Z_{AB}\, k^{AB} - \overline{Z}^{AB}\, k_{AB} \quad \xrightarrow{\mathrm{U}(8)} \quad \sum_{k=1}^{4} \rho_k k_k \quad \text{(normal form of } \mathbf{28}_{+1}\text{)}$$

  Notice θ rotated away! Seed geodesic is a 4-parameter solution

- Seed geodesic is a p-charge solution within (arXiv:0806.2310)

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- True for all V_0-diagonalizable cases. V_0 non-diagonalizable (e.g. extremal solutions) only geodesics originating from regular solutions ($A_H > 0$).

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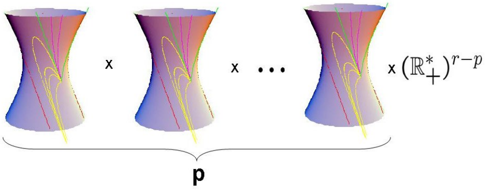

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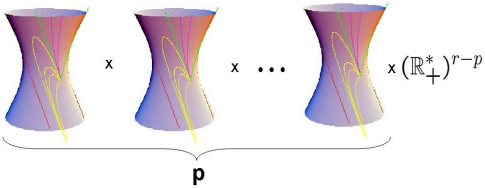

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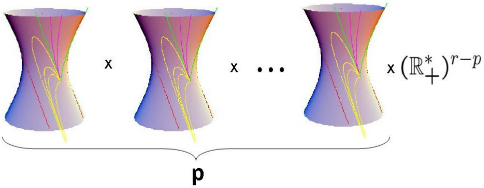

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- Construction of the seed geodesic within a universal submanifold common to a broad class of models. E.g. $p = 4$: $N = 8$, $N = 2$ with rank-3 symmetric SK manifolds (STU) etc.

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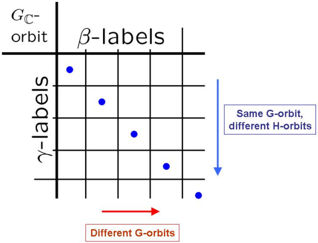

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- $D = 4$, $N = 2$ SUGRA coupled to 1 vector multiplet
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- Explicit matrix representation: $J = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $k = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\mathcal{H} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

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- Solution in terms of $U, t, \mathcal{Z}^M$:

$$e^{-2U} = \sqrt{\mathbf{H}_0 (\mathbf{H}_1)^3}\,;\;\; t = -i\sqrt{\frac{\mathbf{H}_0}{\mathbf{H}_1}}\;,\quad \mathcal{Z}^0 = \frac{\varepsilon_0 a_0\,\tau}{\mathbf{H}_0}\,;\;\; \mathcal{Z}_1 = \sqrt{3}\,\frac{\varepsilon_1 a_1\,\tau}{\mathbf{H}_1}$$

$\mathbf{H}_k = 1 - \sqrt{2}\, a_k\,\tau$. Charges are: $e_0 = \varepsilon_0 a_0$, $m^1 = -\varepsilon_1 a_1$

- Regular solution: $a_k > 0$ $\Rightarrow$ β-label $= \gamma$-label.

- At the horizon $\tau \to -\infty$:

$$\varphi_i \to \varphi_i^{fix} \text{ (stable attractor)}\,, \quad e^{-2U} \to \frac{A_H}{4\,\pi}\,\tau^2$$

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$$\frac{A_H}{4\,\pi} = \sqrt{4\,a_0\,(a_1)^3} = \sqrt{4\,\varepsilon\,e_0\,(m^1)^3} = \sqrt{\varepsilon I_4(e,m)}$$

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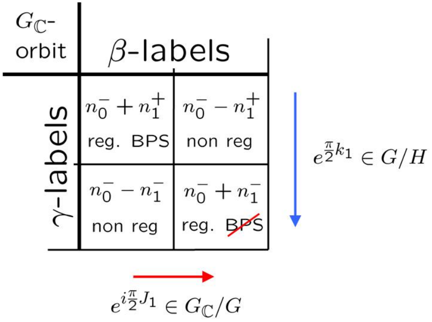

Other Orbits and Tensor Classifiers

- Other two orbits with $I_4 = 0$, X step-2 nilpotent:
  - **a** $\mathcal{O}_1$: $m^1 \to 0$ (*doubly-critical*) *small-bh*
  - **b** $\mathcal{O}_2$: $e_0 \to 0$ (*lightlike*) *small-bh*
- Last orbit: X step-7 nilpotent. No regular single-center, regular *multicenter* solution [Bossard, Ruef, 1106.5806]
- Describe orbits through H-invariants from Lax matrix. Define symmetric H-covariant tensors whose signature is H-invariant [Fre', Sorin, M.T., 1103.0848]

Tensor Classifiers

$V_0 = \Delta^{\alpha A} K_{\alpha A}, \qquad \Delta^{\alpha A} \in (\mathbf{2}, \mathbf{4})$ of $H = \mathrm{SL}(2,\mathbb{R})_1 \times \mathrm{SL}(2,\mathbb{R})_2$

$$\mathcal{T}^{xy} = \epsilon_{\alpha\beta} \Delta^{\alpha A} \Delta^{\beta B} \Pi^{xy}_{AB} \in (\mathbf{1},\mathbf{3}) \times_s (\mathbf{1},\mathbf{3})$$
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where $\mathfrak{sl}(2,\mathbb{R})_1 = \mathrm{Span}\{s_a\}$, $\mathfrak{sl}(2,\mathbb{R})_2 = \mathrm{Span}\{t_x\}$. BPS solution $\Leftrightarrow \mathcal{T}^{xy} \equiv 0$

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