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Derivation of the exact NSVZ beta-function in N=1 SQED, regularized by higher derivatives, by direct summation of Feynman diagrams

NSVZ β-function

The β-function in supersymmetric theories is related with the anomalous dimensions of the matter superfields via the relation

$$\beta(\alpha) = -\frac{\alpha^2\Big[3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha)/r\Big)\Big]}{2\pi(1 - C_2\alpha/2\pi)}.$$

V.Novikov, M.A.Shifman, A.Vainshtein, V.I.Zakharov, Nucl.Phys. B 229, (1983), 381; Phys.Lett. 166B, (1985), 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. B 277, (1986), 456; M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, JETP Lett. 42, (1985), 224; Phys.Lett. 166B, (1986), 334.

This NSVZ β-function was obtained from different arguments: instantons, anomalies etc. With the dimensional reduction in the $\overline{MS}$-scheme it agrees with the explicit calculations

S.Ferrara, B.Zumino, Nucl.Phys. **B79** (1974) 413; D.R.T.Jones, Nucl.Phys. **B87** (1975) 127; L.V.Avdeev, O.V.Tarasov, Phys.Lett. **112 B** (1982) 356; I.Jack, D.R.T.Jones, C.G.North, Phys.Lett **B386** (1996) 138; Nucl.Phys. **B 486** (1997) 479; R.V.Harlander, D.R.T.Jones, P.Kant, L.Mihaila, M.Steinhauser, JHEP **0612** (2006) 024.

in only the two-loop approximation. In the higher loops it is necessary to perform a special redefinition of the coupling constant.

Higher covariant derivative regularization and factorization of integrands into total derivatives

NSVZ β-function relates the β-function in n-th loop with the β-function and the anomalous dimensions in the previous loops. It is convenient to investigate this relation using the higher covariant derivative regularization.

A.A.Slavnov, Nucl.Phys., **B31**, (1971), 301; Theor.Math.Phys. **13**, (1972), 1064.
V.K.Krivoshchekov, Theor.Math.Phys. **36**, (1978), 745; P.West, Nucl.Phys. B268, (1986), 113.

Then the loop integrals are integrals of total derivatives

A.Soloshenko, K.S., hep-th/0304083.

and even integrals of double total derivatives

A.V.Smilga, A.I.Vainshtein, Nucl.Phys. **B 704**, (2005), 445.

This allows to calculate one of the loop integrals analytically and reduce a n-loop integral to $(n-1)$-loop integrals.

Let us prove this for $N=1$ SQED exactly in all loops and derive the exact NSVZ β-fucntion by the direct summation of Fenman diagrams.

$N=1$ supersymmetric electrodynamics (SQED), regularized by higher derivatives

The N=1 SQED in the massless case is described by the action

$$S = \frac{1}{4e^2}\text{Re}\int d^4x\, d^2\theta\, W_a C^{ab} W_b + \frac{1}{4}\int d^4x\, d^4\theta\,\Big(\phi^* e^{2V}\phi + \widetilde{\phi}^* e^{-2V}\widetilde{\phi}\Big),$$

where ϕ_i and $\widetilde{\phi}$ are chiral matter superfields, V is a real gauge superfield, and

$$W_a = \frac{1}{4}\bar{D}^2 D_a V.$$

We add the term with higher derivatives

$$S_{reg} = \frac{1}{4e^2}\text{Re}\int d^4x\, d^2\theta\, W_a C^{ab} R(\partial^2/\Lambda^2) W_b + \frac{1}{4}\int d^4x\, d^4\theta\,\Big(\phi^* e^{2V}\phi + \widetilde{\phi}^* e^{-2V}\widetilde{\phi}\Big)$$

where $R(\partial^2/\Lambda^2)$ is a regulator, e.g. $R = 1 + \partial^{2n}/\Lambda^{2n}$.

The higher derivative regularization and quantization

The gauge is fixed by adding:

$$S_{gf} = -\frac{1}{64e^2}\int d^4x\, d^4\theta\, \Big(V R D^2\bar{D}^2 V + V R \bar{D}^2 D^2 V\Big).$$

After adding the term with the higher derivatives divergences remain only in the one-loop approximation. In order to remove them we insert in the generating functional the Pauli–Villars determinants.

L.D.Faddeev, A.A.Slavnov, Gauge fields, introduction to quantum theory, Benjamin, Reading, 1990.

$$Z[J,\mathbf{\Omega}] = \int D\mu \prod_I \Big(\det PV(V, M_I)\Big)^{c_I} \exp\Big\{iS_{reg} + \mbox{Sources}\Big\},$$

$\sum\limits_I c_I = 1$; $\sum\limits_I c_I M_I^2 = 0$; $M_I = a_I\Lambda$. (Λ is the only dimensionful parameter.)

$$\det PV(V,M) = \Big(\int D\Phi^* D\Phi e^{iS_{PV}}\Big)^{-1},$$

$$S_{PV} = \frac{1}{4}\int d^4x\, d^4\theta\, \Big(\Phi^* e^{2V}\Phi + \widetilde{\Phi}^* e^{-2V}\widetilde{\Phi}\Big) + \Big(\frac{1}{2}\int d^4x\, d^4\theta\, M\Phi\widetilde{\Phi} + ..\Big).$$

Calculation of the β-function

The notation is

$$\Gamma^{(2)} = \int \frac{d^4p}{(2\pi)^4}\, d^4\theta\,\Big(-\frac{1}{16\pi}\mathbf{V}(-p)\,\partial^2\Pi_{1/2}\mathbf{V}(p)\, d^{-1}(\alpha,\mu/p) +$$

$$+\frac{1}{4}(\phi^*)^i(-p,\theta)\,\phi_j(p,\theta)\,(ZG)_i{}^j(\alpha,\mu/p)\Big).$$

We calculate

$$\frac{d}{d\ln\Lambda}\Big(d^{-1}(\alpha_0,\Lambda/p) - \alpha_0^{-1}\Big)\Big|_{p=0} = -\frac{d}{d\ln\Lambda}\alpha_0^{-1}(\alpha,\mu/\Lambda) = \frac{\beta(\alpha_0)}{\alpha_0^2}$$

The main result: (It was obtained as the equality of some well defined integrals due to the factorization of integrands into total derivatives)

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{1}{\pi}\Big(1 - \frac{d}{d\ln\Lambda}\ln G(\alpha_0,\Lambda/q)\Big|_{q=0}\Big) = \frac{1}{\pi} + \frac{1}{\pi}\frac{d}{d\ln\Lambda}\Big(\ln ZG(\alpha,\mu/q)$$

$$-\ln Z(\alpha,\Lambda/\mu)\Big)\Big|_{q=0} = \frac{1}{\pi}\Big(1-\gamma\Big(\alpha_0(\alpha,\Lambda/\mu)\Big)\Big).$$

(Without any redefinition of the coupling constant.)

Three-loop calculation for SQED

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = 2\pi \frac{d}{d\ln\Lambda}\Bigg\{ \sum_I c_I \int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \frac{\ln(q^2+M^2)}{q^2} + 4\pi \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{e^2}{k^2 R_k^2}$$

$$\times \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \Bigg(\frac{1}{q^2(k+q)^2} - \sum_I c_I \frac{1}{(q^2+M_I^2)((k+q)^2+M_I^2)} \Bigg) \Bigg[R_k \Bigg(1 + \frac{e^2}{4\pi^2} \ln\frac{\Lambda}{\mu} \Bigg)$$

$$-2e^2 \Bigg(\int \frac{d^4t}{(2\pi)^4} \frac{1}{t^2(k+t)^2} - \sum_J c_J \int \frac{d^4t}{(2\pi)^4} \frac{1}{(t^2+M_J^2)((k+t)^2+M_J^2)} \Bigg) \Bigg]$$

$$+4\pi \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} \frac{e^4}{k^2 R_k l^2 R_l} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \Bigg\{ \Bigg(- \frac{2k^2}{q^2(q+k)^2(q+l)^2(q+k+l)^2}$$

$$+ \frac{2}{q^2(q+k)^2(q+l)^2} \Bigg) - \sum_I c_I \Bigg(- \frac{2(k^2+M_I^2)}{(q^2+M_I^2)((q+k)^2+M_I^2)((q+l)^2+M_I^2)}$$

$$\times \frac{1}{((q+k+l)^2+M_I^2)} + \frac{2}{(q^2+M_I^2)((q+k)^2+M_I^2)((q+l)^2+M_I^2)} - \frac{1}{(q^2+M_I^2)^2}$$

$$\times \frac{4M_I^2}{((q+k)^2+M_I^2)((q+l)^2+M_I^2)} \Bigg) \Bigg\}$$

Some useful tricks

Two main purposes:

1. How the factorization of the integrands into total derivatives can be proven exactly in all loops?

2. How one can obtain NSVZ β-function exactly to all loops?

In order to simplify the calculations (in the limit $p \to 0$) and find the β-function it is possible to substitute

$$\mathbf{V} \to \bar\theta^a \bar\theta_a \theta^b \theta_b$$

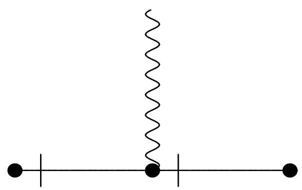
An integral of a total derivative in the coordinate representation is given by

$$\mbox{Tr}\Big([x^\mu, \mbox{Something}]\Big) = 0.$$

We will try to reduce the sum of diagrams to such commutators.

Summation of subdiagrams

In order to extract integrals of total derivatives we consider the following sum of subdiagrams:

 $+$ 

$$= -\theta^a\theta_a\bar\theta^b\frac{\bar D_b D^2}{4\partial^2} + \theta^a\theta_a\frac{D^2}{4\partial^2}$$

$$+ i\bar\theta^b(\gamma^\mu)_b{}^a\theta_a\frac{\bar D^2 D^2\partial_\mu}{\partial^4} - i\theta^a(\gamma^\mu)_a{}^b\frac{\bar D_b D^2\partial_\mu}{4\partial^4} + \frac{\bar D^2 D^2}{16\partial^4}$$

Only the terms written by the blue color give nontrivial contributions to the two-point function of the gauge superfield.

Really, finally it is necessary to obtain

$$\int d^4\theta\,\theta^a\theta_a\bar\theta^b\bar\theta_b,$$

and calculating the θ-part of the graph can not produce powers of θ or $\bar\theta$.

Effective Feynman rules

Let us formally perform Gaussian integration over the matter superfields:

$$Z = \int DV \prod_I \Big(\det PV(V, M_I)\Big)^{c_I}$$

$$\times \exp\left\{ i \int d^8x \,\Big(\frac{1}{4e^2} V\partial^2 R(\partial^2/\Lambda^2) V - j\frac{D^2}{4\partial^2} * \frac{\bar{D}^2}{4\partial^2} j^* - \widetilde{j}\frac{D^2}{4\partial^2}\widetilde{*}\frac{\bar{D}^2}{4\partial^2}\widetilde{j}^* \Big) \right\},$$

where

$$* \equiv \frac{1}{1-(e^{2V}-1)\bar{D}^2 D^2/16\partial^2}, \qquad \widetilde{*} = \frac{1}{1-(e^{-2V}-1)\bar{D}^2 D^2/16\partial^2}$$

encode chains of propagators and vertexes.

$$\Delta\Gamma^{(2)}_{\mathbf{V}} = \Big\langle -2i\Big(\mathrm{Tr}(\mathbf{V}J_0 *)\Big)^2 - 2i\mathrm{Tr}(\mathbf{V}J_0 * \mathbf{V}J_0 *) - 2i\mathrm{Tr}(\mathbf{V}^2 J_0 *)\Big\rangle$$

$+$terms with $\widetilde{*} + (PV)$.

where $J_0 = e^{2V}\dfrac{\bar{D}^2 D^2}{16\partial^2}$ is the effective vertex.

External lines are attached to different matter loops

A sum of diagrams in that the external lines are attached to different matter loops is given by

$$-2i\frac{d}{d\ln\Lambda}\Big\langle\Big(\mathrm{Tr}\Big(-2\theta^c\theta_c\bar\theta^d[\bar\theta_d,\ln(*)-\ln(\widetilde{*})]+i\bar\theta^c(\gamma^\nu)_c{}^d\theta_d[y^*_\nu,\ln(*)-\ln(\widetilde{*})]\Big)$$
$$+(PV)\Big)^2\Big\rangle,$$

where $y^*_\mu = x_\mu - i\bar\theta^a(\gamma_\mu)_a{}^b\theta_b$.

It is easy to see that this expression is a double total derivative and vanishes as a trace of a commutator.

External lines are attached to a single matter loop

If the external lines are attached to a single matter loop, it is also possible to extract double total derivatives using a special algebraic identity.

External lines are attached to a single matter loop

If A, B, and C are operators constructed from the supersymmetric covariant derivatives and usual derivatives which do not explicitly depend on θ and $\bar\theta$, then

$$\mbox{Tr}\Big(\theta^a\theta_a\bar\theta^b\bar\theta_b\Big((\gamma_\mu)^{ab}[y_\mu^*, A][\bar\theta_b, B\}[\theta_a, C\} + (\gamma_\mu)^{ab}(-1)^{P_A}[\theta_a, B\}[\bar\theta_b, C\}$$

$$\times[y_\mu^*, A] - 4i[\theta^a, [\theta_a, A\}\}[\bar\theta^b, B\}[\bar\theta_b, C\}\Big)\Big) + \mbox{cyclic perm. of } A,\ B,\ C$$

$$= \frac{1}{3}\mbox{Tr}\Big(\theta^a\theta_a\bar\theta^b\bar\theta_b(\gamma_\mu)^{ab}\Big[y_\mu^*, A[\bar\theta_b, B\}[\theta_a, C\} + (-1)^{P_A}[\theta_a, B\}[\bar\theta_b, C\}A\Big]\Big)$$

$$+\mbox{cyclic perm. of } A,\ B,\ C$$

The sum of diagrams in that the external lines are attached to a single matter loop is given by

$$i\frac{d}{d\ln\Lambda}\mbox{Tr}\Big\langle\theta^4\Big[y_\mu^*, \Big[(y^\mu)^*, \ln(*) + \ln(\widetilde{*})\Big]\Big]\Big\rangle + (PV) - \mbox{terms with a } \delta\mbox{-function},$$

This expression is evidently an integral of a double total derivative.

Obtaining the exact NSVZ β-function

Thus, the sum of diagrams in that the external lines are attached to a single matter loop is given by the integral of double total derivatives, but does not vanish due to δ-functions. These δ-functions come from the identity

$$[x^\mu, \frac{\partial_\mu}{\partial^4}] = [-i\frac{\partial}{\partial p_\mu}, -\frac{ip^\mu}{p^4}] = -2\pi^2\delta^4(p_E) = -2\pi^2 i\delta^4(p).$$

Qualitatively these δ-functions correspond to cutting the matter loop

A.V.Smilga, A.I.Vainshtein, Nucl.Phys. **B 704**, (2005), 445.

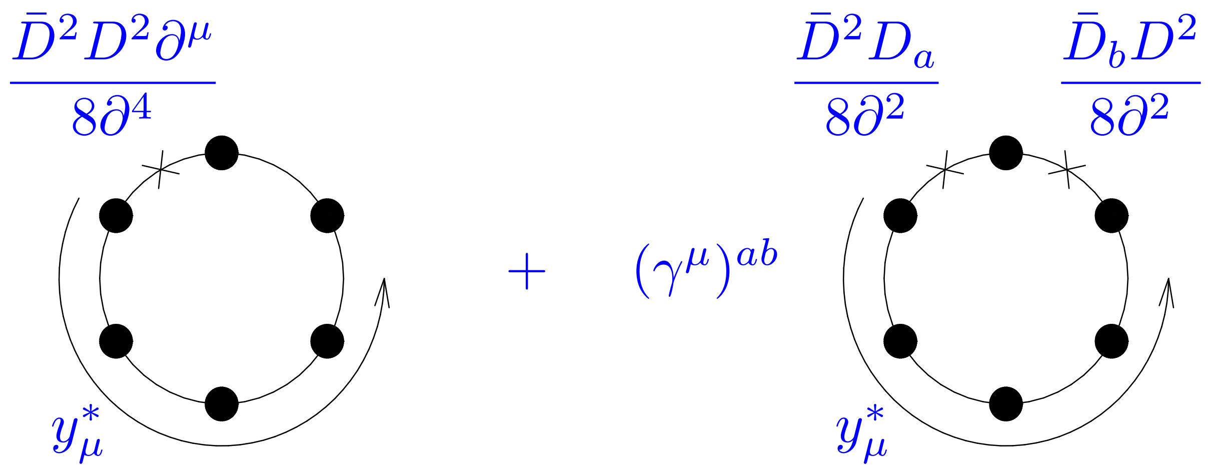
It is possible to calculate all contributions of δ-functions

K.S., ArXiv:1102.3772 [hep-th].

and compare them with the two-point Green function of the matter superfield. The result is the exact NSVZ β-function

$$\beta(\alpha) = \frac{\alpha^2}{\pi}\Big(1-\gamma(\alpha)\Big).$$

Obtaining the exact NSVZ β-function



(for $p = 6$)

$$G^{-1} = (1+\Delta G)^{-1} = \sum_{p=0}^{\infty} (-1)^p (\Delta G)^p$$

$$-\sum_{p=1}^{\infty} \frac{(-1)^p (p-1)}{p} (\Delta G)^p$$

$$1 + \sum_{p=1}^{\infty} \frac{(-1)^p}{p} (\Delta G)^p = 1 - \ln G$$

Non-Abelian $N = 1$ supersymmetric theories

N=1 supersymmetric Yang-Mills theory with matter in the massless case is described by the action

$$S = \frac{1}{2e^2}\mathrm{Re}\,\mathrm{tr}\int d^4x\, d^2\theta\, W_a C^{ab} W_b + \frac{1}{4}\int d^4x\, d^4\theta\, (\phi^*)^i (e^{2V})_i{}^j \phi_j +$$
$$+\Big(\frac{1}{6}\int d^4x\, d^2\theta\, \lambda^{ijk}\phi_i\phi_j\phi_k + \mathrm{h.c.}\Big),$$

where ϕ_i are chiral scalar matter superfields, V is a real scalar gauge superfield, and the supersymmetric gauge field stress tensor is given by

$$W_a = \frac{1}{8}\bar{D}^2\Big[e^{-2V} D_a e^{2V}\Big].$$

The action is invariant under the gauge transformations

$$e^{2V} \to e^{i\Lambda^+} e^{2V} e^{-i\Lambda}; \qquad \phi \to e^{i\Lambda}\phi$$

if $(T^A)_m{}^i \lambda^{mjk} + (T^A)_m{}^j \lambda^{imk} + (T^A)_m{}^k \lambda^{ijm} = 0.$

Higher derivative regularization

For the calculation we use the background field method.

The gauge is fixed by adding the following term:

$$S_{gf} = -\frac{1}{32e^2}\,\text{tr}\int d^4x\, d^4\theta\,\Big(V\boldsymbol{D}^2\bar{\boldsymbol{D}}^2 V + V\bar{\boldsymbol{D}}^2\boldsymbol{D}^2 V\Big).$$

To regularize the theory we add the following term with the higher covariant derivatives:

$$S_\Lambda = \frac{1}{2e^2}\text{tr}\,\text{Re}\int d^4x\, d^4\theta\, V\,\frac{(\boldsymbol{D}_\mu^2)^{n+1}}{\Lambda^{2n}}V + \frac{1}{4}\int d^4x\, d^4\theta\,(\phi^*)^i\Big[e^{\boldsymbol{\Omega}^+}\frac{(\boldsymbol{D}_\mu^2)^m}{\Lambda^{2m}}e^{\boldsymbol{\Omega}}\Big]_i{}^j\phi_j.$$

where $\boldsymbol{D}$, $\bar{\boldsymbol{D}}$, and $\boldsymbol{D}_\mu$ are background covariant derivatives.

In order to regularize the remaining one-loop divergences, it is necessary to introduce Pauli-Villars determinants into the generating functional. As earlier, we assume that $M_I = a_I\Lambda$, where a_I are constants. (Therefore, there is the only dimensionful parameter Λ.)

Two-loop β-function for $N=1$ supersymmetric Yang-Mills theory

Two-loop calculation gives the following result:

$$\beta(\alpha) = -\frac{3\alpha^2}{2\pi}C_2 + \alpha^2 T(R) I_0 + \alpha^3 C_2^2 I_1 + \frac{\alpha^3}{r} C(R)_i{}^j C(R)_j{}^i I_2 +$$
$$+\alpha^3 T(R) C_2 I_3 + \alpha^2 C(R)_i{}^j \frac{\lambda^*_{jkl}\lambda^{ikl}}{4\pi r} I_4 + \ldots,$$

where we do not write the integral for the one-loop ghost contribution and the integrals I_0–I_4 are given below, and the following notation is used:

$$\mathrm{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}; \qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$
$$f^{ACD} f^{BCD} \equiv C_2 \delta^{AB}; \qquad r \equiv \delta_{AA}.$$

Taking into account Pauli–Villars contributions,

$$I_i = I_i(0) - \sum_I I_i(M_I), \qquad i = 0, 2, 3$$

where I_i are given by

$$I_0(M) = -\pi \int \frac{d^4q}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left\{ \frac{1}{q^2} \ln \Big(q^2(1+q^{2m}/\Lambda^{2m})^2 + M^2 \Big) \right\};$$

$$I_1 = -12\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial k^\mu} \frac{\partial}{\partial k_\mu} \left\{ \frac{1}{k^2(1+k^{2n}/\Lambda^{2n})q^2(1+q^{2n}/\Lambda^{2n})} \right.$$

$$\left. \times \frac{1}{(q+k)^2(1+(q+k)^{2n}/\Lambda^{2n})} \right\};$$

$$I_2(M) = 8\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left\{ \frac{1}{k^2(1+k^{2n}/\Lambda^{2n})} \right.$$

$$\left. \times \frac{(1+q^{2m}/\Lambda^{2m})(1+(q+k)^{2m}/\Lambda^{2m})}{(q^2(1+q^{2m}/\Lambda^{2m})^2+M^2)((q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})^2+M^2)} \right\};$$

$$I_3(M) = 8\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial k_\mu} \left\{ \frac{1}{(k+q)^2(1+(q+k)^{2n}/\Lambda^{2n})} \right.$$

$$\left. \times \frac{(1+k^{2m}/\Lambda^{2m})(1+q^{2m}/\Lambda^{2m})}{(k^2(1+k^{2m}/\Lambda^{2m})^2+M^2)(q^2(1+q^{2m}/\Lambda^{2m})^2+M^2)} \right\};$$

$$I_4 = -8\pi^2 \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d}{d\ln\Lambda} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \left\{ \frac{1}{k^2(1+k^{2m}/\Lambda^{2m})q^2(1+q^{2m}/\Lambda^{2m})} \right.$$

$$\left. \times \frac{1}{(q+k)^2(1+(q+k)^{2m}/\Lambda^{2m})} \right\}.$$

Two-loop β-function for $N = 1$ supersymmetric Yang-Mills theory

The integrals can be calculated using the identity

$$\int \frac{d^4q}{(2\pi)^4} \frac{\partial}{\partial q^\mu} \frac{\partial}{\partial q_\mu} \Big(\frac{f(q^2)}{q^2}\Big) = \lim_{\varepsilon\to 0} \int\limits_{S_\varepsilon} \frac{dS_\mu}{(2\pi)^4} \frac{(-2) q^\mu f(q^2)}{q^4} = \frac{1}{4\pi^2} f(0)$$

where f is a nonsingular function, which rapidly decreases at the infinity. It is equivalent to the identity

$$\int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2} \frac{d}{dq^2} f(q^2) = \frac{1}{16\pi^2} \Big(f(\infty) - f(0)\Big) = -\frac{1}{16\pi^2} f(0).$$

(This is a total derivative in the four-dimensional spherical coordinates.)

The result for the two-loop β-function is given by

$$\beta(\alpha) = -\frac{\alpha^2}{2\pi}\Big(3C_2 - T(R)\Big) + \frac{\alpha^3}{(2\pi)^2}\Big(- 3C_2^2 + T(R) C_2 +$$
$$+\frac{2}{r} C(R)_i{}^j C(R)_j{}^i\Big) - \frac{\alpha^2 C(R)_i{}^j \lambda^*_{jkl} \lambda^{ikl}}{8\pi^3 r} + \ldots .$$

Two-loop β-function for $N = 1$ supersymmetric Yang-Mills theory

Comparing the result with the one-loop anomalous dimension

$$\gamma_i{}^j(\alpha) = -\frac{\alpha C(R)_i{}^j}{\pi} + \frac{\lambda^*_{ikl}\lambda^{jkl}}{4\pi^2} + \ldots,$$

gives the exact NSVZ β-function in the considered approximation.

$$\beta(\alpha) = -\frac{\alpha^2\Big[3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha)/r\Big)\Big]}{2\pi(1 - C_2\alpha/2\pi)}.$$

V.A.Novikov, M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, Nucl.Phys. B 229, (1983), 381; Phys.Lett. 166B, (1985), 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. B 277, (1986), 456; M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, JETP Lett. 42, (1985), 224; Phys.Lett. 166B, (1986), 334.

(The result also agrees with the DRED calculations.)

D.R.T.Jones, Nucl.Phys. **B87** (1975) 127.

Thus, factorization of integrands into double total derivatives seems to be a general feature of supersymmetric theories.

Conclusion and open questions

- ✓ It is possible to prove that all integrals defining the β-function in $N=1$ SQED, regularized by higher derivatives, are integrals of double total derivatives. This allows to calculate one of the loop integrals analytically.
- ✓ The factorization of integrands into total derivatives allows to obtain the exact NSVZ β-function without redefinition of the coupling constant.
- ✓ Possibly, the factorization of integrands into double total derivatives is a general feature of supersymmetric theories. At least, this takes place for a general renormalizable $N=1$ supersymmetric theory at the two-loop level.

Thank you for the attention!