Supergravities with positive potentials and chiral reductions

K.S. Stelle

Imperial College London

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T. Pugh, E. Sezgin & K.S.S., 1008.0726
P.S. Howe, T. Pugh, C. Strickland-Constable & K.S.S., 1104.4387
C.N. Pope, T. Pugh & K.S.S., in preparation

Outline

- Noncompact gaugings in supergravity theories
- $D = 7/D = 6$ Hořava-Witten constructions
- Boundary supermatter and hypermultiplet target space geometry
- Ectoplasm with an edge & relative cohomology
- Chiral and non-chiral reduction alternatives

Prelude: Supergravity with a positive potential – The Salam-Sezgin model

The Salam-Sezgin model (1984) is based upon $D = 6$, $(1,0)$ supergravity coupled to a multiplet containing one antisymmetric tensor gauge field with an anti-self-dual field strength, $G^{-}_{\mu\nu\rho}$. This combines with the self-dual field strength $G^{+}_{\mu\nu\rho}$ of the (1,0) supergravity multiplet to form an unconstrained 3-form field strength $G_{[3]} = dB_{[2]}$. The bosonic part of the combined multiplet includes a vector and a dilatonic scalar. The bosonic sector of the model has the Lagrangian

$$\mathcal{L}_{SS} = \frac{1}{2}R - \frac{1}{4}e^{\sigma}F_{\mu\nu}F^{\mu\nu} - \frac{1}{6}e^{-2\sigma}G_{\mu\nu\rho}G^{\mu\nu\rho} - \frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - g^2 e^{-\sigma}$$

The vector field is used to gauge an Abelian R-symmetry in the fermionic sector. This is accompanied in the bosonic sector by the *positive* potential for the dilaton σ, depending on the coupling constant g. Generalisations of this model including additional $D = 6$ vector multiplets have scalar target spaces $\mathrm{SO}(p,q)/(\mathrm{SO}(p) \times \mathrm{SO}(q))$.

- Owing to the scalar potential, the model does not admit $D = 6$ flat space as a solution, but it does have an $S^2 \times \mathbb{R}^4$ electrovac solution with monopole charge ± 1:

$$\begin{aligned} ds^2 &= \eta_{\mu\nu} dx^\mu dx^\nu + a^2(d\theta^2 + \sin^2\theta d\phi^2) \\ A_m dy^m &= (n/2g)(\cos\theta \mp 1)d\phi \end{aligned}$$

- This electrovac solution provides a natural compactification from $D = 6$ to $D = 4$ preserving $N = 1$, $D = 4$ supersymmetry.
- "Rugby ball" solutions deform the round S^2 by cutting out a wedge along lines of longitude, thus producing conical singularities at opposite poles of the S^2. These conical singularities correspond to positive-tension 3-branes. Supersymmetry is then broken by the brane tensions. These may be imagined to arise from $D = 4$ vacuum energies of fields propagating only on the 3-branes.

  Aghababaie, Burgess, Parameswaran & Quevedo
- The deformed solution nonetheless remains flat in the $\mathbb{R}^4$ directions. This exemplifies a general theorem about such solutions. Assuming just axial symmetry in the compactified directions, both supersymmetric and non-supersymmetric solutions to the classical field equations remain flat in the $\mathbb{R}^4$ directions. Cvetič, Gibbons & Pope

General noncompact gaugings in supergravity theories

Hull & Warner 1988: Compact and noncompact supergravity R-symmetries can be promoted to gauged symmetries using vector fields of the theory.

- $D = 4$ toroidal reduction of $D = 11$ SG reduced on T^7 has an $\mathrm{SL}(8, \mathbb{R})$ symmetry of the action (extended to $\mathrm{E}_{7,7}$ for the equations of motion). Cremmer & Julia 1979
- The $SO(8)$ subgroup of $\mathrm{SL}(8, \mathbb{R})$ preserves an internal metric δ_{AB} and can be gauged (with a coupling constant g) using the 28 vector fields of the $N = 8$ theory: this yields the standard $SO(8)$ gauged maximal supergravity de Wit & Nicolai which can also be obtained by dimensional reduction from $D = 11$ on S^7.
  - ⋆ $SO(8)$ gauging breaks the $\mathrm{SL}(8, \mathbb{R})$ symmetry by g-dependent terms.
- One may then deform the model by analytic continuation using the *non-symmetric* $\mathrm{SL}(8, \mathbb{R})$.
  - ⋆ The g-independent terms in the action remain, however, $\mathrm{SL}(8, \mathbb{R})$ invariant, so the ∂^2 kinetic terms are left unchanged: no ghosts appear.
- One may thus obtain gaugings that preserve an $SO(p, q)$ internal metric $\eta_{AB} = \mathrm{diag}(\underbrace{+ + \ldots +}_{\text{p times}}, \underbrace{- - \ldots -}_{\text{q times}})$.

- These gaugings can be classified by the embedding-tensor formalism. de Wit, Samtleben & Trigiante
- Models with noncompact gaugings can be obtained directly via dimensional reduction, but these require noncompact reduction spaces.
- An example of such a reduction & corresponding noncompact gauging is the reduction of M-theory on the noncompact space $S^1 \times \mathcal{H}^{(2,2)}$. Cvetič, Gibbons & Pope
- The space $\mathcal{H}^{(2,2)}$ is a hyperbolic space of Euclidean $(+++)$ signature embedded into $\mathbb{R}^4$ by the condition $\mu_1^2 + \mu_2^2 - \mu_3^2 - \mu_4^2 = 1$. This embedding condition is SO(2, 2) invariant, but the embedding $\mathbb{R}^4$ space has SO(4) symmetry, so the isometries of this space are given by $\mathrm{SO}(2,2) \cap \mathrm{SO}(4) = \mathrm{SO}(2) \times \mathrm{SO}(2)$. The cohomogeneity-one $\mathcal{H}^{(2,2)}$ metric is $ds_3^2 = \cosh 2\rho d\rho^2 + \cosh^2 \rho d\alpha^2 + \sinh^2 \rho d\beta^2$.
- The SO(2, 2) nonetheless gives a local symmetry of the reduced theory. But this happens only via the agency of scalar-field vielbeins.

The $D=7$ bosonic Lagrangian of the reduced theory is given by

$$\mathcal{L}_7 = R*\mathbb{1} - \tfrac{5}{16}\Phi^{-2}*d\Phi\wedge d\Phi - *p_{\alpha\beta}\wedge p^{\alpha\beta} - \tfrac{1}{2}\Phi^{-1}*H_{(3)}\wedge H_{(3)}$$
$$-\tfrac{1}{2}\Phi^{-1/2}\pi_{\bar{A}}{}^{\alpha}\pi_{\bar{B}}{}^{\beta}\pi_{\bar{C}}{}^{\alpha}\pi_{\bar{D}}{}^{\beta}*F_{(2)}^{\bar{A}\bar{B}}\wedge F_{(2)}^{\bar{C}\bar{D}} - \frac{1}{g}\Omega - V*\mathbb{1}$$

where $\pi_{\bar{A}}{}^{\alpha}$ ($\bar{A}$ & $\alpha = 1\ldots 4$) are scalar vielbeins describing $9 = 16$ - 1 (det) - 6 (gauge) degrees of freedom.

- The rigid "composite" group structure is revealed in the covariant derivative

$$p_{\alpha\beta} = \pi^{-1}{}_{(\alpha}{}^{\bar{A}}\,[\delta_{\bar{A}}{}^{\bar{B}}\,d + g\,A_{(1)\bar{A}}{}^{\bar{B}}]\,\pi_{\bar{B}}{}^{\gamma}\,\delta_{\beta)\gamma}$$

  whose α, β indices are always raised and lowered with $\delta_{\alpha\beta}$, showing that there are no scalar ghosts.
- The local gauge symmetry (gauge field $A_{(1)\bar{A}}{}^{\bar{B}}$) acts on the $\bar{A}$, $\bar{B}$ indices, preserving a metric $\eta_{\bar{A}\bar{B}}$. If $\eta_{\bar{A}\bar{B}} = \delta_{\bar{A}\bar{B}}$, one has a local SO(4) symmetry. Nastase, Vaman & Van Nieuwenhuizen
- If $\eta_{\bar{A}\bar{B}} = \mathrm{diag}(++--)$, then one has a local SO(2,2) symmetry. Cvetič,Gibbons & Pope

- The scalar field potential is given by

$$V = \frac{1}{2} g^2 \, \Phi^{1/2} \, (2 M_{\alpha\beta} \, M_{\alpha\beta} - (M_{\alpha\alpha})^2)$$

  built from the unimodular matrix

$$M_{\alpha\beta} = \pi^{-1}{}_{\alpha}{}^{\bar{A}} \, \pi^{-1}{}_{\beta}{}^{\bar{B}} \, \eta_{\bar{A}\bar{B}}$$

  Note again that the α, β indices are contracted with an ordinary Kronecker $\delta_{\alpha\beta}$. The positivity properties of such potentials depend on the gauged subgroup chosen, with a corresponding invariant $\eta_{\bar{A}\bar{B}}$.
- This $D = 7$ theory can form the starting point for construction of a chiral $D = 6$ extension of the Salam-Sezgin model, built to be anomaly-free via a Hořava-Witten construction

$D = 7/D = 6$ Hořava-Witten constructions

Pugh, Sezgin & K.S.S. 1008.0726

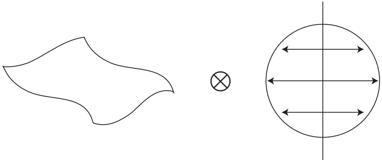
In order to provide an origin for the construction of chiral $D = 6$ models and a mechanism for achieving anomaly freedom, one may repeat the Hořava-Witten $D = 11/D = 10$ construction, but starting now in $D = 7$ with minimal (16 supercharge) supergravity coupled to n $D = 7$ vector multiplets.

In this starting theory, there are $3n$ scalar fields taking their values in an $\frac{\mathrm{SO}(n,3)}{\mathrm{SO}(n)\times\mathrm{SO}(3)}$ target space.

- The n à priori Abelian vector fields can be used to gauge subgroups of $\mathrm{SO}(n, 3)$. Demanding that the subsequent reduction to $D = 6$ preserve some form of R-symmetry gauging in $D = 6$ requires the gauging in $D = 7$ of a noncompact subgroup of $\mathrm{SO}(n, 3)$.

  Bergshoeff, Jong & Sezgin
- The gauged $\mathrm{SO}(2, 2)$ model obtained by reduction from $D = 11$ on $S^1 \times \mathcal{H}^{2,2}$ falls into this class, giving rise to a $U(1)$ gauged R-symmetry model upon reduction to $D = 6$ and chiral truncation. This yields the original Salam-Sezgin model. Cvetič, Gibbons & Pope

The basic framework for a Hořava-Witten construction in dimension D is a spacetime including a line interval, giving two boundary spacetimes of dimension $(D-1)$. In our case, the spacetime can be realised as $\mathcal{M}_6 \otimes S^1/\mathbb{Z}_2$:



$\mathcal{M}_6$ $\qquad$ $S^1/\mathbb{Z}_2$

- Instead of performing a reduction down to $\mathcal{M}_6$, however, in a Hořava-Witten construction one retains full $D=7$ dependence, but imposes $\mathbb{Z}_2$-symmetry boundary conditions at the line-interval ends.

- This construction breaks translation invariance in the x^7 direction, and this in turn gives rise to a corresponding halving of the unbroken supersymmetry, yielding a chiral (1,0) supersymmetry in $D = 6$:

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \text{translations} \qquad \Rightarrow \qquad \gamma^7 \epsilon = \epsilon$$

- Correspondingly, one imposes boundary conditions on parity-odd fields (and on ∂_7 derivatives of even fields) at the $\mathbb{Z}_2$ fixed points $\leftrightarrow$ line-interval endpoints. In the first instance, these quantities are set to zero at the endpoints.
- One needs to take care, however, to include appropriate Gibbons-Hawking-York terms in the action to ensure that the $\mathbb{Z}_2$ boundary conditions are consistent with the Euler-Lagrange variational principle.
- *e.g.* the Einstein-Hilbert action needs to be modified to

$$S_{EH} + S^0_{GHY} = \frac{1}{2\kappa^2} \int_{\mathcal{M}_7} d^7 x e R + \frac{1}{\kappa^2} \int_{\partial \mathcal{M}_7} d^6 x \sqrt{-h} h^{mn} K_{mn}$$

where h_{mn} is the metric induced on the boundary $\partial\mathcal{M}_7$ and K_{mn} is the extrinsic curvature tensor on the boundary.

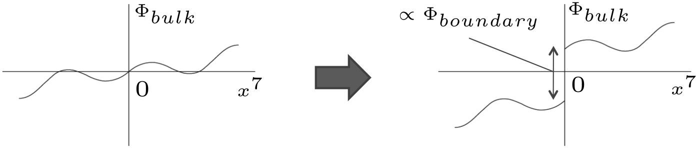
The $D = 7$ bulk Lagrangian, up to terms quadratic in fermions, is

$$
\begin{aligned}
&S_{SG(7)\,\mathrm{bulk}} = \\
&\quad \frac{1}{2\kappa^2}\int d^7x\hat{e}\Big\{\frac{1}{2}\hat{R} - \frac{1}{4g^2}e^{\hat{\sigma}}\hat{F}^i_{MN}\hat{F}^{MNi} - \frac{1}{4g^2}e^{\hat{\sigma}}\hat{F}^{\hat{r}}_{MN}\hat{F}^{MN\hat{r}} - \frac{1}{48}e^{-2\hat{\sigma}}\hat{F}_{MNRS}\hat{F}^{MNRS} - \frac{1}{24\sqrt{2}g^2}\hat{\varepsilon}^{MNRSTUV}\hat{A}_{MNR}\hat{F}^{\hat{r}}_{ST}\hat{F}^{\hat{r}}_{UV} \\
&\qquad - \frac{5}{8}\partial_M\hat{\sigma}\partial^M\hat{\sigma} - \frac{1}{2}\hat{P}^{i\hat{r}}_M\hat{P}^{Mi\hat{r}} - \frac{1}{4}g^2e^{-\hat{\sigma}}\left(C^{i\hat{r}}C^{i\hat{r}} - \frac{1}{9}C^2\right) \\
&\qquad - \frac{i}{2}\hat{\bar{\psi}}_M\hat{\gamma}^{MNR}\hat{D}_N\hat{\psi}_R - \frac{5i}{2}\hat{\bar{\chi}}\hat{\gamma}^M\hat{D}_M\hat{\chi} - \frac{i}{2g^2}\hat{\bar{\lambda}}^{\hat{r}}\hat{\gamma}^M\hat{D}_M\hat{\lambda}_{\hat{r}} - \frac{5i}{4}\hat{\bar{\chi}}\hat{\gamma}^M\hat{\gamma}^N\hat{\psi}_M\partial_N\hat{\sigma} - \frac{1}{2g}\hat{\bar{\lambda}}^{\hat{r}}\sigma^i\hat{\gamma}^M\hat{\gamma}^N\hat{\psi}_M\hat{P}^{i\hat{r}}_N \\
&\qquad + \frac{i}{96\sqrt{2}}e^{-\hat{\sigma}}\hat{F}_{MNRS}\left(\hat{\bar{\psi}}_{[L}\hat{\gamma}^L\hat{\gamma}^{MNRS}\hat{\gamma}^T\hat{\psi}_{T]} + 4\hat{\bar{\psi}}_L\hat{\gamma}^{MNRS}\hat{\gamma}^L\hat{\chi} - 3\hat{\bar{\chi}}\hat{\gamma}^{MNRS}\hat{\chi} + \frac{1}{g^2}\hat{\bar{\lambda}}^{\hat{r}}\hat{\gamma}^{MNRS}\hat{\lambda}^{\hat{r}}\right) \\
&\qquad + \frac{1}{8g}e^{\frac{\hat{\sigma}}{2}}\hat{F}^i_{MN}\left(\hat{\bar{\psi}}_{[L}\sigma^i\hat{\gamma}^L\hat{\gamma}^{MN}\hat{\gamma}^T\hat{\psi}_{T]} - 2\hat{\bar{\psi}}_L\sigma^i\hat{\gamma}^{MN}\hat{\gamma}^L\hat{\chi} + 3\hat{\bar{\chi}}\sigma^i\hat{\gamma}^{MN}\hat{\chi} - \frac{1}{g^2}\hat{\bar{\lambda}}^{\hat{r}}\sigma^i\hat{\gamma}^{MN}\hat{\lambda}^{\hat{r}}\right) - \frac{i}{4g^2}e^{\frac{\hat{\sigma}}{2}}\hat{F}^{\hat{r}}_{MN}\left(\hat{\bar{\psi}}_L\hat{\gamma}^{MN}\hat{\gamma}^L\hat{\lambda}^{\hat{r}} + 2\hat{\bar{\chi}}\hat{\gamma}^{MN}\hat{\lambda}^{\hat{r}}\right) \\
&\qquad - \frac{i\sqrt{2}}{24}ge^{-\frac{\hat{\sigma}}{2}}C\left(\hat{\bar{\psi}}_M\hat{\gamma}^{MN}\hat{\psi}_N + 2\hat{\bar{\psi}}_M\hat{\gamma}^M\hat{\chi} + 3\hat{\bar{\chi}}\hat{\chi} - \frac{1}{g^2}\hat{\bar{\lambda}}^{\hat{r}}\hat{\lambda}^{\hat{r}}\right) + \frac{1}{2\sqrt{2}}e^{-\frac{\hat{\sigma}}{2}}C^{i\hat{r}}\left(\hat{\bar{\psi}}_M\sigma^i\hat{\gamma}^M\hat{\lambda}^{\hat{r}} - 2\hat{\bar{\chi}}\sigma^i\hat{\lambda}^{\hat{r}}\right) + \frac{1}{2g}e^{-\frac{\hat{\sigma}}{2}}C^{\hat{r}\hat{s}i}\hat{\bar{\lambda}}^{\hat{r}}\sigma^i\hat{\lambda}^{\hat{s}}\Big\}
\end{aligned}
$$

Inclusion of $D = 6$ boundary supermatter

A *raison d'être* for the inclusion of boundary matter is to provide sources of anomaly compensation. The $D = 7$ bulk theory has gravitational and gauge-symmetry anomalies when projected onto the $D = 6$ boundaries. Inclusion of boundary supermatter affects the anomaly calculation in two ways: 1) directly via their quantum loops 2) "classically" by anomaly inflow.

Anomaly inflow contributions arise by generalising the initial (parity odd) $= 0$ boundary conditions to allow for a non-zero RHS dependent on boundary fields:



$$\Phi_{bulk}\big|_{\partial M} = 0 \qquad\qquad \Phi_{bulk}\big|_{\partial M} \propto \Phi_{boundary}$$

To prepare the setup for anomaly cancellation, one needs to include couplings to $D = 6$ boundary vector multiplets, hypermultiplets and antisymmetric tensor multiplets. Some specific features of this $D = 7/D = 6$ construction are:

- The $D = 7$ on-shell supersymmetry algebra produces compensating gauge transformations from the commutator of two supersymmetry transformations. Consequently, the willful breaking of gauge invariance on the boundaries in preparation for gauge anomaly inflow also provokes supersymmetry breaking. Accordingly, one cannot fully demand supersymmetry of the classical-level construction. The best one can do is to impose the Wess-Zumino consistency conditions. (Seen also directly in $D = 6$. Ferrara, Riccioni & Sagnotti)
- Hypermultiplet coupling to $D = 6$ supergravity requires a quaternionic target space. Bagger & Witten But one cannot simply add quaternionic spaces. Reduction of the $D = 7$ bulk Lagrangian to $D = 6$ correctly produces a quaternionic scalar target manifold. Bergshoeff, Jong & Sezgin But the inclusion of boundary hypermultiplets is then tricky.

- The coupling of boundary hypermultiplets can be addressed by introducing a partial set of auxiliary fields. $D = 6$ (1,0) supersymmetry, like $D = 4$, $N = 2$ supersymmetry, naturally involves an Sp(1) auxiliary vector field V_M^i. de Wit, van Holten & Van Proeyen
In the bulk $D = 7$ supergravity, this would have the field equation $V_M^i = \frac{1}{2}\epsilon^{ijk} Q_M^{jk}$, where Q_M^{jk} is an Sp(1) connection built from the bulk scalars.
- Upon coupling to $D = 6$ boundary hypermultiplets, the auxiliary field equation relates V_M^i to the boundary Sp(1) connection, $-\frac{i}{2} V_\mu^i \sigma^{i\,AB}\big|_{\partial\mathcal{M}} = Q_\mu^{AB}$. Thus, one obtains a requirement relating bulk and boundary composite Sp(1) connections:
$-\frac{i}{4}\epsilon^{ijk} Q_\mu^{jk} \sigma^{i\,AB}\big|_{\partial\mathcal{M}} = Q_\mu^{AB}$.
- Anomalies are encoded in an 8-form anomaly polynomial. An example of a chiral anomaly-free system has $SO(2,1)$ $D = 7$ bulk gauge symmetry and $n_V = 78 + 133$ boundary vector multiplets for a gauge group $E_6 \times E_7$, together with 2 boundary tensor multiplets and $n_H = 5 \times (27,1) + 5 \times (1,56)$ boundary hypermultiplets.

Aside: ectoplasm with an edge

Howe, Pugh, Strickland-Constable & K.S.S. 1104.4387

The "ectoplasm" formulation gives a supersymmetric invariant in d spacetime dimensions as the integral of a pull-back via the projection map s to the superspace "body" of a closed superspace d-form, $dJ_d = 0$.

Gates, Grisaru, Knut-Whelau, & Siegel; Berkovits & Howe; Bossard, Howe & K.S.S.

$$S = \int_{s(\mathcal{M}_0)} J_d = \int_{\mathcal{M}_0} s^* J_d$$

In the absence of boundaries, such invariants depend only on the de Rham cohomology class of J_d: $J_d \sim J_d + d\lambda_{d-1}$.

In the presence of boundaries, this cohomological formulation naturally changes to one involving relative cohomology. Given a pair of forms A_d and B_{d-1} on a d dimensional manifold $\mathcal{M}$ and on a $(d-1)$ dimensional submanifold $\mathcal{N}$, one has the relative exterior derivative $d^{(\iota)}(A_p, B_{p-1}) = (dA_p, \iota^* A_p - dB_{p-1})$ and the relative integral $\int_{(\mathcal{M},\mathcal{N})}(A_d, B_{d-1}) = \int_{\mathcal{M}} A_d - \int_{\mathcal{N}} B_{d-1}$, with a corresponding Stokes' theorem.

One can define the "superboundary" $\partial\mathcal{M}$ of a (d, n) dimensional supermanifold $\mathcal{M}$ to be a manifold with $(d-1)$ bosonic dimensions and $\frac{n}{2}$ fermionic dimensions given by the locus of the boundary $\partial\mathcal{M}_0$ under the surviving $\frac{n}{2}$ supersymmetry transformations.

Let the embedding map for $\partial\mathcal{M}_0$ into $\mathcal{M}_0$ be c and the embedding map of $\partial\mathcal{M}$ into $\mathcal{M}$ be $\tilde{c}$. Then one has a commutative diagram of maps

$$\begin{array}{ccc} \partial\mathcal{M}_0 & \xrightarrow{\;c\;} & \mathcal{M}_0 \\ \Big\downarrow{\scriptstyle \tilde{s}} & & \Big\downarrow{\scriptstyle s} \\ \partial\mathcal{M} & \xrightarrow{\;\tilde{c}\;} & \mathcal{M} \end{array}$$

The action including boundary contributions is then simply

$$S = \int_{(\mathcal{M}_0, \partial\mathcal{M}_0)} (s^* J_d, \tilde{s}^* I_{d-1})$$

Reproduces "supersymmetry without boundary conditions" Belyaev & Van Nieuwenhuizen

where I_{d-1} is the boundary superform on $\partial\mathcal{M}$ of the relatively closed form (J_d, I_{d-1}).

Chiral dimensional reduction C.N. Pope, T. Pugh & K.S.S., in preparation

The original Salam-Sezgin model was presented as yielding a chiral $D = 4$ theory after dimensional reduction on the sphere & monopole background. This is in a sense true, but this is not chirality in a very physically interesting sense. The original model had just a gauged $U(1)_R$ symmetry, according to which the spinor fields of the reduced $N = 1$, $D = 4$ theory carry opposite $U(1)_R$ charges. Moreover, the $U(1)_R$ gauge vector becomes massive via a Stueckelberg mechanism. When one refers to a chiral theory, one generally means that the spinors carry complex representations with respect to some standard (not R-symmetry) unbroken gauge symmetry.

An example of such a $D = 6$ extended chiral system was given with $E_6 \times E_7 \times U(1)$ gauge symmetry and the hyperfermions in the $\underline{912}$ of E_7 by Randjbar-Daemi, Salam, Sezgin & Strathdee .

An important question is whether this system may be reduced to a chiral theory in $D = 4$, as was claimed in that original work.

- A serious problem was raised concerning the chiral reduction of the $E_6 \times E_7 \times U(1)$ system by Gibbons & Pope. The question concerns which sets of $D = 4$ fluctuations may be retained in a consistent Kaluza-Klein truncation corresponding to the sphere & monopole background. Reductions on spheres with fluxes turned on are familiar from a number of supergravity dimensional reductions, *e.g.* the S_7 reduction of $D = 11$ supergravity with F_4 flux turned on. Freund & Rubin

- Sphere reductions may yield massless fields corresponding to the isometry group of the reduction sphere, which in this S^2 reduction would yield an $SU(2)$ Yang-Mills gauge symmetry in the $D = 4$ reduced theory. At the same time, consistency of that reduction required that the dilatonic scalar of the $D = 6$ theory be given a fixed relation to the Kaluza-Klein scalar normally obtained from such a dimensional reduction.

- The $SU(2)$ gauged reduction for the basic Salam-Sezgin theory involves the ansatz

$$
\begin{aligned}
d\hat{s}^2 &= e^{\frac{1}{2}\phi}\, ds_4^2 + e^{-\frac{1}{2}\phi}\, g_{mn}\,(dy^m + 2g\, A^i\, K_i^m)(dy^n + 2g\, A^j\, K_j^n) \\
\hat{e}^\alpha &= e^{\frac{1}{4}\phi}\, e^\alpha\,, \qquad \hat{e}^a = e^{-\frac{1}{4}\phi}\,(e^a + 2g\, A^i\, K_i^a) \\
\hat{F}_{(2)} &= 2g\, e^{\frac{1}{2}\phi}\, \epsilon_{ab}\, \hat{e}^a \wedge \hat{e}^b - \mu_i\, F^i \\
\hat{H}_{(3)} &= H_{(3)} - 2g\, F^i \wedge K_i^a\,(e^a + 2g\, A^j\, K_j^a) \\
\hat{\phi} &= -\phi
\end{aligned}
$$

  where K_i^m, $i = 1, 2, 3$ are the $SU(2)$ Killing vectors of the round S^2; $K_i^a = K_i^m\, e_m^a$. The μ_i are the $SU(2)$ triplet of lowest non-trivial harmonics on S^2.
- The $D = 4$ massless reduced theory consists of $N = 1$ supergravity coupled to an $N = 1$, $SU(2)$ Yang-Mills vector multiplet and a scalar multiplet. Note, however, that this is not a chiral theory: the chiral structure has been lost in the S^2 reduction.

Chiral reduction with gauged $U(1)_R$

Despite the failure of the $SU(2)$ gauged reduction from 6 to 4 dimensions to be chiral, it is nonetheless possible for a consistent dimensional reduction to $D = 4$ inherit the chiral structure of the $D = 6$ theory. It turns out that there is a bifurcation of consistent reduction ansätze. Preserving chirality, however, requires turning off the $SU(2)$ gauge fields. Here is the second branch of the bifurcated ansatz:

$$\begin{aligned} \hat{e}^\alpha &= e^{\frac{1}{4}(\phi+\varphi)} e^\alpha \,, \qquad \hat{e}^a = e^{-\frac{1}{4}(\phi+\varphi)} e^a \\ \hat{F}_{(2)} &= \frac{1}{2g}\Omega_{(2)} - F_{(2)} \\ \hat{H}_{(3)} &= H_{(3)} - \frac{1}{2g} A_{(3)} \wedge \Omega_{(2)} \\ \hat{\phi} &= \varphi - \phi \end{aligned}$$

where $\Omega_{(2)}$ is the volume form of the S^2. Note that this ansatz retains two fluctuation scalars ϕ and φ, but there are no massless $SU(2)$ vector modes.

The effective action for this second ansatz is of a type that can't be obtained directly by insertion into the higher dimensional action. Nonetheless, the truncation of the $D = 6$ theory implemented by the ansatz is consistent: solutions of the reduced $D = 4$ field equations yield *exact* solutions of the $D = 6$ theory. And one can integrate these $D = 4$ field equations back to obtain a $D = 4$ effective action. The $D = 4$ theory contains a 2-form gauge field, descended from the 2-form in $D = 6$. Dualising this to a scalar, the bosonic part of this effective action becomes

$$
\begin{aligned}
\mathcal{L}_B &= R * 1 - \frac{1}{2} * d\phi \wedge d\phi - \frac{1}{2} * d\varphi \wedge d\varphi - \frac{1}{2} e^{2\phi} * d\sigma \wedge d\sigma \\
&\quad - \frac{1}{2} e^{-\phi} * F_{(2)} \wedge F_{(2)} - 8g^2 e^{2\varphi} * A_{(1)} \wedge A_{(1)} + \frac{1}{2} \sigma F_{(2)} \wedge F_{(2)} \\
&\quad - 8g^2 e^{\phi} (1 - e^{\varphi})^2 * 1
\end{aligned}
$$

Note that we now have two scalars, ϕ and φ, retaining both the descendant of the $D = 6$ dilaton and also the $6 \to 4$ Kaluza-Klein scalar φ. The $SU(2)$ Yang-Mills vectors have not been retained, but there is now a massive vector $A_{(1)}$.

Preservation of chirality

The fermionic sector of the theory reduced according to the second ansatz is chiral, but only in a minimal sense. The gauged $U(1)_R$ of the $D = 6$ theory gives rise to gauge couplings for the reduced $D = 4$ spinor fields. Moreover, the $D = 4$ supersymmetry parameter ϵ inherits chirality from $D = 6$. So the $D = 4$ theory cannot naturally be written in terms of Majorana spinor fields. However, the couplings insuring chirality are gauge couplings to the $U(1)_R$ gauge field $A_{(1)}$, which has become massive. The broken $U(1)_R$ may be restored by Stueckelberg couplings to an additional auxiliary scalar ρ. But this is not what one generally considers a chiral theory in the sense of the Standard Model.

In order to have a more physically relevant chirality in $D = 4$, one needs to include more in the starting model, for example the $D = 6$ gauge fields for $E_6 \times E_7 \times U(1)$ plus 456 hypermultiplets of Randjbar-Daemi, Salam, Sezgin & Strathdee.

- Figuring out what survives at massless level of the $E_6 \times E_7 \times U(1)$ model involves some complicated group theory. The 456 hypermultiplet scalars take their values in a target coset space $Sp(456, 1)/(Sp(456) \otimes Sp(1))$. The E_7 is the gauged part of the rigid $Sp(456)$ within the rigid $Sp(456, 1)$ such that the irreducible $\underline{912}$ of $Sp(456)$ remains an irreducible $\underline{912}$ of E_7.
- In the consistent reduction of this model on the sphere & monopole background, the rigid $Sp(456)$ reduces to $SU(456)$. So the surviving part of E_7 is that part that also lies in $SU(456)$.
- This is tricky to work out. However, one can consider a simpler model starting with just the Salam-Sezgin model coupled to 28 hypermultiplets, together with 133 vector multiplets gauging the E_7 subgroup of $Sp(28)$ (chosen such that the $\underline{56}$ of $Sp(28)$ remains irreducible under E_7). In this case, a brute-force evaluation of all the generators using a computer program of Cacciatori, Piazza & Scotti shows that the surviving symmetry in that case is just $SU(8)$. This is also anticipated for the $E_6 \times E_7 \times U(1)$ model, with surviving fermions in the complex $\underline{420} \oplus \underline{36}$ of $SU(8)$, preserving chirality.

Supersymmetry in Large Extra Dimensions

One motivation for reconsidering such supergravity models with gauged noncompact R-symmetries comes from the SLED program: Supersymmetry in Large Extra Dimensions (Aghababaie, Burgess, Parameswaran & Quevedo). This starts from the observation (Arkani-Hamed, Dimopoulos & Dvali) that if one has just 2 extra dimensions, the scale r of these could be as large as 0.1 mm $= 100\mu$m without running into conflict with measurements of the gravitational inverse-square law.

- Letting vacuum energies on the $D = 4$ subspace be of order $M \sim 10$ TeV and taking r to be of order 10μm, one can obtain a bulk (cosmological constant)$^{-1/4}$ corresponding to an energy scale $M^2/\mathcal{M}_P$, *i.e.* $\sim 10^{-2}$ eV, the correct scale for the observed cosmological constant.

- The SLED scenario fits naturally with the class of supergravity models with gauged R-symmetries. Depending on the details of the scalar potential and gauging, these models may not admit flat spacetime solutions in their original spacetime dimensions D, but they can naturally admit solutions with flat spacetimes of lower dimensionality.
- An example of such a robust compactification scenario to flat spacetimes is provided by the "rugby ball" electrovac compactifications from $D = 6$ to flat $D = 4$ with $U(1)$ flux turned on. Carroll & Guica, Aghababaie, Burgess, Parameswaran & Quevedo
- A basic example of a model admitting such "self-tuning" reductions to flat space is provided by the Salam-Sezgin model.
- At the quantum level, the above 'self-tuning' mechanism is modified, and an effective $D = 4$ cosmological constant does arise as a result of vacuum energies, but this is greatly reduced by the classical self-tuning mechanism. For a 10μm scale of the S^2 directions and vacuum energies M in the 10 TeV range, one can end up with an effective (cosmological constant)$^{1/4}$ of the order of $M^2/M_P \sim 10^{-2}$ eV.