Matrix models, noncommutative gauge theory, and gravity

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Motivation

- "fundamental" d.o.f. for quantum gravity might be different from macroscopic ones
  "emergence", quantum structure of space-time
- cosm. const. problem: need new mechanism ?

candidate: $\boxed{\text{Matrix Models}}$

- related to string theory ("nonpert. def.")
- describes dynamical NC space-time (branes)
- accessible, novel tools

NC gauge theory $\leftrightarrow$ gravity

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Matrix Models

IKKT (IIB) model Ishibashi, Kawai, Kitazawa and Tsuchiya 1996

$$S[X] = -Tr\left([X^a, X^b][X^{a'}, X^{b'}]g_{aa'}g_{bb'} + \bar{\Psi}\gamma_a[X^a, \Psi]\right)$$

$$X^a = X^{a\dagger} \in Mat(N, \mathbb{C}), \qquad a = 0, ..., 9$$

$$N \to \infty$$

gauge symmetry $X^a \to UX^aU^{-1}$, $SO(9,1)$, SUSY, etc.

as $\begin{cases} \text{1) nonpert. def. of IIB string theory (on } \mathbb{R}^{10}) \\ \text{2) } \mathcal{N} = 4 \text{ SUSY Yang-Mills gauge thy. on } \mathbb{R}^4_\theta \end{cases}$ (*IKKT*)

dynamical NC branes $\mathcal{M} \subset \mathbb{R}^{10}$

carry (NC) gauge theory coupled to gravity (H.S. 2007 ff)

(brane-world scenarios)

Space-time & gravity from matrix models:

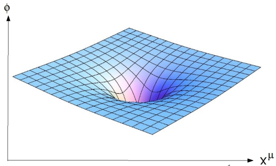
e.o.m.: $\delta S = 0 \;\Rightarrow\; [X^a,[X^{a'},X^{b'}]]\eta_{aa'} = 0$

solutions:

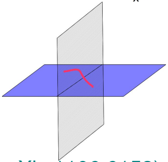
- $[X^a, X^b] = i\theta^{ab}\,\mathbb{1},$ "quantum plane" $\mathbb{R}^4_\theta$
- $[X^a, X^b] \sim i\theta^{ab}(x),$ generic quantum space

  $\rightarrow$ space-time as 3+1-dim. brane solution

$$X^a \sim x^a : \quad \mathcal{M}^4 \hookrightarrow \mathbb{R}^{10}$$



- intersecting branes, stacks (as in string theory)



- compact extra dim $\mathcal{M}^4 \times T^2$, etc.

(H.S., arXiv:1106.6153)

Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$

$$[\bar{X}^\mu, \bar{X}^\nu] = i\bar{\theta}^{\mu\nu}\,\mathbb{1}, \qquad \mu,\nu = 1,...,4$$

... Heisenberg algebra $\mathcal{A} = Mat(\infty, \mathbb{C})$ $=$ space of functions on $\mathbb{R}^4_\theta$

$$\Delta\bar{X}^\mu \Delta\bar{X}^\nu \geq |\bar{\theta}^{\mu\nu}|$$

relation with classical $\mathbb{R}^4$:

$$\phi(x) = \int d^4k\,\tilde{\phi}(k) e^{ik_\mu x^\mu} \;\leftrightarrow\; \int d^4k\,\tilde{\phi}(k) e^{ik_\mu \bar{X}^\mu} =: \Phi(\bar{X}) \;\in Mat(\infty, \mathbb{C})$$

$\Phi(\bar{X}^\mu) \in Mat(\infty, \mathbb{C})$ ... general function on $\mathbb{R}^4_\theta$

$\bar{X}^\mu \in Mat(\infty, \mathbb{C})$ ... quantized coordinate functions on $\mathbb{R}^4_\theta$

$$[\bar{X}^\mu, \Phi(\bar{X})] =: i\bar{\theta}^{\mu\nu}\partial_\nu \Phi(\bar{X}) \sim i\bar{\theta}^{\mu\nu}\partial_\nu \phi(x) \;\to$$ NC field theory

Quantized symplectic manifolds

$(\mathcal{M}, \theta^{\mu\nu}(x))$ ... $2n$-dim. manifold with Poisson structure

Its quantization is NC algebra $\mathcal{A}$ such that

$$\begin{array}{rcl} \mathcal{I}:\ \mathcal{C}(\mathcal{M}) & \to & \mathcal{A} \subset \mathcal{L}(\mathcal{H}) \\ f(x) & \mapsto & \hat{f}(X) \end{array}$$

such that

$$\begin{array}{rcl} \hat{f}\,\hat{g} & = & \mathcal{I}(fg) + O(\theta) \\ [\hat{f}, \hat{g}] & = & \mathcal{I}(i\{f,g\}) + O(\theta^2) \end{array}$$

(“nice“) $\Phi \in Mat(\infty, \mathbb{C}) \quad \leftrightarrow \quad$ quantized function on $\mathcal{M}$

examples:

$$\left.\begin{array}{ll} \text{fuzzy sphere} & S^2_N \\ \text{fuzzy torus} & T^{2n}_N \\ \text{etc.} & \end{array}\right\} \ldots \text{built-in UV cutoff,} \quad \dim\mathcal{H} \sim \mathrm{Vol}(\mathcal{M})$$

small deformation of $\mathbb{R}^4_\theta$:

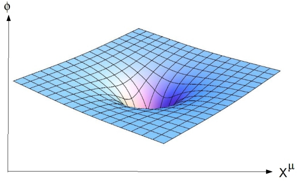
$$X^a = \bar{X}^a + A^a = \begin{pmatrix} \bar{X}^\mu + A^\mu(\bar{X}^\mu) \\ \phi^i(\bar{X}^\mu) \end{pmatrix}$$

- "old" interpretation: $U(1)$ NC gauge theory on $\mathbb{R}^4_\theta$
- more appropriate interpretation:

$$X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$$ quantized embedding map

$$[X^\mu, X^\nu] \sim i\{x^\mu, x^\nu\}$$ ...Poisson bracket

... defines embedded NC space $\mathcal{M}$



consider scalar field φ ("test particle") on generic NC brane

$$X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$$

$\varphi \in \mathcal{A}$, action

$$\begin{aligned} S[\varphi] &= Tr\,[X^a,\varphi][X^b,\varphi]\,\eta_{ab} \qquad \text{(gauge inv.!)} \\ &\sim \int d^4x\,\sqrt{|\theta^{-1}_{\mu\nu}|}\,\theta^{\mu'\mu}\partial_{\mu'}x^a\partial_\mu\varphi\,\theta^{\nu'\nu}\partial_{\nu'}x^b\partial_\nu\varphi\,\eta_{ab} \\ &\sim \int d^4x\,\sqrt{|G_{\mu\nu}|}\,G^{\mu\nu}(x)\,\partial_\mu\varphi\partial_\nu\varphi \end{aligned}$$

(use $[f,\varphi] \sim i\theta^{\mu\nu}(x)\partial_\mu f\,\partial_\nu\varphi$)

where

$$\boxed{\begin{aligned} G^{\mu\nu}(x) &= e^{-\sigma}\theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\,g_{\mu'\nu'}(x) \quad \text{effective metric (cf. open string m.)} \\ g_{\mu\nu}(x) &= \partial_\mu x^a\partial_\nu x^b\eta_{ab} \quad \text{induced metric on } \mathcal{M}^4_\theta \text{ (cf. closed string m.)} \end{aligned}}$$

$$e^{-2\sigma} = \frac{|\theta^{-1}_{\mu\nu}|}{|g_{\mu\nu}|}$$

fluctuations of matrices X^a, Ψ around background
$\to$ gauge fields, scalar fields, fermions on $\mathcal{M}^4$ (NOT 10 dim!)

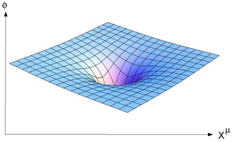
all fields couple to metric $G^{\mu\nu}(x)$
determined by $\theta^{\mu\nu}(x)$, embedding (scalar fields)
dynamical $\Rightarrow$ ("emergent") gravity

can show

$$\Box := [X^a, [X^b, .]] g_{ab} \sim e^{\sigma} \Box_G$$

matrix e.o.m $[X^a, [X^{a'}, X^b]] \eta_{aa'} = 0 \quad \Longleftrightarrow$

$$\boxed{\begin{aligned} \Box_G x^a &= 0, \qquad \text{"minimal surface"} \\ \nabla^\mu (e^\sigma \theta^{-1}_{\mu\nu}) &= e^{-\sigma} G_{\rho\nu} \theta^{\rho\mu} \partial_\mu \eta \\ &\qquad \eta \sim G^{\mu\nu} g_{\mu\nu} \end{aligned}}$$



covariant formulation in semi-classical limit (H.S. 2007/08)

Gauge fields

basic obervation:

M.M. is gauge invariant:

$$X^a \to U^{-1} X^a U$$

$\to$ fluctuations $X^\mu = \bar{X}^\mu + \theta^{\mu\nu} A_\nu$ transform as
$A_\mu \to U^{-1} A_\mu U + U^{-1} \partial_\mu U$ (nonabelian) gauge field!

note

$$\begin{aligned}[][X^\mu, X^\nu] &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'}\left(\partial_{\mu'} A_{\nu'} - \partial_{\nu'} A_{\mu'} + [A_{\mu'}, A_{\nu'}]\right) \\ &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'}\, F_{\mu'\nu'}\end{aligned}$$

(use $[\bar{X}^\mu, \phi] \sim i\theta^{\mu\nu}\partial_\nu \phi$)

<u>however:</u>
carefully distinguish $U(1)$ sector (geometry) from $SU(n)$ sector (YM)!

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$su(n)$ gauge fields: same model, new vacuum

$$Y^a = \begin{pmatrix} Y^\mu \\ Y^i \end{pmatrix} = \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n \end{pmatrix}$$

(n coinciding branes)

include fluctuations:

$$Y^a = (1 + \mathcal{A}^\rho \partial_\rho) \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n + \Phi^i \end{pmatrix}$$

where

$$\begin{aligned} \mathcal{A}^\mu &= -\theta^{\mu\nu} A_{\nu,\alpha} \otimes \lambda^\alpha, \qquad \lambda^\alpha \in \mathfrak{su}(n) \\ \Phi^i &= \Phi^i_\alpha \otimes \lambda^\alpha \end{aligned}$$

$\Rightarrow$ effective action:

$$\boxed{S_{YM} = \int d^4x \sqrt{G}\, e^\sigma\, G^{\mu\mu'} G^{\nu\nu'} \, tr\, F_{\mu\nu} F_{\mu'\nu'} + 2 \int \eta(x)\, tr\, F \wedge F}$$

(H.S., JHEP 0712:049 (2007), JHEP 0902:044,(2009))

... $\mathfrak{su}(n)$ Yang-Mills coupled to metric $G^{\mu\nu}(x)$

note:

- $SU(n)$ fluctuations ... nonabelian gauge fields
- $U(1)$ fluctuations $\rightarrow$ geometry, metric $G \sim \theta\theta g$

$$\left.\begin{array}{l} F_{\mu\nu}(x) \\ \phi^i(x) \end{array}\right\} \rightarrow \left\{\begin{array}{l} \theta^{\mu\nu}(x), \\ g_{\mu\nu}(x) = \eta_{\mu\nu} + \partial_\mu\phi^i\partial_\nu\phi_i \end{array}\right.$$

- generic 4D metric on $\mathcal{M}^4 \subset \mathbb{R}^{10}$ (almost-Kähler)
- explains & takes advantage of UV/IR mixing

  (= new divergences in all NC models except IKKT)

Quantization

Quantization of matrix model:

$$Z = \int dX^a d\Psi \, e^{-S[X]-S[\Psi]} \; = \; e^{-S_{eff}} \qquad \text{“path integral”}$$

2 interpretations:

1. as NC gauge theory on $\mathbb{R}^4_\theta$: $\quad X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu$

   $\rightarrow$ new divergences (UV/IR mixing) in $U(1)$ sector
   except $\mathcal{N} = 4$ SUSY (≡IKKT)

2. on $\mathcal{M}^4 \subset \mathbb{R}^{10}$: $\quad U(1)$ absorbed in $\theta^{\mu\nu}(x)$, $g_{\mu\nu}$

   $\rightarrow$ induced gravity action (Sakharov 1967)

$$S_{eff} \sim \int d^4x \sqrt{|G|} \, (\Lambda^4 + c\Lambda_4^2 \, R[G] + \ldots)$$

- explanation UV/IR mixing, $U(1)$ entanglement (=gravity)
- IKKT model $\Rightarrow$ good quantum theory ! ($\mathcal{N} = 4$ SUSY)

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2 interpretations:

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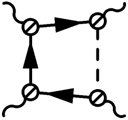
example: 1-loop eff. action due to fermions:
(all terms of dim ≤ 6):



$$
\begin{aligned}
\Gamma_{\rm eff} \;=\; & \frac{\Lambda^4}{\Lambda_{\rm NC}^4}\int\frac{d^4x}{(2\pi)^2}\Big(g^{\alpha\beta}D_\alpha\varphi^i D_\beta\varphi_i \\
& -\tfrac{1}{2}\Lambda_{\rm NC}^4\big(\bar\theta^{\mu\nu}F_{\nu\mu}\bar\theta^{\rho\sigma}F_{\sigma\rho}+(\bar\theta^{\sigma\sigma'}F_{\sigma\sigma'})(F\bar\theta F\bar\theta)\big) \\
& -2\bar\theta^{\nu\mu}F_{\mu\alpha}g^{\alpha\beta}\partial_\nu\varphi^i\partial_\beta\varphi_i+\tfrac{1}{2}(\bar\theta^{\mu\nu}F_{\mu\nu})g^{\alpha\beta}\partial_\beta\varphi^i\partial_\alpha\varphi_i+\text{h.o.}\Big) \\
& +\frac{\Lambda^2}{\Lambda_{\rm NC}^4}\int\frac{d^4x}{(2\pi)^2}\Big(-\tfrac{11}{2}F_{\rho\eta}\Box_g F_{\sigma\tau}\bar G^{\rho\sigma}\bar G^{\eta\tau}-12\Box_g\varphi^i\Box\varphi_i \\
& \quad +\tfrac{1}{2}\Lambda_{\rm NC}^4(\bar\theta^{\mu\nu}F_{\mu\nu})\bar\Box_G(\bar\theta^{\rho\sigma}F_{\rho\sigma})\quad+\ldots\Big) \\
& +\frac{\Lambda^6}{\Lambda_{\rm NC}^8}\int\frac{d^4x}{(2\pi)^2}(\ldots)+\ldots
\end{aligned}
$$

(all of this is due to UV/IR mixing !)

(D. Blaschke, H.S., M. Wohlgenannt JHEP 1103 (2011))

effective generalized matrix model:

re-assemble as matrix model: $X^a = \begin{pmatrix} \bar{X}^\mu \\ 0 \end{pmatrix} + \begin{pmatrix} -\bar{\theta}^{\mu\nu} A_\nu \\ \phi^i \end{pmatrix}$

$$\Gamma_L[X] = \mathrm{Tr} \frac{L^4}{\sqrt{\frac{1}{2}H^2 - H^{ab}H_{ab} + \frac{1}{L^2}\mathcal{L}_{10,\mathrm{curv}}[X] + \ldots}} \sim \int d^4x\, \Lambda^4(x)\sqrt{g(x)}$$

$$\mathcal{L}_{10,\mathrm{curv}}[X] = c_1[X^c, H^{ab}][X_c, H_{ab}] + c_2 H^{cd}[X_c,[X^a,X^b]][X_d,[X_a,X_b]] + \ldots$$

$$H^{ab} = [X^a, X^c][X^b, X_c] + (a \leftrightarrow b), \qquad H = H^{ab}\eta_{ab}$$

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$SO(D)$ manifest, broken by background (e.g. $\mathbb{R}^4_\theta$)

$\Rightarrow$ highly non-trivial predictions for (NC) gauge theory

expect generalization to nonabelian $\mathcal{N} = 4$ SYM: full $SO(9,1)$!

effective generalized matrix model
= powerful tool for (NC) gauge theory and (emergent) gravity

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(1-loop) finiteness of IKKT model

background field method $X^a \to X^a + Y^a$:

$$
\begin{aligned}
\Gamma_{1-\text{loop}} &= \tfrac{1}{2}\mathrm{Tr}\Big(\log(\mathbb{1} + \Sigma^{(10)}_{ab}\Box^{-1}[\Theta^{ab}, .]) - \tfrac{1}{2}\Big(\log(\mathbb{1} + \Sigma^{(16)}_{ab}\Box^{-1}[\Theta^{ab}, .]\Big)\Big) \\
&= O(\mathrm{Tr}(\Sigma_{ab}\Box^{-1})^4), \qquad \text{due to } \mathcal{N} = 4 \\
\Box &= [X^a, [X^a, .]] \\
\Theta^{rs} &= [X^r, X^s], \qquad \Sigma_{rs} \ldots SO(9,1) \text{ generator}
\end{aligned}
$$

fully $SO(9,1)$ covariant (IKKT, Chepelev & Tseytlin)

background $\mathbb{R}^4_\theta$: $\equiv \mathcal{N} = 4$ SYM on $\mathbb{R}^4_\theta$, no UV div.

$SO(9,1)$ invariant formalism, broken spontaneously through $\mathbb{R}^4_\theta$
NC essential.

background $\mathcal{M}^4 \times K$: $\mathcal{N} = 4$ broken

induced gravity on $\mathcal{M}$ below Λ_K

(work in progress, D. Blaschke, H.S.)

higher-order terms $\mathcal{L}_{\rm curv}[X]$: curvature

$$\begin{aligned} H^{ab} &:= [X^a, X^c][X^b, X_c] \\ T^{ab} &:= H^{ab} - \tfrac{1}{4}\eta^{ab} H, \qquad H := H^{ab}\eta_{ab}\,, \\ \Box X &:= [X^b, [X_b, X]] \end{aligned}$$

result:

for 4-dim. $\mathcal{M} \subset \mathbb{R}^D$ with $g_{\mu\nu} = G_{\mu\nu}$:

$$Tr\left(2T^{ab}\Box X_a \Box X_b - T^{ab}\Box H_{ab}\right) \sim \frac{2}{(2\pi)^2}\int d^4x \sqrt{g}\, e^{2\sigma} R$$

$$Tr([[X^a, X^c], [X_c, X^b]][X_a, X_b] - 2\Box X^a \Box X^a)$$
$$\sim \frac{1}{(2\pi)^2}\int d^4x \sqrt{g}\, e^{\sigma}\left(\tfrac{1}{2} e^{-\sigma}\theta^{\mu\eta}\theta^{\rho\alpha} R_{\mu\eta\rho\alpha} - 2R + \partial^\mu\sigma\partial_\mu\sigma\right)$$

Blaschke, H.S. CQG 27 (2010)

(cf. Arnlind, Hoppe, Huisken 2010, 2011)

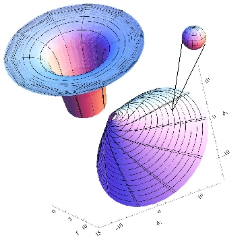
cf. Einstein-Hilbert, pre-geometric

issues for (emergent) gravity

- complicated dynamics, not well understood
- bare M.M. action:
  NC $U(1)$ gauge fields $\partial^\mu F_{\mu\nu} = 0 \quad \Rightarrow \quad R_{\mu\nu}[\bar{G} + h] = 0$
  (Rivelles 2002)
  compactif. extra dim $\Rightarrow$ additional Ricci-flat d.o.f.
- 1-loop
  $\Rightarrow$induced E-H action,
  also modification of bulk metric:
  e.g. $AdS^5 \times S^5$ near stack of branes, cf. DBI, $\mathcal{N} = 4$ SYM
  (Tseytlin, Maldacena ...)
- Minkowski signature:
  either complexified $\theta^{\mu\nu}$
  or G, g have different causality structures

illustrative examples:

- 1) “harmonic branch”
  near-realistic cosmolog. solutions (big bounce)
  (NC) minimal surfaces $\mathcal{M} \subset \mathbb{R}^D$ D. Klammer, H.S. PRL 102 (2009)
  insensitive to vacuum energy, SN Ia without fine-tuning !
- 2) ”Einstein branch“
  Example: Schwarzschild geometry
  Blaschke, H.S. CQG 27 (2010)
  embedding $\mathcal{M}^4 \subset \mathbb{R}^{10}, \quad \theta^{\mu\nu}(x)$

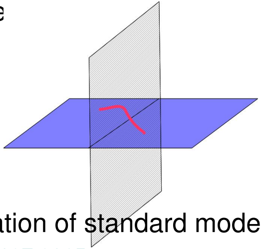


Relation with particle physics

intersecting brane solutions

chiral fermions at intersection = 4D space
(as in string theory)

$$\begin{pmatrix} X^a_{(11)} & \psi_{(12)} \\ \psi_{(21)} & X^a_{(22)} \end{pmatrix}$$

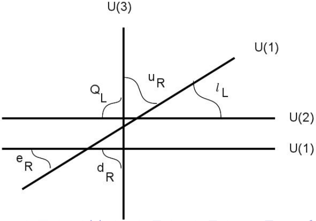


stacks of intersecting branes $\rightarrow$ realization of standard model

A. Chatzistavrakidis, H.S., G. Zoupanos arXiv:1107.0265

cf. string theory
clear-cut, predictive framework

1-loop $\rightarrow$ intersecting branes can form bound system!



realization of standard model

4 intersecting D7 branes $\rightarrow$ $U(3)_C \times U(2)_L \times U(1) \times U(1)$

intersections $M^4 \times K^2_{ab}$, flux on K^2_{ab}, chiral bifund. fermions

Intersection	Representation	Particle	flux
$D_a \cap D_b$	$(\bar{3},2)(-1,1,0,0)$	Q_L	$N'_\beta - N_\beta$
$D_a \cap D_c$	$(\bar{3},1)(-1,0,1,0)$	d_R	$N''_\beta - N_\beta$
$D_a \cap D_d$	$(\bar{3},1)(-1,0,0,1)$	u_R	$N'_\alpha - N_\alpha$
$D_d \cap D_b$	$(1,2)(0,1,0,-1)$	l_L	$N_\gamma - N''_\gamma$
$D_d \cap D_c$	$(1,1)(0,0,1,-1)$	e_R	$N'_\gamma - N''_\gamma$

correct chiral particle spectrum, families from fluxes

1-loop effective action

$\rightarrow$ intersecting branes may form bound state (dep. on flux)

summary and outlook

- matrix models $Tr\Big([X^a, X^b][X_a, X_b] + \overline{\Psi} \not{D} \Psi\Big)$

  dynam. NC spaces (gravity), gauge theory & matter
- NC gauge theory ($\mathcal{N} = 4$ SYM) $\leftrightarrow$ "stringy" (IKKT)

  $\rightarrow$ candidate for quantum theory of fund. interactions
- QFT methods for gravity / "string theory"!
  (integration over geometries)
- intersecting branes $\rightarrow$ particle physics:
  brane-world scenarios
- ... lots of interesting issues!

references

- H. Steinacker, "Emergent Gravity from Noncommutative Gauge Theory". *JHEP* **12** (2007) 049. `[arXiv:0708.2426v1 (hep-th)]`
- H. Steinacker, "Emergent Gravity and Noncommutative Branes from Yang-Mills Matrix Models," *Nucl. Phys.* **B 810**:1-39,2009. arXiv:0806.2032 [hep-th].
- H. Steinacker, "Covariant Field Equations, Gauge Fields and Conservation Laws from Yang-Mills Matrix Models,", *JHEP* **0902**: 044,2009 `[arXiv:0812.3761 [hep-th]].`
- H. Steinacker, "On the Newtonian limit of emergent NC gravity and long-distance corrections," arXiv:0909.4621 [hep-th], *JHEP* **0912**:024,2009.
- D. Klammer and H. Steinacker, "Fermions and noncommutative emergent gravity II: Curved branes in extra dimensions," arXiv:0909.5298 [hep-th]. *JHEP* **100**2:074,2010.
- D. N. Blaschke and H. Steinacker, "Curvature and Gravity Actions for Matrix Models," arXiv:1003.4132 [hep-th].*Class. Quant. Grav.* **27**:165010,2010.
- **H. Steinacker, "Emergent Geometry and Gravity from Matrix Models: an Introduction," arXiv:1003.4134 [hep-th]. topical review, *Class. Quantum Grav.* 27 (2010) 133001.**

$U(1)$ gauge fields as gravitons (fix embedding)

$$G^{\mu\nu}(x) = \overline{\eta}^{\mu\nu} - h^{\mu\nu} \qquad (+O(F^2))$$

$F_{\mu\nu}(x)$ ... $\mathfrak{u}(1)$ field strength

find

$$\boxed{h_{\mu\nu} = \bar\eta_{\nu\nu'}\bar\theta^{\nu'\rho} F_{\rho\mu} + \bar\eta_{\mu\mu'}\bar\theta^{\mu'\eta} F_{\eta\nu} - \tfrac{1}{2}\bar\eta_{\mu\nu}\,(\bar\theta^{\rho\eta}F_{\rho\eta})}$$

... linearized metric fluctuation

e.o.m:

$$\begin{aligned}
[X^\mu,[X^\nu,X^{\mu'}]]\eta_{\mu\mu'} &= 0 \\
\Rightarrow\ \partial^\mu F_{\mu\nu} &= 0 \\
\Rightarrow\ R_{\mu\nu}[G] &= 0 \quad (\partial^\mu h_{\mu\nu} = 0 \ldots \text{harm. gauge})
\end{aligned}$$

cf. Rivelles [hep-th/0212262]

while $R_{\mu\nu\rho\eta} \neq 0$

$\Rightarrow$ on-shell d.o.f. of gravitational waves on Minkowski space

i.e.: NC $U(1)$ on $\mathbb{R}^4_\theta$ as gravitons

Geometrical e.o.m.

assume effective action

$$S = \int d^4x \sqrt{|g|}\,(-2\Lambda^4 + \Lambda_4^2 R) + S_{\rm matter}$$

e.o.m.

$$\begin{aligned}\delta S &= \int d^4x \sqrt{|g|}\,\delta g_{\mu\nu}(-\Lambda^4 g^{\mu\nu} + 8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu}) \\ &= -2\int \delta\phi^i \partial_\mu(\sqrt{|g|}\,(-\Lambda^4 g^{\mu\nu} + 8\pi T^{\mu\nu} - \Lambda_4^2\mathcal{G}^{\mu\nu})\partial_\nu\phi^i)\end{aligned}$$

since $g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu\phi^i\partial_\nu\phi^i$

1. “Einstein branch”

$$\Lambda^4 g^{\mu\nu} + \Lambda_4^2\mathcal{G}^{\mu\nu} = 8\pi T^{\mu\nu}$$

2. “harmonic branch”

$$\Lambda^4 \Box_g \phi = (8\pi T^{\mu\nu} - \Lambda_4^2\mathcal{G}^{\mu\nu})\nabla_\mu\partial_\nu\phi$$

prototype: flat space $\mathbb{R}^4_\theta \subset \mathbb{R}^{10}$, even for $\Lambda \gg 0$!