Superstrings in AdS superbackgrounds and their integrability

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arXiv:1009.3498*, D.S. & L. Wulff*
arXiv:1104.1793, *D.S., A. Tseytlin, L. Wulff & K. Zarembo*

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Principal examples of AdS backgrounds

- Type IIB string on $AdS_5 \times S^5$ – maximally supersymmetric (32 susy) - PSU(2,2|4) isometry gives rise to the AdS_5/CFT_4 correspondence
- Type IIA string on $AdS_4 \times CP^3$ – preserves 24 (of 32) supersymmetries - OSp(6|4) isometry holographically dual to the D=3, N=6 superconformal Chern-Simons theories
- Type IIB strings on $AdS_3 \times S^3 \times T^4$ – preserve 16 (of 32) susy – PSU(1,1|2) $\times$ PSU(1,1|2) give rise to AdS_3/CFT_2 dualities (poorly understood in the case of the RR backgrounds)
- Type IIA and IIB strings on $AdS_2 \times S^2 \times T^6$ – preserve 8 (of 32) susy - PSU(1,1|2) isometry are related to the theory of extremal 4d Reissner-Nordstrom black holes and AdS_2/CFT_1 correspondence (less studied and less understood)
- Other $AdS \times M$ backgrounds with less number of supersymmetries

Green-Schwarz superstring

in a generic supergravity background

$$S = -\frac{1}{2}\int d^2\xi \sqrt{-h}\, h^{ij} G_{ij}(X,\Theta) - \int B_2(X,\Theta)$$

$$Z^{\mathcal{M}} = (X^M, \Theta^{\underline{\alpha}}), \qquad M = 0,1,\ldots,9; \qquad \underline{\alpha} = 1,\ldots,32$$

$B_2(X^M, \Theta^{\underline{\alpha}})$ - worldsheet pullback of the NS-NS 2-form guage field

$h_{ij}(\xi)$ - intrinsic worldsheet metric

$G_{ij} = E_i^A E_j^B \eta_{AB}$ - induced worldsheet metric

$E_i^A = \partial_i Z^{\mathcal{M}}(\xi)\, E_{\mathcal{M}}{}^A(X,\Theta)$ - pullback of the vector supevielbein of $D = 10$ sugra

$A = 0,1,\ldots,9;$

$E^{\underline{\alpha}} = dZ^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{\alpha}}(X,\Theta)$ - spinor supervielbein of $D = 10$ sugra

$$E_M^A(X,\Theta) = e_M^A(X) + \Psi_M(X)\Gamma^A\Theta + \omega_M^{BC}(X)\,\Theta\Gamma^A\Gamma_{BC}\Theta + H_{MBC}(X)\,\Theta\Gamma^A\Gamma^{BC}\Theta$$
$$+ e^{\Phi(X)} F_{BC}(X)\,\Theta\Gamma^A\Gamma^{BC}\Gamma_M\Theta + F_{BCDK}(X)\,\Theta\Gamma^A\Gamma^{BCDK}\Gamma_M\Theta + \cdots$$

Explicit form of the Green-Schwarz superstring action

- The supervielbeins $E^{\mathcal{A}}(X,\Theta)$ and the NS-NS 2-form $B_2(X,\Theta)$ can be found explicitely when the superbackground is a coset superspace K=G/H, as in the case of $AdS_5 \times S^5$ (Metsaev & Tseytlin 98)

$$K = \frac{PSU(2,2|4)}{SO(1,4)\times SO(5)}, \qquad K(X,\Theta) = e^{X^M P_M} e^{\Theta^{\alpha I} Q_{\alpha I}},$$

$$M = 0,1,\ldots,9, \quad \alpha = 1,\ldots,16, \quad I = 1,2$$

$$K^{-1}dK = E^A(X,\Theta)P_A + E^{\alpha I}(X,\Theta)Q_{\alpha I} + \Omega^{AB}(X,\Theta)M_{AB}$$

Sigma-model action on *PSU*(*2,2*|*4*)/SO(1,4)x*SO*(5):

$$S = -\frac{1}{2}\int d^2\xi \left(\sqrt{-h}\,h^{ij}E_i^A E_j^B \eta_{AB} - \varepsilon^{ij} E_i^{\alpha I} C_{\alpha I,\beta J} E_j^{\beta J}\right)$$

Green-Schwarz superstring action in other AdS superbackgrounds

- Type IIA string on $AdS_4 \times CP^3$ – preserves 24 (of 32) supersymmetries - OSp(6|4) isometry.

  The associated supercoset OSp(6|4)/SO(1,3) x U(3) has only 24 fermionic directions. The GS action reduces to the coset sigma-model only upon partial gauge fixing of the fermionic kappa-symmetry by putting to zero 8 fermionic modes. But such a gauge is not always admissible.
  - The complete $AdS_4 \times CP^3$ superspace with 32 fermionic directions is not a supercoset but rather a kind of fermionic fiber bundle over OSp(6|4)/SO(1,3) x U(3)
    J. Gomis, D.S. & L. Wulff, 2008

- Type IIB strings on $AdS_3 \times S^3 \times T^4$ – preserve 16 (of 32) susy – PSU(1,1|2) $\times$ PSU(1,1|2). The associated supercoset PSU(1,1|2) $\times$ PSU(1,1|2)/SO(1,2)xSU(2) has 16 fermionic directions and does not include T^4. Kappa-symmetry can be used to reduce GS action to the supercoset (but not for all the classical configurations of the string)
  - The complete superspace and the GS action are unknown

$AdS_2 \times S^2 \times T^6$ superstrings

- preserve 8 (of 32) susy and have PSU(1,1|2) x $U(1)^6$ isometry. The associated supercoset PSU(1,1|2)/SO(1,1)xU(1) has only 8 fermionic directions and 4 bosonic coordinates.

- No enough (16-parameter) kappa-symmetry to reduce the GS action to the supercoset model. At least 8 string fermionic modes remain which do not belong to the supercoset

- The non-coset fermionic modes and the T^6 – bosonic sector do not decouple from the supercoset model (D.S., A. Tseytlin, L. Wulff & Z. Zarembo, 2011)

- The presence of the non-coset fermions associated with the supersymmetries broken by the AdS backgrounds makes the study of the superstring theory in the non-maximally supersymmetric AdS backgrounds much more complicated than e.g. in the $AdS_5 \times S^5$ case.

  In particular, the proof of the integrability of these theories requires the development of more general (or alternative) methods for the construction of a zero-curvature Lax connection than those used in the case of the supercoset sigma-models.

Classical Integrability of 2d dynamical systems

- The existence of ∞ # of conserved charges (integrals of motion)
- The charges are generated by the Lax connection $\mathscr{L}$

$\mathscr{L}(\xi, z)$ – 2d one-form which depends on a spectral parameter z,

takes values in a symmetry algebra and

has zero curvature: $d\mathscr{L} + \mathscr{L} \wedge \mathscr{L} = 0$ (on the mass-shell)

The integrability is proven if one manages to construct $\mathscr{L}(\xi, z)$

No generic prescription exists how to do this

Classical Integrability of 2d dynamical systems

- G/H supercoset sigma-models with Z_4-grading, e.g.

- $AdS_5 \times S^5$ superstring, $SU(2,2|4)/SO(1,4) \times SO(5)$, *Bena, Roiban, Polchinski '03*

- sigma-models on $\frac{OSp(6|4)}{SO(1,3)\times U(3)}$, $\frac{PSU(1,1|2)}{SO(1,1)\times U(1)}$, ...

Z_4-grading of the Cartan forms:

$$K^{-1}dK = \Omega(x,\vartheta)\, M_0 + E^2(x,\vartheta)\, P_2 + E^1(x,\vartheta)\, Q_1 + E^3(x,\vartheta)\, Q_3$$

$$[M_0,M_0]=M_0, \quad [P_2,P_2]=M_0, \quad \{Q_1,Q_1\}=P_2=\{Q_3,Q_3\}, \quad \{Q_1,Q_3\}=M_0$$

Lax connection:

$$\mathcal{L} = \Omega(x,\vartheta) + \ell_1\, E^2(x,\vartheta) + \ell_2{}^* E^2(x,\vartheta) + \ell_3\, E^1(x,\vartheta) + \ell_4\, E^3(x,\vartheta)$$

$$d\mathcal{L} + \mathcal{L} \wedge \mathcal{L} = 0 \longrightarrow$$

on shell

Coefficients $\ell_i = f_i(z)$ are functions of the spectral parameter

Conditions for the integrability of the GS superstring σ-models on generic superbackgrounds with isometries

(D.S. & L. Wulff, ArXiv:1009.3498; D.S., A. Tseytlin, L. Wulff & K. Zarembo, arXiv:1104.1793)

- Purely bosonic sigma-model should be integrable ⟶ bosonic background is a symmetric space
  - Lax connection is constructed out of the Noether currents of the G/H σ- model (*H. Eichenherr & M. Forger, 1979*)

$$S=\int d^2\xi\,\eta^{ij}e_i^A(X)e_{jA}(X)$$

$$j_{\mathcal{B}}(\xi)=e^A(X)K_A(X)=dX^M(\xi)e_M^A(X)K_A(X)\ \text{- Noether current (}G\text{-valued)}$$

$$K_A(X)\ \text{- Killing vector}$$

$$d*j_{\mathcal{B}}=\partial_i j_{\mathcal{B}}^i=0\ \text{- conserved,}\qquad dj_{\mathcal{B}}+2j_{\mathcal{B}}\wedge j_{\mathcal{B}}=0$$

$$\mathcal{L}_{\mathcal{B}}=\left(\frac{2z^2}{1-z^2}e^A+\frac{2z}{1+z^2}*e^A\right)K_A\quad\Rightarrow\quad d\mathcal{L}_{\mathcal{B}}-\mathcal{L}_{\mathcal{B}}\wedge\mathcal{L}_{\mathcal{B}}=0$$

Conditions for the integrability of the GS superstring σ-models on generic superbackgrounds with isometries

- Noether currents of the Green-Schwarz superstring include fermionic modes $\Theta(\xi)$

$J(X,\Theta) = J_{\mathcal{B}} + J_{susy}$ - generates superisometries $PSU(2,2|4)$, $OSp(6|4)$, $PSU(1,1|2)$, $\ldots$

$$J_{\mathcal{B}}(X,\Theta) = j_{\mathcal{B}}(X) + J_1^A(X,\Theta)K_A(X) + J_2^{AB}(X,\Theta)[K_A(X),K_B(X)]$$

$$J_{susy}(X,\Theta) = J^{\alpha}(X,\Theta)\Xi_{\alpha}(\mathrm{X}) \quad - \quad \text{susy current}, \quad \Xi_{\alpha}(\mathrm{X}) \text{ - Killing spinor}$$

Lax connection:

$$\mathcal{L} = \alpha_1 j_{\mathcal{B}}(X) + \alpha_2 * J_{\mathcal{B}}(X,\Theta) + (\alpha_2)^2 J_2 + \alpha_1\alpha_2 * J_2 + \alpha_2(\beta_1 J_{susy} - \beta_2 * J_{susy}) + \mathcal{O}(\Theta^3)$$

$\alpha_{1,2}(z)$, $\beta_{1,2}(z)$ - functions of the spectral parameter z

Conditions for the integrability of the GS superstring σ-models on generic superbackgrounds with isometries

- Zero-curvature condition versus current relations

$$d\mathcal{L} - \mathcal{L} \wedge \mathcal{L} = 0 \quad \Leftrightarrow \quad d * J_{\mathcal{B}} = d * J_{susy} = 0$$

$$\Uparrow$$

$$dJ_{susy} = -2(J_{\mathcal{B}} \wedge J_{susy} + J_{susy} \wedge J_{\mathcal{B}})$$

$$(\nabla J_2^{AC} - J_1^A \wedge j_{\mathcal{B}}^C)[K_A, K_C] = -J_{susy} \wedge J_{susy}$$

$$J_{\mathcal{B}}(X,\Theta) = j_{\mathcal{B}}(X) + J_1^A(X,\Theta)K_A(X) + J_2^{AB}(X,\Theta)[K_A(X), K_B(X)]$$

These relations hold, at least to the second order in Θ, for the superstrings in $AdS_5 \times S^5$, $AdS_4 \times CP^3$, $AdS_3 \times S^3 \times T^4$, $AdS_2 \times S^2 \times T^6$.

Integrable superstrings in non-supersymmetric backgrounds

- When the target-space susy is completely broken $J_{susy} = 0$

- Conditions on the components of the bosonic current to ensure zero curvature

$$(\nabla J_2^{AC} - J_1^A \wedge j_{\mathcal{B}}^C)[K_A, K_C] = 0$$

$$J_{\mathcal{B}}(X,\Theta) = j_{\mathcal{B}}(X) + J_1^A(X,\Theta)K_A(X) + J_2^{AB}(X,\Theta)[K_A(X), K_B(X)]$$

Example of the integrable superstring sigma-model on AdS_4 with broken susy (*D.S. & L. Wulff 2010*)

D=4 target AdS_4 superspace with 8 fermionic directions and no susy obtained by a consistent truncation of the GS superstring on $AdS_4 \times CP^3$

No general symmetry reasons are known for the integrability of the non-susy sectors of the superstring sigma-model

$AdS_2 \times S^2 \times T^6$ type IIA and IIB superstrings

- The background is supported by RR fluxes
  - Type IIA: F_2 and/or F_4 fluxes
  - Type IIB: F_5 flux

- has 8 of 32 target-space supersymmetries forming $PSU(1,1|2)$ isometry, hence worldsheet fermions have different nature:
  - 8 susy fermions $\theta(\xi)$ belong to the supercoset $\dfrac{PSU(1,1|2)}{SO(1,1)\times U(1)}$
  - 24 “broken susy” fermions $\upsilon(\xi)$

- T^6 sector is coupled to the $AdS_2 \times S^2$ supercoset sector via $\upsilon(\xi)$

$$L_{\mathcal{B}} = \sqrt{-h}h^{ij}\left(e_i^{\underline{a}}(x)e_{j\underline{a}}(x) + \partial_i y^{a'}\partial_j y^{a'}\right)$$

$$\begin{aligned} L^{IIA}_{\text{ferm}} &= i\vartheta\left(\sqrt{-h}h^{ij} - \varepsilon^{ij}\Gamma_{11}\right)e_i^{\underline{a}}(x)\Gamma_{\underline{a}}\nabla_j\vartheta \\ &+ i\vartheta\left(\sqrt{-h}h^{ij} - \varepsilon^{ij}\Gamma_{11}\right)\Gamma_{a'}\nabla_j\upsilon\,\partial_i y^{a'} + i\upsilon\left(\sqrt{-h}h^{ij} - \varepsilon^{ij}\Gamma_{11}\right)\Gamma_{a'}\nabla_j\vartheta\,\partial_i y^{a'} \\ &+ \frac{i}{R}\vartheta\left(\sqrt{-h}h^{ij} - \varepsilon^{ij}\Gamma_{11}\right)e_i^{\underline{a}}\Gamma_{\underline{a}}\not{F}\,\Gamma_{a'}\upsilon\,\partial_j y^{a'} + i\upsilon\left(\sqrt{-h}h^{ij} - \varepsilon^{ij}\Gamma_{11}\right)\not{e}_i\nabla_j\upsilon\partial_j y^{b'} \end{aligned}$$

Conclusion

- In the Green-Schwarz formulation the non-coset fermionic modes and the T^6 oscillations do not decouple

- It is an open problem how to generalize the Bethe ansatz techniques to incorporate the non-susy fermionic modes and the T^6 osciallations for computing the complete spectrum of the $AdS_2 \times S^2 \times T^6$ superstring

- To find out whether a relation exists between the Green-Schwarz formulation of the $AdS_2 \times S^2 \times T^6$ superstring and a hybrid model of *Berkovits et. al.* 2000

  The hybrid model is a direct some of a $\frac{PSU(1,1|2)}{SO(1,1)\times U(1)}$ supercost sigma-model and the RNS superstring on T^6 with the inclusion of a ghost sector to make the theory superconformal at the quantum level

  If exists, the relation should involve a highly non-linear (and probably non-local) field redefinition

- Understanding the quantum spectrum of the $AdS_2 \times S^2 \times T^6$ superstring should shed light on the structure of the CFT_1 dual theory.