Another approach to cosmological term problem

D.V. Soroka[1] **and V.A. Soroka**[2]

[1]**E-mail: dsoroka@kipt.kharkov.ua**
[2]**E-mail: vsoroka@kipt.kharkov.ua**

Kharkov Institute of Physics and Technology

1 Akademicheskaya St., 61108 Kharkov, Ukraine

Abstract

An approach to the cosmological term problem is proposed, using the gauge semi-simple tensor extension of the D-dimensional Poincaré group as a basis.

1. Introduction

Recently, de Azcarraga, Kamimura and Lukierski [1] have proposed an approach to the cosmological term problem, based on the tensor extension of the Poincaré algebra with the generators of rotations M_{ab} and translations P_a [2 - 20]

$$[M_{ab}, M_{cd}] = (g_{ad} M_{bc} + g_{bc} M_{ad}) - (c \leftrightarrow d), \qquad (1.1)$$

$$[M_{ab}, P_c] = g_{bc} P_a - g_{ac} P_b, \qquad (1.2)$$

$$[P_a, P_b] = c Z_{ab}, \qquad (1.3)$$

$$[M_{ab}, Z_{cd}] = (g_{ad} Z_{bc} + g_{bc} Z_{ad}) - (c \leftrightarrow d), \qquad (1.4)$$

$$[P_a, Z_{bc}] = 0, \tag{1.5}$$

$$[Z_{ab}, Z_{cd}] = 0. \tag{1.6}$$

Here Z_{ab} is the tensor generator, g_{ab} is the constant Minkovski metric and c is a certain constant.

In our paper we present another approach to the problem, based on the gauge semi-simple tensor extension of the D-dimensional Poincaré group, whose Lie algebra has the following form [14,17,21]:

$$[Z_{ab}, P_c] = \frac{\Lambda}{3c}(g_{bc}P_a - g_{ac}P_b), \tag{1.7}$$

$$[Z_{ab}, Z_{cd}] = \frac{\Lambda}{3c}[(g_{ad}Z_{bc} + g_{bc}Z_{ad}) - (c \leftrightarrow d)], \tag{1.8}$$

whereas the form of the other permutation relations (1.1)-(1.4) remains unchanged. Λ is some constant.

The Lie algebra given by relations (1.1)-(1.4) and (1.7)-(1.8) has the following quadratic Casimir operator:

$$P^a P_a + cZ^{ab} M_{ba} + \frac{\Lambda}{6} M^{ab} M_{ab} \overset{\text{def}}{=} X_k h^{kl} X_l, \qquad (1.9)$$

where $X_k = \{P_a, Z_{ab}, M_{ab}\}$ is a set of generators for the Lie algebra under consideration (1.1)-(1.4) and (1.7)-(1.8) and the tensor h^{kl} is invariant with respect to the adjoint representation

$$h^{kl} = U^k{}_m U^l{}_n h^{mn}. \qquad (1.10)$$

The inverse tensor h_{kl} $(h_{kl} h^{lm} = \delta_k{}^m)$ is invariant with respect to the co-adjoint representation

$$h_{kl} = h_{mn} U^m{}_k U^n{}_l. \qquad (1.11)$$

2. Gauge-invariant Lagrangian

Let us consider the gauge group corresponding to the Lie algebra (1.1)-(1.4) and (1.7) -(1.8). To this end, we introduce the gauge 1-form

$$A = A^k X_k = dx^\mu \left(e_\mu{}^a P_a + \frac{1}{2} \omega_\mu{}^{ab} M_{ab} + \frac{1}{2} B_\mu{}^{ab} Z_{ab} \right) \qquad (2.1)$$

with the the following gauge transformation:

$$A' = G^{-1} dG + G^{-1} A G, \qquad (2.2)$$

where G is the group element corresponding to the Lie algebra (1.1)-(1.4) and (1.7)-(1.8). Here x^μ are the space-time coordinates, $e_\mu{}^a$ is the vierbein, $\omega_\mu{}^{ab}$ is the spin connection and $B_\mu{}^{ab}$ is the gauge field conforming to the tensor generator Z_{ab}.

The contravariant vector F^k of the field strength 2-form

$$F = F^k X_k = dA + A \wedge A = \frac{1}{2} dx^\mu \wedge dx^\nu F_{\mu\nu} \qquad (2.3)$$

is changed homogeneously under the gauge transformation

$$F'^k X_k = U^k{}_l F^l X_k = G^{-1} F^k X_k G. \qquad (2.4)$$

The field strength

$$F_{\mu\nu} = F_{\mu\nu}{}^k X_k = \partial_{[\mu} A_{\nu]} + [A_\mu, A_\nu] \qquad (2.5)$$

has the following expansion:

$$F_{\mu\nu} = F_{\mu\nu}{}^a P_a + \frac{1}{2} R_{\mu\nu}{}^{ab} M_{ab} + \frac{1}{2} F_{\mu\nu}{}^{ab} Z_{ab}. \qquad (2.6)$$

Here we have

$$F_{\mu\nu}{}^a = T_{\mu\nu}{}^a + \frac{\Lambda}{3c} B_{[\mu}{}^{ab} e_{\nu]b}, \qquad (2.7)$$

where

$$T_{\mu\nu}{}^{a} = \partial_{[\mu} e_{\nu]}{}^{a} + \omega_{[\mu}{}^{ab} e_{\nu]b} \qquad (2.8)$$

is the torsion,

$$R_{\mu\nu}{}^{ab} = \partial_{[\mu} \omega_{\nu]}{}^{ab} + \omega_{[\mu}{}^{ac} \omega_{\nu]c}{}^{b} \qquad (2.9)$$

is the curvature tensor and

$$F_{\mu\nu}{}^{ab} = \partial_{[\mu} B_{\nu]}{}^{ab} + \omega_{[\mu}{}^{c[a} B_{\nu]}{}^{b]}{}_{c} + \frac{\Lambda}{3c} B_{[\mu}{}^{ca} B_{\nu]}{}^{b}{}_{c} + c e_{[\mu}{}^{a} e_{\nu]}{}^{b} \qquad (2.10)$$

is the component corresponding to the tensor generator Z_{ab} .

The invariant Lagrangian is written as:

$$L = -\frac{e}{4} h_{kl} F_{\mu\nu}{}^{l} F_{\rho\lambda}{}^{k} g^{\mu\rho} g^{\nu\lambda} \tag{2.11}$$

$$= \frac{e}{4}\left(\frac{1}{c} R_{\mu\nu}{}^{ab} F_{\rho\lambda;ab} + \frac{\Lambda}{6c^2} F_{\mu\nu}{}^{ab} F_{\rho\lambda;ab} - F_{\mu\nu}{}^{a} F_{\rho\lambda;a} \right) g^{\mu\rho} g^{\nu\lambda}.$$

With the use of the relations (2.7)-(2.10) the Lagrangian L (2.11) can be expressed in a more detailed form, which includes the contribution of three terms

$$\left(\frac{1}{2}R+\Lambda-\frac{1}{4}T_{\mu\nu}{}^{a}T^{\mu\nu}{}_{a}\right)e \qquad (2.12)$$

plus a few additional cumbersome terms, whose coefficients are dependent on the negative degrees of the constant c. In expressions (2.11), (2.12) $R=R_{\mu\nu}{}^{ab}e_{a}{}^{\mu}e_{b}{}^{\nu}$ is a scalar curvature, $g^{\mu\nu}=g^{ab}e_{a}{}^{\mu}e_{b}{}^{\nu}$ is a metric tensor, $e=\det e_{\mu}{}^{a}$ is the determinant of the vierbein and Λ is a cosmological constant.

Note that in the limit $c\to\infty$, the algebra (1.1)-(1.4) and (1.7)-(1.8) is reduced by rescaling

$$P_a \to \sqrt{c}P_a \qquad (2.13)$$

to the algebra (1.1)-(1.6) with $c = 1$. In this case, the invariant Lagrangian is

$$\mathcal{L} = \frac{e}{4}\left(2R + R_{\mu\nu}{}^{ab}\tilde{F}^{\mu\nu}{}_{ab} - T_{\mu\nu}{}^{a}T^{\mu\nu}{}_{a}\right), \tag{2.14}$$

where

$$\tilde{F}_{\mu\nu}{}^{ab} = \partial_{[\mu}B_{\nu]}{}^{ab} + \omega_{[\mu}{}^{c[a}B_{\nu]}{}^{b]}{}_{c}. \tag{2.15}$$

3. Another basis

The extended Poincaré algebra (1.1)-(1.4) and (1.7)-(1.8) can be rewritten in the different basis [14,17,21]:

$$[N_{ab}, N_{cd}] = (g_{ad}N_{bc} + g_{bc}N_{ad}) - (c \leftrightarrow d), \qquad (3.1)$$

$$[L_{AB}, L_{CD}] = (g_{AD}L_{BC} + g_{BC}L_{AD}) - (C \leftrightarrow D), \qquad (3.2)$$

$$[N_{ab}, L_{CD}] = 0, \qquad (3.3)$$

where the metric tensor g_{AB} has the following nonzero components:

$$g_{AB} = \{g_{ab}, g_{D+1D+1} = -1\}. \qquad (3.4)$$

The generators

$$N_{ab} = M_{ab} - \frac{3c}{\Lambda} Z_{ab} \qquad (3.5)$$

form the Lorentz algebra $so(D-1,1)$, and the generators

$$L_{AB} = -L_{BA} = \left\{ L_{ab} = \frac{3c}{\Lambda} Z_{ab}, L_{aD+1} = \sqrt{\frac{3}{\Lambda}} P_a \right\} \qquad (3.6)$$

form the algebra $so(D-1,2)$. The algebra (3.1)-(3.3) is a direct sum $so(D-1,1) \oplus so(D-1,2)$ of the D-dimensional Lorentz algebra and D -dimensional anti-de Sitter algebra, correspondingly. The gauge 1-form (2.1) in this basis takes the form

$$A = \frac{1}{2} dx^{\mu} (\omega_{\mu}{}^{ab} N_{ab} + \Omega_{\mu}{}^{AB} L_{AB}), \qquad (3.7)$$

where

$$\Omega_{\mu}{}^{ab} = \omega_{\mu}{}^{ab} + \frac{\Lambda}{3c} B_{\mu}{}^{ab} \qquad (3.8)$$

and

$$\Omega_{\mu}{}^{aD+1} = \sqrt{\frac{\Lambda}{3}} e_{\mu}{}^{a}. \qquad (3.9)$$

The field strength 2-form (2.5) assumes the following form:

$$F_{\mu\nu} = \frac{1}{2}\left(R_{\mu\nu}{}^{ab} N_{ab} + \mathcal{F}_{\mu\nu}{}^{AB} L_{AB}\right), \tag{3.10}$$

where the quantity

$$\mathcal{F}_{\mu\nu}{}^{AB} = \partial_{[\mu}\Omega_{\nu]}{}^{AB} + \Omega_{[\mu}{}^{AC}\Omega_{\nu]C}{}^{B} \tag{3.11}$$

comprises the components

$$\mathcal{F}_{\mu\nu}{}^{ab} = R_{\mu\nu}{}^{ab} + \frac{\Lambda}{3c} F_{\mu\nu}{}^{ab} \tag{3.12}$$

and

$$\mathcal{F}_{\mu\nu}{}^{aD+1} = \sqrt{\frac{\Lambda}{3}} F_{\mu\nu}{}^{a}. \tag{3.13}$$

The quadratic Casimir operator (1.9) is expressed in the following way:

$$\frac{\Lambda}{6}(N_{ab}N^{ab} - L_{AB}L^{AB}) = \frac{1}{4}(N_{ab}h^{ab;cd}N_{cd} + L_{AB}h^{AB;CD}L_{CD}), \quad (3.14)$$

where

$$h^{ab;cd} = \frac{\Lambda}{3}(g^{ac}g^{bd} - g^{ad}g^{bc}) \quad (3.15)$$

and

$$h^{AB;CD} = \frac{\Lambda}{3}(g^{AD}g^{BC} - g^{AC}g^{BD}). \quad (3.16)$$

With the use of the inverse tensor components

$$h_{ab;cd} = \frac{3}{\Lambda}(g_{ac}g_{bd} - g_{ad}g_{bc}) \quad (3.17)$$

and

$$h_{AB;CD} = \frac{3}{\Lambda}(g_{AD}g_{BC} - g_{AC}g_{BD}) \tag{3.18}$$

the invariant Lagrangian L (2.11) takes the following form:

$$\begin{aligned} L &= -\frac{e}{16}\left(h_{ab;cd}R_{\mu\nu}{}^{ab}R^{\mu\nu;cd} + h_{AB;CD}\mathcal{F}_{\mu\nu}{}^{AB}\mathcal{F}^{\mu\nu;CD}\right) \\ &= \frac{3e}{8\Lambda}\left(\mathcal{F}_{\mu\nu}{}^{AB}\mathcal{F}^{\mu\nu}{}_{AB} - R_{\mu\nu}{}^{ab}R^{\mu\nu}{}_{ab}\right). \end{aligned} \tag{3.19}$$

Finally, with the help of relations (2.7), (2.10), (3.12) and (3.13) we come again to expression (2.11) for the Lagrangian L.

4. Resume

Thus, we have presented another approach to the cosmological term problem in comparison with the possibility described in paper [1]. Our approach is based on the gauge semi-simple tensor extension of the Poincaré group.

Acknowledgments

We are greatly indebted to the referee for the constructive advices. We also thank A.A. Zheltukhin for the useful discussion.

References

[1] J.A. de Azcarraga, K. Kamimura and J. Lukierski, [arXiv:1012.4402[hep-th]].

[2] H. Bacry, P. Combe and J.L. Richard, Nuovo Cim. A 67 (1970) 267.

[3] R. Schrader, Fortsch. Phys. 20 (1972) 701.

[4] J. Beckers and V. Hussin, J. Math. Phys. 24 (1983) 1295.

[5] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, Ann. Phys. 185 (1988) 1.

[6] A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, Ann. Phys. 185 (1988) 22.

[7] D. Cangemi and R. Jackiw, Phys. Rev. Lett. 69 (1992) 233.

[8] D. Cangemi, Phys. Lett. B297 (1992) 261 [arXiv:gr-qc/9207004].

[9] D.V. Soroka and V.A. Soroka, Phys. Lett. B607 (2005) 302 [arXiv:hep-th/0410012].

[10] S.A. Duplij, D.V. Soroka and V.A. Soroka, J. Kharkov National Univ. No. 664 (2005), Physical series “Nuclei, Particles, Fields”, Issue 2/27/, p. 12.

[11] S.A. Duplij, D.V. Soroka and V.A. Soroka, J. Zhejiang Univ. SCIENCE A 7 (2006) 629.

[12] D.V. Soroka and V.A. Soroka, Problems of Atomic Science and Technology 3(1) (2007) 76 [arXiv:hep-th/0508141].

[13] D.V. Soroka and V.A. Soroka, Multiplet containing components with different masses. A contribution to the Proceedings of the International Workshop "Supersymmetries and Quantum Symmetries" (SQS’09) July 29 - August 3, 2009, JINR, Dubna, Russia, [arXiv:0909.3624[hep-th]].

[14] D.V. Soroka and V.A. Soroka, Adv. High Energy Phys. 2009 (2009) 234147 [arXiv:hep-th/0605251].

[15] S. Bonanos and J. Gomis, J. Phys. A: Math. Theor. 42 (2009) 145206 [arXiv:0808.2243[hep-th]].

[16] S. Bonanos and J. Gomis, J. Phys. A: Math. Theor. 43 (2010) 015201 [arXiv:0812.4140[hep-th]].

[17] J. Gomis, K. Kamimura and J. Lukierski, JHEP 08 (2009) 039 [arXiv:0910.0326[hep-th]].

[18] G.W. Gibbons, J. Gomis and C.N. Pope, Phys. Rev. D82 (2010) 065002 [arXiv:0910.3220[hep-th]].

[19] S. Bonanos, J. Gomis, K. Kamimura and J. Lukierski, Phys. Rev. Lett. 104 (2010) 090401 [arXiv:0911.5072[hep-th]].

[20] J. Lukierski, [arXiv:1007.3405[hep-th]].

[21] D.V. Soroka and V.A. Soroka, [arXiv:1004.3194[hep-th]].

Supersymmetry among Regge trajectories

D.V. Soroka[1] **and V.A. Soroka**[2]

[1]**E-mail: dsoroka@kipt.kharkov.ua**
[2]**E-mail: vsoroka@kipt.kharkov.ua**

Kharkov Institute of Physics and Technology
1 Akademicheskaya St., 61108 Kharkov, Ukraine

Abstract

A supermultiplet with components lying on the doublet of the Regge trajectories is proposed.

- **1. Introduction**
- **2. Superalgebra**
- **3. Supermultiplet**
- **4. Resume**

1. Introduction

A few years ago in our papers [12,13] a multiplet with components, having the different values of the mass and angular momentum, has been introduced. It is shown that components of this multiplet go into the Regge trajectory.

In the present paper we give a supersymmetric generalization of these results. We build up the supermultiplet, whose components lie on the doublet of the Regge trajectories.

2. Superalgebra

The Lie superalgebra (permutation relations with nonzero right hand side) is [9 – 11]

$$\left[P_+, P_-\right] = -2Z \ , \qquad \left[J, P_\pm\right] = \pm P_\pm$$

$$\left\{Q_1, Q_2\right\} = 2iaZ \ , \qquad (1)$$

$$\left[J, Q_2\right] = \frac{1}{2} Q_2, \qquad \left[J, Q_1\right] = -\frac{1}{2} Q_1.$$

$P_\pm = P_x \pm P_t,\ \ Q_\alpha$ are step-type operators.

Conditions for the "vacuum" state Ψ are

$$P_- \Psi = 0 \ , \qquad Q_1 \Psi = 0 \ ,$$

$$Z\,\Psi = z\,\Psi\,, \qquad J\,\Psi = j\,\Psi\,. \tag{2}$$

The representation for generators is

$$P_- = \partial_{x_-}\,, \qquad P_+ = 2x_-\partial_y - \partial_{x_+}\,, \qquad Z = \partial_y$$

$$J = x_-\partial_{x_-} - x_+\partial_{x_+} + \frac{1}{2}\left(\theta^1\partial_{\theta^1} - \theta^2\partial_{\theta^2}\right), \tag{3}$$

$$Q_1 = -\partial_{\theta^1}\,, \qquad Q_2 = -\partial_{\theta^2} - 2ia\theta^1\partial_y\,.$$

The “vacuum” state has the following form

$$\Psi = \left(\,b + c\;x_+^{-1/2}\theta^2\right) x_+^{-j} e^{zy} \tag{4}$$

where $b\,,\;c$ are arbitrary constants.

3. Supermultiplet

The states of the infinite dimensional supermultiplet are

$$\Psi_k{}^l = (Q_2)^l P_+{}^k \Psi \ ; \ \ l = 0,\ 1; \ \ k = 0,\ 1,\ 2,\ \dots \ . \qquad (5)$$

Supersymmetry transformations:

$$\begin{cases} Q_1 \Psi_k{}^l = 2iazl \Psi_k{}^{1-l}, \\ Q_2 \Psi_k{}^l = (1-l) \Psi_k{}^{1+l}. \end{cases} \qquad (6)$$

The mass square operator is

$$M^2 = P_+ P_- + Z \ . \qquad (7)$$

Eigenvalues of the operators J and M^2 are:

$$J\Psi_k^{\ l} = \mathscr{J}_l\,\Psi_k^{\ l} \ ,$$
$$M^2\Psi_k^{\ l} = \mathscr{M}^2\Psi_k^{\ l} \ , \qquad (8)$$

where

$$\mathscr{J}_l = j + \frac{l}{2} + k \ ,$$
$$\mathscr{M}^2 = (2k+1)z \ . \qquad (9)$$

By excluding k from (9) we come to two Regge trajectories

$$\mathscr{J}_l = \alpha_l(0) + \alpha' \mathscr{M}^2 \qquad (10)$$

with parameters

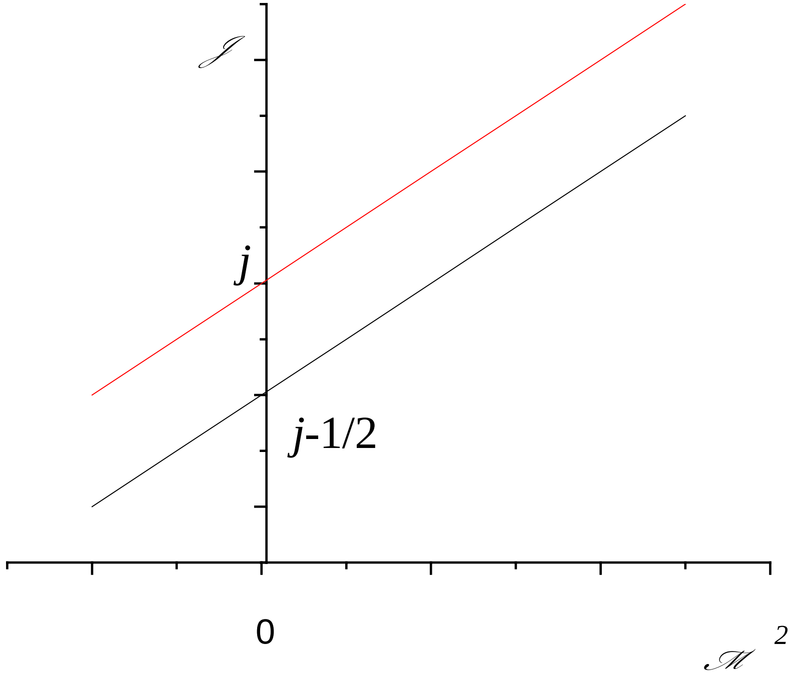
$$\alpha_l(0) = j + \frac{l-1}{2} \ , \ \ \alpha' = \frac{1}{2z} \ . \qquad (11)$$

The supermultiplet consists of the doublet of the Regge trajectories

$$\mathscr{J}_0 = j - \frac{1}{2} + \frac{\mathscr{M}^2}{2z}$$

(12)

$$\mathscr{J}_1 = j + \frac{\mathscr{M}^2}{2z} \quad .$$



4. Resume

Thus, we have constructed the supermultiplet, which consists of two Regge trajectories.

Thank you for attention!