Cubic interaction vertex for higher-spin fields with external electromagnetic field

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Introduction and aim

- There is the general problem of construction of the consistent higher-spin fields theory
- To present time free Lagrangian formulation for higher-spin fields theory is studied well enough and developed the different formalisms: metric-like, frame-like, geometric approach, BRST approach.
- It is known that construction the interacting Lagrangian for higher-spin fields is consistent in AdS space. However, if one exclude gravitational interaction of gauge higher-spin fields one can construct the consistent cubic interaction vertices in flat space.
- In the framework of such consideration we study some aspects of interaction when higher-spin fields coupling to external electromagnetic field.

Aim: The construction of cubic interaction vertex for massless arbitrary integer spin fields coupling to external e/m field

The problem is formulated and discussed in collaboration with
Yu.M. Zinoviev, I.L. Buchbinder

The gauge-invariant description as basic for the investigation of interactions

- For the massless fields the gauge-invariant description is the only possibility to work in manifestly lorenz-covariant form
- The requirement of saving gauge invariance (albeit modified) after including interaction strongly restricts possible form of interaction Lagrangian.
- The gauge-invariant description, which being combined with the perturbation theory in external field, gives a constructive approach to the construction of consistent interaction

Constructive approach to the construction of interaction

- Lagrangian can be represented as series in electromagnetic field strength:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + ...$$

- Analogically for gauge transformations :

$$\delta = \delta_0 + \delta_1 + ...$$

- Then the variation of Lagrangian

$$\delta\mathcal{L} = (\delta_0 + \delta_1 + ...)(\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + ...)$$

is decomposed into the series and demands equality to zero in each order

$$\begin{aligned} \delta_0\mathcal{L}_0 &= 0 \\ \delta_0\mathcal{L}_1 + \delta_1\mathcal{L}_0 &= 0 \\ &... \end{aligned}$$

Free theory for massless charged field

We will work with the real fields, to describe the charged spin-s field we introduce doublet of the symmetric rank-s double traceless tensors $\Phi_s{}^i$, $\tilde{\tilde{\Phi}}_s{}^i = 0$, $i = 1, 2$.
Compact notation

$$\Phi_s{}^i = \Phi_{\mu_1\mu_2...\mu_s}{}^i, \quad (\partial\Phi)_{s-1}{}^i = \partial^{\mu_1}\Phi_{\mu_1 s-1}{}^i, \quad \tilde{\Phi}_{s-2}{}^i = \eta^{\mu_1\mu_2}\Phi_{\mu_1\mu_2 s-2}{}^i$$

$$\eta^{\mu_1\mu_2} = diag(+, -, -, -, ...)$$

- Free Lagrangian is well known and has the form

$$\mathcal{L}_0 = (-1)^s \tfrac{1}{2}[\partial^\mu\Phi^{s,\,i}\partial_\mu\Phi_s{}^i - s(\partial\Phi)^{s-1,\,i}(\partial\Phi)_{s-1}{}^i + \\ + s(s-1)(\partial\Phi)^{\mu_1 s-2,\,i}\partial_{\mu_1}\tilde{\Phi}_{s-2}{}^i - \tfrac{s(s-1)}{2}\partial^\mu\tilde{\Phi}^{s-2,\,i}\partial_\mu\tilde{\Phi}_{s-2}{}^i - \\ - \tfrac{s(s-1)(s-2)}{4}(\partial\tilde{\Phi})^{s-3,\,i}(\partial\tilde{\Phi})_{s-3}{}^i] \qquad (1)$$

- It is invariant under ordinary gauge transformations

$$\delta_0\Phi_s{}^i = \partial_{(\mu_1}\xi_{s-1)}{}^i, \quad \xi^\alpha{}_{\alpha s-3}{}^i = 0$$

- The most general anzac for cubic Lagrangian can be written

$$\mathcal{L}_1 = (-1)^s \varepsilon^{ij} F^{\alpha\beta}[a_1 \partial^\mu \Phi_\alpha{}^{s-1,\, i} \partial_\mu \Phi_{\beta s-1}{}^j + a_2 (\partial\Phi)_\alpha{}^{s-2,\, i} (\partial\Phi)_{\beta s-2}{}^j + \\ + a_3 \partial_\alpha \Phi_\beta{}^{s-1,\, i} (\partial\Phi)_{s-1}{}^j + a_4 (\partial\Phi)_\alpha{}^{s-2,\, i} \partial_\beta \tilde{\Phi}_{s-2}{}^j + \\ + a_5 (\partial\Phi)_\alpha{}^{\mu_1 s-3,\, i} \partial_{\mu_1} \tilde{\Phi}_{\beta s-3}{}^j + a_6 \partial^\mu \tilde{\Phi}_\alpha{}^{s-3,\, i} \partial_\mu \tilde{\Phi}_{\beta s-3}{}^j + \\ + a_7 \partial_\alpha \tilde{\Phi}_\beta{}^{s-3,\, i} (\partial\tilde{\Phi})_{s-3}{}^j + a_8 (\partial\tilde{\Phi})_\alpha{}^{s-4} (\partial\tilde{\Phi})_{\beta s-4}{}^j] \qquad (2)$$

- Linear correction to the gauge transformations

$$\delta_1 \Phi_s{}^i = \gamma \varepsilon^{ij} g_{(\mu_1\mu_2} F^{\alpha\beta} \partial_\alpha \xi_{\beta s-2)}{}^j$$

ε^{ij} is the antisymmetric symbol

- The requirement of gauge invariance in linear approximation over $F_{\mu\nu}$

$$\delta\mathcal{L} = (\delta_0 + \delta_1)(\mathcal{L}_0 + \mathcal{L}_1) = \delta_0\mathcal{L}_0 + (\delta_0\mathcal{L}_1 + \delta_1\mathcal{L}_0) + \delta_1\mathcal{L}_1 = 0$$

The gauge invariance in linear approximation

- Thus we have condition for gauge invariance in linear approximation over $F_{\mu\nu}$

$$\delta_0 \mathcal{L}_1 + \delta_1 \mathcal{L}_0 = 0$$

that give us the system of algebraic linear equations. The solution of this system is

$$a_3 = 2a_1 = \frac{1}{2}\gamma s(d + 2s - 6)$$

$$a_4 = -2a_2 = \frac{1}{2}\gamma s(s-1)(d + 2s - 6)$$

$$a_4 = -2a_6 = 2a_7 = \frac{1}{4}\gamma s(s-1)(s-2)(d + 2s - 6)$$

$$a_8 = -\frac{1}{16}\gamma s(s-1)(s-2)(s-3)(d + 2s - 6)$$

- γ is the arbitrary parameter.

Summary

- Thus, using gauge invariant description, we have constructed consistent interaction for massless arbitrary integer spin field with external electromagnetic field in linear approximation.
- Lagrangian can be written in the form

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int} \tag{3}$$

$\mathcal{L}_{free}$ is free lagrangian (1), $\mathcal{L}_{int}$ - cubic interacting Lagrangian in external electromagnetic field (2).

$$\delta\Phi_s{}^i = \partial_{(\mu_1}\xi_{s-1)}{}^i + \gamma\varepsilon^{ij} g_{(\mu_1\mu_2} F^{\alpha\beta}\partial_\alpha\xi_{\beta s-2)}{}^j$$

In a similar manner can be constructed interaction for massive higher-spin fields theory, where the discussed above vertex play the crucial role

THANKS FOR ATTENTION!