Walls of a massive Kähler sigma model on SO(2N)/U(N)

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published in PRD83(2011)125003

SQS' 2011

July 18, 2011

Plan

- SO(2N)/U(N) as a gauge theory [Higashijima, Nitta 00]
- Moduli matrices for Grassmannian manifold [Isozumi, Nitta, Ohashi, Sakai 04]
- Moduli matrices for SO(2N)/U(N)
- Walls in SO(6)/U(3)

SO(2N)/U(N)

- $\mathcal{N} = 1$ SO(2N)/U(N) in 4D as a gauge theory [Higashijima, Nitta 00]

- Bosonic Lagrangian for a massive SO(2N)/U(N) in 3D

$$\begin{aligned}\mathcal{L}_{\text{bos } 3D} \quad &= \quad -|D_m\phi_a{}^i|^2 - |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b\phi_b{}^i|^2 + |F_a{}^i|^2 + \frac{1}{2}(D_a{}^b\phi_b{}^i\bar{\phi}_i{}^a - D_a{}^a) \\ &\quad + \Big((F_0)^{ab}\phi_b{}^i J_{ij}\phi^{\mathrm{T}j}{}_a + (\phi_0)^{ab}F_b{}^i J_{ij}\phi^{\mathrm{T}j}{}_a + (\phi_0)^{ab}\phi_b{}^i J_{ij}F^{\mathrm{T}j}{}_a + \text{c.c.} \Big)\end{aligned}$$

$$J = \mathbf{1} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad M_j{}^i = \text{diag}(m_1, m_2, \cdots, m_N) \otimes \sigma_3$$

$$\Sigma = \text{diag}(\Sigma_1, \Sigma_2, \cdots, \Sigma_N)$$

- Vacuum condition

$$\phi_a{}^j M_j{}^i - i\Sigma_a{}^b\phi_b{}^i = 0, \quad (\phi_0)^{ab}\phi_b{}^i = 0$$

- Constraints

$$\phi_a{}^i\bar{\phi}_i{}^b - \delta_a{}^b = 0, \quad \phi_a{}^i J_{ij}\phi^{\mathrm{T}j}{}_b = 0$$

Moduli matrices H_0

[Isozumi, Nitta, Ohashi, Sakai 04]

- BPS equation

$$D\phi_a{}^i - (\phi_a^i M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0$$

Assumption

– Fields : static, depends only on the $x_1 \equiv x$

– Poincaré invariance on the two-dimensional world volume of walls to set $v_0 = v_2 = 0$

- Introduce $S_a{}^b(x)$ and $f_a{}^i(x)$ defined by

$$\Sigma_a{}^b - iv_a{}^b = (S^{-1}\partial S)_a{}^b, \quad \phi_a{}^i = (S^{-1})_a{}^b f_b{}^i$$

then the BPS equation and the solutions are

$$\partial f_a{}^i = f_a{}^j M_j{}^i, \quad f_a{}^i = H_{0a}{}^j (e^{Mx})_j{}^i$$

- BPS solution

$$\phi_a{}^i = (S^{-1})_a{}^b H_{0b}{}^j (e^{Mx})_j{}^i$$

- Everything is invariant under

$$S'_a{}^b = V_a{}^c S_c{}^b, \quad H'_{0a}{}^i = V_a{}^c H_{0c}{}^i, \quad V \in GL(N, \mathbf{C})$$

Moduli matrices H_0

Using $\phi_a{}^i = (S^{-1})_a{}^b H_{0b}{}^j (e^{Mx})_j{}^i$

- Vacuum condition : $\phi_a{}^j M_j{}^i - i\Sigma_a{}^b \phi_b{}^i = 0$

$$(\Sigma_1, \Sigma_2, \Sigma_3, \cdots, \Sigma_N) = (\pm m_1, \pm m_2, \pm m_3, \cdots, \pm m_N) \to 2^{N-1} H_0{}' \mathrm{s}$$

Parity has been removed.

- World volume symmetry

$$S'_a{}^b = V_a{}^c S_c{}^b, \quad H'_{0a}{}^i = V_a{}^c H_{0c}{}^i, \quad V \in GL(N, \mathbf{C})$$

The moduli matrix space is an equivalent class of the sets of (S, H_0) defined by V.

- D-term constraint

$$\phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0 \to H_{0a}{}^i (e^{2Mx})_i{}^j H^\dagger_{0j}{}^b = (S\bar{S})_a{}^b \equiv \Omega_a{}^b$$

- F-term constraint

$$\phi_a{}^i J_{ij} \phi^{\mathrm{T}j}{}_b = 0 \to H_{0a}{}^i J_{ij} H^{\mathrm{T}j}{}_b = 0$$

Moduli matrices for walls in the Grassmannian manifold

[Isozumi, Nitta, Ohashi, Sakai 04]

$$S'^{\ b}_{a} = V_a^{\ c} S_c^{\ b}, \quad H'^{\ i}_{0a} = V_a^{\ c} H_{0c}^{\ i}, \quad V \in GL(N, \mathbf{C})$$

$$H_{0a}^{\ i} (e^{2Mx})_i^{\ j} H^{\dagger \ b}_{0j} = (S\bar{S})_a^{\ b} \equiv \Omega_a^{\ b}$$

Walls are constructed algebraically from elementary walls.

- Elementary walls

  : $H_{0\langle A \leftarrow B \rangle} = H_{0\langle A \rangle} e^{a_i(r)}$, $a_i(r) \equiv e^r a_i (r \in \mathbf{C})$

  $\langle A \rangle$ and $\langle B \rangle$ are the vacua in the flavor i and $i+1$ respectively in the same color.

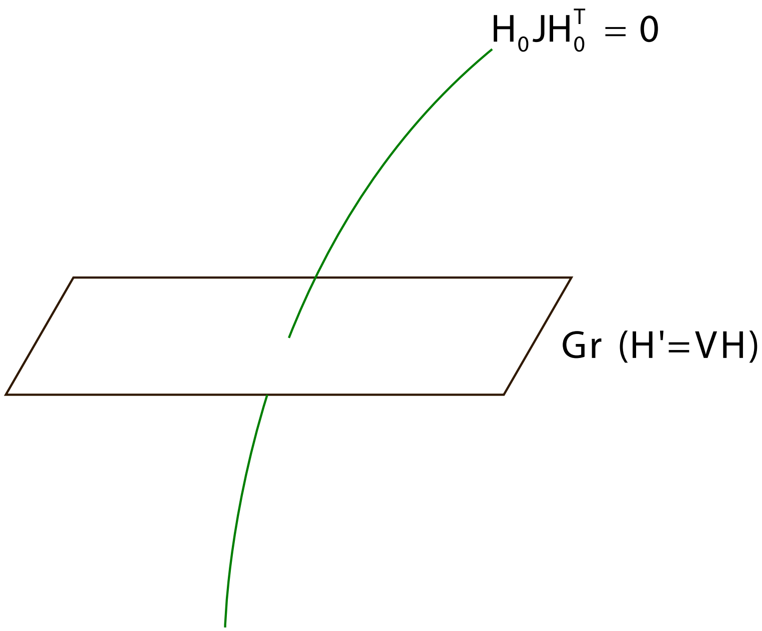
$$[cM, a_i] = c(m_i - m_{i+1}) a_i = T_{\langle i \leftarrow i+1 \rangle} a_i$$

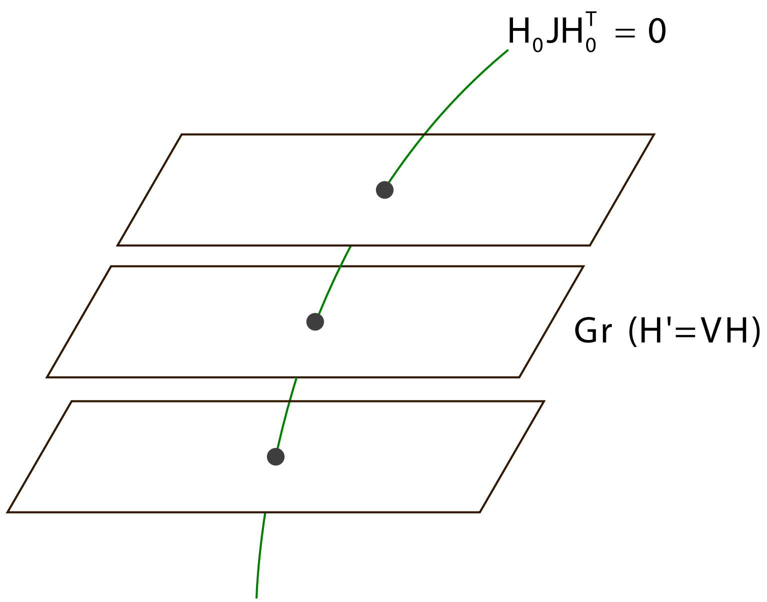
The a_i has a nonzero component only in the $(i, i+1)$-th element.

This cannot be defined in the SO(2N)/U(N) manifold.

We introduce an additional element with an opposite sign into the a_i.

What it means that· · ·





Walls in SO(6)/U(3)

- Vacua

$$H_{0\langle 1\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad H_{0\langle 2\rangle} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$H_{0\langle 3\rangle} = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad H_{0\langle 4\rangle} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Operators generating elementary walls

$$a_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad a_2 = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$a_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Walls in SO(6)/U(3)

- Single walls
  - Elementary walls ($a_i(r) \equiv e^r a_i$)

$$H_{0\langle 1\leftarrow 2\rangle} = H_{0\langle 1\rangle} e^{a_1(r_1)}, \quad H_{0\langle 2\leftarrow 3\rangle} = H_{0\langle 2\rangle} e^{a_2(r_1)}, \quad H_{0\langle 3\leftarrow 4\rangle} = H_{0\langle 3\rangle} e^{a_3(r_1)}$$

  - Compressed walls
    * level one

$$E_1 = [a_1, a_2] \neq 0, \quad E_2 = [a_2, a_3] \neq 0$$

$$H_{0\langle 1\leftarrow 3\rangle} = H_{0\langle 1\rangle} e^{E_1(r_1)}, \quad H_{0\langle 2\leftarrow 4\rangle} = H_{0\langle 2\rangle} e^{E_2(r_1)}$$

    * level two

$$E_3 = [a_1, E_2] = [E_1, a_3] \neq 0$$

$$H_{0\langle 1\leftarrow 7\rangle} = H_{0\langle 1\rangle} e^{E_3(r_1)}$$

Walls in SO(6)/U(3)

- Multiwalls
  - Double walls

$$H_{0\langle 1\leftarrow 2\leftarrow 3\rangle} = H_{0\langle 1\leftarrow 2\rangle} e^{a_2(r_2)}, \quad H_{0\langle 2\leftarrow 3\leftarrow 4\rangle} = H_{0\langle 2\leftarrow 3\rangle} e^{a_3(r_2)}$$

$$H_{0\langle 1\leftarrow 2\leftarrow 4\rangle} = H_{0\langle 1\leftarrow 2\rangle} e^{E_2(r_2)}, \quad H_{0\langle 1\leftarrow 3\leftarrow 4\rangle} = H_{0\langle 1\leftarrow 3\rangle} e^{a_3(r_2)}$$

  - Triple walls

$$H_{0\langle 1\leftarrow 2\leftarrow 3\leftarrow 4\rangle} = H_{0\langle 1\leftarrow 2\leftarrow 3\rangle} e^{a_3(r_3)}$$

Summay

- Walls of a massive Kähler sigma model on SO(2N)/U(N) are studied in the moduli matrix approach.

- 2^{N-1} discrete vacua are observed.

- Elementary walls in the SO(2N)/U(N) manifold can be defined by generating operators with an extra unit component.

- Compressed walls, multiwalls are obtained.

Summay

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Thank you for your attention.