$\mathcal{N}=4$ SYM low-energy effective action in $\mathcal{N}=4$ harmonic superspace

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References

D.V. Belyaev, I.B. Samsonov, JHEP 1104 (2011) 112, arXiv:1103.5070
D.V. Belyaev, I.B. Samsonov, arXiv:1106.0611

$\mathcal{N}=4$ $d=4$ gauge multiplet: $\Phi=(X_A,\Psi^N_\alpha,A_m)$,

- X_A, $A=1,2,3,4,5,6$ six real scalars,
- Ψ^N_α, $N=1,2,3,4$ four Weyl spinors
- A_m one vector field with Maxwell field strength F_{mn}

Low-energy effective action

$$\Gamma=\Gamma[\Phi]$$

- In Coulomb branch, $\langle X_A\rangle\neq 0$
  $\Rightarrow$ gauge symmetry SU(n) is broken down spontaneously to $[\mathrm{U(1)}]^{n-1}$
- Low-energy effective action depends only on the massless fields, $\Phi=(X_A,\Phi^N_\alpha,A_m)$ – massless fields
- We are interested in $\Gamma[\Phi]$ on-shell, i.e., on classical equations of motion,

$$\Box X_A=0\,,\quad \partial\!\!\!/\Psi^N=0\,,\quad \partial^m F_{mn}=0$$

Look for $\Gamma[\Phi]$ within the derivative expansion,

$$\begin{aligned}
\Gamma[\Phi] &= \Gamma_0 + \Gamma_2 + \Gamma_4 + \Gamma_6 + \dots \\
\Gamma_0 &= \int d^4x\, \mathcal{L}_{\rm eff}(\Phi) \equiv 0 \qquad \text{no effective potential} \\
\Gamma_2 &= S_2 = \int d^4x\, \mathcal{L}_2(\partial_m \Phi \partial^m \Phi) \qquad \text{free classical action} \\
\Gamma_4 &= \int d^4x\, \mathcal{L}_4(\partial^m\Phi\partial^n\Phi\partial^r\Phi\partial^s\Phi) \qquad \text{first non-trivial quantum contribution}
\end{aligned}$$

Our aim

Find an expression for $\Gamma_4[\Phi]$ in some superspace, which

1. Depends on all fields X_A, Ψ^N, A_m
2. Contains no more than four derivatives of these fields
3. Is explicitly $\mathcal{N} = 4$ supersymmetric
4. is at least scale invariant (may be superconformal)

Different superspace approaches

- $\mathcal{N}=0$ No full expression for Γ_4 is known; only some terms were found, e.g.,

$$F^4/X^4 \equiv \frac{1}{(8\pi)^2}\int d^4x \frac{(F_{mn}F^{nk}F_{kl}F^{lm} - \frac{1}{4}F_{mn}F^{mn})^2}{(X^A X^A)^2}$$

- $\mathcal{N}=1$ No full expression for Γ_4 is known; only some terms were found
- $\mathcal{N}=2$ The complete expression for Γ_4 was obtained by Buchbinder and Ivanov in 2002
- $\mathcal{N}=3$ Nothing about the effective action is known
- $\mathcal{N}=4$ The expression for Γ_4 is obtained in our works

$\mathcal{N}=2$ harmonic superspace

- q^+_a hypermultiplet
- W vector multiplet

$\mathcal{N}=4$ SYM free (abelian) action

$$S=\frac{1}{2}\int d\zeta^{(-4)}\,q^+_a D^{++}q^{+a}+\frac{1}{4}\int d^4x d^4\theta\,W^2+\frac{1}{4}\int d^4x d^4\bar\theta\,\bar W^2$$

Hidden $\mathcal{N}=2$ supersymmetry

$$\begin{aligned}\delta W &= \bar\epsilon^{\dot\alpha a}\bar D^-_{\dot\alpha}q^+_a\,,\quad \delta\bar W=\epsilon^{\alpha a}D^-_\alpha q^+_a\\ \delta q^+_a &= \epsilon^\beta_a D^+_\beta W+\bar\epsilon^{\dot\alpha}_a\bar D^+_{\dot\alpha}\bar W\end{aligned}$$

Buchbinder and Ivanov 2002

Non-holomorphic potential [Dine and Seiberg 1997; Seiberg 1998]

$$\int d^4x d^8\theta \ln W \ln \bar{W}$$

Unique $\mathcal{N}=4$ supersymmetric generalization of this expression is

$$\Gamma_4 = c \int d^4x d^8\theta du \left[\ln W \ln \bar{W} + \sum_{n=1}^{\infty} \frac{1}{n^2(n+1)} \left(-\frac{q^{+a} q^-_a}{W \bar{W}} \right)^n \right]$$

- This is full expression for the four-derivative part of the effective action, i.e., it contains all component fields and is $\mathcal{N}=4$ supersymmetric
- This expression was obtained latter in [Buchbinder and Pletnev 2004-2006] using direct quantum computations in the $\mathcal{N}=2$ harmonic superspace
- The coefficient $c = 1/(8\pi)^2$ in the case of gauge group SU(2) spontaneously broken down U(1)
- In components it contains, in particular, the F^4/X^4 term in the SO(6) covariant form

Our aim:

To find an analog of this action in $\mathcal{N}=4$ harmonic superspace

Standard $\mathcal{N}=4$ superfield strengths $W^{IJ}=-W^{IJ}$, $I,J=1,2,3,4$

$$D^I_\alpha W^{JK}+D^J_\alpha W^{IK}=0\,,\qquad \bar W_{IJ}=\frac{1}{2}\epsilon_{IJKL}W^{KL} \tag{1}$$

SU(4) harmonics [Hartwell, Howe 1995]

- Introduce harmonic variables $u_I^{(p,q,r)}$ which belong to a coset SU(4)/H where H is a subgroup of SU(4), e.g., U(1)×U(1)×U(1)
- Rotate the SU(4) indices of W^{IJ} with these harmonics, $W^{IJ}\to\mathcal{W}^{(p,q,r)}$
- For these rotated superfield strengths $\mathcal{W}^{(p,q,r)}$ obtain full set of constraints which are equivalent to (1)
  - ⇒ Grassmann analyticity
  - ⇒ Harmonic analyticity
- Under these constraints, all auxiliary fields vanish while the physical fields obey the free equations of motion

$$\Box X_A=0\,,\quad \partial\!\!\!/\Psi^N=0\,,\quad \partial^m F_{mn}=0$$

$\mathcal{N}=4$ USp(4) harmonic superspace

$$D^I_\alpha W^{JK} + D^J_\alpha W^{IK} = 0\,, \qquad \bar W_{IJ} = \frac{1}{2}\epsilon_{IJKL} W^{KL}$$

USp(4) harmonics variables [Buchbinder, Lechtenfeld, Samsonov 2008]

- Introduce the harmonic variables $u_I^{(\pm,0)}$, $u_I^{(0,\pm)}$ which belong to the coset USp(4)/[U(1)×U(1)]
- Contract the superfield strengths with harmonics, e.g.,

$$\mathcal{W} = W^{IJ}(u_I^{(+,0)} u_J^{(-,0)} + u_I^{(0,+)} u_J^{(0,-)})$$

  Obtain six independent superfields,

$$W^{IJ} \to (\mathcal{W}, \mathcal{W}', \mathcal{W}^{(+,+)}, \mathcal{W}^{(-,-)}, \mathcal{W}^{(+,-)}, \mathcal{W}^{(-,+)})$$

- Rewrite the initial constraints in terms of these new superfields.
  - $\Rightarrow$ Grassmann analyticity
  - $\Rightarrow$ Harmonic analyticity
- These constraints eliminate all auxiliary fields from $\mathcal{W}$ and put the physical component field on shell.

Most general four-derivative action with $\mathcal{N}=4$ chargeless superfield

$$\int d^4x d^8\theta du\, H(\mathcal{W})$$

$H(\mathcal{W})$ is some function of $\mathcal{W}$ without derivatives
Note that $d^8\theta \sim (D)^8 \sim (\partial_m)^4$

Require scale invariance:

- $d^4x d^8\theta$ is dimensionless $\Rightarrow$ $H(\mathcal{W})$ is dimensionless, but $[\mathcal{W}]=1$
- Hence, $H(\mathcal{W}) = H(\mathcal{W}/\Lambda)$, $[\Lambda]=1$, $\Lambda = const$
- Scale invariance imply

$$\Lambda \frac{d}{d\Lambda} H(\mathcal{W}/\Lambda) = const \quad \Rightarrow \quad H(\mathcal{W}/\Lambda) = c \ln \frac{\mathcal{W}}{\Lambda}$$

Our main result

$$\Gamma_4 = c \int d^4x d^8\theta du \ln \frac{\mathcal{W}}{\Lambda}$$

$\mathcal{N}=4$ SU(2)×SU(2) harmonic superspace [Belyaev, Samsonov, arXiv:1106.0611]

SU(4) index $I \longrightarrow (i,a)$ – SU(2)×SU(2) indices

$$W^{IJ} \longrightarrow (W^{(ij)}, W^{(ab)})$$

- Introduce the SU(2) harmonic variables $u_i^\pm$ and $v_a^\pm$ which belong to two SU(2)
- Contract the superfield strengths with harmonics, e.g.,

  $$\mathcal{W} = W^{ij}u_i^+u_j^- - W^{ab}v_a^+v_b^-$$

  Obtain six independent superfields,

  $$W^{IJ} \rightarrow (\mathcal{W}, \mathcal{W}', \mathcal{W}^{(++,0)}, \mathcal{W}^{(--,0)}, \mathcal{W}^{(0,++)}, \mathcal{W}^{(0,--)})$$

- Rewrite the $\mathcal{N}=4$ SYM constraints in terms of these new superfields.
  - $\Rightarrow$ Grassmann analyticity
  - $\Rightarrow$ Harmonic analyticity
- These constraints eliminate all auxiliary fields from $\mathcal{W}$ and put the physical component field on shell.
- Scale invariant action

  $$\Gamma_4 = c\int d^4x d^8\theta du dv \,\ln\frac{\mathcal{W}}{\Lambda}$$

There are three different forms of Γ_4:

1. Buchbinder-Ivanov action, has SU(2)×SU(2)×U(1) explicit R-symmetry
2. Our action in USp(4) harmonic superspace
3. Our action in SU(2)×SU(2) harmonic superspace

Question:

Is there a superspace which makes manifest full SU(4) R-symmetry in the $\mathcal{N}=4$ SYM effective action?

Our answer:

No. In any four-dimensional superspace only a subgroup of SU(4) can be manifest in the $\mathcal{N}=4$ SYM effective action

The $\mathcal{N}=4$ SYM effective action contains the Wess-Zumino term for the six scalar fields [Tseytlin, Zarembo 2000]

$$S_{WZ} = -\frac{1}{60\pi^2}\varepsilon^{MNKLP}\varepsilon^{ABCDEF}\int d^5x \frac{X_A\partial_M X_B\partial_N X_C\partial_K X_D\partial_L X_E\partial_P X_F}{|X|^6}$$

- The Wess-Zumino action has manifest SO(6) symmetry only in the five-dimensional formulation.
- It can be rewritten in the four-dimensional form as soon as the WZ action is the integral of the exact differential form

$$S_{WZ} = \int_{M_5}\Omega_5 = \int_{M_5} d\omega_4 = \int_{\partial M_5}\omega_4$$

The form ω_4 is invariant only under one of the following three subgroups of SO(6):

1. SO(5)≃USp(4) ($\mathcal{N}=4$ USp(4) harmonic superspace action)
2. SO(4)×SO(2)≃SU(2)×SU(2)×U(1) ($\mathcal{N}=2$ Buchbinder-Ivanov action)
3. SO(3)×SO(3)≃SU(2)×SU(2) ($\mathcal{N}=4$ bi-harmoic superspace action)

Results:

1. The four-derivative part Γ_4 of the $\mathcal{N}=4$ SYM effective action is constructed in $\mathcal{N}=4$ harmonic superspace in manifestly $\mathcal{N}=4$ supersymmetric form
2. Although the effective action has SO(6) R-symmetry, only one of the three subgroups SO(5), SO(4)×SO(2) or SO(3)×SO(3) can be manifest in a four-dimensional formulation.

Open problems:

- $\mathcal{N}=4$ SYM effective action in $\mathcal{N}=0,1,3$ superspaces
- F^6 and other higher-order contributions to the $\mathcal{N}=4$ SYM action and its comparison with the D3 brane action in the $AdS_5 \times S^5$ background