Critical gravities in $d \geq 3$

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Outline

Introduction

- The gauge/gravity correspondence has become one of the most important analytical tools in addressing strongly coupled gauge field theories.
- Best known form (AdS/CFT): motivated by string theory, allows one to study strongly coupled CFTs by studying weakly coupled (classical) gravity theories on AdS backgrounds.
- Here : another (mild) extension of the usual AdS/CFT correspondence, where the field theory side concerns logarithmic CFTs.
- Log CFTs are non-unitary CFTs that were developed by condensed matter physicists (for e.g. percolation, turbulence, critical phenomena,...).
- In this talk : discuss $(d+1)$-dim. conjectured gravity duals of (strongly coupled) d-dim. log CFTs : critical gravities.

Critical gravity

- Critical gravities are special instances of 'massive gravity' theories involving higher curvature/derivative terms.
- Typically, extensions of ordinary EH-gravity with higher curvature/derivative terms up to four derivatives. Three kinds of terms :
  1. Einstein-Hilbert
  2. cosm. const.
  3. higher curvature terms
- New propagating massive degrees of freedom, apart from the usual massless ones.
- Motivation to add R^2: Quantum Gravity, e.g. in $d = 4$: render gravity generically renormalizable ($G_N m^2$ is dimensionless), but non-unitary (Stelle).

Critical gravity

- These theories come with a certain parameter space (‘coupling constants’):
  1. Sign in front of EH-term
  2. Λ
  3. m^2

  Typically, the spectrum of linearized excitations consists of massless + massive d.o.f. (non-critical).

- At ‘critical’ points in parameter space, the spectrum changes and the massive d.o.f. disappear : ‘critical gravities’.

- Example : New Massive Gravity (NMG) (Bergshoeff, Hohm, Townsend; Lu, Pope)

$$S = \frac{1}{\kappa^2}\int \mathrm{d}^d x \sqrt{-g}\left\{\sigma R - 2\lambda m^2 + \frac{1}{m^2} G^{\mu\nu} S_{\mu\nu}\right\} ,$$

with

$$S_{\mu\nu} = \frac{1}{(d-2)}\left(R_{\mu\nu} - \frac{1}{2(d-1)} R g_{\mu\nu}\right) .$$

Critical gravity

- A linearized analysis ($g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$) is dependent on whether

$$\bar{\sigma} = \sigma - \frac{\Lambda}{m^2} \frac{1}{d-1}$$

  is zero or not. Consider first $\bar{\sigma} \neq 0$ (non-crit.).

- Upon using aux. field, linearized Lagrangian becomes equivalent to:

$$\mathcal{L}_2 \quad = \quad \bar{\sigma}\, L_{EH}(\bar{h}) - \frac{4}{m^4(d-1)^2(d-2)^2\bar{\sigma}}\, L_{FP}(k; -m^2(d-2)\bar{\sigma})\,.$$

- First part : linearized Einstein-Hilbert : propagates massless gravitons ($d > 3$) or nothing ($d = 3$).

- Second part : linearized Fierz-Pauli Lagrangian : propagates correct number of helicity ($d = 3$)/spin ($d > 3$) states of a massive graviton (with $M^2 = -m^2(d-2)\bar{\sigma}$).

- Non-unitary (except in $d = 3$).

Critical gravity

- For $\bar{\sigma} = 0 : M^2 \rightarrow 0$: massive gravitons become massless! Critical case.
- Original e.o.m. still fourth order at critical point $\Rightarrow$ number of solutions can not change at critical point.
- Massive gravitons are replaced by new modes : logarithmic modes (Grumiller, Johansson; Alishahiha, Fareghbal; Bergshoeff, Hohm, Rosseel, Townsend).
- Linearized e.o.m.:

$$\mathcal{G}_{\mu\nu}(\mathcal{G}(h)) = 0$$

is solved by

1. massless gravitons : $\mathcal{G}_{\mu\nu}(h) = 0$
2. h for which $\mathcal{G}_{\mu\nu}(h) = 2\nabla_{(\mu}A_{\nu)}$: 'Proca log modes'
3. h for which $\mathcal{G}_{\mu\nu}(h) = k^{\perp}_{\mu\nu}$, with $k^{\perp}_{\mu\nu}$ a non-trivial solution of the Einstein equations : 'spin 2 log modes'

Critical gravity

- Log modes have different properties than massive/massless gravitons.
  - Massive/massless modes obey 2nd order equations

$$\left(\Box - \tfrac{2}{3}\Lambda - M^2\right) h_{\mu\nu} = 0\,.$$

Log modes obey 4th order equations

$$\left(\Box - \tfrac{2}{3}\Lambda\right)^2 h_{\mu\nu} = 0\,.$$

  - Log modes have a more general boundary behaviour than massive/massless modes.
  - Massive/massless modes can be constructed as eigenstates of AdS energy operator H

$$H\, h_{\mu\nu} = E_0\, h_{\mu\nu}\,.$$

No longer true for log modes

$$H \begin{pmatrix} h^{\log} \\ h \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} h^{\log} \\ h \end{pmatrix}.$$

H is not diagonalizable.

Logarithmic CFTs

- Log CFTs are conformal field theories for which the correlators can depend on the logarithm of the separation of operators (Gurarie).
- The Hamiltonian is non-diagonalizable. Operators form pairs $(\psi, \psi^{\log})$ such that

$$\begin{aligned} \left[H, \psi^{\log}\right] &= E_0\, \psi^{\log} + \psi \\ [H, \psi] &= E_0\, \psi\,. \end{aligned}$$

- Correlators involving 'log' operators:

$$\begin{aligned} < \psi^{\log}(x)\, \psi(0) > &= \frac{b}{2|x|^{2\Delta}} \\ < \psi^{\log}(x)\, \psi^{\log}(0) > &= -\frac{b \ln(m|x|^2)}{|x|^{2\Delta}} \end{aligned}$$

- Although non-unitary, used in condensed matter physics : percolation, turbulence,...

The critical gravity/log CFT correspondence

- Critical gravities are conjectured to be gravity duals of log CFTs.
- According to AdS/CFT correspondence : bulk modes are dual to CFT operators. AdS energy = conformal energy.
- In the bulk : log modes form pairs with massless modes (or pure gauge modes) such that

$$H\begin{pmatrix} h^{\log} \\ h \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} h^{\log} \\ h \end{pmatrix}.$$

  On the bulk side H is not diagonalisable. This is the bulk dual of having a non-diagonalizable H on the CFT side $\Rightarrow$ log CFT!
- Can be checked in more detail through a gravity calculation of the 2- and 3-point functions (Grumiller, Sachs; Skenderis, Taylor, van Rees; Grumiller, Hohm; Alishahiha, Naseh; Grumiller, Johansson, Zojer).

Conclusions

- Critical gravities are special instances of ‘massive gravities’, that are extensions of EH-gravity involving higher curvature terms.
- For critical gravities no massive gravitons are propagated, instead logarithmic modes.
- Existence and properties of log modes leads to conjecture that critical gravities are dual to log CFTs.
- Critical gravity/log CFT gives a mild extension of usual AdS/CFT.
- log CFTs are non-unitary but find their place in condensed matter physics.
- Useful to see whether e.g. critical NMG is useful for condensed matter and whether one can find other critical gravity duals of useful log CFTs.