BRST-inspired Lagrangian formulations for arbitrary free HS fields on d-dimensional Minkowski background

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Dubna, SQS'11

based on J.Buchbinder, A.A. to appear soon

Outline

1. Motivations and setting of the problems
   - HS formulations on $\mathbb{R}^{1,d-1}$ & (A)dS, SFT
   - BFV-BRST for inverse problem of LF construction
   - Basic aims to solve
2. Integer HS mixed-symmetry fields on $\mathbb{R}^{1,d-1}$
   - Integer HS symmetry algebra $\mathcal{A}(Y(k), \mathbb{R}^{1,d-1})$
   - Additive conversion for $A(Y(k), \mathbb{R}^{1,d-1})$
   - Verma module for $sp(2k)$
   - Oscillator Realization for $V(sp(2k))$
   - BRST for HS symmetry algebra $\mathcal{A}_c(Y(k), \mathbb{R}^{1,d-1})$
   - Unconstrained Lagrangian formulation
3. Notes on Half-Integer HS MS fields on $\mathbb{R}^{1,d-1}$

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HS formulations on $\mathbb{R}^{1,d-1}$ & (A)dS, SFT

Problems of HS field theory (M. Fierz, W. Pauli; L. Singh, C. Hagen; C. Fronsdal) have attracted significant attention ($k=1$ row in Young tableau (YT)), ($k>1$) $\mathbf{s}=(n_1+1/2, n_2+1/2, ...)$ $\mathbf{s}=(s_1, s_2, ...)$ (massive and massless:$m=0$) HS fields :

$$\Phi_{(\mu)_{s_1},(\nu)_{s_2},\ldots,(\rho)_{s_k}} \longleftrightarrow \begin{array}{|c|c|c|c|c|c|c|c|c|}\hline \mu_1 & \mu_2 & \cdot \quad \cdot \quad \cdot & \cdot & \cdot & \cdot & \cdot & \mu_{s_1} \\ \hline \nu_1 & \nu_2 & \cdot \quad \cdot \quad \cdot & \cdot & \cdot & \nu_{s_2} & \multicolumn{2}{c}{} \\ \cline{1-6} \cdots & \cdots & \cdots \quad \cdot \quad \cdot & \cdot & \cdots & \multicolumn{3}{c}{} \\ \cline{1-5} \end{array} = Y(s_1, \ldots, s_k)$$

in view of connection to SuperString Field Theory (SFT): (E. Witten (1986); C. Thorn(1989)) through special tensionless limit for intercept ($\alpha' \to 0$): (A. Sagnotti, M. Tsulaia, (2004)).

$\Longrightarrow$ SFT $\xrightarrow{\alpha'\to 0}$ $\{\infty\}$ set of HS fields in s/string spectrum

From cosmological research $\Longrightarrow$ an exceptional role of (Anti-)de-Sitter [(A)dS] space for consistent propagation of free (J. Fang, C. Fronsdal (1980); M. Vasiliev (1988)) and interacting (E. Fradkin, M. Vasiliev (1987, 2001), R. Metsaev (2005), E.Skvortsov, Yu.Zinoviev (2011)) HS fields due to:

- natural dimensional parameter – square inverse radius r of d-dimensional (A)dS space,
- connection of HS fields on AdS(d) space to AdS/CFT correspondence among $\mathcal{N} = 4$ SYM and superstring theory on $AdS_5 \times S_5$ RR background,$k \leq \left[\frac{d-1}{2}\right] = 2$.

While LF for free HS fields subject to arbitrary $Y(s_1, ..., s_k)$ within constrained frame-like formulation (M. Vasiliev) was found (Yu. Zinoviev, Arxiv:0809.3287, Arxiv:0904.0549, E.Skvortsov NPB, 2009, 2011), the same problem in unconstrained metric-like formulation HAVE NOT BEEN SOLVED except for $YT(2)$ on $\mathbb{R}^{1,d-1}$.

Within stringy-inspired BRST-BFV approach (S. Ouvry, J. Stern, A. Bengtsson, A. Pashnev, M. Tsulaia, J. Buchbinder, V. Krykhtin, A.R.) this problem meets OBSTACLES in constructing:

1) Verma module for $sp(2k)$, $osp(k|2k)$ (additive conversion);

2) but not BFV-BRST operator (as it was in AdS(d) case)

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BFV-BRST for direct problem

Whereas, the direct BFV-BRST prescription to quantize an initial degenerate field theory given in LF

degenerate LF $(S(q^i), \delta q^i)$ $\xrightarrow{\text{Dirac-Bergmann}}$ t-local HF $(H_0, o_a)(t)$ $\{o_a, o_b\} = f^c_{ab}(q,p)o_c + \Delta_{ab}$

$\xrightarrow[\text{Batalin, Tyutin}]{\text{conversion}}$ converted HF$(H_0, O_a)(t)$ $\{O_a, O_b\} = F^c_{ab}(q,p,\zeta)O_c$

$\xrightarrow{\text{BFV method}}$ BFV-BRST charge $Q(t), Q^2 = 0$ $Q(t) = C^a O_a + \frac{1}{2} C^b C^a F^c_{ab} \mathcal{P}_c + \text{more}$

$\xrightarrow[\text{Kugo, Ojima}]{\text{quantization}}$ $|\Psi\rangle \in \mathcal{H}_{\text{phys}} : Q|\Psi\rangle = 0, \text{gh}(|\Psi\rangle) = 0$ gauge transfs: $|\Psi'\rangle = |\Psi\rangle + Q|\Lambda\rangle$

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BFV-BRST for inverse problem

CONSTRUCTION OF LF FOR HS FIELD WITH GIVEN (m, s)

Irreps conditions
ISO(1,d-1), SO(2,d-1)

$\xrightarrow{\text{SFT}}$

(Super)algebra $\{o_I(x)\} : \mathcal{H}$
$[o_I, o_J\} = f^K_{IJ}(o)o_K + \Delta_{ab}(g_0)$

$\xrightarrow[\text{Burdik, Pashnev}]{\text{conversion}}$

$O_I = o_I + o'_I : \mathcal{H} \bigotimes \mathcal{H}'$
$[O_I, O_J\} = F^K_{IJ}(o', O)O_K$

$\xrightarrow[\text{Henneaux}]{\text{BFV}}$

BRST operator for $\{O_I\}$: $Q'(x)$
$Q' = C^I O_I + \frac{1}{2} C^I C^J F^K_{IJ} \mathcal{P}_K + \text{more}$

$\xrightarrow{\text{LF}}$

$Q' = Q + (g_0 + h + \text{more})C_g + \ldots : Q^2 = 0$
mass-shell : $Q|\Psi\rangle = 0, \text{gh}(|\Psi\rangle) = 0$
spin: $(g_0 + \text{more})(|\Psi\rangle, |\Lambda\rangle, \ldots) = -h(|\Psi\rangle, |\Lambda\rangle, \ldots)$
gauge transfs: $\delta|\Psi\rangle = Q|\Lambda\rangle, \delta|\Lambda\rangle = Q|\Lambda^1\rangle, \ldots$

At 2-3rd steps the Stuckelberg and gauge fields are appeared

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Basic aims to solve

1. on Verma Modules construction & its oscillator realization for $sp(2k)$ ($osp(k|2k)$) (super)algebra underlying (fermionic) bosonic HS fields on $\mathbb{R}^{1,d-1}$ space subject to YT$(s_1, ..., s_k)$;
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Derivation of HS symmetry algebra $\mathcal{A}(Y(k),\mathbb{R}^{1,d-1})$

The m of g. spin $s=(s_1,...,s_k)$ $ISO(1,d-1)$ group irrep

$$\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},...,(\mu^k)_{s_k}} \longleftrightarrow \begin{array}{|c|c|c|c|c|c|c|c|}\hline \mu^1_1 & \mu^1_2 & \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot & \cdot & \mu^1_{s_1} \\ \hline \mu^2_1 & \mu^2_2 & \cdot\ \cdot\ \cdot & \cdot & \cdot & \mu^2_{s_2} \\ \cline{1-6} \cdot & \cdot & \cdot\ \cdot\ \cdot & \cdot & \cdot & \cdot \\ \cline{1-6} \mu^k_1 & \mu^k_2 & \cdot\ \cdot\ \cdot & \cdot & \mu^k_{s_k} \\ \cline{1-5} \end{array}\ ,$$

$$\left[\partial^2+m^2\right]\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},...}=0, \tag{1}$$

$$\partial^{\mu^i_{l_i}}\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},...,(\mu^k)_{s_k}}=0,\quad i,j=1,...,k;\ l_i,m_i=1,...,s_i\,, \tag{2}$$

$$\eta^{\mu^i_{l_i}\mu^i_{m_i}}\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},...,(\mu^k)_{s_k}}=\eta^{\mu^i_{l_i}\mu^j_{m_j}}\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},...,(\mu^k)_{s_k}}=0,\quad l_i<m_i, \tag{3}$$

$$\Phi_{(\mu^1)_{s_1},...,\{(\mu^i)_{s_i}\underbrace{,...,\mu^j_1\cdots}\mu^j_{l_j}\}...\mu^j_{s_j},...(\mu^k)_{s_k}}=0,\quad i<j,\ 1\leq l_j\leq s_j, \tag{4}$$

We want to find the LF for given HS field on more wider $\mathcal{M}$:

Primary constraints

$$\text{SFT} \quad \Longrightarrow \mathcal{H} : [a^i_\mu, a^{j+}_\nu] = -\eta_{\mu\nu}\delta^{ij},$$

An arbitrary "string-like" vector $|\Phi\rangle \in \mathcal{H}$

$$|\Phi\rangle = \sum_{s_1=0}^{\infty}\sum_{s_2=0}^{s_1}\cdots\sum_{s_k=0}^{s_{k-1}} \Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2},\ldots,(\mu^k)_{s_k}}(x)\prod_{i=1}^{k}\prod_{l_i=1}^{s_i} a_i^{+\mu^i_{l_i}}|0\rangle,, \tag{5}$$

permits to realize $\Longleftrightarrow$ Eqs. (1 - 4) as constraints on $|\Phi\rangle$.

Then the constraints

$$\boxed{(l_0, l_i, l_{ij}, t_{ij})|\Phi\rangle = \vec{0}} \Longleftrightarrow \texttt{irreps (1)- (4) for each } s_1,\ldots,s_k$$

All constraints, $sp(2k)$, $\mathcal{A}(Y(k),\mathbb{R}^{1,d-1})$

$$l_0=\partial^2+m^2,\ l_{ij}=\frac{1}{2}a_i^{\mu}a_{j\mu}\,,\quad l_i=-\imath a_i^{\mu}\partial_{\mu},\ t_{ij}=a_i^{+\mu}a_{\mu j}\theta^{ji},\theta^{ji}=1(0),j>(<)i$$

which form together with $(l_i^+,l_{ij}^+,t_{i_1j_1}^+,g_0^i)$ integer HS symmetry algebra $\mathcal{A}(Y(k),\mathbb{R}^{1,d-1})$ w.r.t. [,].
Subalgebra of operators

$$\{l^{ij},t^{i_1j_1},g_0^i,l_{ij}^+,t_{i_1j_1}^+\}\overset{\texttt{Howe duality}}{\simeq} sp(2k).$$

For $m=0$ the only o_I from upper and lower triangular subalgebras in $sp(2k)$ compose an invertible matrix:

$\|[\theta_{\mathbf{a}},\theta_{\mathbf{b}}]\|=\|\Delta_{\mathbf{ab}}(g_0^i)\|+(o_I)$,

for $m\neq 0$ its number k^2 increase on $2k$ items l^i,l_I^+

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note on additive conversion procedure

To convert $\mathcal{A}(Y(k),\mathbb{R}^{1,d-1})$ with 2nd C.C. we have used the general procedure of additive conversion
$o_I \to O_I = o_I + o'_I : [o_I, o'_J] = 0$, so that $[O_I, O_J] \sim O_K$,
$\Rightarrow$ if $[o_I, o_J] = f^K_{IJ} o_K$, then $[o'_I, o'_J] = f^K_{IJ} o'_K$ & $[O_I, O_J] = f^K_{IJ} O_K$.
But, it's sufficient to convert only subalgebra $sp(2k)$ for $\{o_a\}$.
So that the algebra of O_I is the same $\mathcal{A}_c(Y(k),\mathbb{R}^{1,d-1}) = \mathcal{A}(Y(k),\mathbb{R}^{1,d-1})$ as for o_I, but for o'_I - $sp(2k)$.

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Verma module for $sp(2k)$

Cartan decomposition

$$sp(2k)=\left\{\overbrace{l'^{ij+},t'^{+}_{nm}}\right\}\oplus\left\{\overbrace{g'^{i}_{0}}\right\}\oplus\left\{\overbrace{l'^{ij},t'_{nm}}\right\}\equiv\mathcal{E}^{-}_{k}\oplus H_k\oplus\mathcal{E}^{+}_{k}$$

Requirement: boundary conditions for o'_I from Cartan subalgebra:

$$g^{i}_{0}\to g'^{i}_{0}(h^{i})=h^{i}+\ldots,$$

So that, following the result by C.Burdik 1985 we start with highest weight vector $|0\rangle_V$ & construct following Poincare–Birkhoff–Witt theorem

$$V(sp(2k))=U(\mathcal{E}^{-}_{k})\otimes(|0\rangle_V):\ \mathcal{E}^{+}_{k}|0\rangle_V=0\,,\ g'^{i}_{0}|0\rangle_V=h^{i}|0\rangle_V\,,$$

to find $\{o'_I\}=\{o'_I(b_{ij},b^{+}_{ij},\overbrace{b_i,b^{+}_i}^{m\neq 0},d_{ln},d^{+}_{ln})\},$

Explicit obtaining of the $V(sp(2k))$ meet the technical obstacle because of not commuting of t'^{+}_{ln}, l'^{+}_{ij} with each other $\mathcal{E}^{-}_{k}$. The general $V(sp(2k))$ vector

$$\left|\vec{n}_{ij},\vec{p}_{rs}\right\rangle_V = \left|n_{11},\ldots,n_{1k},n_{22},\ldots,n_{2k},\ldots,n_{kk};p_{12},\ldots,p_{1k},p_{23},\ldots,p_{2k},\ldots,p_{k-1k}\right\rangle$$

$$\left|\vec{n}_{ij},\vec{p}_{rs}\right\rangle_V \equiv |\vec{N}\rangle_V \equiv \prod_{i\le j}^{k}(l'^{+}_{ij})^{n_{ij}}\prod_{r,r<s}^{k}(t'^{+}_{lm})^{p_{rs}}|0\rangle_V, \qquad (6)$$

$$g'_{0i}|\vec{N}\rangle_V = \Big(\sum_{l}(1+\delta_{il})n_{il}-\sum_{s>i}p_{is}+\sum_{r<i}p_{ri}+h^{i}\Big)\left|\vec{N}\right\rangle_V ,$$

$$t'^{+}_{r's'}|\vec{N}\rangle_V = \left|\vec{n}_{ij},\vec{p}_{rs}+\delta_{r's',rs}\right\rangle_V - \sum_{k'=1}^{r'-1}p_{k'r'}\left|\vec{n}_{ij},\vec{p}_{rs}-\delta_{k'r',rs}+\delta_{k's',rs}\right\rangle_V$$

$$-\sum_{k'=1}^{k}(1+\delta_{k'r'})n_{r'k'}\left|\vec{N}-\delta_{r'k',ij}+\delta_{s'k',ij}\right\rangle_V ,$$

explicit construction of $V(sp(2k))$

$$l'^{+}_{i'j'}|\vec{N}\rangle_V = \Big|\vec{N}+\delta_{i'j',ij}\rangle_V\,, \qquad \texttt{for "-" root vectors} \in \mathcal{E}_k^-$$

where $AB^n = \sum_{k=0}^{n}\frac{n!}{k!(n-k)!}B^{n-k}\mathrm{ad}_B^k A\,,\ \mathrm{ad}_B^k A = [[\ldots[A,\overbrace{B\},\ldots],B}^{k\text{ times}}],$

To get the action of E^{α_i} on $|\vec{N}\rangle_V$ we get the recurrent relation

$$t'_{l'm'}|\vec{0}_{ij},\vec{p}_{rs}\rangle_V = \Big|C^{l'm'}_{\vec{p}_{rs}}\rangle_V - \sum_{n'=1}^{l'-1}p_{n'm'}\Big|\vec{0}_{ij},\vec{p}_{rs}-\delta_{n'm',rs}+\delta_{n'l',rs}\rangle_V$$

$$+\sum_{k'=l'+1}^{m'-1}p_{l'k'}\Big[\prod_{r'<l',s'>r'}\ \prod_{r'=l',m'>s'>r'}(t'^{+}_{r's'})^{p_{r's'}-\delta_{l'k',r's'}}\Big]t'_{k'm'}\Big|\vec{0}_{ij},\vec{p}_{q't'}\rangle_V\,,\quad \vec{p}_{rs}>$$

The solution of the above Eq. exists, so that the explicit form of $t'_{l'm'}$ action on the vector $|\vec{N}\rangle_V$ has the final form

$$\begin{aligned} t'_{l'm'}|\vec{N}\rangle_V \;=\; & -\sum_{k'=1}^{k}(1+\delta_{k'm'})n_{k'm'}\big|\vec{n}_{ij}-\delta_{k'm',ij}+\delta_{k'l',ij},\vec{p}_{rs}\rangle_V \\ & + \sum_{p=0}^{m'-l'-1}\sum_{k'_1=l'+1}^{m'-1}\cdots\sum_{k'_p=l'+p}^{m'-1}\prod_{j=1}^{p} p_{k'_{j-1}k'_j}\Big|C^{k'_p m'}_{\vec{n}_{ij},\vec{p}_{rs}-\sum_{j=1}^{p}\delta_{k'_{j-1}k'_j,rs}}\Big\rangle \\ & -\sum_{n'=1}^{l'-1} p_{n'm'}\,\big|\vec{n}_{ij},\vec{p}_{rs}-\delta_{n'm',rs}+\delta_{n'l',rs}\rangle_V\ . \end{aligned}$$

Analogously, the action of the rest E^{α_i}: $l'_{l'm'}$ on $|\vec{N}\rangle_V$ is determined with help of the "basic-block" vector $\big|C^{l'm'}_{\vec{p}_{rs}}\rangle_V$

$\Longrightarrow$ **$V(sp(2k))$ is explicitly found!**

Making use of the mapping (C. Burdik, 1985)

$$|\vec{n}_{ij},\vec{p}_{rs}\rangle_V \leftrightarrow |\vec{n}_{ij},\vec{n}_s\rangle = \prod_{i,j\geq i}^{k}(b_{ij}^{+})^{n_{ij}}\prod_{r,s,s>r}^{k}(d_{rs}^{+})^{p_{rs}}|0\rangle \in \mathcal{H}',$$

$$[b_k, b_l^{+}] \overset{m\neq 0}{=} \delta_{kl}, \quad [b_{ij}, b_{lk}^{+}] = \delta_{il}\delta_{jk}, \; i\leq j, k\leq l, \qquad [d_{r_1s_1}, d_{r_2s_2}^{+}] = \delta_{r_1r_2}\delta_{s_1s_2},$$

Theorem

The polynomial oscillator realization for the $V(sp(2k))$ over Heisenberg-Weyl algebra $A_{k\times k}$ exists in the form

$$C(b_{ij}, b_{lk}^{+}, d_{r_1s_1}, d_{r_2s_2}^{+}), \quad C\in\{t'_{l'm'}, t'^{+}_{l'm'}, l'_{i'j'}, l'^{+}_{i'j'}g_0^{\prime i}\}. \qquad (7)$$

explicit form of **basic block** $C^{lm}(d^+,d) \to \left|C^{lm}_{\vec{p}_{rs}}\right\rangle_V$

$$
\begin{aligned}
C^{lm}(d^+,d) \;\equiv\; & \Big(h^l - h^m - \sum_{n=m}^{k}(d^+_{ln}d_{ln} + d^+_{mn}d_{mn}) + \sum_{n=l+1}^{m-1} d^+_{nm}d_{nm} - d^+_{lm}d_{lm}\Big)d_{lm} \\
& - \sum_{n=l+1}^{m-1} d^+_{ln}d_{nm} + \sum_{n=m+1}^{k}\Big\{d^+_{mn} - \sum_{n'=1}^{m-1} d^+_{n'n}d_{n'm}\Big\}d_{ln}
\end{aligned}
$$

so that, f.i. for t'_{lm}:

$$
\begin{aligned}
t'_{lm} \;=\; & -\sum_{n=1}^{l-1} d^+_{nl}d_{nm} + \sum_{p=0}^{m-l-1}\ \sum_{k_1=l+1}^{m-1} \cdots \sum_{k_p=l+p}^{m-1} C^{k_p m}(d^+,d)\prod_{j=1}^{p} d_{k_{j-1}k_j} \\
& - \sum_{n=1}^{k}(1+\delta_{nm})b^+_{nl}b_{nm}\,, \qquad k_0 \equiv l, \qquad (8)
\end{aligned}
$$

. Thus, the additive conversion of o_I into the 1st class O_I is realized! (It completely applicable for massive HS fields as well)

BRST operator for Lie algebra $\mathcal{A}(Y(k),\mathbb{R}^{1,d-1})$

The BRST operator Q' for Lie algebra $\mathcal{A}_c(Y(k),\mathbb{R}^{1,d-1})$ is constructed by the standard rules of BFV- method (without difficulties as in AdS(d) case J.Buchbinder,P.Lavrov 2007, A.R. arxiv:0812.2329).

$$Q' = O_I\mathcal{C}^I + \frac{1}{2}\mathcal{C}^I\mathcal{C}^J f^K_{JI}\mathcal{P}_K, \quad Q'^2 = 0 \quad \texttt{where} \quad (\varepsilon, gh)Q' = (1,1), \tag{9}$$

$\mathcal{C}^I = (\vartheta,\eta,\vartheta^+,\eta^+), \mathcal{P}_K$ - ghost coordinates and momenta with of opposite Grassmann parity to O_I with following non-vanishing C.R.

$$\begin{aligned} &\{\vartheta_{rs},\lambda^+_{tu}\} = \{\lambda_{tu},\vartheta^+_{rs}\} = \delta_{rt}\delta_{su}, && \{\eta_i,\mathcal{P}^+_j\} = \{\mathcal{P}_j,\eta^+_i\} = \delta_{ij}\,, \\ &\{\eta_{lm},\mathcal{P}^+_{ij}\} = \{\mathcal{P}_{ij},\eta^+_{lm}\} = \delta_{li}\delta_{jm}\,, && \{\eta_0,\mathcal{P}_0\} = \imath,\ \{\eta^i_{\mathcal{G}},\mathcal{P}^j_{\mathcal{G}}\} = \imath\delta^{ij}\,; \end{aligned} \tag{10}$$

and $gh(\mathcal{C}^I) = -gh(\mathcal{P}_I) = 1$.

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Explicit form of Q'

$$
\begin{aligned}
Q' \;=\;& \tfrac{1}{2}\eta_0 L_0 + \eta_i^+ L^i + \sum_{l\le m}\eta_{lm}^+ L^{lm} + \sum_{l<m}\vartheta_{lm}^+ T^{lm} + \tfrac{1}{2}\eta_G^i G_i + \tfrac{\imath}{2}\textstyle\sum_l \eta_l^+\eta^l\mathcal{P}_0 \\
& - \sum_{i<l<j}(\vartheta_{lj}^+\vartheta_i^{+l} - \vartheta_{il}^+\vartheta^{+l}{}_j)\lambda^{ij} - \tfrac{\imath}{2}\sum_{l<m}\vartheta_{lm}^+\vartheta^{lm}(\mathcal{P}_G^m - \mathcal{P}_G^l) - \sum_{l<m,n}\vartheta_{lm}^+\vartheta^l{}_n\lambda^{nm} \\
& + \sum_{n<l<m}\vartheta_{lm}^+\vartheta_n{}^m\lambda^{+nl} - \sum_{n,l<m}(1+\delta_{ln})\vartheta_{lm}^+\eta^{l+}{}_n\mathcal{P}^{mn} + \sum_{n,l<m}(1+\delta_{mn})\vartheta_{lm}^+\eta^m{}_n\mathcal{P}^{+ln} \\
& + \frac{\imath}{8}\sum_{l\le m}(1+\delta_{lm})\eta_{lm}^+\eta^{lm}(\mathcal{P}_G^l + \mathcal{P}_G^m) + \sum_{l\le m}(1+\delta_{lm})\eta_G^l\eta_{lm}^+\mathcal{P}^{lm} \\
& + \Big[\tfrac{1}{2}\sum_{n,l<m}\eta_{nm}^+\eta^n{}_l + \sum_{l<m}(\eta_G^m - \eta_G^l)\vartheta_{lm}^+\Big]\lambda^{lm} \\
& - \Big[\tfrac{1}{2}\sum_{l\le m}(1+\delta_{lm})\eta^m\eta_{lm}^+ + \sum_{l<m}\vartheta_{lm}\eta^{+m} + \sum_{m<l}\vartheta_{ml}^+\eta^{+m} + \textstyle\sum_l \eta_G^l\eta_l^+\Big]\mathcal{P}^l + Herm.C.
\end{aligned}
$$

$Q'^+K = KQ'$, in $\mathcal{H}_{tot} = \mathcal{H}\otimes\mathcal{H}'\otimes\mathcal{H}_{gh}$ due to $V(sp(2k))$ osc.realization .

The obtaining of resulting LF takes standard character
As usual, we extract the spin operator from the Q':

$$\Longrightarrow Q' = Q + \eta^i_G(\sigma^i + h^i) + \mathcal{A}^i\mathcal{P}^i_G,$$
$$\sigma^i = G^i_0 - h^i - \eta_i\mathcal{P}^+_i + \eta^+_i\mathcal{P}_i + \sum_m (1+\delta_{im})(\eta^+_{im}\mathcal{P}^{im} - \eta_{im}\mathcal{P}^+_{im})$$
$$+ \sum_{l<i}[\vartheta^+_{li}\lambda^{li} - \vartheta^{li}\lambda^+_{li}] - \sum_{i<l}[\vartheta^+_{il}\lambda^{il} - \vartheta^{il}\lambda^+_{il}],$$
$$[Q,\sigma_i] = 0\,,.$$

The same applies to a scalar physical and gauge vectors $|\chi^0\rangle, |\chi^s\rangle \in \mathcal{H}_{tot}$ i.e. $\partial(|\chi^0\rangle)/\partial\eta^i_G = 0$: gh$(|\chi^0\rangle, |\chi^s\rangle)$ = $(0,-s)$

$|\chi\rangle = |\Phi\rangle + |\Phi_A\rangle, |\Phi_A\rangle_{\{(b,b^+,d,d^+) = \mathcal{C} = \mathcal{P} = 0\}} = 0$ with $|\Phi\rangle$–basic HS f.

and with the use of the BFV–BRST EQUATION $Q'|\chi^0\rangle = 0$ that determines the physical states and a sequence of reducible gauge transformations,

$$Q|\chi\rangle = 0, \quad (\sigma^i + h^i)|\chi\rangle = 0, \quad (\varepsilon, gh)\,(|\chi\rangle) = (0,0), \tag{11}$$

$$\delta|\chi\rangle = Q|\chi^1\rangle, \quad (\sigma^i + h^i)|\chi^1\rangle = 0, \quad (\varepsilon, gh)\,(|\chi^1\rangle) = (1,-1), \tag{12}$$

$$\delta|\chi^1\rangle = Q|\chi^2\rangle, \quad (\sigma^i + h^i)|\chi^2\rangle = 0, \quad (\varepsilon, gh)\,(|\chi^2\rangle) = (2,-2), \tag{13}$$

$$\ldots \qquad \ldots \qquad \ldots$$

$$\delta|\chi^{s-1}\rangle = Q|\chi^{(s)}\rangle, \quad (\sigma^i + h^i)|\chi^s\rangle = 0, \quad (\varepsilon, gh)\,(|\chi^s\rangle) = ((s \,mod\, 2, -s). \tag{14}$$

the middle Eqs. determines the spectrum of spin values for $|\chi\rangle$ and gauge pars. $|\chi^i\rangle$, $i = 1, ..., k(k+1)$, the corresponding proper eigenvalue and eigenvectors,

$$-h^i \quad = \quad n^i + \frac{d-2-4i}{2}\ , \quad i = 1,..,k\ , \quad n_1, ..., n_{k-1} \in \mathbb{Z}, n_k \in \mathbb{N}_0\ ,\ ,\ |\chi\rangle_{(s_1,...,s_k)}$$

where n_i must be associated with s_i from basic $|\Phi\rangle$: $n_i = s_i$.

$\Longrightarrow$ The equations of motion and the sequence of reducible gauge transformations for the field with given $\mathbf{s}=(s)_k$:

$$Q_{(s)_k}|\chi^0\rangle_{(s)_k}=0,\delta|\chi^l\rangle_{(s)_k}=Q_{(s)_k}|\chi^{l+1}\rangle_{(s)_k},\ \delta|\chi^{k(k+1)}\rangle_{(s)_k}=0, l=0,\dots,k^2,$$

for $|\chi^0\rangle\equiv|\chi\rangle$, and can be obtained from the LAGRANGIAN ACTION

$$\mathcal{S}_{(s)_k}=\int d\eta_0\ {}_{(s)_k}\langle\chi^0|K_{(s)_k}Q_{(s)_k}|\chi^0\rangle_{(s)_k},\ K_{(s)_k}=K|_{-h^i=n^i+\frac{d-2-4i}{2}},$$

The corresponding LF of a bosonic field with a specific value of spin **s** subject to $Y(s_1,\dots,s_k)$ is an UNCONSTRAINED REDUCIBLE GAUGE THEORY OF MAXIMALLY $L=k(k+1)$-TH STAGE OF REDUCIBILITY

Corollary: the result contains as a particular case LF for bosonic HS subject to $Y(s_1),Y(s_1,s_2)$ (Buchbinder, Krycktin, 2005, Burdik, Pashnev, Tsulaia, 2001)

The analogous programm of LF construction may be fulfilled in this case with only inessential peculiarities of spin-tensor and fermionic nature.

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Summary

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- A GI reducible unconstrained LF for mixed-symmetry integer HS fields subject to $YT(k)$ in $\mathbb{R}^{1,d-1}$ space is developed;
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Outlook

- Consideration free LFs for integer HS fields with $Y(s_1, s_2, ..., s_k), k \geq 1$, on $\mathbb{R}^{1,d-1}$ with off-shell algebraic constraints;
- unconstrained and constrained LF construction for arbitrary fermionic HS fields on $\mathbb{R}^{1,d-1}$;
- Transition of the above results on frame-like formalism.

Thank you for attention

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