Hidden Conformal & Poincaré Symmetries in General Relativity

V.Pervushin (JINR, Dubna)

with the participation of A. Arbuzov,(JINR, Dubna) B. Barbashov (JINR, Dubna), A. Borowiec (ITP, University of Wrocław, Poland), R. Nazmitdinov (JINR, Dubna, and Department de Física, Universitat de les Illes Balears, Palma de Mallorca, Spain) K. N. Pichugin, (Kirensky Institute of Physics, Krasnoyarsk, Russia), A. Zakharov (ITEP, Moscow, Russia).

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CONTENT

- $A(4) \bigotimes C$ GRAVITATIONAL THEORY
  LINEAR FORMS, TANGENT SPACE-TIME, & DILATON
- $A(4) \bigotimes C$ COSMOLOGY
- EMPTY UNIVERSE and SNe Ia data
- PLANCK'S EPOCH HIERARCHY
- PARTICLE CREATION and WMAP data
- PREDICTION of SUSY of GR & SM?

ORIGIN: Conformal & Poincaré Symmetries in GR:

V.Fock, *Geometrization of Dirac's theory of the electron,* Z.Phys.**57**(1929)261. $ds^2 = \omega_{\overline{\alpha}}^{P}\omega_{\overline{\beta}}^{P}\eta^{\overline{\alpha}\overline{\beta}},\ \eta^{\overline{\alpha}\overline{\beta}} = (1, -1, -1, -1)$

V.I.Ogievetsky, *Infinite-dimensional algebra of general covariance group as the closure of finite-dimensional algebras of conformal and linear groups,* Lett. Nuovo Cim. **8** (1973) 988.

A. B. Borisov and V. I. Ogievetsky, *Theory of Dynamical Affine and Conformal Symmetries as Gravity Theory,* Theor. Math. Phys. **21** (1975) 1179 $G = e^{iR\cdot h}e^{iP\cdot x}, \quad GdG^{-1} = i\sum_{\hat{J}=\hat{P}_{(4)}\hat{L}_{(6)},\hat{R}_{(10)}} \omega^J \cdot \hat{J}, \quad W_{\rm GR}[\omega^J]$

P.A.M.Dirac,*Long range forces and broken symmetries,* Proc.Roy.Soc.Lond.A**333**(1973)403.

"*a new action principle was set up, much simpler than Weyl's, but requiring a scalar field function*" (dilaton D) "*to describe the gravitation field, in additional to $g_{\mu\nu}$*"

SUMMARY: Nonlinear realization of the affine group $A(4)$ as the finite-parameter group of all linear transformation of the four-dimensional manyfold

$$\widetilde{x}^{\mu} = x^{\mu} + y^{\mu} + L_{[\mu\nu]}x^{\nu} + R_{\{\mu\nu\}}x^{\nu},$$

where $L_{[\mu\nu]}$ and $R_{\{\mu\nu\}}$ are antisymmetric and symmetric matrixes respectively excludes z-factor from intervals.

$$\begin{aligned}
G &= e^{iR\cdot h}e^{iP\cdot x}, \qquad GdG^{-1} = i\sum_{\hat{J}=\hat{P}_{(4)}\hat{L}_{(6)},\hat{R}_{(10)}} \omega^{J}\cdot\hat{J} \\
W_{\rm GR}[\omega^{J}] &= \int d^{4}x\left[-\frac{\sqrt{-\widetilde{g}}e^{-2D}}{6}\widetilde{R}^{(4)} + e^{-D}\partial_{\mu}\left(\sqrt{-\widetilde{g}}\widetilde{g}^{\mu\nu}\partial_{\nu}e^{-D}\right)\right] \\
ds^{2} &= e^{-2D}\widetilde{g}_{\mu\nu}dx^{\mu}dx^{\nu} = \omega_{\overline{\alpha}}^{P}\omega_{\overline{\beta}}^{P}\eta^{\overline{\alpha}\overline{\beta}}, \\
D &= -\ln(1+z) + \overline{D} \qquad \text{is DILATON} \\
& \eta^{\overline{\alpha}\overline{\beta}} \qquad \text{is TANGENT Minkowskian space time metrics}
\end{aligned}$$

$$\hbar = c = M_{\rm Pl}\sqrt{3/(8\pi)} = 1 \quad [e^{\langle D\rangle} = 1+z = a^{-1}], \quad d\tau = \frac{dt}{a^3} = \frac{d\eta}{a^2}$$

$$\boxed{\widetilde{\omega}_{\overline{0}} = e^{-2D}Ndx^0} \quad \int\limits_{V_0}\frac{d^3x}{V_0}\underbrace{N^{-1}}_{\sqrt{-g}g^{00}}\partial_0\ln(1+z) = \partial_\tau \ln(1+z)$$

$$\boxed{\widetilde{\omega}_{\overline{b}} = \mathbf{e}_{\overline{b}i}dx^i + N_{\overline{b}}dx^0}, \qquad |\widetilde{g}^{(3)}_{ij}| = 1 \qquad \text{Lichnerowicz 1944},$$

$$W_{\rm Hilbert} = W_{\rm Universe} + W_{\rm graviton} + W_{\rm potential},$$

$$W_{\rm Universe} = \int\limits_{\tau_I}^{\tau_0} d\tau[-V_0\left(\partial_\tau \ln(1+z)\right)^2 - \mathrm{H}_{\rm Casimir\ \ energy}]$$

$$W_{\rm graviton} = \int d^4x\frac{N}{6}\left[v^2_{ab} - e^{-4D}R^{(3)}(\mathbf{e})\right]$$

$$W_{\rm potential} = \int d^4xN\left[-\underbrace{v^2_{\overline{D}}}_{v_{\overline{D}}=0} - \underbrace{\frac{4}{3}e^{-7D/2}\triangle^{(3)}e^{-D/2}}_{\rm Schwarzschild, r_g = Mm/M^2_{Pl}}\right]$$

In EMPTY Universe Casimir Energy determines the geometry

$$W_{\rm Hilbert}[1+z=a^{-1}\to\infty] \quad\simeq\quad W_{\rm Universe}$$

✠ A. Friedmann, Z. Phys. **10**, 377 (1922); *ibid*, **21**, 306 (1924);

1. $ds^2 = g_{\mu\nu}dx^\mu dx^\nu \approx dt^2 - a^2dr^2 = 0 \to \qquad dr = \dfrac{dt}{a} \equiv d\eta = a^2 d\tau$

2. $\dfrac{\delta W_{\rm H}}{\delta g_{00}} = 0 \to \left[\dfrac{da}{d\eta}\right]^2 = \rho(a) \to \qquad \boxed{r_{\rm hor}(a) = \int\limits_{a_I\to 0}^{a} d\overline{a}\frac{1}{\sqrt{\rho(\overline{a})}}}$

$$\rho_{\Lambda CDM}(a) \quad=\quad \underbrace{\frac{3M_{\rm Pl}^2H_0^2}{8\pi}}_{\rho_{\rm cr}}\left[\underbrace{a^4\Omega_\Lambda}_{\textbf{INFLATION ?}} + \underbrace{\Omega_{\rm rad}+a\Omega_{\rm M}}_{\text{Friedmann ``dust'' ?}} + \right]$$

$$\rho_{\rm CC}(a) \quad=\quad \rho_{\rm cr}\left[\underbrace{\frac{\Omega_{\rm rigid}}{a^2}}_{\text{CASIMIR ENERGY !}} + \underbrace{\Omega_{\rm rad}+a\Omega_{\rm M}}_{\text{Vacuum Creation of Matter !}}\right]$$

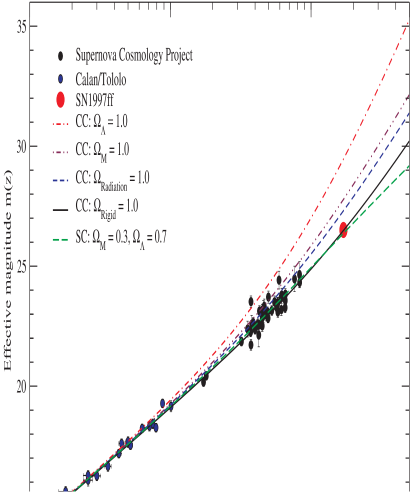
✠ CASIMIR ENERGY in EMPTY UNIVERSE

$$\Omega_{\rm Cas}(a) = \frac{1}{\rho_{\rm cr}} V_0{}^{-1} \sum_{\mathbf{k},\rm p} \frac{\sqrt{\mathbf{k}}^2}{2} = \frac{1}{H_0 d_{\rm Cas}(a)}$$

$$H_0 d_{\rm Cas}(a) = 2 \int_{a=0}^{a} d\overline{a}\ \Omega_{\rm Cas}^{-1/2}(\overline{a}),$$

Solution: $\Omega_{\rm Cas}(a) = \frac{1}{a^2} = (1+z)^2$

CASIMIR ENERGY [$\mathrm{H}_{\rm Casimir\ energy} = V_0 H_0^2$]
I. explains SNe Ia data and
II. forms Matter content of the Universe via Higgs boson-dilaton interaction in SM.



DILATON long space interval

explains **long Supernovae Distances** $\boxed{\uparrow}$ $R_{\rm SNeIa}$ **at** z $\boxed{\rightarrow}$ **via dominant Casimir energy**

$$r_{\rm horizon}(z) = H_0^{-1}(1+z)^{-2}$$

[SEE BLACK LINE];

ΛCDM **model with**

short space interval $R = ra$

requires Inflation to explain long Supernovae Distances

$R_{\rm SNeIa} = R_{\Omega_\Lambda=0.7,\Omega_M=0.3}$

[SEE GREEN LINE].

D. Behnke, *et al.* Phys. Lett. **B 530** (2002) 20;

A. Zakharov, V. Pervushin, Int. J. Mod. Phys. **D19** (2010) No.9

SNe Ia DATA IN CC REVEAL scale $z_{\rm Planck} \simeq 10^{15}$ via

PLANCK LEAST ACTION POSTULATE

$$W_{\rm GR} = \rho_{\rm cr} V_h^{(4)}(a_{\rm Pl}) = \frac{M_{\rm Pl}^2}{H_0^2}\frac{(1+z_{\rm Pl})^{-8}}{32} = 2\pi. \quad \hbar = c = 1$$

$$W_{\rm SM} = \lambda\phi_0^4 a_{\rm SM}^4 V_h^{(4)}(a_{\rm SM}) = 2\pi.$$

$$W_{\rm QED} = k_0^4 V_h^{(4)}(a_{\rm QED}) = 2\pi.$$

$$a_{\rm Pl}^{-1} = (1+z_{\rm Pl}) \approx \left[\frac{M_{\rm Pl}}{H_0}\right]^{1/4} \simeq 1\cdot 10^{15} \simeq a_{\rm SM}^{-1} \simeq a_{\rm QED}^{-1}$$

Conformal Weights of Poincaré representations with respect to dilaton ENERGIES: $d\tau = d\eta/a^2 = dt/a^3$

$\omega_\tau = a^2\sqrt{\mathbf{k}^2 + a^2 M_0^2} \to \, <\omega_{\rm n}(a)> = a^n a_{\rm Pl}^{-n} H_0$,

reveal common conformal symmetry breaking in both GR & SM

$<\omega_{\rm n}(a)_{\rm Pl}> = H_0$ **for all n=0,1,2,3,4**

There are coincidence of two scales
I. the electroweak (EW) scale

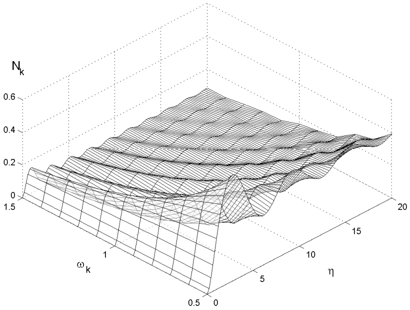
$$1 + z_{\rm EW} = [M_{0\,EW}/H_0]^{1/3} \simeq 0.4 \cdot 10^{15}, \qquad (1)$$

when the horizon contained only a single EW boson with the mass $M_{0\,EW} \simeq 100$ **GeV**;

II. the scale of the Cosmic Microwave Background (CMB)

$$\begin{aligned} 1 + z_{\rm CMB} &= [\langle\omega\rangle_{\rm CMB}/H_0]^{1/2} \\ &= [10^{-29} \cdot 2.35/1.5]^{1/2} \simeq 0.4 \cdot 10^{15}, \end{aligned} \qquad (2)$$

when the horizon contained only a single CMB photon with the mean one photon energy $\langle\omega\rangle_{\rm CMB}$ (*i.e.* the present day value of its temperature $T_{0\rm CMB}$)
$\langle\omega\rangle_{\rm CMB} = T_{0\rm CMB} = 2.35 \cdot 10^{-13}$ **GeV**.



This Figure, where

$\nearrow$ is time-axis

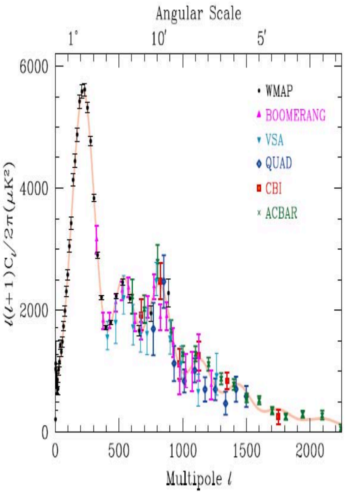
$\uparrow$ is number of bosons $N_{W,Z,h}$,

$\searrow$ is their momentum, shows us creation of $N_h = 10^{90}$ Higgs particles at $1 + z_{\rm Pl} \sim 10^{15}$ during the first 10^{-12}sec.

$$N_h \to N_\gamma \simeq \alpha_W^2 10^{90} = 10^{87} \to T_{\rm CMB} \sim 3\mathrm{K}\ \mathrm{Termolization.}$$

A. Arbuzov, *et al.*,Phys.Lett.B v. 691, p. 230 (2010)

$|\triangle T/T|$ WMAP peaks at multipole values: 220, 446, 800 MEAN 2γ processes.



CMB peaks $\frac{\triangle T_{\rm CMB}}{T_{\rm CMB}}$ $\boxed{\uparrow}$ at multipoles ℓ $\boxed{\rightarrow}$

reflect 2γ processes

$$\boxed{h \to 2\gamma, \;\; \ell \simeq 220}$$

$$\boxed{W^+W^{\to}2\gamma, \;\; \ell \simeq 546}$$

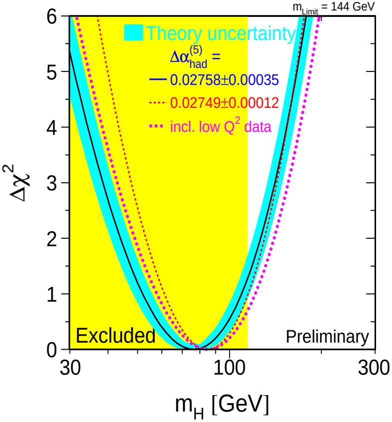
$$\boxed{Z\;Z \to 2\gamma, \;\; \ell \simeq 800}$$

and predict the SM boson spectrum [1]:

$$\boxed{\frac{M_Z}{M_W}{=}1.134|_{\rm hep} \approx \left(\frac{800}{546}\right)^{1/3}{=}1.136}$$

$$\boxed{M_h{=}2M_W\left[\frac{220.1}{546}\right]^{1/3}{=}120{\rm GeV}}$$

[5] **A.B. Arbuzov *et al.*, Physics of Atomic Nuclei,v.72,p.744(2009); A.B. Arbuzov *et al.*, G&C,v. 15,p.199(2009).**



Blue Band prediction

114.4 GeV $< M_H <$ 144 GeV

by Standard Model of 1967 without heavy fermions, SUSY, Λ and Cold Dark Matter means here to believe the conformal symmetry of unified SM & GR.

✠ Baryon asymmetry goes from γ_5-anomaly in WW processes (YaF-2004). They determine the size of **Large Scale Structure**

$\ell_W = 546$, $R_{LSS} \sim 10^{28}/546 \sim 5 \cdot 10^{25}$cm

In this region Newtonian dynamics is modified

$$v^2_{\rm luminocity} = \underbrace{v^2_{\rm N}}_{r_g/(2R)} + \gamma \underbrace{v^2_{\rm H}}_{(RH_0)^2}$$

Newtonian velocity $\sqrt{r_g/(2R)}$ becomes greater less than the Hubble one RH_0

$$\gamma_{CC} = +2 > 0; \qquad \gamma_{\Lambda CDM} \simeq -2 < 0$$

A. Einstein and E. G. Straus, *The influence of the expansion of space on the gravitation fields surrounding the individual stars* Rev. Mod. Phys. 17 (1945) 120.

A. A. Gusev, V. N. Pervushin, S. I. Vinitsky, and A. G. Zorin, *Cold dark matter as cosmic evolution of galaxies in relative units,* Astrophysics 47 (2004) 242

Physical theories as realizations of FINITE PARAMETER SYMMETRIES PLAYED FUNDAMENTAL ROLE:
ALL THESE SYMMETRIES.
GALILEY [CEL.M.], POINCARÉ + INTERNAL [SM],
A(4)[GR] are SUBGROUPS OF

superaffine group of “fundamental superspaces”
$\boxed{A[(2_b + 2_f)x(2_b + 2_f)] = A[8_b + 8_f]}$ L.B.Litov, V.N.Pervushin, Quantum supertwistors and fundamental superspaces Physics Letters B, 147, 76 (1984). J.Lukierski(1981)Wroclaw:N 534; Yu.S. Vladimirov and A.N. Gubanov, “Unification of gravi-electroweak and strong interactions in an 8- dimensional theory”, G&C,v.5,p.277(1999). $R^{(4)} \to R^{(8)}$.
Thanks