Supersymmetric Galileon Theory

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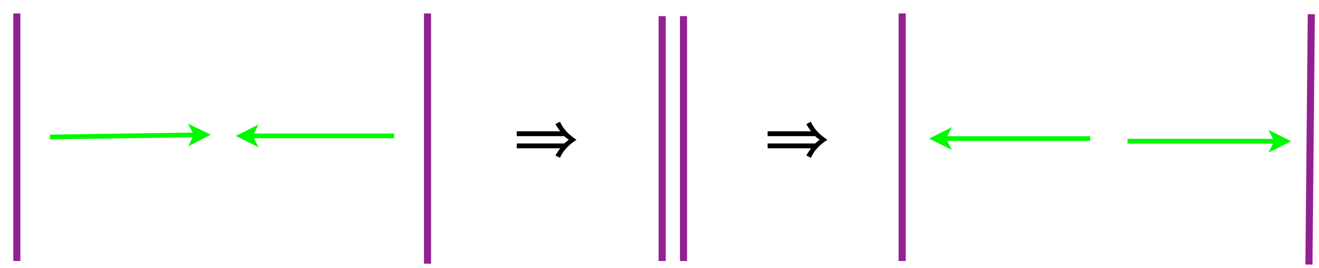
Heterotic M-Theory:

Wrapped N=1 supersymmetric 5-brane collision

d=5



$\Rightarrow$N=1 supersymmetric "Ekpyrotic" cosmology

But- how does one violate the NEC?

Non-supersymmetric "New Ekpyrotic" cosmology:

d=4 Effective field theory with two real scalars ϕ , χ

NEC violated by "ghost condensate" in field ϕ

But- how do we restore the N=1 supersymmetry!

Supersymmetric Ghost Condensate

Review of Bosonic Ghost Condensation:

Single real scalar field- ϕ

Flat space- $\eta_{\mu\nu}$

Higher-derivative Lagrangian

$$\mathcal{L} = P(X) \quad , \quad X \equiv -\frac{1}{2m^4}(\partial\phi)^2 = \frac{1}{2m^4}(\dot{\phi}^2 - \phi^{,i}\phi_{,i})$$

Set m=1. For $\phi = \phi(t)$ the equation of motion is

$$\frac{d}{dt}\left(P_{,X}\dot{\phi}\right) = 0$$

with solution

$$\phi = ct$$

In cosmological FRW context with $a = a(t) \Rightarrow$

$$P_{,X} = 0$$

Ghost Condensate Solution:

$$\phi = ct \quad \text{with} \quad P_{,X} = 0$$

$\longleftarrow$ spontaneously breaks Lorentz invariance

Expanding in fluctuations $\phi = c\,t + \delta\phi(t, \vec{x})$, to quadratic order the Lagrangian becomes

$$\mathcal{L}_{\text{quad}} = \boxed{X P_{,XX}} \cdot (\delta\dot{\phi})^2 - \boxed{0} \cdot \delta\phi^{,i}\delta\phi_{,i}$$

a) Temporal gradient: No ghost $\Rightarrow$ take

$$P_{,XX} > 0$$

$\Rightarrow$ ghost condensate local minimum of P(X).

b) Spatial gradient: Vanishes $\Rightarrow$ on verge of instability! Add higher-derivative terms such as

$$-\frac{(\Box\phi)^2}{M^2}$$

Ghost condensate still a solution, but improves the dispersion relation to

$$\omega^2 \sim \frac{k^4}{M^2}$$

Prototypical Condensate Action:

Near the condensate P(X) is approximately quadratic. $\Rightarrow$

Without loss of generality can take

$$\mathcal{L} = -X + X^2 = +\frac{1}{2}(\partial\phi)^2 + \frac{1}{4}(\partial\phi)^4$$

$\Rightarrow$ to quadratic order

$$\mathcal{L}_{\text{quad}} = 1(\delta\dot{\phi})^2 - 0 \cdot \delta\phi^{,i}\delta\phi_{,i}$$

Supersymmetric Ghost Condensation:

Extend to an N=1 chiral supermultiplet

$$\phi \longrightarrow (A, \psi^\alpha, F)$$

where

$$A = \frac{1}{\sqrt{2}}(\phi + i\chi)$$

ψ^α is a two-component Weyl spinor and F is a complex auxiliary field. These can be written in as a chiral superfield

$$\Phi = \boxed{A} + i\theta\sigma^\mu\bar{\theta}A_{,\mu} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A + \theta\theta\boxed{F} + \sqrt{2}\underset{\substack{\uparrow\\ \text{anticommuting spinor coordinate}}}{\theta}\boxed{\psi} - \frac{i}{\sqrt{2}}\theta\theta\psi_{,\mu}\sigma^\mu\bar{\theta}$$

The Supersymmetric Prototypical Action:

We find that

$$\mathcal{L}^{\text{SUSY}} = \left(-\Phi\Phi^\dagger + \frac{1}{16}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\right)\bigg|_{\theta\theta\bar{\theta}\bar{\theta}}$$

where $D_\alpha = \dfrac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$. In component fields

$$\begin{aligned}\mathcal{L}^{\text{SUSY}} &= \boxed{\frac{1}{2}(\partial\phi)^2 + \frac{1}{4}(\partial\phi)^4} + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}(\partial\phi)^2(\partial\chi)^2 + (\partial\phi\cdot\partial\chi)^2\\ &- \frac{i}{2}(\psi_{,\mu}\sigma^\mu\bar{\psi} - \psi\sigma^\mu\bar{\psi}_{,\mu}) - \frac{i}{4}(\partial\phi)^2(\psi_{,\mu}\sigma^\mu\bar{\psi} - \psi\sigma^\mu\bar{\psi}_{,\mu})\\ &- \phi_\mu\phi_{,\nu}\frac{i}{2}(\psi^{,\nu}\sigma^\mu\bar{\psi}\psi\sigma^\mu\bar{\psi}^{,\nu}) + \ldots\end{aligned}$$

No derivatives on F ⇒ remains auxiliary field.

Ghost Condensate Solution:

$$\phi = ct, \;\; \chi = 0 \;\; \text{and} \;\; \psi^\alpha = 0 \longleftarrow$$ spontaneously breaks Lorentz and supersymmetry invariance

Setting c=1 and expanding in fluctuations

$$\phi = t + \delta\phi(t,\vec{x}) \,, \quad \chi = \delta\chi(t,\vec{x}) \,, \quad \psi = \delta\psi(t,\vec{x})$$

to quadratic order the Lagrangian becomes

$$\begin{aligned}\mathcal{L}^{\text{SUSY}}_{\text{quad}} &= (\delta\dot{\phi})^2 - 0 \cdot \delta\phi^{,i}\delta\phi_{,i} \\ &+ 0 \cdot (\delta\dot{\chi})^2 + \delta\chi^{,i}\delta\chi_{,i} \\ &+ \frac{i}{4}\left(\delta\psi_{,0}\sigma^0\delta\bar{\psi} - \delta\psi\sigma^0\delta\bar{\psi}_{,0}\right) - \frac{i}{4}\left(\delta\psi_{,i}\sigma^i\delta\bar{\psi} - \delta\psi\sigma^i\delta\bar{\psi}_{,i}\right)\end{aligned}$$

The ϕ Fluctuations:

$$(\delta\dot{\phi})^2 - 0 \cdot \delta\phi^{,i}\delta\phi_{,i}$$

Same gradient instability as in pure bosonic condensate case.

In the supersymmetric case, solved by adding

$$-\frac{1}{2^{11}} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger \left(\{D,\bar{D}\}\{D,\bar{D}\}(\Phi+\Phi^\dagger)\right)^2\Big|_{\theta\theta\bar{\theta}\bar{\theta},\ \mathrm{quad}} = -(\Box\delta\phi)^2$$

Note, does not contain other fields at all to this order.

The χ Fluctuations:

$$\boxed{0}\cdot(\delta\dot{\chi})^2 \boxed{+} \delta\chi^{,i}\delta\chi_{,i}$$

Two serious problems- a) marginal temporal ghost and

b) deep wrong sign spatial gradient.

Adding supersymmetric terms can solve both problems. These are

$$\Big[\frac{8}{16^2} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger \left(\{D,\bar{D}\}(\Phi-\Phi^\dagger)\{D,\bar{D}\}(\Phi^\dagger-\Phi)\right)$$
$$-\frac{4}{16^3} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger \left(\{D,\bar{D}\}(\Phi+\Phi^\dagger)\{D,\bar{D}\}(\Phi-\Phi^\dagger)\right)^2\Big]\Big|_{\theta\theta\bar{\theta}\bar{\theta},\ \mathrm{quad}}$$
$$= -2(\partial\phi)^4(\partial\chi)^2 - (\partial\phi)^4(\partial\phi\cdot\partial\chi)^2$$

Adding to $\mathcal{L}^{SUSY}$ and expanding to quadratic order around the ghost condensate ⇒ the correct sign Lorentz-covariant expression

$$\mathcal{L}^{\rm SUSY}_{\rm quad} = \ldots + (\delta\dot{\chi})^2 - \delta\chi^{,i}\delta\chi_{,i} + \ldots$$

Useful to analyze in more detail. Add the above to the 2nd term in $\mathcal{L}^{SUSY}$ ⇒

$$\Big[\frac{8}{16^2}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\left(\{D,\bar{D}\}(\Phi-\Phi^\dagger)\{D,\bar{D}\}(\Phi^\dagger-\Phi)\right)$$
$$-\frac{4}{16^3}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\left(\{D,\bar{D}\}(\Phi+\Phi^\dagger)\{D,\bar{D}\}(\Phi-\Phi^\dagger)\right)^2 + \boxed{\frac{1}{16}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger}\Big]\Big|_{\theta\theta\bar{\theta}\bar{\theta},\rm quad}$$
$$= \left[-2(\partial\phi)^4(\partial\chi)^2 - (\partial\phi)^4(\partial\phi\cdot\partial\chi)^2\right] + \left[-\frac{1}{2}(\partial\phi)^2(\partial\chi)^2 + (\partial\phi\cdot\partial\chi)^2\right]$$

Evaluated around the ghost condensate gives Lorentz-covariance

$$\left[-2(\partial\chi)^2 - \cancel{(\dot{\chi})^2}\right] + \left[\frac{1}{2}(\partial\chi)^2 + \cancel{(\dot{\chi})^2}\right] = -\frac{3}{2}(\partial\chi)^2$$

with correct sign. Adding to first term in $\mathcal{L}^{SUSY}$ gives

$$\left[\frac{1}{2}(\partial\chi)^2\right] + \left[-\frac{3}{2}(\partial\chi)^2\right] = -(\partial\chi)^2$$

with canonical normalization.

The ψ^α Fluctuations:

$$\frac{i}{4}\left(\delta\psi_{,0}\sigma^0\delta\bar{\psi}-\delta\psi\sigma^0\delta\bar{\psi}_{,0}\right)-\frac{i}{4}\left(\delta\psi_{,i}\sigma^i\delta\bar{\psi}-\delta\psi\sigma^i\delta\bar{\psi}_{,i}\right)$$

a) Temporal gradient: No ghost

b) Spatial gradient: Deep wrong sign spatial gradient.

Analogous to the χ case. $\Rightarrow$ Need to add appropriate supersymmetric terms.

"However, within the context of the supersymmetric extension of the *pure $P(X)$ theory*, we are unable to find a fermionic analog of this mechanism"

$\Rightarrow$ fermion spatial kinetic term has wrong sign.

Not necessary unacceptible.

Be that as it may, one can ask a different question: by

"modifying the bosonic theory so that it is no longer purely a $P(X)$ theory, can one find a supersymmetric extension that is free of both ghost-like and gradient-like instabilities in *all* of its component fields? The answer, as we will see, is yes, and leads to another interesting class of higher-derivative Lagrangians — the conformal Galileon theories."

Supersymmetric Galileons

Curing the Fermion Gradient Instability:

To proceed, we must modify the original Lagrangian

$$\mathcal{L}^{\rm SUSY} = \left(-\Phi\Phi^\dagger + \frac{1}{16}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\right)\Big|_{\theta\theta\bar{\theta}\bar{\theta}}$$

Recall that

$$\begin{aligned}\frac{1}{16}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\Big|_{\theta\theta\bar{\theta}\bar{\theta}} &= \frac{1}{4}(\partial\phi)^4 - \frac{i}{4}(\partial\phi)^2(\psi_{,\mu}\sigma^\mu\bar{\psi} - \psi\sigma^\mu\bar{\psi}_{,\mu}) \\ &- \frac{i}{2}\phi_{,\mu}\phi_{,\nu}(\psi^{,\nu}\sigma^\mu\bar{\psi} - \psi\sigma^\mu\bar{\psi}^{,\nu}) + \dots\end{aligned}$$

Let us modify this to

$$\left[\boxed{\frac{1}{4(\Phi+\Phi^\dagger)^4}} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger\right]\Bigg|_{\theta\theta\bar{\theta}\bar{\theta}} = \frac{1}{4\phi^4}(\partial\phi)^4 - \overbrace{\frac{i}{4\phi^4}(\partial\phi)^2(\psi_{,\mu}\sigma^\mu\bar{\psi} - \psi\sigma^\mu\bar{\psi}_{,\mu})}^{\text{Lorentz} - \text{covariant}}$$
$$- \underbrace{\frac{i}{2\phi^4}\phi_{,\mu}\phi_{,\nu}(\psi^{,\nu}\sigma^\mu\bar{\psi} - \psi\sigma^\mu\bar{\psi}^{,\nu})}_{\text{Lorentz} - \text{violating}} + \ldots$$

Note that setting

$$\psi^\alpha = 0 \quad \Rightarrow \quad \text{reduces to} \quad \frac{X^2}{\phi^4}$$

⇒ the modified term is the supersymmetric extension of a $P(X,\phi)$ type bosonic Lagrangian. Can we find a supersymmetric interaction that will cancel the Lorentz-violating fermion term?

Consider

$$\left[\frac{-1}{24(\Phi+\Phi^\dagger)^3}\left(D\Phi D\Phi\bar{D}^2\Phi^\dagger + \text{h.c.}\right)\right]\Bigg|_{\theta\theta\bar{\theta}\bar{\theta}} = -\frac{1}{6\phi^3}\Box\phi(\partial\phi)^2$$
$$-\frac{i}{6\phi^3}\phi_{,\mu}(\psi_{,\nu}\sigma^\nu\bar{\psi}^{,\mu} - \psi^{,\mu}\sigma^\nu\bar{\psi}_{,\nu}) + \frac{i}{12\phi^3}\Box\phi(\psi_{,\nu}\sigma^\nu\bar{\psi} - \psi\sigma^\nu\bar{\psi}_{,\nu})$$
$$-\frac{i}{12\phi^3}\phi_{,\mu}(\psi\sigma^\mu\Box\bar{\psi} - \Box\psi\sigma^\mu\bar{\psi}) - \boxed{\frac{i}{4\phi^4}(\partial\phi)^2(\psi_{,\nu}\sigma^\nu\bar{\psi} - \psi\sigma^\nu\bar{\psi}_{,\nu})} \Leftarrow \frac{1}{(\Phi+\Phi^\dagger)^3}$$

Integrating by parts and dropping terms which vanish on ghost condensate background ⇒

$$
\begin{aligned}
\left[\frac{-1}{24(\Phi+\Phi^\dagger)^3}\left(D\Phi D\Phi \bar{D}^2\Phi^\dagger + \text{h.c.}\right)\right]\Bigg|_{\theta\theta\bar\theta\bar\theta} &= \overset{\text{not } P(X,\phi) \text{ form}}{-\frac{1}{6\phi^3}\Box\phi(\partial\phi)^2} \\
&+ \underset{-1\times\ \text{Lorentz}-\text{violating term}}{\frac{i}{2\phi^4}\phi_{,\mu}\phi_{,\nu}(\psi^{,\nu}\sigma^\mu\bar\psi - \psi\sigma^\mu\bar\psi^{,\nu})}
\end{aligned}
$$

Adding this to the above gives

$$
\begin{aligned}
&\left[\frac{-1}{24(\Phi+\Phi^\dagger)^3}\left(D\Phi D\Phi \bar{D}^2\Phi^\dagger + \text{h.c.}\right) + \frac{1}{4(\Phi+\Phi^\dagger)^4}D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger\right]\Bigg|_{\theta\theta\bar\theta\bar\theta} \\
&= -\frac{1}{6\phi^3}\Box\phi(\partial\phi)^2 + \left[\frac{1}{4\phi^4}(\partial\phi)^4 - \frac{i}{4\phi^4}(\partial\phi)^2(\psi_{,\mu}\sigma^\mu\bar\psi - \psi\sigma^\mu\bar\psi_{,\mu})\right] + \ldots
\end{aligned}
$$

Note that the Lorentz-violating fermion kinetic term has cancelled.

Integrating by parts⇒

$$
-\frac{1}{6\phi^3}\Box\phi(\partial\phi)^2 = -\frac{1}{6\phi^4}(\partial\phi)^4 + \frac{1}{18\phi^2}(\partial_\mu\partial_\nu\phi)^2 - \frac{1}{18\phi^2}(\Box\phi)^2
$$

It follows that

$$\left[\frac{-1}{24(\Phi+\Phi^\dagger)^3}\left(D\Phi D\Phi\bar{D}^2\Phi^\dagger+\text{h.c.}\right)+\frac{1}{4(\Phi+\Phi^\dagger)^4}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\right]\Bigg|_{\theta\theta\bar{\theta}\bar{\theta}}$$
$$=\frac{1}{12\phi^4}(\partial\phi)^4+\frac{1}{18\phi^2}(\partial_\mu\partial_\nu\phi)^2-\frac{1}{18\phi^2}(\Box\phi)^2-\frac{i}{4\phi^4}(\partial\phi)^2(\psi_{,\mu}\sigma^\mu\bar{\psi}-\psi\sigma^\mu\bar{\psi}_{,\mu})+\ldots$$

Three fundamental conclusions: " 1) the fermion kinetic term is Lorentz-covariant and, for any purely time-dependent background, of the correct sign — that is, *ghost-free with correct-sign spatial gradient*; 2) the first term is simply $X^2/3\phi^4$ and is manifestly of the $P(X,\phi)$ type; 3) the remaining ϕ terms are of a different differential form and *not* of the $P(X,\phi)$ type. Thus, by moving away from purely $P(X,\phi)$ theory we have solved the problem of the fermion gradient instability "

Must now add this to an appropriate "$\frac{1}{\phi^4}$" modification of the quadratic term $-\Phi\Phi^\dagger$. Defining

$$K(\Phi,\Phi^\dagger)=\frac{2}{3(\Phi+\Phi^\dagger)^2}$$

the correct modification is

$$-K(\Phi,\Phi^\dagger)\Big|_{\theta\theta\bar\theta\bar\theta} = \frac{1}{2\phi^4}(\partial\phi)^2 - \frac{i}{2\phi^4}\overbrace{(\psi_{,\mu}\sigma^\mu\bar\psi - \psi\sigma^\mu\bar\psi_{,\mu})}^{\text{Lorentz}-\text{covariant}}$$

where we suppress irrelevant χ and F contributions.

Note that setting

$$\psi^\alpha = 0 \quad\Rightarrow\quad \text{reduces to} \quad -\frac{X}{\phi^4}$$

$\Rightarrow$ the modified term is the supersymmetric extension of a $P(X,\phi)$ type bosonic Lagrangian.

Putting everything together, choose $\mathcal{L}^{SUSY}$ to be

$3\times(\text{above})$

$$\left[-K(\Phi,\Phi^\dagger) - \frac{1}{8(\Phi+\Phi^\dagger)^3}\left(D\Phi D\Phi\bar D^2\Phi^\dagger + \text{h.c.}\right) + \frac{3}{4(\Phi+\Phi^\dagger)^4}D\Phi D\Phi\bar D\Phi^\dagger\bar D\Phi^\dagger\right]\Bigg|_{\theta\theta\bar\theta\bar\theta}$$

$$= \frac{1}{2\phi^4}(\partial\phi)^2 + \frac{1}{4\phi^4}(\partial\phi)^4 + \frac{1}{6\phi^2}(\partial_\mu\partial_\nu\phi)^2 - \frac{1}{6\phi^2}(\Box\phi)^2$$

$$-\frac{i}{2\phi^4}\left(1+\frac{3}{2}(\partial\phi)^2\right)(\psi_{,\mu}\sigma^\mu\bar\psi - \psi\sigma^\mu\bar\psi_{,\mu}) + \dots$$

$$= \frac{1}{\phi^4}(\underbrace{\boxed{-X+X^2}}_{P(X,\phi)}\!\!\!\overset{\text{prototype}}{}) + \frac{1}{6\phi^2}\underbrace{\left((\partial_\mu\partial_\nu\phi)^2 - (\Box\phi)^2\right)}_{P(\cancel{X},\phi)} + \frac{i}{4\phi^4}(\psi_{,\mu}\sigma^\mu\bar\psi - \psi\sigma^\mu\bar\psi_{,\mu}) + \dots$$

Note that although no longer a pure $P(X,\phi)$ theory, it still admits a "ghost condensate" soluton

$$\phi = ct \ , \ \ \chi = 0 \ \ \text{where } c = 1$$

The fermion coefficient is evaluated there.

Final Conclusion: The price one pays to obtain a ghost free, spatially stable, Lorentz-covariant fermion kinetic term is that the bosonic ϕ Lagrangian is no longer pure $P(X,\phi)$!

What theory is it? Answer:

Conformal Galileons

Scalar Conformal Galileons:

Consider real scalar field π

The unique set of Lagrangians symmetric under the infinitesimal dilation and special conformal transformations

$$\delta_c \pi = c\,(1 + x^\mu \partial_\mu \pi)$$
$$\delta_v \pi = v_\mu x^\mu - \partial_\mu \pi \left(\frac{1}{2} v^\mu x^2 - (v\cdot x) x^\mu\right)$$

and leading to second order equations of motion are

$$\begin{aligned}
\mathcal{L}_2 &= -\frac{1}{2} e^{2\pi} (\partial\pi)^2 \\
\mathcal{L}_3 &= -\frac{1}{2} (\partial\pi)^2 \Box\pi - \frac{1}{4} (\partial\pi)^4 \\
\mathcal{L}_4 &= e^{-2\pi} (\partial\pi)^2 \Big[-\frac{1}{2} (\Box\pi)^2 + \frac{1}{2} \pi^{,\mu\nu}\pi_{,\mu\nu} \\
&\quad + \frac{1}{5} (\partial\pi)^2 \Box\pi - \frac{1}{5} \pi^{,\mu}\pi^{,\nu}\pi_{,\mu\nu} - \frac{3}{20} (\partial\pi)^4 \Big] \\
\mathcal{L}_5 &= e^{-4\pi} (\partial\pi)^2 \Big[-\frac{1}{2} (\Box\pi)^3 - \pi^{,\mu\nu}\pi_{,\nu\rho}\pi^{,\rho}{}_{\mu} \\
&\quad + \frac{3}{2} \Box\pi \pi^{,\mu\nu}\pi_{,\mu\nu} + \frac{3}{2} (\partial\pi)^2 (\Box\pi)^2 - \frac{3}{2} (\partial\pi)^2 \pi^{,\mu\nu}\pi_{,\mu\nu} \\
&\quad - \frac{15}{7} (\partial\pi)^4 \Box\pi + \frac{15}{7} (\partial\pi)^2 \pi^{,\mu}\pi^{,\nu}\pi_{,\mu\nu} - \frac{3}{56} (\partial\pi)^6 \Big]
\end{aligned}$$

To compare to $P(X,\phi)$ theories, change variables to $\phi \equiv e^{-\pi}$

Then

$$
\begin{aligned}
\boxed{\mathcal{L}_2 = -\frac{1}{2\phi^4}(\partial\phi)^2} \\
\boxed{\mathcal{L}_3} &= \frac{1}{2\phi^3}\Box\phi(\partial\phi)^2 - \frac{3}{4\phi^4}(\partial\phi)^4 \\
&\boxed{= -\frac{1}{4\phi^4}(\partial\phi)^4 - \frac{1}{6\phi^2}(\partial_\mu\partial_\nu\phi)^2 + \frac{1}{6\phi^2}(\Box\phi)^2} \\
\mathcal{L}_4 &= -\frac{1}{2\phi^2}(\partial\phi)^2(\Box\phi)^2 + \frac{1}{2\phi^2}(\partial\phi)^2\phi^{,\mu\nu}\phi_{,\mu\nu} + \frac{4}{5\phi^3}(\partial\phi)^4\Box\phi \\
&\quad -\frac{4}{5\phi^3}(\partial\phi)^2\phi^{,\mu}\phi^{,\nu}\phi_{,\mu\nu} - \frac{3}{20\phi^4}(\partial\phi)^6 \\
\mathcal{L}_5 &= (\partial\phi)^2\Bigg[\frac{1}{2\phi}(\Box\phi)^3 + \frac{1}{\phi}\phi^{,\mu\nu}\phi_{,\nu\rho}\phi^{,\rho}{}_{\mu} \\
&\quad -\frac{3}{2\phi}\Box\phi\phi^{,\mu\nu}\phi_{,\mu\nu} - \frac{3}{4\phi^2}\partial_\mu(\partial\phi)^2\partial^\mu(\partial\phi)^2 + \frac{3}{\phi^2}\Box\phi\phi^{,\mu\nu}\phi_{,\mu}\phi_{,\nu} \\
&\quad +\frac{6}{7\phi^3}(\partial\phi)^2\phi^{,\mu\nu}\phi_{,\mu}\phi_{,\nu} - \frac{6}{7\phi^3}(\partial\phi)^4\Box\phi - \frac{3}{56\phi^4}(\partial\phi)^8\Bigg] \\
&= (\partial\phi)^2\Bigg[\frac{1}{2\phi}(\Box\phi)^3 + \frac{1}{\phi}\phi^{,\mu\nu}\phi_{,\nu\rho}\phi^{,\rho}{}_{\mu} \\
&\quad -\frac{3}{2\phi}\Box\phi\phi^{,\mu\nu}\phi_{,\mu\nu} - \frac{3}{4\phi^2}(\partial\phi)^2(\Box\phi)^2 + \frac{3}{4\phi^2}(\partial\phi)^2\phi^{,\mu\nu}\phi_{,\mu\nu} \\
&\quad +\frac{9}{14\phi^3}(\partial\phi)^4\Box\phi - \frac{9}{14\phi^3}(\partial\phi)^2\phi^{,\mu}\phi^{,\nu}\phi_{,\mu\nu} - \frac{3}{56\phi^4}(\partial\phi)^8\Bigg]
\end{aligned}
$$

Note that $-\mathcal{L}_2$ and $-\mathcal{L}_3$ are exactly the ϕ part of the supersymmetric Lagrangians derived earlier!

Conclusion:

“ although the bosonic Galileon Lagrangians $\mathcal{L}_2$ and $\mathcal{L}_3$ were introduced for entirely different reasons, they are *precisely* of the form required by a quadratic and cubic supersymmetric theory to have a ghost condensate vacuum with *Lorentz-covariant* and *canonical sign* fermion kinetic energy ”

Aside: Note that if we add

$$\left[\frac{-1}{24(\Phi+\Phi^\dagger)^3}\left(D\Phi D\Phi \bar{D}^2\Phi^\dagger + \text{h.c.}\right) + \frac{(1+\Delta)}{4(\Phi+\Phi^\dagger)^4} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger\right]\Big|_{\theta\theta\bar{\theta}\bar{\theta}}$$

$$= \frac{1}{12\phi^4}(\partial\phi)^4 + \frac{1}{18\phi^2}(\partial_\mu\partial_\nu\phi)^2 - \frac{1}{18\phi^2}(\Box\phi)^2 - \frac{(1+\Delta)\, i}{4\phi^4}(\partial\phi)^2(\psi_{,\mu}\sigma^\mu\bar{\psi} - \psi\sigma^\mu\bar{\psi}_{,\mu}) +$$

$$\frac{\Delta}{4\phi^4}(\partial\phi)^4 - \frac{i\Delta}{2\phi^4}\,\phi_{,\mu}\phi_{,\nu}(\psi^{,\nu}\sigma^\mu\bar{\psi} - \psi\sigma^\mu\bar{\psi}^{,\nu}) + \ldots$$

dilatation invariant

special conformal violating

Lorentz – violating

$\Rightarrow$ leads to supersymmetric theory of generalized

Dilatation Galileons

Supersymmetric Extension of $\mathcal{L}_4$:

Not unique. One choice is

$$\begin{aligned}
\hat{\mathcal{L}}_4^{\rm SUSY} &= \Big(\frac{1}{64(\Phi+\Phi^\dagger)^2}\{D,\bar{D}\}(D\Phi D\Phi)\{D,\bar{D}\}(\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger) \\
&- \frac{1}{128(\Phi+\Phi^\dagger)^2}\Big[\{D,\bar{D}\}(\Phi+\Phi^\dagger)\{D,\bar{D}\}(D\Phi D\Phi)\bar{D}^2\Phi^\dagger + \text{h.c.}\Big] \\
&- \frac{1}{5\times 64(\Phi+\Phi^\dagger)^3}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\{D,\bar{D}\}\{D,\bar{D}\}(\Phi+\Phi^\dagger) \\
&+ \frac{6}{5\times 64(\Phi+\Phi^\dagger)^3}(D\Phi D\Phi\bar{D}^2\Phi^\dagger + \text{h.c.})\{D,\bar{D}\}\Phi\{D,\bar{D}\}\Phi^\dagger \\
&- \frac{9}{2^8\times 5(\Phi+\Phi^\dagger)^4}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\Big(\{D,\bar{D}\}(\Phi+\Phi^\dagger)\{D,\bar{D}\}(\Phi-\Phi^\dagger)\Big)^2 \\
&+ \frac{1}{2^8(\Phi+\Phi^\dagger)^2}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\{D,\bar{D}\}D^2\Phi\{D,\bar{D}\}\bar{D}^2\Phi^\dagger \\
&- \frac{1}{2^9(\Phi+\Phi^\dagger)^2}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\Big|\{D,\bar{D}\}\Phi\{D,\bar{D}\}D^2\Phi\Big|^2\Big)\Big|_{\theta\theta\bar{\theta}\bar{\theta}}
\end{aligned}$$

In component fields on condensate background

$$\begin{aligned}
\hat{\mathcal{L}}_{4,\,\text{quad},\,X=\text{const}}^{\rm SUSY} &= \overbrace{-\frac{1}{4\phi^2}\partial_\mu(\partial\phi)^2\partial^\mu(\partial\phi)^2 + \frac{1}{\phi^2}\Box\phi\phi^{,\mu}\phi^{,\nu}\phi_{,\mu\nu} - \frac{1}{4\phi^3}(\partial\phi)^4\Box\phi}^{\mathcal{L}_4\ \text{by parts}} \\
&+\frac{9}{10\phi^4}(\partial\phi)^4(\partial\chi)^2 + \underbrace{\frac{3}{\phi^3}(\partial\phi)^4F^*F}_{F\ \text{auxiliary}} + \underbrace{\frac{9i}{5\phi^4}(\partial\phi)^4(\psi_{,\nu}\sigma^\nu\bar{\psi} - \psi\sigma^\nu\bar{\psi}_{,\nu})}_{\text{Lorentz}-\text{covariant}}
\end{aligned}$$

Physical Conclusions:

Putting everything together, for

$$\mathcal{L}^{\rm SUSY} = c_2\mathcal{L}_2^{\rm SUSY} + c_3\mathcal{L}_3^{\rm SUSY} + c_4\mathcal{L}_4^{\rm SUSY}$$

we find (setting c= H_0)

$$\begin{aligned} c_2 - \frac{3}{2}c_3H_0^2 + \frac{3}{2}c_4H_0^4 &= 0 && \Rightarrow \text{ ghost condensate solution} \\ c_2 - 3c_3H_0^2 + \frac{9}{2}c_4H_0^4 &> 0 && \Rightarrow \text{ stable boson fluctuations} \\ c_2 + \frac{3}{2}c_3H_0^2 + \frac{3}{2}c_4H_0^4 &< 0 && \Rightarrow \text{ NEC violation} \\ c_2 - \frac{3}{2}c_3H_0^2 + \frac{18}{5}c_4H_0^4 &> 0 && \Rightarrow \text{ correct sign fermion fluctuations} \end{aligned}$$

Can be simultaneously satisfied as long as

$$c_2 < \frac{3}{2}c_3H_0^2 < 0$$

⇒ supersymmetric conformal Galileon theory that a) admits Lorentz-violating ghost condensate vacuum, b) is ghost free with no gradient instability and c) violates the NEC!